MAR 23 1964

MASTER

CEAL- 1010

STABILITY OF BEAM

IN RADIOFREQUENCY SYSTEM

by

Kenneth W. Robinson

Facsimile Price \$

1.60

Microfilm Price \$

.80

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

February 27, 1964

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CAMBRIDGE ELECTRON ACCELERATOR

HARVARD UNIVERSITY
42 OXFORD STREET
CAMBRIDGE 38, MASS.

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STABILITY OF BEAM IN RADIOFREQUENCY SYSTEM

ABSTRACT

The stability of the circulating beam in a radiofrequency system with large beam loading is investigated. The results are applied to the 3-Bev electron-positron storage ring under design at the Cambridge Electron Accelerator. It is found that for this type of system, in which there are no large transients, it should be possible to use most of the radiofrequency power to supply the radiation losses of the circulating beam, and only dissipate a small fraction of the power in the rf cavities.

STABILITY OF BEAM IN RADIOFREQUENCY SYSTEM

In the 3-Bev electron-positron storage ring to be operated in conjunction with the Cambridge Electron Accelerator, it is hoped that most of the radiofrequency power can be used to supply the radiation loss of the circulating beam, with only a small fraction of the radiofrequency power being dissipated in the rf cavities. For this reason it is necessary to investigate the stability of radiofrequency systems with very large beam loadings.

The basic equations for a radiofrequency-beam system are obtained by extending the equations derived in CEA-11 to the condition of large amplitude deviations from the equilibrium conditions. The equations are:

$$\frac{dV}{dt} = -\alpha V + \alpha V_g \cos \phi_v - \alpha V_B \sin \phi_B$$

$$\frac{d\phi_v}{dt} = -\alpha \tan \phi_Y - \alpha \frac{V_g}{V} \sin \phi_v + \alpha \frac{V_B}{V} \cos \phi_B$$

$$\frac{d\phi_B}{dt} = \frac{2\pi f \alpha' k}{U_0} \Delta U + \alpha \tan \phi_Y + \alpha \frac{V_g}{V} \sin \phi_v - \alpha \frac{V_B}{V} \cos \phi_B$$

$$\frac{d\Delta U}{dt} = f_0 (V \sin \phi_B - V_R)$$

V is the peak amplitude of the rf voltage per turn

V_g is the amplitude of the voltage that the rf power source would produce, if there were no beam, and if the system were tuned to resonance

V_B is the amplitude of the voltage that the beam would induce on the rf system, if the rf power were reduced to zero

while maintaining the same impedance for the rf power source, and the system were tuned to resonance. It is equal to twice the beam current divided by the conductance of the rf system.

ϕ_V is the phase angle between V and the phase angle that the rf power source would produce if there were no beam, and the system were tuned to resonance.

ϕ_Y is the phase angle of tuning of the admittance of the rf system, including the admittance of the rf power source

ϕ_B is the phase angle of the beam relative to the rf voltage, such that the energy delivered to the circulating beam

is $V \sin \phi_B$

α is the decay rate of the rf system including the conductance of the rf power source:

$$\alpha = \frac{\pi f}{Q}$$

f_0 is the orbital frequency

k is the harmonic number

α' is the average compaction factor

U_0 is the particle energy

ΔU is the energy deviation from the synchronous energy

V_R is the radiation loss per turn

The relation between the quantities is shown in the vector diagram.

See next page

These equations neglect radiation damping of synchrotron oscillations, which would make the system slightly more stable than the results obtained by neglecting radiation damping.

Also these equations assume that the length of a bunch is short compared to the rf wavelength, such that it may be treated as if it were concentrated at one point in phase. This assumption seems quite well justified for an electron-positron storage ring.

The steady state condition for the radiofrequency-beam system is obtained immediately from the first two equations by setting $\frac{dV}{dt} = 0$ and $\frac{d\phi_v}{dt} = 0$.

$$\tan \phi_v = \frac{V_B \cos \phi_B - V \tan \phi_\gamma}{V_B \sin \phi_B + V} \quad (2)$$

$$V_g = \frac{V_B \sin \phi_B + V}{\cos \phi_v}$$

For the steady state condition ϕ_B is the synchronous phase angle. $\phi_B = \sin^{-1} \frac{V_R}{V}$. In order to make V_g minimum the system would be tuned so that the reactance of the rf system compensates for the reactance of the beam.

