Theoretical Predictions for Alpha Particle Spectroscopic Strengths

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Multinucleon transfers induced in heavy-ion reactions of the type (${}^6\text{Li}, d$) furnish a selective probe with which to study the interplay between rotational and clustering phenomena so characteristic of the structure of the light sd-shell nuclei. For these nuclei (for p-shell results see ref. 3c), theoretical predictions for inter-band as well as intra-band transfer strengths can be made using recently tabulated results for angular momentum dependent $SU_3 \supset R_3$ relative spectroscopic strengths and angular momentum independent $SU_6 \supset SU_3$ coefficients of fractional parentage. The pure SU_3 (oscillator)- SU_4 (supermultiplet) symmetry limit agrees well with results obtained using available eigenfunctions determined in large shell model calculations. In particular, the scalar nature of a transferred "alpha"-cluster insures that the effect of spatial symmetry admixtures in the initial and final states of the target and residual nuclei are minimized. Sum rule quantities provide a measure of the probable effects of symmetry breaking. Strength variations within a band are expected; transfers to core excited states are often favored. Results extracted from exact finite range DWBA analyses of (${}^6\text{Li}, d$) data on ${}^{16,18}\text{O}$, ${}^{20,21,22}\text{Ne}$, ${}^{24,25}\text{Mg}$ show some anomalies in our understanding of the structure and/or reaction mechanisms.

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Introduction

As indicated in Figure 1, a large fraction of the activity in heavy-ion (HI) physics has been experimental, following for the most part logical extensions to earlier light-ion (LI) studies.¹ Closely linked with this there has been a tremendous effort devoted to understanding the complex reaction mechanisms effective in HI induced processes.² Some theoretical work has been done on the spectroscopy of multiparticle stripping and pickup strengths.³ If there is a need in the program as it is now emerging, it is for detailed theoretical-experimental comparisons, thereby testing our understanding of both the structure and reaction mechanisms. Bridging that gap is difficult. The problem is a standard one: In contrast with cross sections which drop⁴ as $\sim 1/A^3 \sim 1/R$, shell-model dimensionalities grow⁵ as $\sim \binom{N}{m} \sim N^m$. Here A is the number of nucleons and m and N are, respectively, the number of particles and single-particle levels outside the appropriate closed core. Such dimensionalities preclude microscopic calculations even without including multiparticle, multihole configurations which undoubtedly contribute strongly in HI induced transfer processes.⁶

With these considerations in mind, we adopted a "keep it simple" or "first things first" philosophy and set ourselves to the task of making theoretical predictions for α -particle transfer strengths among the light sd-shell nuclei ($16 \leq A \leq 28$). The reasons behind this particular choice are spelled out in Figure 2: First of all, shell-model calculations are feasible (The SU_3 (oscillator)- SU_4 (supermultiplet) model suggests useful approximations); Secondly, experimental studies are straightforward (For example, high resolution ($^6\text{Li}, d$) and ($d, ^6\text{Li}$) data are easily acquired); And third, the reaction channels are simple

{An α -cluster is a spin-isospin scalar}. In addition, of course, such studies allow one to probe the interplay between rotational and clustering degrees of freedom so characteristic of the structure of these nuclei.⁷ Most of the results I will be presenting were obtained through active collaboration with experimentalists at the Nuclear Structure Research Laboratory of the University of Rochester. As shown in Figure 3, this includes H. E. Gove, N. Anantaraman, R. M. DeVries, and J. Töke. Valuable theoretical support was received from K. T. Hecht (Michigan) and P. Manakos (Darmstadt/Stony Brook).

I. The Model

Our starting premise is that the SU_3 (oscillator)- SU_4 (supermultiplet) coupling scheme (LS as compared to jj) affords good quantum numbers for eigenstates of the light sd-shell nuclei.⁸ This has been tested in a variety of ways, perhaps the most critical being recent large shell-model calculations carried out by the theoretical group at Darmstadt.⁹ Rather than dwelling on the inadequacies of this assumption, and there are some, let me counter by saying there are selection rules which, especially for α -transfer processes, greatly enhance its credibility. The simplifications are tremendous! And, as I hope you will be persuaded, the end does justify the means!

A. Reaction Picture

To start from a proper perspective, consider Figure 4. The distorted-wave-Born-approximation (DWBA) structure factor is given in terms of two spectroscopic amplitudes, one for each of the interaction vertices.¹⁰ The sum is over internal state quantum numbers of the transferred group

of nucleons (angular momentum couplings being tacitly understood).^{3d} Each amplitude in this sum is determined by three factors. The first, which is trivial, comes from a generalized Talmi-Moshinsky, relative to center-of-mass coordinate transformation.¹¹ The second is the usual statistical factor.¹² And the third is itself a sum involving the product of a shell-model coefficient of fractional parentage (cfp) and an overlap factor for x -nucleons in a state Ψ_x with x -nucleons having an internal wavefunction Φ_x and motion relative to the well center given by $\Phi_{nl}(\vec{r})$ ($n = 2N + l$).^{3b} The sum is over all Ψ_x consistent with Φ_x and $\Phi_{nl}(\vec{r})$. If the transferred nucleons are in a single pure configuration with zero intrinsic angular momentum, as for the Os internal motion of an unexcited α -cluster, each sum reduces to a simple product. Relative strengths are then determined solely by the appropriate shell-model cfp. This is the simple cluster assumption; we assume it to be valid.

