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Argonne National Laboratory

AN ANALOGUE SOLUTION OF A
REACTOR PERTURBATION INITIATED BY A
METALLURGICAL PHASE TRANSFORMATION
OF THE FUEL

by

Lawrence T. Bryant, Joseph C. Carter,
and Marion J. Janicke

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ABSTRACT

This study is directed toward the analysis of a reactor model that is thermally nonlinear. The significant sources of feedback in the closed-loop model are dilational changes of the fuel due to axial expansion caused by temperature and metallurgical phase changes. The dynamics of this phenomena have been represented on the PACE electronic analogue computer.

The model was subjected to unusually large inputs of sinusoidal power and did not show any drastic tendencies toward being unstable.

I. INTRODUCTION

This report is a presentation of the equations and their analog transformation equivalents necessary for the investigation of a reactor perturbation initiated by a metallurgical phase transformation. Also included is an introduction to the phenomena that made this investigation applicable to a reactor kinetics study.

An accounting of all movements of core material is important in a kinetics analysis of a reactor. The movements of fuel are of special significance in the concentrated systems of fast reactors. The many mechanisms which provide a driving force for these fuel movements have been thoroughly investigated. One phenomenon, however, that has not been investigated as thoroughly as others is the study of metallurgical fuel phase transformations on reactor response.

Without doubt the most investigated and widely reported fuel material is pure uranium. Other fuel alloys are usually quite specialized and are of lesser general interest. This study, therefore, is directed toward a pure uranium model. Experimental investigations of pure uranium indicate that the isothermal transformation initially involves bursts of transformation activity and proceeds in its later stages very slowly to an apparent end of transformation.^(7,15) A curve which describes the kinetics of the transformation from beta to gamma uranium is shown in Figure 1-0. Generally,

reactor cores are not constructed with pure uranium. Alloys are usually added to enhance the resistance to radiation damage. The addition of alloys to pure uranium changes its transformation kinetics characteristics. There are other factors which affect the transformation but not as significantly; a list of such influences includes:

1. alloy agents,
2. thermal history,
3. geometry,
4. rate of heat transfer,
5. radiation history, and
6. stress history.

Figure 1-0

Rate of isothermal $\beta \rightarrow \gamma$ transformation for pure uranium.

In heterogeneous reactors the heat from the fuel alloy is transferred to the coolant through a protective cladding material. Heat transfer to the clad is usually accomplished by metallurgically bonding the clad to the fuel or by introducing a liquid heat transporting material between the fuel and the clad. For example, see Figures 2-1 and 2-2.

Figure 2-1

Metallurgical bonding between clad and fuel element.

Figure 2-2

Liquid clad-to-uranium heat transport bond

In Figure 2-1, the metallurgical bond suppresses the "free" movement of the fuel. The axial fuel movement, which affects the magnitude and phase of feedback reactivity, is a function of the coefficients of expansion (α) and the moduli of elasticity (E) of both the fuel and clad. For a geometrical description of the fuel-clad arrangement that does not restrain the axial movement of the fuel, see Figure 2-2. This fuel construction makes this phase transformation analysis particularly applicable to the EBR-II fast reactor core element.

The specific heat is a measure of the heat that is required to be transferred per unit mass of material to produce a unit change in temperature. A metal with a constant specific heat to be heated at a constant temperature rate would have a plot of the specimen temperature versus time represented by Figure 4-1; the slope of the curve would equal $\Delta T / \Delta t$. In actual metals, the specific heat is rarely constant. If the metal undergoes a phase transformation a thermal arrest occurs; the time required for the arrest is a measure of the amount of heat absorbed. This value is

the heat of transformation and can be calculated once the rate of heat input is known⁽³⁾ (see Figure 4-2).⁽⁸⁾ Since

$$dH = c_p dT,$$

it follows that

$$\frac{dH}{dt} = c_p \frac{dT}{dt}$$

Figure 4-1

Curve for a material having a constant specific heat and undergoing no phase transformation

Figure 4-2

Curve for a material having variable specific heat and undergoing a phase transformation

Investigations by Feuerstein and Smith⁽¹⁾ have clearly indicated that the elevations of the phase-transformation temperatures occur in steel upon imposing high heating rates. For example, when annealed AISI 1020 was tested at a rate of 2400°F/sec, the A_{C_3} critical temperature was shown to increase 300°F above that which would normally be expected. Their investigation also showed that previous heat treatment and stressing influenced the critical temperature during experiments at high heating rates.