$$V_B \cos \phi_B = V \tan \phi_Y \quad (3)$$

then $\phi_V = 0$ and $V_g = V_B \sin \phi_B + V$

If the equations for the radiofrequency-beam system are expanded for the condition of small oscillations about the equilibrium condition, the equations of CEA-11 are obtained. These equations are solved in CEA-11 for the roots of the form $e^{\gamma t}$. The equation for the roots is shown to be:

$$\gamma^4 + 2\alpha\gamma^3 + \left[\omega_s^2 + \alpha^2 + \alpha^2 \tan^2 \phi_Y \right] \gamma^2 + 2\alpha\omega_s^2 \gamma + \dots \quad (4)$$

$$\dots \left[\alpha^2 + \alpha^2 \tan^2 \phi_Y - \alpha^2 \frac{V_B}{V} \frac{\tan \phi_Y}{\cos \phi_B} \right] \omega_s^2 = 0$$

$$\omega_s = \left[\frac{2\pi f_0^2 \alpha^2 V \cos \phi_B}{U_0} \right]^{1/2} \text{ is the angular frequency of}$$

free synchrotron oscillations.

By means of Routh's criterion this equation is tested for roots with positive real parts. The condition for no roots to have positive real parts, which is the condition for stability, is found to be:

$$0 < \sin 2\phi_Y < 2 \frac{V}{V_B} \cos \phi_B \quad (5)$$

Then a radiofrequency-beam system is always unstable if it is tuned such that the phase angle of the admittance is negative. This condition corresponds to the rf system being inductive, or the applied frequency being below the resonant frequency of the cavity. This represents tuning the system

on the opposite side of resonance to that required to compensate for the reactance of the beam, so there seems to be no reason to operate a system tuned in this manner.

The equations and results obtained here apply to an accelerator or storage ring operating above the transition energy. For operation below the transition energy, the signs of the stability criterion are reversed, such that it is unstable when the applied frequency is above the resonant frequency. However the direction of tuning required to compensate for the reactance of the beam below transition is also reversed, so that there is also no reason to operate in this manner below transition.

If the beam loading of the system is large such that $2\frac{V}{V_B} \cos\phi_B < 1$, the system may be unstable when tuned such that ϕ_Y is positive. Under these conditions there will be two regions of stability for tuning of the radiofrequency system:

$$0 < \phi_Y < \frac{1}{2} \sin^{-1} \left(2\frac{V}{V_B} \cos\phi_B \right)$$

and $\frac{\pi}{2} - \frac{1}{2} \sin^{-1} \left(2\frac{V}{V_B} \cos\phi_B \right) < \phi_Y < \frac{\pi}{2}$ (6)

and the region

$$\frac{1}{2} \sin^{-1} \left(2\frac{V}{V_B} \cos\phi_B \right) < \phi_Y < \frac{\pi}{2} - \frac{1}{2} \sin^{-1} \left(2\frac{V}{V_B} \cos\phi_B \right)$$

will be unstable.

It may be seen that as the beam loading (V_B) is increased, the regions of stability become smaller.

In the design of the 3-Bev electron-positron storage ring to be operated in conjunction with the Cambridge Electron Accelerator, it is planned to supply the required 2.4 Mev per turn with a waveguide structure about 15 feet long operating at 475 m.c. This structure will dissipate about

50 kw of power. It is planned to have an rf power source of 500 kw capability. Then the largest possible beam loading conditions will have 90% of the rf power absorbed by the circulating beams and radiated as synchrotron radiation, and 10% of the rf power dissipated in the cavities. Therefore, we investigate this condition, as other conditions with less power absorbed by the beam will be more stable.

For purposes of this analysis the electron and positron beams circulating in opposite directions may be considered equivalent to a single beam of twice the intensity.

If we assume that the tuning of the cavity is adjusted to compensate for the reactive component of the circulating beam, and that the coupling between the rf power source and the cavity is adjusted to match the power source when the circulating beam absorbs a power of P_{BM} , then $\frac{V}{V_B}$ and $\tan\phi_Y$ may be written:

$$\frac{V}{V_B} = \left(\frac{P_D}{P_B}\right) \left[2 + \frac{P_{BM}}{P_D}\right] \sin\phi_B$$

$$\tan\phi_Y = \frac{\frac{P_B}{P_D} \cot\phi_B}{2 + \frac{P_{BM}}{P_D}}$$

P_B is the power absorbed by the beam, and P_D is the power dissipated in the cavity.

If the coupling is adjusted so the rf power source is matched at the beam power P_B , then the condition for stability becomes:

$$\sin 2 \tan^{-1} \frac{\cot \phi_B}{\frac{P_D}{2P_B} + 1} < 2 \left[\frac{P_D}{2P_B} + 1 \right] \cos \phi_B \sin \phi_B \quad (8)$$

In the proposed 3-Bev storage ring the radiation loss is 470 kv per turn and the rf voltage will be 2.4 Mev per turn, giving a synchronous phase angle of 11.3° .

When the rf power source is matched to the system, it is found that the system is stable for all values of $\frac{P_D}{P_B}$. For large values of P_B the system is always operating in the region of stability with ϕ_Y near 90° .