B. Structure Simplifications

For pure symmetry eigenstates, the relative spectroscopic amplitude can be factorized into two parts. This is shown in Figure 5. The first, dubbed the inter-band factor, carries all the spatial $\{[f]\}$ symmetry and spin-isospin $\{[\tilde{f}]\}$ dependence. The second or intra-band factor gives the expected strength variation within a rotational band of specific oscillator ($\lambda\mu$) symmetry. This factor carries all the dependence on angular momentum quantum numbers. The detailed expressions are given to display the simplicity of the theory! Programs for evaluating A^I and A^{II} exist. For sd-shell nuclei, numerical tabulations are available in the literature.^{3f,g}

Why should the pure symmetry limit be expected to give a reasonable

first approximation for α -transfer strengths? Certainly the $[f]$ and $(\lambda\mu)$ quantum numbers are approximately good (typically ~75% purity for sd-shell nuclei). States with maximum spatial symmetry are strongly favored. An α -cluster is space symmetric and spin-isospin antisymmetric so it fits right in! Beyond that however, is the fact that an α -cluster is an SU_4 scalar and therefore the $[f]$ couplings are all uniquely determined. A large component in the target couples only to the same (module 4) large component in the residual nucleus; similarly a small component only couples with the same (module 4) small component. Cross-symmetry connections vanish identically. This is true only for the transfer of an α -cluster. For the transfer of other clusters, be they totally space symmetric or not, cross-symmetry connection will in general contribute enhancing the possibility of small components in a wavefunction producing strong coherent effects in the transfer amplitude. In general, SU_3 couplings provide no comparable rigorous selection rule. Because of the strong correlation¹³ between the Hamiltonian and $-Q \cdot Q$ (the Casimir invariant of SU_3) however, the tendency is definitely towards re-enforcing this behavior. Sum rules can be used to give a quantitative measure of the effect. A typical example is $^{24}\text{Mg}([44](84)) \rightarrow ^{28}\text{Si}([444](\lambda\mu))$ for which the oblate shape $(\lambda\mu) = (0,12)$ is favored over $(\lambda\mu) = (66)$ by a factor of $9.0/2.5 = 3.6$.

II. Predicted Strengths

I now want to present some pure symmetry predictions, noting features of special interest. We will compare selected examples with more detailed calculations later.

A. Pure Symmetry Limit

The results shown in Figure 6 are for stripping and pickup strengths from even A targets to band members of the same $(\lambda\mu)$ symmetry. The numbers in parenthesis are ground state to ground state predicted strengths relative to one for $^{16}\text{O} + \alpha + ^{20}\text{Ne}$. All other numbers are relative to the predicted ground state strength and are therefore to be compared with unity. Note one striking prediction: In $^{24}\text{Mg} + ^{20}\text{Ne} + \alpha$ pickup, the 2^+ member of the ground state band is expected to be more weakly populated by about a factor of ten than other members of the band. As we shall see, calculations taking into account representation mixing do not change this result significantly. And indeed, spectroscopic strengths extracted from $(d, ^6\text{Li})$ data taken by Comfort, et al.¹⁴ on the Princeton cyclotron confirm this prediction. A similar result is predicted for $^{22}\text{Ne} \rightarrow ^{18}\text{O} + \alpha$. The canonical question is, "Why?", "What is the physics behind such an intriguing result?" I can only give what is at best an unsatisfactory answer; namely, it is in the coupling! Note also that in $^{20}\text{Ne} + \alpha \rightarrow ^{24}\text{Mg}$ the $J^\pi = 4^+$ of the $K_J^\pi = 2^+$ band is predicted to be strongly favored over the 4^+ of the ground state $K_J^\pi = 0^+$ band. As we shall see, this is also what one observes experimentally.

The quantity $\Sigma(\kappa)$ is the percentage of the fixed $(\lambda\mu)$ sum rule strength which goes into each band. Transitions to members of the ground state band are not necessarily expected to be favored. In $^{20}\text{Ne} + \alpha \rightarrow ^{24}\text{Mg}$ it is only 15% of the total! A partitioning of the strength into parts associated with a particular L-transfer, $\Sigma(L)$, is also shown. For example, as compared to $^{16}\text{O} + \alpha + ^{20}\text{Ne}$, only a small part of the total strength in $^{24}\text{Mg} + \alpha \rightarrow ^{28}\text{Si}$ is expected in the L=8 channel (38% as compared to 6%). This shift in strength from high to

low L-transfers with increasing target mass is a characteristic feature of the predictions.

In Figure 7 the predicted stripping strengths to the $J^\pi = 0^+, 2^+, 4^+$ members of the ground state band are plotted as a function of mass number. Relative to the $J^\pi = 0^+$, the $J^\pi = 2^+$ and 4^+ strengths are expected to fall sharply. The 6^+ and 8^+ members, though not shown, behave similarly. Experimental data seem to confirm this result. A similar effect has been noted in p-shell studies.^{3c} The higher the J the faster the fall. This interpretation applies only to members of the ground state band. It is neither predicted nor observed to hold for members of $K_J \neq 0$ excited bands. Plotted as a dashed curve are the predicted ground state to ground state strengths relative to one for $^{16}\text{O} + \alpha \rightarrow ^{20}\text{Ne}$. It falls to a minimum for $^{22}\text{Ne} + \alpha \rightarrow ^{26}\text{Mg}$ and rises again for $^{24}\text{Mg} + \alpha \rightarrow ^{28}\text{Si}$. One possible interpretation of this behavior is that it reflects blocking of the $d_{5/2}$ configurations with increasing mass of the target and then the subsequent availability of $s_{1/2}$ states. An example suggesting the value of such an interpretation will be discussed later. As shown in the inset, for fixed target state symmetry, the final $(\lambda'\mu')$ considered soaks up a large percentage of the total transfer strength.