This superheating effect was also observed in uranium. Duwez⁽²⁾ noted a marked superheating effect in uranium to occur at the alpha-to-beta transformation with a heating rate of approximately 75°C/sec.

Klepfer and Chiotti⁽⁷⁾ describe the isothermal transformation in uranium as one involving (a) initial bursts of transformation activity followed by (b) a very slow approach to an apparent end of transformation. The maximum initial rate of transformation was observed to increase rapidly with an increasing degree of supercooling or superheating. The increase in maximum rate of transformation was increased by a factor of 10 for a 3°C increase in superheat for the beta-to-gamma transformation experiments.

A first impression of standard⁽⁶⁾ phase-transformation activity studies implies that a phase transformation of uranium is rather lethargic, and an increase in heating rate would only tend to decrease the manifestations of the phase transformation. However, an increased heating rate magnifies the initial transformation activity in uranium. The kinetics of the phase transformation, therefore, are influenced by the rate of heat transfer and directly affect the kinetics of the reactor core.

Another important phenomenon that can influence reactor operation is phase reversal. A kinetic study of phase reversal through experimental investigations^(4,5) has shown that uranium-molybdenum alloys transform from the stable to a metastable phase upon neutron bombardment. This phenomenon has been explained on the basis of the smoothing out of concentration gradients due to the action of "displacement spikes" generated within the sample. Phase reversion is a phenomenon that can influence a fuel alloy to transform into a "higher" phase, say (A)→(B), at a lower temperature than anticipated. Therefore, a core designed to operate at a temperature slightly lower than the transformation (B) temperature may well be in that phase (B) temperature area.

Generally, a phase transformation (A)→(B) during a temperature rise denotes a volume increase in material or a decrease in reactor reactivity; a (B)→(A) transformation decreases the volume and an increased reactivity is associated with an increase in temperature.

II. NOMENCLATURE

A	Liquid metal heat transfer analog constant [cal/(sec)(°C)]
AC ₃	Temperature at which the transformation of ferrite to austenite is completed during heating.
C _i	Flux density of i'th delayed-neutron group
C _{Na} [*]	Coefficient of reactivity associated with volumetric expansion of coolant = $0.0246 \frac{\Delta k}{k} / \text{cm}^3 / \text{cm}^3$
C _{Na}	$\beta_{Na} C_{Na}^*$
C _{pt} [*]	$C_u^* \lambda$
C _{pt} = C _{pt} [*]	Coefficient of reactivity associated with the phase transformation of the uranium fuel element
C _u [*]	Coefficient of reactivity associated with the axial expansion of uranium = $0.2796 \frac{\Delta k}{k} / \text{cm} / \text{cm}$
C _u	αC_u^*
c _p	Specific heat of coolant [cal/(gm)(°C)]
$\left(\frac{\Delta k}{k}\right)_{Na}$	Reactivity associated with the volumetric expansion of the coolant
$\left(\frac{\Delta k}{k}\right)_{pt}$	Reactivity associated with the longitudinal expansion of the uranium fuel element due to phase transformation
$\left(\frac{\Delta k}{k}\right)_u$	Reactivity associated with the longitudinal expansion of the uranium fuel element
f	Frequency (cps)
h	Heat transfer constant [cal/(cm ²)(sec)(°C)]
k _{eff}	$k_{ex} - C_i (\Delta k/k)_i$
k _{ex}	Excess reactivity
ℓ	Prompt-neutron lifetime
n	Prompt-neutron flux density
Q	$(1.357 n/n_0 q''' + \sum m_i T_i) = \text{heat in cal/sec, where the } m_i \text{ are constant coefficients}$
q'''	Average power density [cal/(sec)(cm ³)]
s	Surface area of the cladding (cm ²)
T _c	Average coolant temperature (°C)

T_{c1}	Inlet coolant temperature ($^{\circ}\text{C}$)
T_{c2}	Outlet coolant temperature ($^{\circ}\text{C}$)
T_h	Temperature of the homogenized bond and clad ($^{\circ}\text{C}$)
T_s	Temperature of the surface of the homogenized bond and clad ($^{\circ}\text{C}$)
T_{u1}	Temperature of the first uranium section ($^{\circ}\text{C}$)
T_{u2}	Temperature of the second uranium section ($^{\circ}\text{C}$)
t	Time
W	Mass flow rate of coolant (gm/sec)
α	Longitudinal expansion of uranium = $18.21 \times 10^{-6} \text{ cm}/(\text{cm})(^{\circ}\text{C})$
β_i	β_i , the fraction of prompt neutrons entering all delayed groups
β_i	The fraction of prompt neutrons entering the i 'th delayed-neutron group
β_{Na}	Volumetric coefficient of sodium = $0.2428 \times 10^{-3} \text{ cm}^3/\text{cm}^3/^{\circ}\text{C}$
η	A constant controlling the amount of excess reactivity applied to the system
λ	$3.7698 \times 10^{-5} \text{ cm}/\text{cm}/\text{cal}$
λ_i	Decay constant of neutrons in the i 'th delayed group
$\omega = 2\pi f$	A variable depending on the rate at which excess reactivity is to be injected into the system (rad/sec)

