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SAMPLE-SIZE EFFECTS IN
FAST-NEUTRON GAMMA-RAY
PRODUCTION MEASUREMENTS:
SOLID-CYLINDER SAMPLES

by

Donald L. Smith
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TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT.	3
1. INTRODUCTION.	4
2. NUCLEAR DATA FOR COMPUTATIONS	7
3. EFFECTS OF GEOMETRY AND ABSORPTION.	8
3.1 Mathematical Formalism.	9
3.2 Results of Numerical Studies.	14
4. EFFECTS OF NEUTRON MULTIPLE SCATTERING.	20
4.1 Mathematical Formalism.	20
4.2 Results of Numerical Studies.	25
5. COMPARISON OF EXPERIMENTAL AND COMPUTED RESULTS FOR NATURAL IRON SAMPLES.	27
6. CONCLUSIONS	29
ACKNOWLEDGEMENTS.	31
APPENDIX A: COHERENT PHOTON SCATTERING	32
APPENDIX B: LISTING OF CODE GAMSCT	34
REFERENCES.	45
TABLES.	46
FIGURES	51

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ABSTRACT

The effects of geometry, absorption and multiple scattering in $(n, X\gamma)$ reaction measurements with solid-cylinder samples are investigated. Both analytical and Monte-Carlo methods are employed in the analysis. Geometric effects are shown to be relatively insignificant except in definition of the scattering angles. However, absorption and multiple-scattering effects are quite important; accurate microscopic differential cross sections can be extracted from experimental data only after a careful determination of corrections for these processes. The results of measurements performed using several natural iron samples (covering a wide range of sizes) confirm validity of the correction procedures described herein. It is concluded that these procedures are reliable whenever sufficiently accurate neutron and photon cross section and angular distribution information is available for the analysis.

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1. INTRODUCTION

A recent report describes the facility which has been developed at Argonne National Laboratory's FNG for $(n, X\gamma)$ reaction studies [1,2]. The geometry is shown in Figs.1-6 of Ref. 1 and it is recommended that the reader refer to this earlier report in conjunction with the present one.

Experience with this facility has indicated that the precision of raw data obtained generally improves with increased sample size. The relative importance of background decreases and the statistical accuracy of the significant data improves under these conditions. Furthermore, in measurements with relatively large samples, it is possible to exploit the advantages of longer flight paths (improved time-of-flight resolution) and massive detector shielding. The penalty involved in measurements with large samples is that corrections to the raw data for effects of absorption and multiple scattering can be quite large. Accurate determination of these corrections requires knowledge of neutron and photon cross sections and their angular distributions and use of complex computational procedures. Clearly, a compromise is necessary. The objective of this report is to present the results of a detailed study of sample-size effects which was conducted in the course of developing the data processing routines which are employed in the reduction of experimental data acquired with this facility.

The results of a literature survey were disappointing. There are relatively few readily available articles on the subject of sample corrections [3-13]. Most of these references deal with experiments in which neutrons (not gamma rays) are detected. These articles provided guidance, but were not a basis for the present work.

The analysis presented in this report employs only a few simplifying assumptions and therefore adheres to a

realistic representation of the physical problem.

The ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction is usually used as a neutron source at this laboratory for measurements in the region of interest for current $(n,X\gamma)$ studies ($E_n \lesssim 5$ MeV). Natural lithium metal is evaporated onto a thin tantalum backing to form a target. The proton-beam spot on target is defined by slits and is essentially rectangular. The lithium films are relatively thin ($\Delta E_p \lesssim 0.1$ MeV). The analysis presented in this report assumes an infinitesimally thin square target; however, the routines actually used for data processing take cognizance of realistic target thicknesses by superimposing contributions from several very thin layers. Target thickness is an important consideration whenever the cross section varies rapidly with neutron energy or for proton energies near the resonance in the lithium source reaction at ~ 2.3 MeV. The angular distribution of neutrons from the source reaction is taken into account. Neutrons from the ${}^7\text{Li}(p,n){}^7\text{Be}^*$ and ${}^7\text{Li}(p,n){}^3\text{He}{}^4\text{He}$ reactions are considered in the analysis for proton energies above their production thresholds.

Gamma rays from $(n,X\gamma)$ reactions are detected with a Ge(Li) detector and the yields of full-energy-peak events are recorded. Since the Ge(Li) detector has a diameter of ~ 5 cm and is placed ~ 100 - 150 cm from the sample, it is assumed that the only photons which are capable of producing full-energy pulses in the detector are those which either experience no interaction in the sample after production via $(n,X\gamma)$ reactions or are coherently scattered in the sample. Therefore, the total photon cross section is assumed for sample absorption calculations and a small correction for coherent scattering is applied when necessary (see Appendix A).

The scattering of neutrons in the sample by elastic and inelastic scattering through discrete levels is treated.

The effects of more complicated reactions such as $(n;2n)$, $(n;n,p)$, $(n;n,\alpha)$, etc. are insignificant for $E_n \lesssim 5$ MeV so they are ignored in the computations. Kinematic effects are considered, and energy-dependent cross sections and angular distributions are employed.

The scattering sample is assumed to be a uniform right-circular cylinder centered on the beam line with axis normal to the scattering plane defined by the beam line and the detector. Macroscopic cross sections are used in absorption calculations, and samples consisting of either single- or multiple-isotope elements, compounds or mixtures can be treated.

Let

Y_{TOT} = total observed gamma-ray yield for a particular geometry,

Y_0 = gamma-ray yield produced by unscattered neutrons,

Y_l = gamma-ray yield produced by neutrons which have scattered " l " times in the sample before initiating $(n,X\gamma)$ reactions,

k = highest order of scattering considered,

then

$$Y_{TOT} \lesssim Y_0 + \sum_{l=1}^k Y_l = Y_0 \left[1 + \sum_{l=1}^k (Y_l/Y_0) \right]. \quad (1)$$

If

$$\alpha_l = (Y_l/Y_0), \quad (2)$$

$$\alpha_{TOT} = \sum_{l=1}^k \alpha_l, \quad (3)$$

then

$$Y_{TOT} \lesssim Y_0 (1 + \alpha_{TOT}). \quad (4)$$

The symbol $\sum$ represents summation to avoid confusion with Σ used for macroscopic cross sections elsewhere in this report. The quantity α_{TOT} is called the multiple-

scattering parameter and α_l is the l -th component. For most samples,

$$\alpha_{TOT} < 1 \text{ and } \alpha_{l+1} \ll \alpha_l, \quad (5)$$

and the sums in Eqs. 1 and 3 converge rapidly. Acceptable accuracy is obtained in practice for $k \leq 3$.

The evaluation of Y_0 and α_{TOT} are treated separately. The quantity Y_0 is determined analytically while α_{TOT} is deduced by Monte-Carlo methods.

The acquisition of nuclear data required for the computations is considered in Section 2; this is a problem which must be addressed before any sample-size correction factors can be computed. Section 3 of this report deals with evaluation of Y_0 and its relationship to the $(n, X\gamma)$ reaction differential cross section which is sought from the measurements. Section 4 treats the subject of multiple scattering and determination of α_{TOT} . In Section 5, the results of computations are compared with experimental data and a simplified computational procedure is explored.

2. NUCLEAR DATA FOR COMPUTATIONS

The methods for computation of sample correction factors described in this report are powerful in principle; however, the quality of the results obtained is only as good as the accuracy of the nuclear data utilized in the analysis. Thus, the experimenter must exercise judgement in selection of the sample size for an experiment. If the available cross section and angular distribution information is uncertain, it is necessary to use smaller samples and thereby sacrifice sensitivity and statistical accuracy in order to minimize the magnitude of the corrections which must be computed. Actually, the quality of available nuclear data is gradually improving, and high-

speed digital computers are accessible to most researchers. Therefore, it appears worthwhile to develop the sophisticated computational tools required to determine realistic corrections and exploit the experimental advantages of using relatively large samples whenever possible.

The existence of pronounced resonance structure in nuclear data complicates many aspects of nuclear science and technology; the present topic is no exception. It has been found convenient to smooth all energy-dependent nuclear data used in correction calculations with resolution functions which approximate the experimental conditions. The smoothed excitation functions can usually be represented with sufficient accuracy by a relatively small number of parameters. Fig. 1 demonstrates the concept. The use of smoothed cross sections for sample absorption and multiple-scattering calculations is an approximation, the validity of which must be investigated carefully prior to use in applications. One method is to compare the results of small- and large-sample measurements in regions where strong resonances are present in the cross sections.