If the system is not matched to the power source, but is tuned to compensate for the reactance of the beam, the criterion for stability becomes:

$$\sin 2 \tan^{-1} \frac{\frac{P_B}{P_D} \cot \phi_B}{2 + \frac{P_{BM}}{P_D}} < 2 \frac{P_D}{P_B} \left[2 + \frac{P_{BM}}{P_D} \right] \sin \phi_B \cos \phi_B \quad (9)$$

It is found that if the coupling is adjusted so that the power is matched at a larger beam power ($P_{BM} < P_B$), then the system is always stable. It is then possible to adjust the coupling so that the rf power source is matched at the maximum beam current, and the system will always be stable at lower values of beam current without adjusting the coupling. However, there will then be large reflected power at lower beam currents, which may be disadvantageous to the rf power source or isolator if there is one.

If the coupling of the rf power source and the tuning of

the cavity are adjusted as the current is varied so as to keep the rf power source matched, equation 9 gives the tolerance on the adjustment of the coupling to maintain stability. For all conditions the rf power source may be over-coupled by any amount ($P_{BM} < P_B$), and the system will be stable. At 3-Bev with 10% of the power dissipated in the cavities, P_{BM} may be about 25% less than P_B before the system becomes unstable. For filling an operation at 2-Bev, with a current of 1 ampere in each beam, the radiation loss is 93 kv per turn, and the fraction of power dissipated in the cavity is about 18%. Under these conditions P_{BM} may be about 20% less than P_B before the system becomes unstable.

The tolerance of the tuning of θ_Y to avoid instability may be investigated using equation 5 in conjunction with equation 7. For 3-Bev operation with 10% of the power dissipated in the cavity, the value of θ_Y to compensate for the reactance of the beam is 76.4° . It is found that the system is always stable when θ_Y is larger than this value, and that θ_Y may be reduced to 76.1° before the system becomes unstable. This represents a change in tuning of about .05 bandwidths. It is probably better to operate with a value of θ_Y somewhat greater than that required to compensate for the reactance of the beam in order to increase the tolerance on the tuning.

At lower beam energies the region of stability becomes smaller. At 2-Bev with a current of one ampere in each beam, the region of stability is from $\theta_Y = 86.77^\circ$ to $\theta_Y = 90^\circ$, and the separation of θ_Y from the stability limit

when the system is matched is only $.006^\circ$, In these cases it seems to be necessary to adjust the tuning somewhat further from the stability limit, than for the matched condition.

The roots of equation 4 have been calculated for some cases. In the normal region of operation:

$$\frac{\pi}{2} - \frac{1}{2} \sin^{-1} \left(2 \frac{V}{V_B} \cos \phi_B \right) \ll \phi_Y < \frac{\pi}{2}$$

the roots give two damped sinusoidal oscillations. For the unstable case:

$$\frac{1}{2} \sin^{-1} \left(2 \frac{V}{V_B} \cos \phi_B \right) < \phi_Y < \frac{\pi}{2} - \frac{1}{2} \sin^{-1} \left(2 \frac{V}{V_B} \cos \phi_B \right)$$

the roots are a damped sinusoidal oscillation; and two real exponential solutions, one growing and one decreasing.

In the stable region,

$$0 < \phi_Y < \frac{1}{2} \sin^{-1} \left(2 \frac{V}{V_B} \cos \phi_B \right)$$

the roots are a damped sinusoidal oscillation, and two real decreasing exponential solutions.

In the unstable region $\phi_Y < 0$, the solutions are two sinusoidal oscillations, one growing and one damped.

For the particular case of 3-Bev energy with the system matched and 10% of the power dissipated in the cavity, with $\alpha = 6.6 \times 10^5 \text{ sec}^{-1}$, $[\omega_s = 5 \times 10^5 \text{ sec}^{-1}$,
 $\gamma = -1.99 \times 10^4 \pm 7.33 \times 10^4 i, -6.4 \times 10^5 \pm 2.77 \times 10^6 i$

The longest damping time is then about 50 microseconds.

A computer program was made to solve the differential equations (1) exactly. It was found that the small amplitude

solutions agreed with the calculated roots. For the case of 3-Bev energy and 10% of the power dissipated in the cavities, the beam would fall out of synchronism when started more than 12° from the phase stable position, due to large variations produced in the cavity voltage.

Since the storage ring is filled slowly, and all changes such as energy are made slowly, there should be nothing to produce large transients of this nature, so it should be possible to use most of the rf power to supply the radiation loss of the beam and only dissipate a small fraction in the cavity.

It would probably be possible to devise a feedback system sensing the phase position of the beam and modifying the amplitude or phase of the rf power so as to change the regions of stability. However, this would be a more complicated system, and does not seem to be necessary.

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