B. Shell-model Comparisons

A number of these simple pure symmetry predictions have now been tested by direct comparison with detailed shell-model calculations. One such comparison is made in Figure 8. The detailed calculations were carried out by M. Conze at Darmstadt.¹⁵ The ^{20}Ne wavefunctions used in the calculation were determined in the full sd-shell-model space. For ^{24}Mg all symmetries which contribute strongly in the full space calculation for $J^\pi = 0^+$ states (dimension = 325) were included in the calculations for

higher J-values. Results are shown for the interactions of Kuo¹⁶, Preedom-Wildenthal¹⁷, and a Darmstadt version¹⁸ of the Kuo interaction obtained by fitting energy levels of nuclei through mass 21. The results speak for themselves. For low spins ($\zeta J^\pi = 6^+$) the pure symmetry prediction is in excellent agreement with the more realistic calculations. More often than not, the latter vary more among themselves than with the simple result. For higher spins the SU_3 predictions provide an upper bound. This is reasonable because the pure symmetry limit represents, at least for strong transitions, a prediction for states having a maximum coherence for α -clustering.

Another example, this time for odd A so multiple L-transfers are in general allowed, is given in Figure 9. Here the pure symmetry predictions are compared with the corresponding numbers obtained using realistic wavefunctions for ²¹Ne calculated by Arima and Strottman,¹⁹ The PW and Scalar labels refer to calculations for ²⁵Mg wavefunctions²⁰ obtained using the Preedom-Wildenthal¹⁷ and scalar interaction of Akiyama, Arima and Sebe²¹. The predictions are again in excellent agreement with one another. Where there are discrepancies, the structure of the calculated eigenstates was sensitive to the interaction (in ²⁵Mg there are two low-lying $K_J = \frac{1}{2}^+$ so strong mixing is expected and observed).

Before moving on to direct comparisons with experimental results, note that similar analyses can, and indeed have, been carried out for transfers to core-excited states³⁸ such as members of the 5.78 MeV, $K_J^\pi = 0^-$ band in ²⁰Ne. Low-lying negative parity excitations proceed either totally or predominantly via the transfer of 3 particles into sd-shell orbitals and one into an pf-orbital. In this case $2N + l = 9$ rather than $2N + l = 8$ as for pure sd-shell transfers. It follows that

such processes are favored by kinematic factors in the reaction process; essentially, the cluster hangs further out and is therefore more accessible. In some cases these processes are also favored by spectroscopic factors! Two examples are given in Figure 10. For $^{20}\text{Ne} + \alpha \rightarrow ^{24}\text{Mg}$, the sum rule strength to the $(11,3)^-$, $(sd)^3(pf)^1$ negative parity states is larger than the $(84)^+$, $(sd)^4$ strength by a factor of 1.45. The $(14,2)^+$, $(sd)^2(pf)^2$ is favored by 1.28. Relative to the ground state to ground state prediction, strengths to members of the various bands are as shown. Note that although the summed strength may seem large, strengths to individual states of interest may in fact be small. One cannot live by sum rules alone!

III. Experimental Findings

We have carried out $(^6\text{Li},d)$ studies on targets of $^{16,18}\text{O}$, $^{20,21,22}\text{Ne}$, $^{24,25}\text{Mg}$. The outgoing deuterons were momentum analyzed in a split-pole magnetic spectrograph and then, in most cases, detected in photographic emulsions. Typical distributions were taken in the angular range $\theta_{\text{lab}} = 5^\circ$ to 60° . A typical spectra is shown for $^{24}\text{Mg}(^6\text{Li},d)^{28}\text{Si}$ in Figure 11. Resolution was ≤ 70 keV. Note that the unnatural parity 3^+ state at 6.28 MeV is only seen weakly, suggesting that two-step processes, although present, are weak. DWBA calculations using the exact-finite-range code LOLA²² were carried out using a set of optical model parameters²³ determined from elastic α -scattering on ^{40}Ca . Equally good results have since been obtained using folded potentials to generate the appropriate form factors.³⁴ Typical fits to the data are shown in Figure 12a for $^{24}\text{Mg}(^6\text{Li},d)^{28}\text{Si}$. As is characteristic of a direct transfer, the $L=0$ curves are strongly peaked at forward angles. The $L=2$ and $L=4$

curves in Figure 12b peak further out. Since the angular distribution are characteristic of the transferred L they can, in favorable cases, be used to assign spin values.

A. Relative Strengths

A comparison of spectroscopic strengths extracted from $^{20}\text{Ne} + \alpha \rightarrow ^{24}\text{Mg}$ data with the corresponding theoretical predictions are shown in Figure 13. As mentioned previously, the agreement among the theoretical predictions is excellent. What is more striking is that there is also agreement with the experimental results. Note that as predicted, the second $J^\pi = 4^+$ is seen much more strongly than the first. Differences of a factor of two in the absolute predictions we consider quite tolerable.

For odd A nuclei more than one L-transfer will in general contribute. However, by using characteristic L-shapes from the neighboring even A nuclei or, perhaps less reliably from the DWBA itself, one can extract the relevant L-dependent spectroscopic strengths. Results for low-lying states in $^{21}\text{Ne} + \alpha \rightarrow ^{25}\text{Mg}$ are shown in Figure 14. As noted previously, there is excellent agreement among the theoretical predictions. And agreement with experiment is again quite acceptable. In the ground state band the lowest allowed L-transfer is predicted to dominate and it is observed to do so. It should be noted, with all due humility of course, that the theoretical results were first presented at the Washington meeting a year ago²⁵ -- the experimental results are now about a month old! The first $K_J = \frac{1}{2}^+$ band is of considerable interest because transfer to the 3/2 member is predicted to be dominated by L=2 even though the L=0 is not ruled out by angular momentum selection. This is what one observes. Figure 15 shows the relevant angular

distribution. The $J^\pi = 3/2^+$ shape does not display the strong forward peaking of a pure $L=0$ transfer. More conclusively, the pure $L=2$ shape of the $J^\pi = 1/2^+$ transfer fits the $3/2^+$ angular distribution.