Neutron cross section and angular distribution information is obtained from the evaluated neutron data file, ENDF/B-IV [14]; photon cross sections are obtained from an evaluation by Storm and Israel [15].

3. EFFECTS OF GEOMETRY AND ABSORPTION

The dominant features of the observed gamma-ray yield from (n, γ) reactions in the sample are determined by geometry and the absorption of neutron and gamma radiation. These features are predicted by the response of Y_0 in Eq. (4) to variation of the experimental conditions. Multiple scattering, represented by α_{TOT} in Eq. (4), yields a less significant correction to this behavior. This section describes the procedure used to compute Y_0 and presents the results of calculations designed to explore the sensitivity

of Y_0 to various parameters.

3.1 Mathematical Formalism

The yield for the entire sample is computed by summing the contributions from various portions of the sample. The neutron source and the sample are represented as described below. Fig. 2. illustrates the geometry.

Let

d_n = distance from a particular neutron source point to a particular sample point,

d_γ = distance from a particular sample point to the gamma-ray detector (assumed to be a single point),

δ_n = distance through the sample which the neutron must penetrate to reach the particular sample point,

δ_γ = distance through the sample which the gamma ray must penetrate to reach the gamma-ray detector,

R_s = radius of the sample,

H = height of the sample,

D_n = distance from the center of the neutron source to the center of the sample (which is also the pivot for the gamma-ray detector),

D_γ = distance from the center of the sample to the gamma-ray detector,

θ_n = incident-neutron angle relative to the beam line,

θ_{DET} = angle of gamma-ray detector relative to the beam line,

$\theta_{n\gamma}$ = angle of emission of the gamma ray relative to the incident neutron,

F_n = absolute neutron-source strength of a uniform square source (neutrons/sr),

a = dimension of the square neutron source,

- $(d\sigma/d\Omega)_\gamma$ = differential gamma-ray production cross section for the $(n,X\gamma)$ reaction,
 (x_T, y_T, z_T) = coordinates for a point on the neutron-source surface,
 (x_1, y_1, z_1) = coordinates for a point S_1 in the sample,
 (x_D, y_D, z_D) = coordinates for the gamma-ray detector,
 N_S = number of atoms per unit volume of the sample which can contribute to $(n,X\gamma)$ reactions,
 Σ_{nT} = neutron macroscopic total cross section for the sample material,
 $\Sigma_{\gamma T}$ = photon macroscopic total cross section for the sample material.

Two-body neutron-producing reactions are assumed in the present analysis. The neutron fluence F_n is a function of the reaction parameters, incident energy, and emission angle. Kinematics governs the variation of neutron energy with angle. The cross sections $(d\sigma/d\Omega)_\gamma$, Σ_{nT} and $\Sigma_{\gamma T}$ are energy-dependent; $(d\sigma/d\Omega)_\gamma$ also varies with angle $\theta_{n\gamma}$.

The grid systems for the neutron source and the sample are illustrated in Fig. 2. The sample grid system used yields more uniformly sized elements than a standard cylindrical-coordinate grid system. The parameters which define these grid systems are

- n_H = mesh for the sample height (the sample is divided into n_H layers of height H/n_H along the x-axis),
 n_R = radial mesh (the sample is divided into a series of n_R-1 shells of thickness R_S/n_R plus a central cylinder with radius R_S/n_R),
 n_ϕ = fundamental angular mesh (the central cylinder, $j=1$, is divided into n_ϕ wedges while

the j th shell has $j n_\phi$ segments, $j=2, \dots, n_R$),
 n_T = neutron-source mesh (the square source is
divided into n_T^2 sections each with area
 a^2/n_T^2).

The total number of elements in the sample is $\frac{1}{2} n_H n_\phi n_R (n_R + 1)$.
The variation in volume of these elements depends only on
the radial variable. Therefore

$$V_j = \frac{\pi R_S^2 H}{n_R n_H n_\phi} \left(\frac{2j-1}{j} \right), \quad j=1, \dots, n_R. \quad (6)$$

A constraint on the angular mesh n_ϕ , required for the
type of sample grid used, is $n_\phi \geq 2$.

The intercept of the beam line and neutron source
plane is selected as the origin of coordinates. The
coordinates of the center of each neutron-source element
are given by $(x_{T\alpha}, y_{T\beta}, z_T)$ where

$$x_{T\alpha} = \frac{a}{2n_T} (2\alpha - n_T - 1), \quad \alpha = 1, \dots, n_T, \quad (7)$$

$$y_{T\beta} = \frac{a}{2n_T} (2\beta - n_T - 1), \quad \beta = 1, \dots, n_T, \quad (8)$$

and $z_T = 0$ for all elements.

The coordinates of the gamma-ray detector are given
by the formulas

$$y_D = D_\gamma \sin \theta_{DET}, \quad (9)$$

$$z_D = D_n + D_\gamma \cos \theta_{DET}, \quad (10)$$

and $x_D = 0$.

The coordinates of the center of each sample element
are given by the formulas

$$x_{1i} = \frac{H}{2n_H} (2i - n_H - 1), \quad i=1, \dots, n_H, \quad (11)$$

$$y_{1jk} = r_j \sin \phi_{jk}, \quad j=1, \dots, n_R \text{ and } k=1, \dots, j n_\phi, \quad (12)$$

$$z_{ijk} = D_n + r_j \cos \phi_{jk}, \quad j=1, \dots, n_R \text{ and } k=1, \dots, jn_\phi, \quad (13)$$

and

$$r_j = \frac{R_S}{2n_R} (2j-1), \quad j=1, \dots, n_R, \quad (14)$$

$$\phi_{jk} = \frac{\pi}{jn_\phi} (2k-1), \quad j=1, \dots, n_R \text{ and } k=1, \dots, jn_\phi \quad (15)$$

Analytic geometry is applied in derivation of formulas for δ_n and δ_Y . The expression for δ_n is

$$\delta_n = d_n - s_n, \quad (16)$$

where

$$d_n = [(x_1 - x_T)^2 + (y_1 - y_T)^2 + (z_1 - z_T)^2]^{1/2}, \quad (17)$$

$$s_n = \frac{[B_n(y_0 - y_T) + C_n(z_0 - z_T)]}{(B_n^2 + C_n^2)} - \frac{\{(B_n^2 + C_n^2)R_S^2 - [C_n(y_T - y_0) - B_n(z_T - z_0)]^2\}^{1/2}}{(B_n^2 + C_n^2)}, \quad (18)$$

$$B_n = (y_1 - y_T)/d_n, \quad (19)$$

$$C_n = (z_1 - z_T)/d_n, \quad (20)$$

and (x_0, y_0, z_0) are the coordinates of the center of the sample ($x_0 = y_0 = 0, z_0 = D_n$). The expression for δ_Y is

$$\delta_Y = d_Y - s_Y, \quad (21)$$

where

$$d_Y = [(x_1 - x_D)^2 + (y_1 - y_D)^2 + (z_1 - z_D)^2]^{1/2}, \quad (22)$$

$$s_Y = \frac{[B_Y(y_0 - y_D) + C_Y(z_0 - z_D)]}{(B_Y^2 + C_Y^2)} - \frac{\{(B_Y^2 + C_Y^2)R_S^2 - [C_Y(y_D - y_0) - B_Y(z_D - z_0)]^2\}^{1/2}}{(B_Y^2 + C_Y^2)}, \quad (23)$$

$$B_Y = (y_1 - y_D)/d_Y, \quad (24)$$

$$C_Y = (z_1 - z_D)/d_Y. \quad (25)$$

Let

η_n = neutron absorption factor,

η_γ = gamma-ray absorption factor,

then

$$\eta_n = \exp(-\Sigma_{nT} \delta_n), \quad (26)$$

$$\eta_\gamma = \exp(-\Sigma_{\gamma T} \delta_\gamma). \quad (27)$$

If

ϵ_{DET} = efficiency of the gamma-ray detector,
defined as the ratio of detected (in the
full-energy peak) to incident gamma rays,

then

$$Y_0 = \sum_{\alpha, \beta, i, j, k}^{n_T, n_H, n_R, n_\phi} \frac{F_n}{n_T^2} \left(\frac{\eta_n}{d_n^2} \right) \left(\frac{\eta_\gamma}{d_\gamma^2} \right) N_S (d\sigma/d\Omega)_\gamma V_j \epsilon_{DET}. \quad (28)$$