III. DESCRIPTION OF THE MODEL

The analogue of the core was represented by an average element which was constructed of one axial section and two radial uranium regions, a radial homogenized bond and clad, and a radial region of flowing sodium coolant. Equivalent heat generation and reactivity worth of the average rod regions were evaluated at EBR-II design operating conditions and are expressed in terms of reactivity $\Delta k/k$.

For the large sinusoidal inputs, nonlinear reactor kinetic equations were used. The assumption was made that the power remained directly proportional to the neutron density variations.

Fission heat is transported to the uranium surface by conduction; it is then conducted through the non-heat-generating bond and clad to the outer shell surface. The heat is then transported to the coolant by convection and conduction. The small percentage of heat liberated due to axial transfer and by gamma and decay heating were neglected in this model.

Rate of dilatation and time for completion of the phase transformation were taken as a function of heat input. The average temperature of the two uranium radial regions constituted the isothermal temperature at which transformation occurred. Functions which associate $\Delta k/k$ coefficients to phase transformation and heat input rate are developed in Section IV.

These physical phenomena are represented in block diagram form in Figure 3-1. The diagram is self-explanatory; however, one feature worth noting in the diagram is the switch for directing heat to the thermal equations or to the equations which express heat of transformation with respect to the thermal equilibrium during the phase transformation.

Mathematical expressions describing the physical events are transformed into electronic representation and are illustrated in Figure 3-2.

Figure 3-1 Analog Flow Diagram

Block * represents heat addition for the thermal equations without phase transformation and heat required for heat balance at the isothermal state during transformation plus the heat required for transformation.

The right-hand side of the equations in the thermal section are shown in box ** for completeness.

Figure 3-2. Analog circuit diagram

IV. EQUATION DEVELOPMENT*

A. The Kinetics Equation

$$\frac{dn}{dt} = \frac{1-\beta}{\ell} k_{ex} n - \frac{\beta}{\ell} n + \sum \lambda_i C_i \quad (1)$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\ell} k_{ex} n + \frac{\beta_i}{\ell} n - \lambda_i C_i \quad (2)$$

When the system is in equilibrium,

$$\frac{dn}{dt} = 0; \quad \frac{dC_i}{dt} = 0; \quad k_{ex} = 0.$$

One then has, from (2),

$$\frac{\beta_i}{\ell} n(0) = \lambda_i C_i(0) \quad (3)$$

Since $\sum \beta_i = \beta$, Eq. (1) is also satisfied. A convenient arbitrary value, usually 1.000, is assumed for $n(0)$; the $C_i(0)$'s are then calculated from Eq. (3). Since the terms in Eqs. (1) and (2) differ considerably in magnitude, it is desirable to introduce new variables, defined as follows:

$$\left. \begin{aligned} n_1 &= n(t)/n(0) \\ C_{1i} &= C_i(t)/C_i(0) \\ k_{1ex} &= k_{ex}/\beta \end{aligned} \right\} \quad (4)$$

In many reactor systems ℓ/β is so small that $(\ell/\beta) (dn_1/dt)$ can be neglected. This is always true where, for this term to be significant, the capacity of the machine would have to be exceeded.

Substitution of Eqs. (4) into Eqs. (1) and (2) leads to

$$\frac{\ell}{\beta} \frac{dn_1}{dt} = (1-\beta) k_{1ex} n_1 - n_1 + \sum \frac{\beta_i}{\beta} C_{1i} \quad (5)$$

and

$$\frac{dC_{1i}}{dt} = \lambda_i \beta k_{1ex} n_1 + \lambda_i n_1 - \lambda_i C_{1i} \quad (6)$$

*Reference is made to ANL-6027, Analogue Computer Solution of the Nonlinear Reactor Kinetics Equations, by Lawrence T. Bryant and N. Frank Morehouse, Jr., July, 1959. Definition of symbols will be the same as that used in the above-mentioned report. The equations have been repeated for completeness.