We have equally interesting data for $^{16,18}\text{O}$ and $^{24,25}\text{Mg}$ targets. Since most of that has already been published,²⁶ we will not pursue it further except to summarize our findings. In all cases the angular distributions are characteristic if the transferred L is 0, 2 or 4. At higher energies this should extend to higher L -values. For the neon isotopes a discrepancy exists between theory and experiment. This is shown by the comparisons made in Figure 16. The 2^+ and 4^+ states are not observed to be as strongly populated as predicted. The ^{16}O experiment has since been repeated at other energies ($E_{\text{Li}}=20$ MeV and 38 MeV, the results shown are from 32 MeV data) with much improved agreement between theory and experiment. Changing beam energy does not however affect the $^{20}\text{Ne} + \alpha + ^{24}\text{Mg}$ results. This is, to say the least, puzzling. Several possibilities, including extensions to the simple cluster DWBA reaction assumption, and the effect of particle-hole admixtures in the target, were cursively explored, but none proved satisfactory. No simple SU_3 prediction explains the $^{25}\text{Mg} + \alpha + ^{29}\text{Si}$ results either.²⁶ In this case, however, the obvious symmetry assignments $\{(66) \rightarrow (1,11)\}$ exhaust only about 14% of the sum rule strength. This is to be compared to $\geq 50\%$ for the lighter sd-shell nuclei, and in particular 60% for $^{24}\text{Mg} + \alpha + ^{28}\text{Si} \{(84) \rightarrow (0,12)\}$. The lack of agreement in this case is therefore not surprising.

B. Absolute Strengths

As a final comparison, in Figure 17 the predicted ground state to ground state strengths are plotted as a function of target mass. For

even A targets this means the strongest sd-shell transfer but for odd A targets it need not. The odd A strengths are shifted down by a factor of five to ten from the neighboring even A values. This represents a fragmentation of L-transfer strength due to spin couplings. The crosses are the measured values relative to $^{16}\text{O} + \alpha + ^{20}\text{Ne}$. The agreement is excellent. Only for $^{20}\text{Ne} + \alpha + ^{24}\text{Mg}$ is there any significant difference and even this we consider consistent to within experimental uncertainties.

I have restricted comparisons with data to low-lying positive parity ($^6\text{Li}, d$) stripping results. That obviously reflects a prejudice! There is, of course, other data with which one can compare. In particular, ($d, ^6\text{Li}$) pickup results are available.²⁷ I must apologize for not referring to those studies. Some comparisons were presented at the Maryland Conference on Clustering.²⁹

Conclusion

The natural question is, "Can calculations such as these I have described be extended to other mass regions?" The answer is yes, in principle it is possible to do so. Just because the pure symmetry limit breaks down it does not mean the techniques are no longer valid. It does mean however, that detailed shell-model calculations will be required so there is the old problem of being able to diagonalize huge matrices. Some hope is provided by earlier successes of the so-called pseudo SU_3 model for the strongly deformed nuclei. With the available technology, the problems are in principle no greater here than for calculations in the mass 16-28 region. This approach is being investigated.²⁹

I would like to suggest an alternative approach which seems promising. It uses statistical procedures and is therefore not limited by dimensionality considerations. The full potential of such a scheme has only

recently been demonstrated for evaluating E2 strengths.³⁰ It is given in its most elementary form in Figure 18. The zeroth order strength is a sum over configurations of the product of two configuration intensity functions, one for the target and one for the final nucleus, weighted by the average strength with which the excitation operator couples the pair. If $\chi^\Gamma = (A \times A)^\Gamma$ is a coupled two-nucleon creation operator, then for α -transfers $\theta = (\chi_N^\Gamma \times \chi_P^\Gamma)^\Gamma \alpha$. Each configuration intensity is for fixed J and T. They depend upon the excitation energy and are uniquely determined by the system Hamiltonian.³¹ The histograms display cases of interest in $^{24}\text{Mg} + \alpha + ^{28}\text{Si}$. Given the averages $\langle\langle\theta^+\theta\rangle\rangle$, which are nothing more than averages of four-particle coefficients of fractional parentage, the sum is easy to evaluate. For example, starting from the $(d_{5/2})^8$ configuration in ^{24}Mg (17% intensity in ground state) and adding an $(s_{1/2})^4$ cluster carries one to the $(d_{5/2})^8(s_{1/2})^4$ configuration in ^{28}Si (24% for $J^\pi = 0^+$, 9% for $J^\pi = 2^+$, and 8% for $J^\pi = 4^+$), etc. The transfer is even presented in a form convenient for calculating a microscopic form factor!

The zeroth order approximation is somewhat like the direct term in a DWBA analysis. Interference effects enter naturally in higher order. The Gaussian or Central Limit Theorem (CLT) result³² requires, for example, three additional terms, one linear in E with coupling $\langle\langle\theta^+\theta H\rangle\rangle$, one linear in E' with coupling $\langle\langle\theta^+ H \theta\rangle\rangle$, and one bilinear in EE' with coupling $\langle\langle\theta^+ H \theta H\rangle\rangle$. These terms introduce two-step correlations. The formally exact expansion include all order correlations.³⁰ I am confident that the CLT expansion with fixed $\bar{m}$, J, T partitioning, of which this lowest (zeroth) order approximation is but the first of four terms, will suffice to provide accurate limits for sum rule strengths. How well it can do for individual states is at this point only a matter of speculation. These questions are being investigated.