Y_0 is a function of E_n and $\theta_{n\gamma}$. The energy E_n lies in the range $(0, E_{n, \max})$ while the scattering angle $\theta_{n\gamma}$ is in the range $(0, \pi)$. Therefore, it is possible to define resolution functions $\mathcal{E}(E_n)$ and $\mathcal{A}(\theta_{n\gamma})$ such that

$$Y_0 = \int_0^{E_{n, \max}} \mathcal{E}(E_n) dE_n, \quad (29)$$

$$Y_0 = \int_0^\pi \mathcal{A}(\theta_{n\gamma}) d\theta_{n\gamma}. \quad (30)$$

These resolution functions can be used to compute the average neutron energy $\langle E_n \rangle$ and the average scattering angle $\langle \theta_{n\gamma} \rangle$ according to the formulas

$$\langle E_n \rangle = Y_0^{-1} \int_0^{E_{n, \max}} E_n \mathcal{E}(E_n) dE_n, \quad (31)$$

$$\langle \theta_{n\gamma} \rangle = Y_0^{-1} \int_0^\pi \theta_{n\gamma} \mathcal{A}(\theta_{n\gamma}) d\theta_{n\gamma}. \quad (32)$$

The relationship between Y_0 and the differential cross section $(d\sigma/d\Omega)_\gamma$ is indicated in Eq. (28). Let

$\langle (d\sigma/d\Omega)_\gamma \rangle$ = value of the differential cross section
corresponding to neutron energy $\langle E_n \rangle$ and
scattering angle $\langle \theta_{n\gamma} \rangle$,

and define

$$\xi_\gamma = (d\sigma/d\Omega)_\gamma / \langle (d\sigma/d\Omega)_\gamma \rangle, \quad (33)$$

then Eq. (28) can be rewritten in the form

$$Y_0 = \langle (d\sigma/d\Omega)_\gamma \rangle \sum_{\alpha, \beta, i, j, k}^{n_T, n_H, n_R, n_\phi} \frac{F_n}{n_T^2} \left(\frac{n_n}{d_n} \right) \left(\frac{n_\gamma}{d_\gamma} \right)^{N_S \xi_\gamma V_j \epsilon_{DET}} \quad (34)$$

which explicitly relates the gamma-ray yield to the differential cross section for a specific neutron energy and scattering angle. This formalism requires an approximate knowledge of ξ_γ (the shape of the differential cross section function in terms of neutron energy and scattering angle). In practice, most of the contributions to Y_0 come from limited ranges of neutron energy and scattering angle. Therefore, one estimates the behavior of ξ_γ for the regions of interest and applies this estimate in computations. Improved accuracy can be achieved by the process of iteration. Experience has shown no more than two passes are required for most applications.

3.2 Results of Numerical Studies

It is worthwhile to factor the gross solid-angle and sample-volume dependence from the expressions for Y_0 . This can be achieved through definition of the quantity $\bar{Y}_0$ as follows:

$$Y_0 \equiv (\pi R_S^2 H D_n^{-2} D_\gamma^{-2}) \bar{Y}_0. \quad (35)$$

In this section, sample geometry and absorption effects are investigated solely in terms of the behavior of $\bar{Y}_0$.

A Systems Engineering Laboratories Model 840 MP digital computer was utilized in computation of $\bar{Y}_0$ for various experimental conditions. A nominal parameter set for these calculations appears in Table I. The variation of $\bar{Y}_0$ in response to departures from the conditions represented by these parameters is investigated in the present section.

Selection of an appropriate set of mesh parameters n_T , n_H , n_R and n_ϕ is an important consideration. Coarse meshes lengthen the computation time unnecessarily. The computations were least sensitive to n_T and most sensitive to n_R as expected. The mesh parameters listed in Table I appear to be satisfactory for most practical applications.

The effects of radiation absorption were investigated by computing $\bar{Y}_0$ for four sets of parameters which differ from each other only in the assumed values for Σ_{nt} and $\Sigma_{\gamma T}$: i) neutron and gamma-ray absorption (Table I), ii) gamma-ray absorption only ($\Sigma_{nT} = 0$), iii) neutron absorption only ($\Sigma_{\gamma T} = 0$), and iv) no absorption ($\Sigma_{nT} = \Sigma_{\gamma T} = 0$). The effects of absorption are a reduction of gamma-ray yield and a distortion of the observed angular distribution. The relative gamma-ray yields at $\theta_{DET} = 90^\circ$ for these four cases are: i) 0.34, ii) 0.50, iii) 0.68, and iv) 1.00. The induced anisotropy for each situation is shown in Fig. 3. These distortions appear to be well represented by the expression

$$\bar{Y}_0(\theta_{DET}) \approx \bar{Y}_0(90^\circ) (1 - \Delta_0 \cos \theta_{DET}) \quad (36)$$

with Δ_0 assuming the following positive values for the four cases considered: i) 0.172, ii) 0.0658, iii) 0.0056,

and iv) 0.0021. Geometric effects and neutron attenuation alone produce very little distortion; however, gamma-ray absorption (particularly in combination with neutron absorption) skews the observed angular distribution about $\theta_{\text{DET}} = 90^\circ$ so that the back-angle yield exceeds the forward-angle yield.

The relative contributions to $\bar{Y}_0$ for $\theta_{\text{DET}} = 90^\circ$ from the midplane sample elements are presented for each of the four cases considered as follows: i) Fig. 4, ii) Fig. 5, iii) Fig. 6, and iv) Fig. 7. In conjunction with the results presented in Figs. 4-7, it is worthwhile to consider the ratios of yields from larger segments of the sample. Define the "back" of the sample as that half of the cylinder which is farthest from the neutron source, and the "front" of the sample as the opposite half. Similarly, label as "far" the sample half farthest from the gamma-ray detector. The opposite half is labelled as "near". Computed "front"-to-"back" and "near"-to-"far" ratios are listed in Table II.

The parameter D_n was varied over the range 5-50 cm with other parameters fixed at the values given in Table I. The value of $\bar{Y}_0$ for $\theta_{\text{DET}} = 90^\circ$ increased by only 3.3% as D_n increased from 5 to 50 cm. This would seem to imply that a parallel neutron beam approximation is warranted. However, variation in the distortion of the angular distribution was more pronounced as Δ_0 decreased from 0.268 for $D_n = 5$ cm to 0.121 for $D_n = 50$ cm. For small values of distance D_n , the sample subtends a sizeable solid angle so that the average scattering angle becomes quite sensitive to D_n . Therefore, use of the parallel neutron beam approximation is not recommended.

The parameter D_y was varied over the range 30-200 cm with other parameters fixed at the values given in Table I. The value of $\bar{Y}_0$ for $\theta_{\text{DET}} = 90^\circ$ decreased by only 1.2%

as D_Y increased from 30 to 200 cm, and the distortion parameter Δ_0 decreased from 0.190 to 0.168. A parallel gamma ray approximation could be justified for these calculations.

Variation of the sample height H produces very little effect on $\bar{Y}_0$; however, the gamma-ray yield and distortion of the angular distribution depend critically upon the sample radius R_S . Computations were made for $R_S = 0.635, 0.95, 1.27, 1.59, 1.905$ and 2.54 cm with the other parameters fixed at the values listed in Table I. The results of these calculations appear in Table III. The near constancy of $R_S \bar{Y}_0$ for large values of R_S implies that, in this domain, the total gamma-ray yield from the sample increases more or less linearly with sample radius rather than as the square of the radius. The difference is due to absorption. Clearly, the total gamma ray yield from the sample also increases linearly with sample height. A set of calculations was made assuming no neutron or gamma-ray absorption and using values of $H = 2 R_S$ ("square" samples). As R_S increases from 0.635 cm to 2.54 cm, $\bar{Y}_0$ decreases by 1.7%. This result indicates that the yield per unit volume depends only slightly on the size of the sample in the absence of absorption.