The form of the equations used in this report are found by introducing the deviations from equilibrium as new variables, as follows:

$$\left. \begin{aligned} \Delta n_1 &= \frac{n(t) - n(0)}{n(0)} = n_1(t) - 1 \\ \Delta C_{1i} &= \frac{C_i(t) - C_i(0)}{C_i(0)} = C_{1i}(t) - 1 \end{aligned} \right\} \quad (7)$$

Substitution of Eq. (7) in Eqs. (5) and (6), there is obtained

$$\frac{\ell}{\beta} \frac{d\Delta n_1}{dt} = (1 - \beta) k_{1ex} \Delta n_1 - \Delta n_1 + \sum \frac{\beta_i}{\beta} \Delta C_{1i} + (1 - \beta) k_{1ex} \quad (8)$$

and

$$\frac{d\Delta C_{1i}}{dt} = \lambda_i \beta k_{1ex} \Delta n_1 + \lambda_i \Delta n_1 - \lambda_i \Delta C_{1i} + \lambda_i \beta k_{1ex} \quad (9)$$

These equations differ from Eqs. (5) and (6) only by the forcing function terms, $(1 - \beta) k_{1ex}$ and $\lambda_i \beta k_{1ex}$, which appear on the right-hand side. Equations (8) and (9) with $(\ell/\beta)(d\Delta n/dt)$ neglected become

$$0 = (1 - \beta) k_{1ex} \Delta n_1 - \Delta n_1 + \sum (\beta_i/\beta) \Delta C_{1i} + (1 - \beta) k_{1ex} \quad (10)$$

and

$$\frac{d\Delta C_{1i}}{dt} = \lambda_i \beta k_{1ex} \Delta n_1 + \lambda_i \Delta n_1 - \lambda_i \Delta C_{1i} + \lambda_i \beta k_{1ex} \quad (11)$$

B. The Machine Equations

Even though the transformations (4) reduce the problem of scaling considerably, it is still necessary to set up an equivalence between machine voltages and the problem variable as follows:

$$\left. \begin{aligned} a t &= t' \\ b \Delta n_1 &= \Delta n'_1 \\ d_i C_{1i} &= C'_{1i} \\ e k_{1ex} &= k'_{1ex} \end{aligned} \right\} \quad , \quad (12)$$

where the primes denote the machine voltages corresponding to the unprimed variable.

The kinetics equations then become

$$0 = \frac{1-\beta}{e} k'_{\text{lex}} \Delta n'_1 - \Delta n'_1 + \sum \frac{b\beta_i}{d_i\beta} \Delta C'_{1i} + \frac{(1-\beta)}{e} b k'_{\text{lex}} \quad (13)$$

and

$$\frac{d\Delta C'_{1i}}{dt} = \frac{\lambda_i \beta d_i}{abe} k'_{\text{lex}} \Delta n'_1 + \frac{\lambda_i d_i}{ab} \Delta n'_1 - \frac{\lambda_i}{a} \Delta C'_{1i} + \frac{\lambda_i \beta d_i}{ae} k'_{\text{lex}} \quad . \quad (14)$$

Initial Conditions:

Initially when the reactor is in steady state,

$$\left. \begin{aligned} \frac{d\Delta n_1}{dt} &= \frac{d\Delta C_{1i}}{dt} = k_{\text{lex}} = 0 \\ \Delta n_1 &= 0 \\ \Delta C_{1i} &= 0 \end{aligned} \right\} \quad . \quad (15)$$

C. Thermal Equations for EBR-II Geometry

In the following equations note that the sodium bond and the clad have been homogenized:

$$\frac{dT_{u_1}}{dt} = 1.357 (n/n_0) q''' - 26.6 (T_{u_1} - T_{u_2}) \quad (a)$$

$$\frac{dT_{u_2}}{dt} = 1.357 (n/n_0) q''' + 26.6 (T_{u_1} - T_{u_2}) - 56.82 (T_{u_2} - T_h) \quad (b)$$

$$\frac{dT_h}{dt} = 60.24 (T_{u_2} - T_h) - 129.2 (T_h - T_s) \quad (c)$$

$$0 = 1.503 (T_h - T_s) - A(T_s - T_c) \quad (d)$$

$$T_{c2} = (hs/Wc_p)(T_s - T_c) + T_{c1} \quad (e)$$

$$T_c = \frac{1}{2} T_{c1} + \frac{1}{2} T_{c2} \quad . \quad (f)$$

The following equations hold for the uranium radial sections during the phase transformation:

$$\frac{dT_{u_1}}{dt} = 0 \quad ; \quad \frac{dT_{u_2}}{dt} = 0 \quad ; \quad \frac{dT_h}{dt} = 0 \quad ,$$

and Q is stored or released in accordance with the reactivity equation $(\Delta k/k)_{pt}$ shown on page 20.