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Figure Captions

1. A characterization of heavy-ion physics ~ 1975.
2. Why study α -transfer among the light sd-shell nuclei?
3. Experimental and theoretical collaboration.
4. The DWBA structure factor (angular momentum couplings quietly understood). The first sum involves the product of two spectroscopic amplitudes, one for each vertex, and is over the internal state quantum numbers of the transferred cluster. Each amplitude in turn involves the product of a shell-model cfp and the overlap of a shell model wavefunction ψ_x with the internal state ϕ_x and motion relative to the well-center given by $\Phi_{nl}(\vec{r})$. Here we use $n = 2N + l$ for the principle quantum number. The first multiplicative factor is from a generalized Talmi-Moshinsky bracket for the relative to center of mass transformation and the second is the usual statistical factor. For the transfer of a cluster ($x \leq 4$) in a single pure configuration with zero intrinsic angular momentum each sum reduces to a simple product. Relative strengths are then determined solely by the shell-model cfp.
5. Factorization of spectroscopic amplitudes for pure spatial $[f]$ {spin-isospin $[\tilde{f}]$ } and oscillator $(\lambda\mu)$ symmetries. The total spectroscopic amplitude can be expressed as a product of an inter-band factor which carries all the space and spin-isospin dependence and an intra-band factor which displays the total dependence on angular momentum quantum numbers. The detailed expressions are calculable; numerical results for sd-shell nuclei have been tabulated. For the transfer of an α -particle $[\tilde{f}] = [1^4] = [0]$ so all $[f]([\tilde{f}])$ couplings are uniquely determined. Because states of maximum spatial symmetry are favored, large components in the target couple only to large components in the residual nucleus. Sum rule measures show that the $(\lambda\mu)$ couplings enhance this selectivity.

6. Spectroscopic strengths for α -transfer among stable even A sd-shell nuclei with $16 \leq A \leq 28$. The pure symmetry predictions.
7. Predicted ground state band systematics for α -stripping. Relative to $J^\pi = 0^+$, the strength falls with increasing spin. The higher the J, the faster the fall. The absolute (relative to unity for $^{16}\text{O} + \alpha + ^{20}\text{Ne}$) $J^\pi = 0^+$ strengths are given on the dashed curve. The predicted fall and subsequent rise can be related to configuration occupancies. The inset percentages indicated what fraction of the total α -strength is represented by the assigned final state ($\lambda'\mu'$).
8. A comparison of theoretical predictions for α -strengths in $^{20}\text{Ne} + \alpha \rightarrow ^{24}\text{Mg}$. SU(3) is the pure symmetry prediction. The other results were obtained by M. Conze at Darmstadt using wavefunctions for ^{20}Ne calculated in the full space and for ^{24}Mg calculated using all important symmetries found in a full calculation for $J^\pi = 0^+$. The effective interactions are - Q·Q (SU(3)), Kuo, Freedom-Wildenthal, and a Darmstadt version of Kuo-Brown obtained by fitting levels through A=21.
9. A comparison of theoretical predictions for α -strengths in $^{21}\text{Ne} + \alpha + ^{25}\text{Mg}$. SU(3) is the pure symmetry prediction. Wavefunctions for ^{21}Ne were taken from the compilation of Arima and Strottman. Scalar and PW refer to ^{25}Mg wavefunctions as obtained by Draayer using the scalar interaction of Akiyama, Arima and Sebe and the Freedom-Wildenthal interaction. The asterisk indicates value obtained from eigenfunction in ^{25}Mg which were sensitive to the choice of residual interaction and hence not expected to be a reliable prediction.
10. Spectroscopic strengths to core-excited states in $^{20}\text{Ne} + \alpha + ^{24}\text{Mg}$. The sum rule strength to states of leading $(sd)^3(pf)^1$ and $(sd)^2(pf)^2$ SU(3) symmetry are favored over the $(sd)^4(84)$ symmetry by 1.45 and 1.28, respectively. The observed favoring of low ℓ -transfers in

$\kappa_J^\pi = 0^+$ bands is seen to extend to core-excited representations. A strong α -transfer to the $J^\pi = 3^-$ member of the $\kappa_J^\pi = 1^-$ band is expected whereas the 3^- band head should only be populated very weakly.

11. Typical experimental spectra. The unnatural parity state at 6.28 MeV is only weakly excited, suggesting that multistep processes, though present, do not dominate. Resolution is typically ≤ 70 keV.
12. Angular distributions for $^{24}\text{Mg}(^6\text{Li},d)^{28}\text{Si}$ together with EFR (LOLA) fits. The ℓ -curves are characteristic and hence can, in favorable cases, be used to assign spin values.
13. Experimental-theoretical comparisons of α -particle stripping strengths on ^{20}Ne . As predicted, the $J^\pi = 4_1^+$ states at 4.12 MeV is more weakly populated by about a factor of ten than the $J^\pi = 4_2^+$ state at 6.01 MeV. Agreement of absolute strengths within a factor of two is considered within experimental uncertainties.
14. Experimental-theoretical comparisons of α -particle stripping strengths on ^{21}Ne . As predicted, the $J^\pi = 3/2^+$ (975 keV) transfer is strongly populated via an $\ell=2$ transfer, even though the $\ell=0$ is not ruled out by angular momentum couplings.
15. Angular distributions for α -transfer to members of the $\kappa_J^\pi = 1/2^+$ band in ^{25}Mg . Note that the $1/2^+$ and $3/2^+$ shapes are identical indicating a near pure $\ell=2$ transfer for the $3/2^+$ member ($\ell=0$ is allowed once $J_{\text{g.s.}}^\pi = 3/2^+$ in ^{21}Ne).
16. A comparison of theory and experiment using the low-lying $J^\pi=0^+, 2^+, 4^+$ states of the light sd shell nuclei. The observed $J^\pi = 2^+$ and 4^+ strengths for the oxygen targets are substantially weaker than predicted. At 20 MeV and 38 MeV (the results quoted are from 32 MeV data) much improved agreement is obtained. Changing beam energy does not change the ^{24}Mg and ^{28}Si results significantly. Several possible explanations that were cursively explored, including

coupled-channels effects and particle-hole admixtures in the oxygen ground states, proved unsatisfactory in explaining the discrepancy.