The most significant geometric effect is the loss of angular resolution which results from the use of large samples. The maximum geometric angular range for $\theta_{n\gamma}$ associated with the conditions of Table I is $\sim 19^\circ$. The contribution to this spread from the neutrons is $\sim 17^\circ$ while that from the gamma rays is $\sim 2^\circ$. The resolution functions $\mathcal{R}(\theta_{n\gamma})$, as defined by Eq. (30), were determined for several values of θ_{DET} using Table I parameters. The results are presented in Fig. 8. Values of $\langle \theta_{n\gamma} \rangle$, computed with these resolution functions, differ significantly from the corresponding detector angles θ_{DET} when θ_{DET} approaches

0° or 180°. This result illustrates the well-known fact that relatively small samples are required for the measurement of differential cross sections near 0° or 180°.

Gamma-ray angular distributions for (n,X γ) reactions are symmetric about $\Theta_{n\gamma} = 90^\circ$. Computations were made using various assumed gamma-ray differential cross section functions $(d\sigma/d\Omega)_\gamma$. These calculations indicate that, for positive values of Δ_0 , the formula

$$\bar{Y}_0 \approx (\text{Constant}) (1 - \Delta_0 \cos \Theta_{DET}) \langle (d\sigma/d\Omega)_\gamma \rangle \quad (37)$$

is valid to a considerable degree of accuracy. Furthermore, the distortion parameter Δ_0 and the average scattering angle $\langle \Theta_{n\gamma} \rangle$ are very insensitive to the shape of the differential cross section $(d\sigma/d\Omega)_\gamma$. Eq. (37) resembles Eq. (34) and it is concluded that the complicated sum given in Eq. (34) has a simple angular dependence. This particular result will be designated the "factorization rule" since it provides a prescription for relating the observed gamma-ray yield to the shape of the differential cross section. Fig. 9 demonstrates the factorization rule. This rule applies reasonably well for most realistic applications even when multiple scattering is taken into consideration (Section 4.2). Application of the factorization rule leads to a significant labor reduction in processing angular distribution data.

Realistically, the neutron field produced by proton bombardment of natural lithium is a mixture of first- and second-group neutrons plus some breakup neutrons at higher bombarding energies (Section 1). The energies and angular distributions of these components differ and this will influence the overall gamma-ray and monitor yields observed. Of concern, however, is the effect of superposition of contributions on the applicability of the factorization rule. To investigate this point, computations were made for several incident energies assuming realistic lithium first-

and second-group neutrons plus some breakup neutrons at higher bombarding energies (Section 1). The energies and angular distributions of these components differ and this will influence the overall gamma-ray and monitor yields observed. Of concern, however, is the effect of superposition of contributions on the applicability of the factorization rule. To investigate this point, computations were made for several incident energies assuming realistic lithium first- and second-group neutron sources as well as the hypothetical isotropic source identified in Table I. These calculations show that the distortion parameter Δ is relatively insensitive to the shape of the neutron-source reaction angular distribution. The parameter Δ varies with neutron energy since it depends on the total cross section. However, since $\Delta \ll 1$ for typical conditions, the energy dependence of Δ does not affect the angular distributions severely. For example, Δ is found to vary by $\sim 30\%$ over the range $E_n = 0.9\text{--}2$ MeV for the sample described in Table I; however, the ratio $\bar{Y}_0(0^\circ)/\bar{Y}_0(90^\circ)$ varies by only $\sim 3\%$ over this range. Therefore, it is often possible to apply the factorization rule for multigroup neutron sources. The formulas suggested by the results of numerical analysis are

$$\bar{Y}_0(\text{multigroup}) \sim (\text{Constant}) (1 - \langle \Delta \rangle \cos \theta_{\text{DET}}) .$$

$$\sum_m G_m \langle (d\sigma/d\Omega) \rangle_m , \quad (38)$$

$$\langle \Delta \rangle = (\sum_m G_m \Delta_m / \sum_m G_m) \quad (39)$$

The parameters G_m depend upon the group intensities and sample absorption properties for neutrons in these groups.

The factorization rule is not a rigorous product of the formalism, but is an empirical concept which has been

distilled from the results of numerical calculations. Caution should be exercised when using this rule in processing data, particularly for measurements involving multigroup neutron sources.

4. EFFECTS OF NEUTRON MULTIPLE SCATTERING

The obvious result of multiple scattering is the enhancement of the observed yield relative to that predicted by the computations of the previous section. The objectives of the present section are to describe the method used for the evaluation of the multiple scattering parameter α_{TOT} and to investigate the dependence of multiple scattering on various experimental factors.

4.1 Mathematical Formalism

The approach taken in this work is to calculate values of Y_ℓ for $\ell = 0, 1, \dots, k$ (see Section 1) by statistical methods and then compute the partial multiple-scattering parameters α_ℓ by means of Eq. (2). The total multiple-scattering parameter α_{TOT} is given by Eq. (3). There are several ways to formulate Monte-Carlo problems [13]. The present approach generally resembles that which is employed in multi-dimensional Monte-Carlo integration. The fundamental assumption of Monte-Carlo integration is that

$$\int_V d\vec{q} f(\vec{q}) \approx \frac{V}{N_{hist}} \sum_{i=1}^{N_{hist}} f(\vec{q}_i) \quad (40)$$

for a sufficiently large number of histories N_{hist} , where $\vec{q}_i$ is selected at random from a region of v -dimensional Cartesian space defined by

$$\vec{q} = (q_1, q_2, \dots, q_v) \quad (41)$$

$$d\vec{q} = (dq_1 dq_2 \dots dq_v) \quad (42)$$

$$V = (q_{1,max} - q_{1,min}) (q_{2,max} - q_{2,min}) \dots (q_{v,max} - q_{v,min}). \quad (43)$$

The region of space defined by the volume V is a v -dimensional rectangle and is selected so that the true region of interest lies entirely within V . The Monte-Carlo process consists of selecting points $\vec{q}_1$ at random in the larger region. If $\vec{q}_1$ falls outside the true region of interest, then $f(\vec{q}_1) = 0$ (a "miss"). This approach does not lead to optimum efficiency, however such wastefulness is usually tolerable with high-speed digital computers and avoids many computational complexities.

Some of the variables used in this analysis are defined in Sections 1 and 3; others are defined at appropriate points in the present section.

First, computation of Y_0 by Monte-Carlo methods is considered (see Fig. 10). The origin of coordinates is the neutron source (assumed here to be a point). The beam line is the z -axis, the sample axis is normal to the $y - z$ plane as in Section 3. The center of the sample is at the coordinates $(0,0,D_n)$. The gamma-ray detector is a point in the $y - z$ plane located a distance D_y from the center of the sample. The first-scattering point S_1 in the sample is identified by the vector $\vec{x}_1$ with coordinates (x_1, y_1, z_1) . A related spherical coordinate system can be defined by the equations

$$x_1 = r_1 \sin \theta_1 \cos \phi_1, \quad (44)$$

$$y_1 = r_1 \sin \theta_1 \sin \phi_1, \quad (45)$$

$$z_1 = r_1 \cos \theta_1. \quad (46)$$

The sample is located entirely within a region of space defined by the following expressions

$$r_{1,\min} \leq r_1 \leq r_{1,\max}, \quad (47)$$

$$r_{1,\min} = D_n - R_s, \quad (48)$$

$$r_{1,\max} = [(D_n + R_s)^2 + \frac{1}{4} H^2 + R_s^2]^{\frac{1}{2}}, \quad (49)$$

$$0 < \phi_1 < 2\pi, \quad (50)$$

$$0 \leq \theta_1 \leq \theta_{1,\max}, \quad (51)$$

$$\theta_{1,\max} = \tan^{-1} [(1/4 H^2 + R_S^2)^{1/2} / (D_n - R_S)]. \quad (52)$$

The energy and angular dependence of all physical parameters is taken into consideration as well as kinematic effects. The gamma-ray yield from $(n, X\gamma)$ reactions initiated by unscattered neutrons is given by

$$Y_0 \approx \frac{V_1}{N_{\text{hist}}} \sum_{i=1}^{N_{\text{hist}}} \Gamma_{0i} \quad (53)$$

with

$$\Gamma_0 = \begin{cases} F_n \eta_1 \sin\theta_1 N_S (d\sigma/d\Omega)_\gamma \left(\frac{\eta_\gamma}{d_\gamma z} \right) \epsilon_{\text{DET}} & \text{if } \vec{x}_1 \text{ is inside the sample,} \\ 0 & \text{otherwise (a "miss"),} \end{cases} \quad (54)$$

$$\eta_1 = \exp(-\delta_n \Sigma_{nT}) , \quad (55)$$

$$V_1 = 2\pi(r_{1,\max} - r_{1,\min})\theta_{1,\max}. \quad (56)$$

For $\vec{x}_1$ to lie inside the sample, it is required that

$$-H/2 < x_1 < H/2 , \quad (57)$$

$$[y_1^2 + (z_1 - D_n)^2]^{1/2} < R_S . \quad (58)$$

Although determination of Y_0 by Monte-Carlo integration is technically simpler than the approach described in Section 3, it is slower since quite a few histories are required for convergence. Furthermore, it is not possible to derive the midplane-yield profile information presented in Figs. 4-7 from a simple Monte-Carlo treatment. However, for $\ell \geq 1$ it is impractical to determine Y_ℓ by any method other than Monte-Carlo analysis.