Since initially we are operating at a fairly high temperature, it would be inadvisable to attempt to vary reactivity in any manner other than by means of fairly large excursions, because the temperature changes would be slight for small changes in reactivity. Therefore we set up the following relationships: Let

$$\begin{aligned}\Delta T_{u_1} &= T_{u_1} - T_{u_1}(0) & \Delta T_h &= T_h - T_h(0) \\ \Delta T_{u_2} &= T_{u_2} - T_{u_2}(0) & \Delta T_s &= T_s - T_s(0) \\ \Delta T_{c_1} &= T_{c_1} - T_{c_1}(0) & \Delta T_{c_2} &= T_{c_2} - T_{c_2}(0)\end{aligned}\quad (g)$$

Eliminating T_c from the Eqs. (a) $\rightarrow$ (e) by substituting $\frac{1}{2} T_{c_1} + \frac{1}{2} T_{c_2}$ from Eq. (f) and replacing the other temperatures by the relationships set up in Eqs. (g), we obtain

$$\frac{d\Delta T_{u_1}}{dt} = 1.357 \Delta n_1 q''' - 26.6 \Delta T_{u_1} + 26.6 \Delta T_{u_2} \quad (h)$$

$$\frac{d\Delta T_{u_2}}{dt} = 1.357 \Delta n_1 q''' + 26.6 \Delta T_{u_1} - 83.42 \Delta T_{u_2} + 56.82 \Delta T_h \quad (i)$$

$$\frac{d\Delta T_h}{dt} = 60.24 \Delta T_{u_2} - 189.44 \Delta T_h + 129.2 \Delta T_s \quad (j)$$

$$\Delta T_s = \frac{1.503}{1.503 + A} \Delta T_h + \frac{A}{2(1.503 + A)} \Delta T_{c_1} + \frac{A}{2(1.503 + A)} \Delta T_{c_2} \quad (k)$$

$$\Delta T_{c_2} = (hs/Wc_p) \Delta T_s - (hs/2Wc_p) (\Delta T_{c_1} + \Delta T_{c_2}) + \Delta T_{c_1} \quad (l)$$

Transforming to machine equations for insertion in the analog computer, let

$$\begin{aligned}a_1 \Delta T_{u_1} &= \Delta T_{u_1}' & a_1 d\Delta T_{u_1} &= d\Delta T_{u_1}' \\ a_2 \Delta T_{u_2} &= \Delta T_{u_2}' & a_2 d\Delta T_{u_2} &= d\Delta T_{u_2}' \\ a_3 \Delta T_h &= \Delta T_h' & a_3 d\Delta T_h &= d\Delta T_h' \\ a_4 \Delta T_s &= \Delta T_s' & kt &= t' \\ a_5 \Delta T_{c_2} &= \Delta T_{c_2}' & kdt &= dt' \\ a_6 \Delta T_{c_1} &= \Delta T_{c_1}' & b\Delta n &= \Delta n_1'\end{aligned}\quad (m)$$

where k and b are the same as in the reactor kinetics ($k = a$ and $b = b$). Substituting Eqs. (m) into Eqs. (h) $\rightarrow$ (l) we get our machine equations.

D. Thermal Model Machine Equations

$$\frac{d\Delta T_{u_1}'}{dt'} = \frac{1.357}{kb} \Delta n_1' q''' a_1 - \frac{26.6}{k} \Delta T_{u_1}' + 26.6 \frac{a_1}{a_2 k} \Delta T_{u_2}' \quad (n)$$

$$\frac{d\Delta T_{u_2}'}{dt'} = \frac{1.357 a_2}{kb} \Delta n_1' q''' + 26.6 \frac{a_2}{a_1 k} \Delta T_{u_1}' - \frac{83.42}{k} \Delta T_{u_2}' + \frac{56.82 a_2}{a_3 k} \Delta T_h' \quad (o)$$

$$\frac{d\Delta T_h'}{dt'} = 60.24 \frac{a_3}{a_2 k} \Delta T_{u_2}' - \frac{189.44}{k} \Delta T_h' + 129.2 \frac{a_3}{a_4 k} \Delta T_s' \quad (p)$$

$$\Delta T_s' = \frac{(1.503)a_4}{(1.503+A)a_3} \Delta T_h' + \frac{Aa_4}{2(1.503+A)a_6} \Delta T_{c1}' + \frac{Aa_4}{2(1.503+A)a_5} \Delta T_{c2}' \quad (q)$$