17. Ground state strengths as a function of target mass. The solid dots with connecting lines are the theoretical predictions. The crosses are the measured values.
18. Distribution approach for calculating transfer strengths. The average occupancies are given by $\bar{n}_{5/2}$, $\bar{n}_{1/2}$, $\bar{n}_{3/2}$. Configuration intensities are represented by the histogram percentages. In lowest order the transfer strength is simply a sum over target and final state configurations of the product of the corresponding intensities weighted by the average strength with which the excitation operator couples the pair. The technique, for all practical purposes, is independent of dimensionality limitations which plague microscopic shell-model calculations.

REACTION

EXPERIMENT

THEORY

GRAZING ANGLE
 Q-VALUES
 CLUSTERS
 TWO-STEP
 STRONGLY DAMPED COLLISIONS
 COULOMB BARRIERS
 (16, 16)
 (12C, 8Be)
 (16O, 12C)
 (23O, 12C)
 (6Li, d)
 (7Li, t)
 A(a, b)B
 ZERO RANGE
 FORM FACTORS
 UNNATURAL PARITY
 Q-SUBP
 WARS
 SEMI-CLASSICAL
 SATURN
 CCBA
 ALPHA CORRELATIONS
 PSEUDO SU(3)
 GENERATOR COORDINATES
 DMB
 MULTI-PARTICLE CFP
 LOLA
 CENTER-OF-MASS
 CLUSTERING
 QUARTET MODEL
 ALPHA-DECAY
 $\lambda_{\alpha}(y' + y + x)$
 $\sigma(q_1)$
 $\{r\}(\lambda\mu)$
 x^*
 SEQUENTIAL TRANSFER

INVERSE ALPHA DECAY
 RECOIL
 JULIE
 FOLDED POTENTIALS
 SU₄-SCALAR
 (16O, 15N)
 EFR
 DRUCK
 25 MV
 $-1/A^3$
 $g_0 = \tau_y / \tau_{\text{EP}}$
 $(\frac{B}{B-K})^4$
 $- \mu^2$

HI PHYSICS

SHELL-MODEL CALCULATIONS FEASIBLE

{The SU_3 (oscillator)- SU_4 (supermultiplet)
model suggests useful approximations}

EXPERIMENTAL STUDIES STRAIGHTFORWARD

----- {For example, high-resolution $(^6\text{Li}, d)$
and $(d, ^6\text{Li})$ data are easily acquired}

REACTION CHANNELS SIMPLE

{An "alpha"-cluster is a spin-isospin scalar}

EXPERIMENTAL:

H. E. Gove
N. Anantaraman
R. M. DeVries
J. Töke

THEORETICAL:

K. T. Hecht (Michigan)
P. Manakos (Darmstadt)

$A(a,b)B$

$$A_{n\ell, n'\ell'} = \int A_{n\ell}(B \rightarrow A + x) A_{n'\ell'}^*(a \rightarrow b + x)$$

$$A_{n\ell}(y' \rightarrow y + x) = \left(\frac{y'}{y}\right)^{n/2} \left(\frac{y'}{y}\right)^{1/2} \int \langle \Psi_y; \Psi_x | \rangle \Psi_{y'} \rangle \langle \Phi_x; \Phi_{n\ell}(\bar{r}) | \Psi_x \rangle$$

$$\Lambda(Y' \rightarrow Y + X) = A^I \text{ (INTER-BAND)} \times A^{II} \text{ (INTRA-BAND)}$$

$$A^I = \left(\frac{Y'}{Y}\right)^{n/2} \quad \text{transform, rel to cm}$$

$$\times G(qi) \quad \text{overlap, } \langle \phi_x; \phi_{nl}(\vec{R}) | \psi_x \rangle$$

$$\times \left[\left(\frac{m}{x} \right) \frac{d[f]}{d[f']} \right]^{1/2} \quad \text{statistical factors}$$

$$\times \langle [f] \alpha(\lambda\mu); [x](n,0) | [f'] \alpha'(\lambda'\mu') \rangle \quad SU_6 \rightarrow SU_3$$

$$\times \langle [\tilde{f}] \beta ST; [1^x]_{St} | [\tilde{f}'] \beta' S' T' \rangle \quad SU_4 \rightarrow SU_2 \times SU_2$$

$$A^{II} = \sum_{\kappa L \kappa' L'} C_{\kappa L} C_{\kappa' L'} \quad \text{state amplitudes}$$

$$\times X \begin{pmatrix} L & S & J \\ l & s & j \\ L' & S' & J' \end{pmatrix} \quad \text{LS-jj recoupling}$$

$$\times \langle (\lambda\mu) \kappa L; (n0) l | (\lambda'\mu') \kappa' L' \rangle \quad SU_3 \rightarrow R_3$$