Next, consider computation of Y_1 (see Fig. 10). The scattering preceding the $(n, X\gamma)$ event can be either elastic or inelastic. The possibility for more than one neutron channel adds an additional complication to the computations. Some neutrons which scatter in the vicinity of point S_1 propagate toward point S_2 defined by the vector $\vec{x}_2$. Assume that there are N_1 distinct scattering processes applicable to the first scattering point S_1 . The j -th process is defined by the parameters A_{1j} , Q_{1j} and $(d\Sigma/d\Omega)_{1j}$ where

$$\begin{aligned} A_{1j} &= \text{mass of the target nucleus,} \\ Q_{1j} &= \text{reaction Q-value,} \\ (d\Sigma/d\Omega)_{1j} &= \text{macroscopic differential neutron-} \\ &\quad \text{scattering cross section.} \end{aligned}$$

For simplicity, the inelastic scattering processes are assumed to be isotropic since they are nearly so in reality. Let ψ_1 be the total macroscopic scattering cross section defined by the equation

$$\psi_1 = \sum_{j=1}^{N_1} (d\Sigma/d\Omega)_{1j}, \quad (59)$$

then the relative probability P_{1j} of each process is given by the equation

$$P_{1j} = (d\Sigma/d\Omega)_{1j} / \psi_1. \quad (60)$$

Since

$$\sum_{j=1}^{N_1} P_{1j} = 1, \quad (61)$$

the unit interval can be divided by a set of N_1 points $\{\rho_{1j}\}$ defined by the equation

$$\rho_{1j} = \sum_{\ell=1}^j P_{1\ell}. \quad (62)$$

A random number R is selected for each history. The neutron is then assumed to propagate from point S_1 to point S_2 by means of the j -th scattering process if

$$\rho_{1,j-1} < R \leq \rho_{1j} . \quad (63)$$

Although the particular scattering process is selected by random sampling in the space of open channels, the macroscopic differential scattering cross section used in the computation is ψ_1 .

A new spherical coordinate system with origin at S_1 is defined. The cartesian coordinates (origin at the neutron source) and spherical coordinates in the new system for point S_2 are related by the equations

$$x_2 = x_1 + r_2 \sin \theta_2 \cos \phi_2 , \quad (64)$$

$$y_2 = y_1 + r_2 \sin \theta_2 \sin \phi_2 , \quad (65)$$

$$z_2 = z_1 + r_2 \cos \theta_2 . \quad (66)$$

The region of space defined by the expressions

$$0 \leq \theta_2 \leq \pi , \quad (67)$$

$$0 \leq \phi_2 \leq 2\pi , \quad (68)$$

$$0 \leq r_2 \leq r_{2,\max} , \quad (69)$$

$$r_{2,\max} = (H^2 + 8 R_S^2)^{1/2} , \quad (70)$$

encompasses the entire sample. Then,

$$y_1 \approx \frac{V_1 V_2}{N_{\text{hist}}} \sum_{i=1}^{N_{\text{hist}}} r_{1i} \quad (71)$$

with

$$r_{1i} = \begin{cases} F_n \eta_1 \sin \theta_1 \psi_1 \eta_2 \sin \theta_2 \cdot \\ \quad \cdot N_S (d\sigma/d\Omega)_Y \left(\frac{n_Y}{d_Y} \right) \epsilon_{\text{DET}} \\ \quad \text{if } \vec{x}_1 \text{ and } \vec{x}_2 \text{ are inside the sample,} \\ 0 \text{ otherwise (a "miss"),} \\ \eta_2 = \exp(-r_2 \Sigma_{nT}), \\ V_2 = 2\pi^2 r_{2,\max} . \end{cases} \quad (72)$$

For $\vec{x}_2$ to lie inside the cylinder, it is required that

$$-H/2 < x_2 < H/2, \quad (73)$$

$$[y_2^2 + (z_2 - D_n)^2]^{1/2} < R_S. \quad (74)$$

Generalization to arbitrary orders of multiple scattering is straightforward. The expression for Y_k is

$$Y_k \sim \frac{v_1 v_2 \dots v_{k+1}}{N_{\text{hist}}} \sum_{i=1}^{N_{\text{hist}}} \Gamma_{ki}, \quad (75)$$

with

$$\Gamma_k = \begin{cases} F_n \left(\pi, \eta_l \sin \theta_l, \psi_l \right) \eta_{k+1} \sin \theta_{k+1} \cdot \\ \quad \cdot N_S(d\sigma/d\Omega) \gamma \left(\frac{\eta_Y}{d_{Y2}} \right) \epsilon_{\text{DET}} \\ \quad \text{if } \vec{x}_1, \dots, \vec{x}_{k+1} \text{ are inside the sample,} \\ \quad 0 \text{ otherwise (a "miss").} \end{cases} \quad (76)$$

The parameters required for computation of all the higher-order scattering contributions resemble those described for computation of Y_1 .

Values of $\Gamma_{0i}, \dots, \Gamma_{ki}$ are computed for every history ($i=1, \dots, N_{\text{hist}}$). Whenever a particular $\Gamma_{li} = 0$ (a "miss"), the higher-order expressions $\Gamma_{l+1,i}, \dots, \Gamma_{ki}$ are automatically equal to zero too. Thus, the efficiency for computation of Y_k declines with increased scattering order k . However,

$$Y_0 \gg Y_1 \gg \dots \gg Y_k, \quad (77)$$

so it is unnecessary to determine the high-order contributions to Y_{TOT} (Eq. 1) as accurately as the low-order contributions.

4.2 Results of Numerical Studies

Multiple-scattering calculations were performed with the SEL 840MP computer using a code named GAMSCT. This

code has been written in FORTRAN IV and a listing of the orders is given in Appendix B.

A nominal set of parameters, which provided a starting point for numerical studies of multiple scattering, is given in Table IV. Experience indicates that for $k=3$, satisfactory accuracy in computation of α_{TOT} is achieved for $N_{hist} \sim 100,000$. This value was selected for all calculations. Typical efficiencies ("hit" percentages) of the Monte-Carlo trials are as follows: Y_0 (54.9%), Y_1 (14.2%), Y_2 (3.7%) and Y_3 (0.8%). Relative values of Y_0 , Y_1 , Y_2 and Y_3 for various θ_{DET} are plotted in Fig. 11. It is seen that the ratio Y_{l+1}/Y_l is more or less independent of l , and furthermore for all l ,

$$Y_l(\theta_{DET}) \sim Y_l(90^\circ) (1 - \Delta_l \cos \theta_{DET}), \quad (78)$$

$$\Delta_0 > \Delta_1 > \Delta_2 > \Delta_3 \sim 0. \quad (79)$$

Intuitively, one expects the effects of geometric anisotropy to be washed out by multiple scattering. Eq. (79) supports this contention. Eq. (78) indicates that the factorization rule applies for an assumed isotropic differential cross section $(d\sigma/d\Omega)_Y$. Actually it also applies reasonably well for most realistic differential cross section functions. However, the factorization rule does fail in extreme cases where $(d\sigma/d\Omega)_Y$ approaches zero for $\theta_{NY} = 0^\circ$ or 180° . Under these conditions, the observed yield for θ_{DET} near 0° or 180° is dominated multiple scattering in a fashion which cannot be explained by a simple rule.

