Intrabeam Scattering Results for a
High Frequency RF System

(Mini-Workshop on RHIC RF Systems)

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Intra beam Scattering Results
for a

High Frequency RF System

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Factors Leading to Growth

$$1) \frac{1}{\sigma} \frac{d\sigma}{dt} \sim (Q^2/A)^2$$

$$2) \frac{1}{\sigma} \frac{d\sigma}{dt} \sim N_b / 6\text{-dimensional Phase Space}$$

$$3) \text{ Distance from "Equilibrium State"}$$

$$\text{Invariant, } \sigma_E^2 - 2 \sigma_x^2 = \text{Constant}$$

$$\sigma_E = X_p \sigma_p$$

holds for $x > x_t$, cells only lattice,
complete coupling, bunched beam

$$a) t \rightarrow \infty, \sigma_E \sim 1.4 \sigma_x$$

$$b) \frac{1}{\sigma_p} \frac{d\sigma_p}{dt} = 2 \left(\frac{\sigma_x}{\sigma_E} \right)^2 \frac{1}{\sigma_x} \frac{d\sigma_x}{dt}$$

If $\sigma_E \ll \sigma_x$ at $t=0$, then σ_E will grow much faster than σ_x until

$$\sigma_E \sim 1.4 \sigma_x$$

Protons may show large growth in σ_E ,
 if $\sigma_E < \sigma_x$ at $t = 0$

$$\frac{1}{\sigma_x} \frac{d\sigma_x}{dt} \approx \frac{\sigma_E^2}{\sigma_E^2 + \sigma_x^2}$$

$$\frac{1}{\sigma_P} \frac{d\sigma_P}{dt} \approx 2 \frac{\sigma_x^2}{\sigma_E^2 + \sigma_x^2}$$

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High Frequency RF system

To get shorter σ_z suggests $f \sim 200 \text{ MHz}$

New Suggested Procedure

V varied with time so that bucket just contains the bunch, $\Delta_B = 2\sigma_p$ at all times.

For fixed σ_p / Δ_B , σ_z is constant.

$$\frac{\sigma_p}{\Delta_B} = \sin \phi/2$$

For $\Delta_B = 2\sigma_p$, $\phi = 60^\circ = \frac{1}{3}\pi$

$$\sigma_z = \frac{1}{3} l_B$$

Proposed RF System

$$f = 160 \text{ MHz}, \quad h = 2052$$

$$\sigma_L = 31 \text{ cm}, \quad A_0 = .3 \text{ eV-sec}$$

$$\Delta_B = 2 \sigma_P$$

frequency dependence

$$\text{Initially, } \sigma_L \sim 1/f, \quad \sigma_P \sim f, \quad V \sim f^3$$

$$\text{For final state, } V \sim f^{1.54} \text{ for one case -}$$

Dependence on $f_B = \Delta_B / \sigma_P$

$$\text{Initially, } \sigma_L \sim 1/f_B, \quad \sigma_P \sim f_B, \quad V \sim f_B^4$$

$$\text{For final state, } V \sim f_B^{2.6} \text{ for one case.}$$

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	Au, 160 MHz		Au, 214 MHz	
	$N_b = 1.1 \times 10^9, \Delta_B = 2\sigma_P$		$N_b = 1.1 \times 10^9, \Delta_B = 2\sigma_P$	
	$h = 2052, A = 3 \text{ ev-sec}, \epsilon_{x_0} = 10$		$h = 2736, A = 3, \epsilon_{x_0} = 10$	
γ	30	100	30	100
$\sigma_{e_0} (\text{cm})$	31	31	23.4	23.4
$\sigma_{p_0} / 10^{-3}$	.827	.248	1.09	.327
$\sigma_e (\text{cm})$	31	31	23.4	23.4
$\sigma_p / 10^{-3}$	2.27	1.58	2.42	1.70
$\sigma_x (\text{mm})$	3.06	1.69	3.24	1.78
$\epsilon_x / 10^{-6}$	34	34	38	38
$V (\text{MV})$	2.4	11.4	3.67	17.7
$\Delta p/p = 2\sigma_P / 10$	4.54	3.16	4.84	3.40
$V_0 (\text{MV})$	.321	.283	.744	.656

	Au, 160 MHz		Protons, 160 MHz	
	$N_b = 1.1 \times 10^9, \Delta_B = 2.5\sigma_P$		$N_b = 1 \times 10^{11}, \Delta_B = 2.5\sigma_P$	
	$h = 2052, A = 3, \epsilon_{x_0} = 10$		$h = 2052, A = 3, \epsilon_{x_0} = 10$	
γ	30	100	30	100
σ_{e_0}	24.4	24.4	24.4	24.4
σ_{p_0}	1.05	.315	1.05	
σ_e	24.4	24.4	24.4	.126
σ_p	2.40	1.68	1.53	.545
σ_x	3.21	1.76	2.14	.754
ϵ_x	37	37	16	17
V	4.22	20.2	.689	2.25
$\Delta p/p$	24.80	3.36	3.06	1.09
V_0	.810	.713	.324	.120

Voltage versus Time

$h = 2052$

Protons

$$\Delta_B = 2.5 \sigma_P$$

$$E_{x_0} = 10, A_0 = .3 \text{ ev-sec}$$

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Dependence on Initial Bunch Area A_u

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INTRABEAM SCATTERING ABOVE TRANSITION*

$$\tau_E^{-1} = \frac{1}{\delta_E} \frac{d\delta_E}{dt} = \left[\frac{\langle \sigma_H \rangle}{\langle X_p \rangle \delta_E} \right]^2 \tau_H^{-1}$$

with

$$\tau_H^{-1} = \frac{27\pi}{2} L_g r_p^2 E_o \frac{N_B}{S \varepsilon_H \varepsilon_v} \frac{\langle X_p \rangle}{\langle \beta \rangle} \frac{1}{\left[1 + \left[\frac{\langle \sigma_H \rangle}{\langle X_p \rangle \delta_E} \right]^2 \right]^{1/2}} \left[\frac{Q^2}{A} \right]^2$$

where

$$L_g \approx 20$$

$$r_p = \frac{\mu_o e^2 c^2}{4\pi E_o}$$

$$\langle \sigma_H \rangle = \left[\frac{\varepsilon_H}{6\pi} \frac{\langle \beta \rangle}{\gamma} \right]^{1/2}$$

$$S = 6\pi \sigma_l \delta_E \gamma E_o / c$$

$\varepsilon_{H,v}$ = normalized transverse emittance

$\langle X_p \rangle$ = averaged dispersion

$\langle \beta \rangle$ = averaged betatron function

*G. Parzen, Nucl. Instr. Meth. A251, p. 220 (1986), A256, p. 231 (1987).