SELECTION RULES (ALPHA-TRANSFER)

$$SU_4: \quad [f_\alpha] = [4] \{ [\tilde{f}_\alpha] = [0] \} \Rightarrow [f'] = [4f] \{ [\tilde{f}'] = [\tilde{f}] \}$$

$$\begin{array}{ccc} [f_1](\text{large}) & \xrightarrow{\quad} & [4f_1](\text{large}) \\ & \text{forbidden} & \\ [f_2](\text{small}) & \xrightarrow{\quad} & [4f_2](\text{small}) \end{array}$$

$$SU_3: \quad H \sim -Q \cdot Q \Rightarrow \text{leading } (\lambda\mu) \text{ favored}$$

$$\frac{{}^{24}_{Mg}([44](84)) \rightarrow {}^{28}_{Si}([444](0,12))}{{}^{24}_{Mg}([44](84)) \rightarrow {}^{28}_{Si}([444](66))} = \frac{9.0}{2.5} = 3.6$$

$[A_{n\lambda}(y' \pm y + \alpha)]^2$ for α -transfers among stable even A sd-shell nuclei with $16 \leq A \leq 28$

STRIPPING ($y+\alpha \rightarrow y'$ with $\kappa_J=J=0$)		(J'=L values for $\kappa_J=0$)						(J'=L values for $\kappa_J=2$)					(J'=L values for $\kappa_J=4$)				$\Sigma(L)$				
$y(\lambda\mu)$	$y'(\lambda'\mu')$	0	2	4	6	8	$\Sigma(\kappa)$	2	4	6	8	$\Sigma(\kappa)$	4	6	8	$\Sigma(\kappa)$	0	2	4	6	8
$^{16}_0(00)$	$^{20}_{Ne}(80)$	(1.00)	1.00	1.00	1.00	1.00	100										2	11	20	29	38
$^{18}_0(40)$	$^{22}_{Ne}(82)$	(0.34)	0.77	0.36	0.04	0.07	27	0.00	0.06	0.33	1.23	73					3	11	11	14	62
$^{20}_{Ne}(80)$	$^{24}_{Mg}(84)$	(0.35)	0.37	0.80	0.15	0.00	15	0.05	0.46	0.71	0.05	43	0.00	0.04	0.80	43	3	6	12	35	44
$^{22}_{Ne}(82)$	$^{26}_{Mg}(48)$	(0.18)	0.24	0.12	0.06	0.03	27	0.05	0.01	0.00	0.00	2	0.75	0.26	0.10	71	6	9	47	25	13
$^{24}_{Mg}(84)$	$^{28}_{Si}(0,12)$	(0.41)	0.23	0.10	0.04	0.02	100										26	30	24	15	6
PICKUP ($y' \rightarrow y+\alpha$ with $\kappa_J=J'=0$)		(J=L values for $\kappa_J=0$)						(J=L values for $\kappa_J=2$)					(J=L values for $\kappa_J=4$)				$\Sigma(L)$				
$y'(\lambda'\mu')$	$y(\lambda\mu)$	0	2	4	6	8	$\Sigma(\kappa)$	2	4	6	8	$\Sigma(\kappa)$	4	6	8	$\Sigma(\kappa)$	0	2	4	6	8
$^{20}_{Ne}(80)$	$^{16}_0(00)$	(1.00)					100										100				
$^{22}_{Ne}(82)$	$^{18}_0(40)$	(0.34)	0.00	2.28			100										31	0	69		
$^{24}_{Mg}(84)$	$^{20}_{Ne}(80)$	(0.35)	0.13	0.80	1.84	1.02	100										21	3	17	38	21
$^{26}_{Mg}(48)$	$^{22}_{Ne}(82)$	(0.18)	0.77	0.11	0.21	1.53	42	1.32	0.23	0.17	3.29	58					12	24	4	4	56
$^{28}_{Si}(0,12)$	$^{24}_{Mg}(84)$	(0.41)	1.20	1.07	0.83	0.45	34	2.10	1.69	1.79	1.93	66	0.40	0.34	0.53	10	8	25	24	22	22

$$[A_{n2}({}^{24}\text{Mg} + {}^{20}\text{Ne} + \alpha)]^2$$

J^π	SU(3)	KUO	PW	KB-F3
$0_1^+ - 0_1^+$	(0.353)	(0.436)	(0.339)	(0.323)
${}^{20}\text{Ne} \quad 2_1^+$	0.13	0.13	0.12	0.16
4_1^+	0.80	0.60	0.71	0.56
6_1^+	1.84	0.88	0.92	0.94
8_1^+	1.02	0.34	0.46	0.34
${}^{24}\text{Mg} \quad 2_1^+$	0.37	0.44	0.43	0.40
4_1^+	0.00	0.04	0.03	0.00
6_1^+	0.15	0.00	0.00	0.07
2_2^+	0.05	0.02	0.04	0.07
4_2^+	0.46	0.29	0.34	0.43

$$[A_{nl}(^{21}\text{Ne} + \alpha + ^{25}\text{Mg})]^2$$

κ_J^π	J	ℓ	SU(3)	SCALAR	PW
$5/2^+$	5/2	2	(0.044)	(0.064)	(0.048)
		4	0.00	0.05	0.03
	7/2	2	2.20	1.81	2.32
		4	0.04	0.00	0.02
	9/2	4	0.48	0.31	0.20
		6	0.00	0.04	0.05
$1/2_1^+$	1/2	2	0.51	0.48	0.73
	3/2	0	0.49	0.64	1.01
		2	2.92	3.26	5.19
	5/2	2	1.28	0.63	1.42
		4	0.01	0.03	0.02
	$7/2^*$	2	0.01	0.08	0.23
		4	1.32	2.78	4.66
$1/2_2^+$	1/2	2	1.27	0.70	0.90
	$3/2^*$	0	0.01	0.27	0.15
		2	0.54	0.19	0.01
	$5/2^*$	2	0.14	0.34	0.49
		4	2.53	0.70	0.13

*Calculated eigenstates do not correspond to well-developed rotational bands.