The effect of sample size was tested by varying R_S and H ($H = 2R_S$) with respect to the values in Table IV. The results are presented in Fig. 12. The contributions from second- and higher-order scattering are negligible for small samples. Variation of D_n and D_Y over realistic ranges produced very little effect on the computed multiple-scattering parameters.

Computations performed with various assumed realistic neutron-source reactions indicate that the multiple-scattering correction parameters are relatively insensitive to the properties of the neutron source. Therefore, it is reasonable to compute α_{TOT} for various energies E_n and angles θ_{DET} assuming an isotropic, monoenergetic neutron source.

The multiple-scattering parameters are relatively insensitive to Σ_{nT} and $\Sigma_{\gamma T}$. However, they depend critically on the magnitudes of the scattering cross sections. The relationship

$$\alpha_L \sim (\text{Constant}) \Sigma_{EL}^L \quad (80)$$

gives a rough indication of this dependence for the simple case of energy-independent elastic scattering. The multiple scattering parameters are considerably less sensitive to the shape of the neutron scattering angular distributions.

5. COMPARISON OF EXPERIMENTAL AND COMPUTED RESULTS FOR NATURAL IRON SAMPLES

Measurements were performed with seven natural iron samples to test the validity of the methods described in this report. The sizes of the samples investigated were $R_S = 0.635, 0.953, 1.27, 1.59, 1.91, 2.22$ and 2.54 cm ($H = 2 R_S$). Realistic energy-averaged cross sections were utilized in the computations (Section 2 and Refs. 14 and 15). The facility described in Ref. 1 was utilized for the irradiations. A 0.1-MeV-thick natural lithium target was bombarded with 3.68-MeV protons. Approximately 90% of the neutrons originated from the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction ($E_n \sim 2$ MeV) and 10% came from the ${}^7\text{Li}(p,n){}^7\text{Be}^*$ reaction ($E_n \sim 1.535$ MeV); the proton energy was slightly

below threshold for the ${}^7\text{Li}(p,n){}^3\text{He}{}^4\text{He}$ breakup reaction. The Ge(Li) detector was situated at $\theta_{\text{DET}} = 90^\circ$. The relative neutron fluence was monitored by time-of-flight techniques using a plastic scintillator.

The full-energy peak yields for the 0.846-MeV line from the ${}^{56}\text{Fe}(n,n'\gamma){}^{56}\text{Fe}$ reaction were divided by the masses of the corresponding samples (proportional to the volume) to determine quantities proportional to the yield per atom. All measurements were performed in identical geometry, so the yields per atom deduced are proportional to $\bar{Y}_0 (1 + \alpha_{\text{TOT}})$ as defined in Sections 1-4.

Measurements for samples with $R_S < 0.635$ cm were not practical because of background problems; however, computations were performed for a wide range of sample sizes including R_S near zero. In the limit of very small samples, geometry, absorption and multiple-scattering effects vanish. The experimental and computed values were normalized so that the yield per atom approaches unity for very small samples.

Four sets of computations were performed. The assumptions made in these calculations are as follows: i) no absorption, geometric corrections only, ii) absorption of neutrons and gamma-rays with the neutron total cross section used for absorption calculations and multiple scattering neglected, iii) identical to (ii) except that the total non-elastic cross section is used for the neutron absorption calculations, and iv) identical to (ii) except multiple scattering is considered (most realistic approach). The results of these calculations are compared with the experimental data in Fig. 13. The calculations labelled (iii) and (iv) both agree well with the experimental results; the agreement of set (iv) values is superior as anticipated. The agreement for such a wide range of sample sizes is very encouraging

(the diameter of the largest sample considered is equivalent to 1.4 mean free path lengths for neutrons and 2.5 mean free path lengths for 0.846-MeV photons).

The assumptions made for the set (iii) calculations were suggested by Day [3]. The Day approximation is widely used by researchers in the analysis of $(n, X\gamma)$ data. It is appealing because it eliminates the necessity for performing multiple-scattering calculations. The results shown in Fig. 13 correspond to $\Theta_{DET} = 90^\circ$. Computations were performed to compare the predictions of the Day approximation with those from the more realistic treatment at other angles. The results of this analysis show that these two approaches yield results which agree within $\sim 3\%$ for $\Theta_{DET} = 90^\circ - 150^\circ$; however, the agreement for $\Theta_{DET} = 30^\circ - 90^\circ$ is only within $\sim 7\%$ (for an iron sample with $R_S = 1.9$ cm). This suggests that for careful work, where accuracies of better than 10% are sought, it is advisable to employ a realistic treatment which includes multiple-scattering analysis. In applications where such accuracy is not sought, or is unfeasible, the Day approximation appears to be warranted since it saves considerable labor.

6. CONCLUSIONS

The formalism described in this report provides a means for determining differential cross section data for $(n, X\gamma)$ reactions from measurements made using relatively large cylindrical samples provided that accurate absorption and scattering cross section data is available.

Geometric effects reduce angular resolution and make it difficult to measure differential cross sections near 0° or 180° ; otherwise, they have a relatively minor influence on the measurements.

The absorption of radiation reduces the overall yield and distorts angular distributions. Under most conditions, this distortion assumes the form $1 - \Delta \cos \Theta_{DET}$

($\Delta > 0$); the shape of the differential cross section can be deduced from the experimental data by factoring out this simple angular dependence. Factorization is possible, even in the presence of multiple scattering, for most realistic situations and this saves considerable labor in processing data.

The Day approximation [3] permits one to avoid making detailed multiple scattering calculations, and appears to be an acceptable approach when accuracies of no better than $\sim 10\%$ are acceptable.

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APPENDIX A

COHERENT PHOTON SCATTERING

Photons which scatter coherently in the sample are indistinguishable from those which suffer no interaction. The coherent scattering cross sections depend upon photon energy E_γ and atomic number Z . They decrease rapidly with E_γ . For this reason, the coherent scattering corrections are relatively small for most materials of interest when E_γ exceeds a few hundred kilovolts.

In order to estimate the correction, a model which assumes a well-defined, distributed gamma-ray source intensity in a solid cylinder is employed. The unscattered and coherently-scattered photon yield at a distant detector is computed as described below.

If $\Sigma_{\gamma, \text{COH}}$ is the integrated macroscopic coherent scattering cross section, then the macroscopic differential scattering cross section is given by

$$(d\Sigma/d\Omega)_{\gamma, \text{COH}} = \frac{3\Sigma_{\gamma, \text{COH}}}{16\pi} (1 + \cos^2 \theta_\gamma) . \quad (81)$$

Assume that the cylinder is divided into a large number of discrete elements. Let

$\vec{x}_i$ = coordinates of the center of the i -th element

ΔV_i = isotropic gamma-ray source strength density
for the i th component (gamma rays/sr/cm³),

$\vec{x}_\Delta$ = coordinates of the detector.

δ_i = distance through sample material which the
photon must penetrate to reach the detector
if it originates at point $\vec{x}_i$. (See Section 3),

Y_U = yield at the detector due to unscattered photons

Y_S = yield due to photons which have scattered once
coherently in the sample.

$$Y_U \approx \sum_I \frac{S_1 \Delta V_1 \exp(-\Sigma_{YT} \delta_1)}{|\vec{x}_{O1} - \vec{x}_D|} \epsilon_{DET} , \quad (82)$$

$$Y_S \approx \sum_{ij} \frac{S_1 \Delta V_1 \Delta V_j \exp[-\Sigma_{YT} (\delta_1 + |\vec{x}_1 - \vec{x}_j|)]}{|\vec{x}_1 - \vec{x}_j|^2 |\vec{x}_j - \vec{x}_D|^2} \cdot (d\Omega/d\Omega)_{Y,COH,ij} \epsilon_{DET} \quad (83)$$

The problem has been formulated in cylindrical coordinates and a code is available for operation on the SEL 840 MP digital computer. A number of calculations were performed assuming a constant value for S_1 throughout the sample. Samples with $R_S = 1.9$ cm and $H = 3.8$ cm, fabricated from Li, Al, Ti, Fe, Zn and Mo, were considered. The detector was assumed to be 130 cm from the sample. The ratio $Y_U / (Y_U + Y_S)$ was computed for $E_Y = 0.1, 0.3, 0.5, 0.8, 1.0, 2.0$ and 5.0 for these samples. The results appear in Table V. The coherent scattering correction is clearly quite small if not negligible for most cases of interest.