$$\Delta T_{c2}' = \frac{(hs/Wc_p)a_5}{[1+(hs/2Wc_p)]a_4} \Delta T_s' - \frac{[1+(hs/2Wc_p)]a_5}{[1+(hs/Wc_p)]a_6} \Delta T_{c1}' \quad ,$$

where

$$hs/Wc_p = 5.8679 \quad . \quad (r)$$

Substituting Eq. (r) into Eq. (q) for $\Delta T_{c2}'$, Eq. (q) becomes

$$\begin{aligned} \Delta T_s' &= \frac{1.503 a_4}{(1.503 + A) \left(1 - \frac{A(hs/Wc_p)}{2[1.503 + A][1+(hs/2Wc_p)]} \right) a_3} \Delta T_h' \\ &= \frac{2(1.503)[1+(hs/2Wc_p)]a_4}{(2[1.503 + A][1+(hs/2Wc_p)] - A[hs/Wc_p]) a_3} \Delta T_h' \end{aligned} \quad (q')$$

Note that since T_{c1} , the inlet temperature, is constant, $\Delta T_{c1} = 0$ at any time t and may therefore be eliminated from all equations.

Initial Conditions:

$$\Delta T_i = T_i - T_i(0) \text{ from Eqs. (g);}$$

then

$$\Delta T_i(0) = T_i(0) - T_i(0)(0) = 0$$

and

$$a_i \Delta T_i = \Delta T_i' = 0,$$

(i.e., all initial conditions are zero).

E. Constants

$$A = 1.396$$

$$q''' = 668$$

$$C_u = 0.5092 \times 10^{-5}$$

$$C_{Na} = 0.5973 \times 10^{-5}$$

$$C_{pt} = 1.054 \times 10^{-5}$$

F. Reactivity Equations

$$k_{ex} = \eta \sin \omega t$$

$$k_{ex}/\beta = (\eta/\beta) \sin \omega t$$

By standard analog methods the above equation is differentiated twice and programmed for the analog computer for generation of the sine wave. The diagram is shown on the analog circuit diagram.

$$\left(\frac{\Delta k}{k}\right)_{Na} = C_{Na} T_c = \frac{C_{Na}}{2} (T_{c1} + T_{c2})$$

$$\left(\frac{\Delta k'}{k}\right)_{Na} = \frac{C_{Na} e}{2 \beta a_{56}} (\Delta T'_{c1} + \Delta T'_{c2})$$

$$\left(\frac{\Delta k}{k}\right)_u = C_u T_u \text{ avg} = \frac{C_u}{2} (T_{u1} + T_{u2})$$

$$\left(\frac{\Delta k'}{k}\right)_u = \frac{C_u e}{2 \beta a_{12}} (\Delta T'_{u1} + \Delta T'_{u2})$$

$$\left(\frac{\Delta k}{k}\right)_{pt} = \lambda \int dQ \text{ where } \int dQ = \int_0^t Q dt \quad (t = \text{time to transform})$$

$$\left(\frac{\Delta k'}{k}\right)_{pt} = \frac{C_{pt} e \int_0^t Q dt}{\beta a_{12}},$$

where a_{56} and a_{12} are the values of the scale factors, a_5 or a_6 and a_1 or a_2 (for which $a_1 = a_2$ and $a_5 = a_6$).