Alpha-Particle Spectroscopic Strengths for Stripping to Core-Excited States

$$\frac{{}^{20}\text{Ne}(80)^+ \rightarrow {}^{24}\text{Mg}(11,3)^-}{{}^{20}\text{Ne}(80)^+ \rightarrow {}^{24}\text{Mg}(84)^+} = 1.45$$

$$\frac{{}^{20}\text{Ne}(80)^+ \rightarrow {}^{24}\text{Mg}(14,2)^+}{{}^{20}\text{Ne}(80)^+ \rightarrow {}^{24}\text{Mg}(84)^+} = 1.28$$

(sd) ⁴			(sd) ³ (pf) ¹			(sd) ² (pf) ²		
(λμ)κ _J ^π J ^π	Percent Sum Rule	A ²	(λμ)κ _J ^π J ^π	Percent Sum Rule	A ²	(λμ)κ _J ^π J ^π	Percent Sum Rule	A ²
(84) 0 ⁺ 0 ⁺	3.0	(0.488)	(11,3)1 ⁻ 1 ⁻	5.5	0.200	(14,2) 0 ⁺ 0 ⁺	2.1	0.917
2 ⁺	5.6	0.374	3 ⁻	21.4	0.780	2 ⁺	7.5	0.639
4 ⁺	0.0	0.001	5 ⁻	19.7	0.720	4 ⁺	4.6	0.221
6 ⁺	5.8	0.149	7 ⁻	2.0	0.074	6 ⁺	0.1	0.003
8 ⁺	0.2	0.004	9 ⁻	9.7	0.354	8 ⁺	3.6	0.092
2 ⁺ 2 ⁺	0.7	0.047	3 ⁻ 3 ⁻	0.1	0.002	10 ⁺	9.8	0.201
4 ⁺	12.3	0.455	5 ⁻	1.5	0.056	2 ⁺ 2 ⁺	0.1	0.007
6 ⁺	27.4	0.708	7 ⁻	9.9	0.362	4 ⁺	2.0	0.096
8 ⁺	2.6	0.052	9 ⁻	30.3	1.107	6 ⁺	10.5	0.344
4 ⁺ 4 ⁺	0.0	0.001				8 ⁺	26.6	0.669
6 ⁺	1.7	0.042				10 ⁺	34.2	0.699
8 ⁺	40.9	0.804						

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E_x (MeV)	J^π	$(^6\text{Li}, d)$	SU(3)	KUO	S_α	PW	KB-F3
0	0_1^+	(0.71)	(0.353)	(0.436)	(0.339)	(0.323)	
1.37	2_1^+	0.42	0.37	0.44	0.43	0.40	
4.12	4_1^+	0.03	0.00	0.04	0.03	0.00	
4.24	2_2^+	0.13	0.05	0.02	0.04	0.07	
5.22	3_1^+	0.03	unnatural parity, no direct coupling				
6.01	4_2^+	0.28	0.46	0.29	0.34	0.43	
6.44	0_2^+		0.86	0.49	0.86	0.93

$E_x(\text{keV})$	π_{K_J}	J	ℓ	$(^6\text{Li}, d)$	S_α		
					SU(3)	Scalar	PW
0	$5/2^+$	$5/2$	2	(0.03)	(0.044)	(0.064)	(0.048)
			4	-	0.00	0.05	0.03
1612		$7/2$	2	1.90	2.20	1.81	2.32
			4	-	0.04	0.00	0.02
585	$1/2^+$	$1/2$	2	0.24	0.51	0.48	0.73
975		$3/2$	0	-	0.49	0.64	1.01
			2	1.94	2.92	3.26	5.19
1965		$5/2$	2	1.45	1.28	0.63	1.42
			4	-	0.01	0.03	0.02
2736		$7/2^*$	2	.98	0.01	0.08	0.23
			4	1.17	1.32	2.78	4.66
2564	$1/2^+$	$1/2$	2	0.37	1.27	0.70	0.90
2801		$3/2^*$	0	2.12	0.01	0.27	0.15
			2	0.88	0.54	0.19	0.01
3901		$5/2^*$	2	0.42	0.14	0.34	0.49
			4	0.26	2.53	0.70	0.13

* Calculated eigenstates do not correspond to well-developed rotational bands.

Relative Alpha Spectroscopic Strengths

Final Nucleus	$S_{\alpha}({}^6\text{Li,d})$			$S_{\alpha}(\text{Theory})$		
	0^{+}	2^{+}	4^{+}	0^{+}	2^{+}	4^{+}
${}^{20}\text{Ne}$	1.00	0.30	0.19	1.00	1.00	1.00
${}^{22}\text{Ne}$	1.00	0.35	0.11	1.00	0.83	0.50
${}^{24}\text{Mg}$	1.00	0.42	0.03	1.00	0.47	0.02
${}^{28}\text{Si}$	1.00	0.21	0.10	1.00	0.23	0.10

G.S. STRENGTH

$$\text{TRANSFER STRENGTH}^* \sim R(E, E) = \sum_{\bar{m}, \bar{m}'} \langle \langle \Theta^+ \Theta(\bar{m}' - \bar{m}) \rangle \rangle_{\bar{m}} \bar{m}' J' T' \bar{m} J T$$

$$\langle \langle \Theta^+ \Theta(\bar{m}' - \bar{m}) \rangle \rangle_{\bar{m}} = \frac{1}{d(\bar{m}') d(\bar{m})} \sum_{J' T'} |\langle \bar{m}' J' T' | \Theta | \bar{m} J T \rangle|^2$$

$$\Theta = (\chi^{\Gamma_N} \times \chi^{\Gamma_P}), \quad \chi^{\Gamma} = (A \times A)^{\Gamma}$$

* LOWEST ORDER APPROXIMATION