APPENDIX B
LISTING OF CODE GAMSCT

Code GAMSCT was developed to perform the multiple-scattering computations discussed in Section 4 of this report. This code is written in ASI Standard FORTRAN IV. All code input is from cards (Unit 4). Output is produced on a teletype (Unit 1) and a line printer (Unit 5). The version of this code listed here is operated on a Systems Engineering Laboratories Model 840 MP digital computer.

C	GMSCT=D.L.SMITH=SEL 840MP	GMSCT 1
C		GMSCT 2
	DIMENSION ENT(25),SIGNT(25),EGP(25),SIGGP(25),NWGP(6),EWGP(6,15),WGMSCT 3	
	1GP(6,15),QNS(6),A2NS(6),NNS(6),ENS(6,25),SIGNS(6,25),MWNS(6),NWNS(GMSCT 4	
	26,10),EWNS(6,10,15),WNS(6,10,15),DSIGNS(6),WT(10),YLD(5),YLDISIM(5)GMSCT 5	
	3,PSI(6),INDEX(4),JNDFX(5),NHIT(5),NLEV(6),A(25),B(25),WORK(25)	GMSCT 6
C		GMSCT 7
	DATA PI/3.14159/	GMSCT 8
C		GMSCT 9
	VALUE(V,VMIN,VMAX)=VMIN+V*(VMAX-VMIN)	GMSCT 10
	SEPAR(X1,Y1,Z1,X2,Y2,Z2)=SORT((X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2)+(Z1-Z2)*(Z1-Z2))	GMSCT 11
		GMSCT 12
C		GMSCT 13
C	CONTROL	GMSCT 14
C		GMSCT 15
	1 READ(4,2) IC	GMSCT 16
	2 FORMAT(11)	GMSCT 17
	GO TO(10,20,30,50),IC	GMSCT 18
	10 PAUSE	GMSCT 19
	GO TO 1	GMSCT 20
C		GMSCT 21
C	READ INTERPOLATION TABLES	GMSCT 22
C		GMSCT 23
	20 READ(4,21) MNT	GMSCT 24
	21 FORMAT(16I5)	GMSCT 25
	READ(4,22) (ENT(I),SIGNT(I),I=1,MNT)	GMSCT 26
	22 FORMAT(8E10,4)	GMSCT 27
	READ(4,21) MGP	GMSCT 28
	READ(4,22) (EGP(I),SIGGP(I),I=1,MGP)	GMSCT 29
	READ(4,21) MWGP	GMSCT 30
	IF(MWGP) 23,25,23	GMSCT 31
	23 DO 24 I=1,MWGP	GMSCT 32
	READ(4,21) NWGP(I)	GMSCT 33
	M=NWGP(I)	GMSCT 34
	24 READ(4,22) (EWGP(I,J),WGP(I,J),J=1,M)	GMSCT 35
	25 READ(4,21) MNS	GMSCT 36
	DO 28 I=1,MNS	GMSCT 37
	READ(4,22) QNS(I),A2NS(I)	GMSCT 38
	READ(4,21) NNS(I)	GMSCT 39
	M=NNS(I)	GMSCT 40
	READ(4,22) (ENS(I,J),SIGNS(I,J),J=1,M)	GMSCT 41
	READ(4,21) MWNS(I)	GMSCT 42
	IF(MWNS(I)) 26,28,26	GMSCT 43
	26 L=MWNS(I)	GMSCT 44
	DO 27 J=1,L	GMSCT 45
	READ(4,21) NWNS(I,J)	GMSCT 46
	M=NWNS(I,J)	GMSCT 47
	27 READ(4,22) (EWNS(I,J,K),WNS(I,J,K),K=1,M)	GMSCT 48
	28 CONTINUE	GMSCT 49
C		GMSCT 50
C	READ AND WRITE BASIC PARAMETERS	GMSCT 51
C		GMSCT 52
	30 READ(4,31) NSCAT,NHIST	GMSCT 53
	31 FORMAT(11,16)	GMSCT 54
	READ(4,22) RS,H,DNO,DGO	GMSCT 55
	READ(4,22) EG,SIGGT,ENTHG	GMSCT 56
	READ(4,22) ALT,A2T,QT	GMSCT 57
	READ(4,32) ET,NWT	GMSCT 58
	32 FORMAT(E10,4,I5)	GMSCT 59