G. Potentiometer Settings

NEUTRON KINETICS					SHEET 1 of 2				
POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING *!	SET			PARAMETERS
DRAWING	MACHINE								
	21	$\lambda_{1/a}$	.0127		.0013				$l = 8 \cdot 10^{-8}$
	22	$\lambda_{2/a}$	.0318		.0032				$\beta = .007347$
	23	$\lambda_{3/a}$	.1153		.0115				$\lambda_1 = .0127$
	24	$\lambda_{4/a}$	.3110		.0311				$\lambda_2 = .0318$
	25	$\lambda_{5/a}$	1.400		.1400				$\lambda_3 = .1153$
	26	$\lambda_{6/a}$	3.870		.3870				$\lambda_4 = .3110$
	27	$\beta_{1/d_1\beta}$	.0316		.0316				$\lambda_5 = 1.400$
	28	$\beta_{2/d_2\beta}$	.1937		.1937				$\lambda_6 = 3.870$
	29	$\beta_{3/d_3\beta}$	.1814		.1814				$\beta_1 = .000232$
	30	$\beta_{4/d_4\beta}$	.4023		.4023				$\beta_2 = .001423$
	31	$\beta_{5/d_5\beta}$	.1526		.1526				$\beta_3 = .001333$
	32	$\beta_{6/d_6\beta}$	.0384		.0384				$\beta_4 = .002956$
	33	$\lambda_1^d_{1/ab}$	.0127		.0025	(5)			$\beta_5 = .001121$
	34	$\lambda_2^d_{2/ab}$	.0318		.0064	(5)			$\beta_6 = .000282$
	35	$\lambda_3^d_{3/ab}$	.1153		.0231	(5)			$(1-\beta) = .99265$
	36	$\lambda_4^d_{4/ab}$	.3110		.0622	(5)			SCALE FACTORS
	37	$\lambda_5^d_{5/ab}$	1.400		.1400	(10)			$a = 1$
	38	$\lambda_6^d_{6/ab}$	3.870		.3870	(10)			$b = d_1 = 10$
	39	$\lambda_1^d_{1\beta/ae}$	.0000		.0000				$e = 10^2$
	40	$\lambda_2^d_{2\beta/ae}$	.0000		.0000				
	41	$\lambda_3^d_{3\beta/ae}$	.0000		.0000				
	42	$\lambda_4^d_{4\beta/ae}$	.0000		.0000				
	43	$\lambda_5^d_{5\beta/ae}$	.0010		.0001				
	44	$\lambda_6^d_{6\beta/ae}$	.0028		.0003				

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*!See p. 23.

THERMAL EQUATIONS

SHEET 1 of 1

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	* SET	PARAMETERS
DRAWING	MACHINE						
	60	$1.357 a_1 q'''/kb$	9.065		.9065	(10)	$a_1=a_2=a_3=a_4$ $=a_5=a_6= 0.1$ $K = 1.0$ $b = 10$ $q = 668$ $A = 1.396$
	61	$26.6/k$	26.60		.2660	(100)	
	62	$26.6 a_1/a_2 k$	26.60		.2660	(100)	
	63	$26.6 a_2/a_1 k$	26.60		.5320	(50)	
	64	$83.42/k$	83.42		.8342	(100)	
	65	$56.82 a_2/a_3 k$	56.82		.5682	(100)	
	66	$60.24 a_3/a_2 k$	60.24		.6024	(100)	
	67	$189.44/k$	189.44		.3788	(500)	
	68	$129.2 a_3/a_4 k$	129.2		.6460	(200)	
	69	$\left\{ \frac{2(1.503)(1+hs/2Wcp)a_4}{2(1.503+A)(1+hs/2Wcp)-Ahs/Wcp} a_3 \right\}$			.3236	(5)	
	70	$\frac{1}{2} [z a_5 / (1+z/2) a_4]$	1.49177		.74589		$c_p = cp$ $\frac{hs}{Wcp} = z$ $= 5.8679$

NEUTRON KINETICS

SHEET 2 of 2

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	* SET	PARAMETERS
DRAWING	MACHINE						
	45	$\lambda_1 \beta d_1 / (1-\beta) ab$	.0001		.0000		
	46	$\lambda_2 \beta d_2 / (1-\beta) ab$	.0002		.0000		
	47	$\lambda_3 \beta d_3 / (1-\beta) ab$	.0008		.0001		
	48	$\lambda_4 \beta d_4 / (1-\beta) ab$	.0023		.0002		
	49	$\lambda_5 \beta d_5 / (1-\beta) ab$	.0104		.0010		
	50	$\lambda_6 \beta d_6 / (1-\beta) ab$	.0286		.0029		
	51	$(1-\beta)b/e$	.0993		.0993		
	52	$(1-\beta) \times 10^2 / e$	.9927		.9927		
	53		.1000		.1000		

REACTIVITY EQUATIONS

SHEET 1 of 1

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING *	SET				PARAMETERS
DRAWING	MACHINE									
THERMAL	57	$C_u e / \beta a_{12}$	.6931		.6931					$C_u = .5092 \times 10^{-5}$ $C_{Na} = .5973 \times 10^{-5}$ $C_{pt} = 1.054 \times 10^{-5}$
	58	$C_{Na} e / 2 \beta a_{56}$	.4065		.4065					
	78	$1/2$	.5000		.5000					
	79	$1/2$	.5000		.5000					
PHASE TRANSFORMATION										
	56	$C_{pt} e / \beta a_{12}$	1.435		.1435	(10)				
$Kex/\beta = \eta \sin \omega t$										
	54	$.1 \omega/a$	ω/a		ω/a					
	55	$.1 \omega/a$	ω/a		ω/a					
	7	η	η		η					

RELAY CONTROL

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	* SETTING	SET	PARAMETERS
DRAWING	MACHINE						
	8	These potentiometers are	set to control				
	9	the temperature at which	the phase trans-				
	10	formation takes place, and to control					
	11	transformation such that no more than					
		the amount of reactivity produced during					
		the transformation is injected into the					
		system. In this analysis the amount of					
		heat stored or liberated = 197.2 cal/					
		3.795 cu. in. fuel pin.					
	71		.5000		.5000		
	72		.5000		.5000		
	73		.5000		.5000		
	74		.5000		.5000		

*¹ It is noted that the feedback capacitors of the integrators in this problem have a value of $0.1 \mu f$, increasing the gain or rate of integration of only the amplifiers used as integrators by a factor of 10. The potentiometer settings whose outputs are connected to the inputs of integrators are reduced by a factor of 10.

V. PRESENTATION OF MODEL PERFORMANCE

Figure 5-1. Response of the system with no phase transformation
 $(k_{ex}/\beta = 0.5 \sin 0.05 t)$

Figure 5-2. Response of the system with phase transformation at
 $\Delta T_{u_{avg}} = 0^\circ\text{C}$ ($k_{ex}/\beta = 0.64 \sin 0.05 t$)

Figure 5-3. Response of the system with phase transformation at $\Delta T_{u_{avg}} = +14^\circ\text{C}$ and 0°C ($k_{ex}/\beta = 0.64 \sin 0.05 t$)

Figure 5-4. Response of the system with phase transformation at $\Delta T_{u_{avg}} = \pm 7^\circ\text{C}$ ($k_{ex}/\beta = 0.64 \sin 0.05 t$)

Figure 5-5. Response of the system with no phase transformation ($k_{ex}/\beta = 0.64 \sin 0.1 t$)

Figure 5-6. Response of the system with phase transformation at $\Delta T_{u_{av}} = 0^\circ\text{C}$ ($k_{ex}/\beta = 0.64 \sin 0.1 t$)

Figure 5-7. Response of the system with phase transformation at $\Delta T_{u_{avg}} = +14^\circ\text{C}$ and 0°C ($k_{ex}/\beta = 0.64 \sin 0.1 t$)

Figure 5-8. Response of system with phase transformation at $\Delta T_{u_{avg}} = \pm 7^\circ\text{C}$ ($k_{ex}/\beta = 0.64 \sin 0.1 t$)

Figure 6-1 to 6-9. Phase-transformation response of the system to various sinusoidal reactivity inputs; with phase transformation initiated at the average uranium temperatures indicated in Figures 5-2 through 5-8, respectively.

Figure 6-1

Figure 6-2

Figure 6-3

Figure 6-4

Figure 6-5

Figure 6-6

Figure 6-7

Figure 6-8

Figure 6-9

VI. GENERAL DISCUSSION

One of the noteworthy results produced by this model of fuel phase transformation is shown typically in Figure 5-2. The k_{eff} curve on this figure indicates that the anticipated power rise of an increasing k_{ex} due to fuel insertion will be masked by the nullifying $\Delta k/k$ interjected by the axial phase transformation expansion. After completion of the transformation, the k_{eff} stops supplying heat for the transformation and directs it to increasing the temperature of the coolant.

The model appears to be a good representation of a pure uranium fuel element of the geometry described herein. Fuels other than this will require experimental results to verify the reactivity associated with its phase change.

The values given in the table below were obtained during the first half-cycle of excess reactivity input.

Temp Condition of Phase Transformation	Input	Amplitude Peak	Time to Amplitude Peak, sec
(+0, -0)	$k_{ex} = 0.64 \sin 0.05 t$	+470°C	38
(+0, -0)	$k_{ex} = 0.64 \sin 0.1 t$	+380°C	42
No phase change	$k_{ex} = 0.64 \sin 0.1 t$	+480°C	43

Since no describing function analysis was made of this nonlinear thermal response, no quantitative speculation was made concerning its dynamic stability.

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APPENDIX

GLOSSARY OF TERMS

Dilatometry - Dilatometry is the measurement of the dimensional change occurring in metals during heating, cooling, or holding at temperature. The linear coefficient of thermal expansion is one of the properties that can be determined by dilatometry.

Thermal Analysis - The determination of equilibrium conditions and phase relationships in metals by means of thermal arrests shown on heating and cooling curves.

Alloy - A substance that has metallic properties and is composed of two or more chemical elements of which at least one is a metal.

Transformation Temperature - The temperature at which a change in phase occurs. The term is sometimes used to denote the limiting temperature of a transformation range.

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