	IF(NWT) 34,34,33	GMSCOT 60
33	READ(4,22) (WT(I),I=1,NWT)	GMSCOT 61
34	WRITE(5,35) NSCAT,NHIST	GMSCOT 62
35	FORMAT(1H1/11HNSCAT,NHIST/I1,16)	GMSCOT 63
	WRITE(5,36) RS,H,DNO,DGO	GMSCOT 64
36	FORMAT(12HRS,H,DNO,DGO/4E10,4)	GMSCOT 65
	WRITE(5,37) EG,SIGGT,ENTHG	GMSCOT 66
37	FORMAT(14HEG,SIGGT,ENTHG/3E10,4)	GMSCOT 67
	WRITE(5,38) A1T,A2T,QT	GMSCOT 68
38	FORMAT(10HA1T,A2T,QT/3E10,4)	GMSCOT 69
	WRITE(5,39) ET,NWT	GMSCOT 70
39	FORMAT(6HET,NWT/E10,4,15)	GMSCOT 71
	IF(NWT) 40,42,40	GMSCOT 72
40	WRITE(5,41)	GMSCOT 73
41	FORMAT(5HWT(I))	GMSCOT 74
	WRITE(5,22) (WT(I),I=1,NWT)	GMSCOT 75
42	WRITE(5,43)	GMSCOT 76
43	FORMAT(/5H,.....)	GMSCOT 77
C		GMSCOT 78
C	READ AND WRITE SCATTERING ANGLE, CONVERT TO RADIANS	GMSCOT 79
C		GMSCOT 80
	50 READ(4,22) THTANK	GMSCOT 81
	WRITE(5,51) THTANK	GMSCOT 82
	51 FORMAT(/7HTHTANK=,E10,4)	GMSCOT 83
	THDET=PI*THTANK/180,0	GMSCOT 84
C		GMSCOT 85
C	PRELIMINARY CALCULATIONS	GMSCOT 86
C		GMSCOT 87
	RSRS=RS*RS	GMSCOT 88
	HH=H*H	GMSCOT 89
	HD2=0.5*H	GMSCOT 90
	R1MIN=DNO*RS	GMSCOT 91
	R1MAX=SQRT(DNO*DNO+2.0*DNO*RS+2.0*RSRS+0.25*HH)	GMSCOT 92
	TH1MAX=ATAN(SQRT(0.25*HH+RSRS)/R1MIN)	GMSCOT 93
	VOL1=2.0*PI*(R1MAX-R1MIN)*TH1MAX	GMSCOT 94
	RMAX=SQRT(HH+8.0*RSRS)	GMSCOT 95
	VOLH=2.0*PI*PI*RMAX	GMSCOT 96
	YD=DGO*SIN(THDET)	GMSCOT 97
	ZD=DNO*DGO*COS(THDET)	GMSCOT 98
	D0 60 I=1,NSCAT	GMSCOT 99
	NHIT(I)=0	GMSCOT100
60	YLDSUM(I)=0,0	GMSCOT101
	D0 61 I=1,MNS	GMSCOT102
61	NLEV(I)=0	GMSCOT103
	IHIST=1	GMSCOT104
C		GMSCOT105
C	START OF HISTORY LOOP	GMSCOT106
C		GMSCOT107
C		GMSCOT108
	100 D0 101 I=1,NSCAT	GMSCOT109
	101 JNDEX(I)=0	GMSCOT110
C		GMSCOT111
C	SELECT SCATTERING POINT S(1)	GMSCOT112
C		GMSCOT113
	R=РАНF(=1)	GMSCOT114
	RR=VALUE(R,R1MIN,R1MAX)	GMSCOT115
	R=РАНF(=1)	GMSCOT116
	TH=VALUE(R,0,0,TH1MAX)	GMSCOT117
	R=РАНF(=1)	GMSCOT118
	PHI=VALUE(R,0,0,2.0*PI)	GMSCOT119

```

      SINTH=SIN(TH)
      X=RR*SINTH*COS(PHI)
      Y=RR*SINTH*SIN(PHI)
      Z=RR*COS(TH)
      IF(X+HD2) 700,700,150
150 IF(X+HD2) 151,700,700
151 ZMDNO=Z-DNO
      TEST=Y*Y+ZMDNO*ZMDNO
      IF(TEST=RSRS) 152,700,700
152 JNDEX(1)=1
      NHIT(1)=NHIT(1)+1
      CALL ANGLE(X,Y,Z,0.0,0.0,0.0,0.0,0.0,DNO,0.0,0.0,0.0,THT)
C
C      CALCULATE NEUTRON ENERGY AND FLUX AT POINT S(1)
C
      CALL KINAM(A1T,A2T,1.0087,QT,ET,THT,EN,EDUM)
      IF(EN=ENTHG) 700,700,160
160 CALL DISTR(WT,THT,FT,NWT,10)
      DELTN=DELTA(0.0,0.0,DNO,0.0,0.0,0.0,X,Y,Z,RS,1)
      CALL INTRPL(MNT,ENT,SIGNT,EN,VSNT)
      ETAN=EXF(-VSNT*DELTN)
      FLUX=FT*ETAN*SIN(TH)*V0L1
C
C      CALCULATE GAMMA PRODUCTION FROM POINT S(1)
C
      DG=SEPAR(X,Y,Z,0.0,YD,ZD)
      CALL INTRPL(MGP,EGP,SIGGP,EN,VSGP)
      IF(MWGP) 170,170,200
170 DSIGGP=VSGP/4.0/PI
      G0 T0 203
200 D0 202 I=1,MWGP
      M=NWGP(I)
      D0 201 J=1,M
      A(J)=EWGP(I,J)
201 B(J)=WGP(I,J)
202 CALL INTRPL(M,A,B,EN,W0RK(I))
      CALL ANGLE(0.0,YD,ZD,X,Y,Z,X,Y,Z,0.0,0.0,0.0,THNG)
      CALL DISTR(W0RK,THNG,SG,MWGP,10)
      DSIGGP=VSGP*SG/4.0/PI
203 DELTG=DELTA(0.0,0.0,DNO,0.0,YD,ZD,X,Y,Z,RS,1)
      ETAG=EXF(-SIGGT*DELTG)
      YLDG=FLUX*DSIGGP*ETAG/DG/DG
C
C      UPDATE YLDSUM(1)
C
      YLDSUM(1)=YLDSUM(1)+YIDG
C
C      CHECK IF MULTIPLE SCATTERING CALCULATIONS ARE REQUESTED.
C      INITIALIZE PARAMETERS IF REQUIRED
C
      IF(NSCAT=1) 240,700,240
240 ISCAT=2
      XSAV1=0.0
      YSAV1=0.0
      ZSAV1=0.0
      XSAV2=X
      YSAV2=Y
      ZSAV2=Z
      D0 250 I=2,NSCAT
      J=I-1

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GMSCT120
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GMSCT179

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250	INDEX(J)=0	GMSC1180
C		GMSC1181
CXXX	START OF MULTIPLE SCATTERING LOOP	GMSC1182
C		GMSC1183
C	SELECT SCATTERING POINT S(ISCAT)	GMSC1184
C		GMSC1185
300	R=RANF(=1)	GMSC1186
	RR=VALUE(R,0.0,RMAX)	GMSC1187
	R=RANF(=1)	GMSC1188
	TH=VALUE(R,0.0,PI)	GMSC1189
	R=RANF(=1)	GMSC1190
	PHI=VALUE(R,0.0,2.0*PI)	GMSC1191
	SINTH=SIN(TH)	GMSC1192
	X=XSAV2+RR*SINTH*COS(PHI)	GMSC1193
	Y=YSAV2+RR*SINTH*SIN(PHI)	GMSC1194
	Z=ZSAV2+RR*COS(TH)	GMSC1195
	IF(X+HD2) 700,700,350	GMSC1196
350	IF(X+HD2) 351,700,700	GMSC1197
351	ZMDNO=Z-DNO	GMSC1198
	TEST=Y*Y+ZMDNO*ZMDNO	GMSC1199
	IF(TEST=RSRS) 352,700,700	GMSC1200
352	JINDEX(ISCAT)=1	GMSC1201
	NHIT(ISCAT)=NHIT(ISCAT)+1	GMSC1202
	CALL ANGLE(X,Y,Z,XSAV2,YSAV2,ZSAV2,XSAV1,YSAV1,ZSAV1,THSCT)	GMSC1203
C		GMSC1204
C	SELECT NEUTRON SCATTERING PROCESS FOR POINT S(ISCAT-1)	GMSC1205
C		GMSC1206
	D0 407 I=1,MNS	GMSC1207
	EB=QNS(I)*(1.0+(1.0087/(A2NS(I)-1.0087))-(0.5*QNS(I)/(A2NS(I)-1.0087)/931.478))	GMSC1208
	IF(EN-EB) 400,400,401	GMSC1209
400	DSIGNS(I)=0.0	GMSC1210
	G0 T0 407	GMSC1211
401	M=NNS(I)	GMSC1212
	D0 402 J=1,M	GMSC1213
	A(J)=ENS(I,J)	GMSC1214
402	B(J)=SIGNS(I,J)	GMSC1215
	CALL INTRPL(M,A,B,EN,VSSCT)	GMSC1216
	IF(MWNS(I)) 403,403,404	GMSC1217
403	DSIGNS(I)=VSSCT/4.0/PI	GMSC1218
	G0 T0 407	GMSC1219
404	M=MWNS(I)	GMSC1220
	D0 406 J=1,M	GMSC1221
	L=NWNS(I,J)	GMSC1222
	D0 405 K=1,L	GMSC1223
	A(K)=EWNS(I,J,K)	GMSC1224
405	B(K)=WNNS(I,J,K)	GMSC1225
406	CALL INTRPL(L,A,B,EN,W0RK(J))	GMSC1226
	CALL DISTR(W0RK,THSCT,SSCT,M,10)	GMSC1227
	DSIGNS(I)=VSSCT*SSCT/4.0/PI	GMSC1228
407	CONTINUE	GMSC1229
	SUMSCT=0.0	GMSC1230
	D0 408 I=1,MNS	GMSC1231
408	SUMSCT=SUMSCT+DSIGNS(I)	GMSC1232
	IF(SUMSCT) 409,700,409	GMSC1233
409	PSI(1)=0.0	GMSC1234
	K=MNS+1	GMSC1235
	PSI(K)=1.0	GMSC1236
	IF(MNS=1) 410,410,411	GMSC1237
		GMSC1238
		GMSC1239

410	INDX=1	GMSC2240
	G0 T0 414	GMSC2241
411	SUMPSI=0.0	GMSC2242
	D0 413 I=2,MNS	GMSC2243
	J=I-1	GMSC2244
	SUMPSI=SUMPSI+DSIGNS(J)	GMSC2245
	PSI(I)=SUMPSI/SUMSCT	GMSC2246
	IF(PHI(I)=1.0) 413,413,412	GMSC2247
412	PSI(I)=1.0	GMSC2248
413	CONTINUE	GMSC2249
	R=RANF(-1)	GMSC2250
	CALL FINDI(PSI,K,6,R,INDX)	GMSC2251
414	NLEV(INDX)=NLEV(INDX)+1	GMSC2252
C		GMSC2253
C	CALCULATE NEUTRON ENERGY AND FLUX AT POINT S(ISCAT)	GMSC2254
C		GMSC2255
500	ENSAV=EN	GMSC2256
	CALL KINAM(1.0087,AZNS(INDX),1.0087,ONS(INDX),ENSAV,THSCT,EN,FDUM)	GMSC2257
	IF(EN=ENTHG) 700,700,501	GMSC2258
501	CALL INTRPL(MNT,ENT,SIGNT,EN,VSNT)	GMSC2259
	ETAN=EXF(-VSNT*RR)	GMSC2260
	FLUX=FLUX+SUMSCT*ETAN*SIN(TH)*V0LH	GMSC2261
C		GMSC2262
C	CALCULATE GAMMA PRODUCTION FROM POINT S(ISCAT)	GMSC2263
C		GMSC2264
	DG=SEPAR(X,Y,Z,0.0,YD,ZD)	GMSC2265
	CALL INTRPL(MGP,EGP,SIGGP,EN,VSGP)	GMSC2266
	IF(MWGP) 502,502,600	GMSC2267
502	DSIGGP=VSGP/4.0/PI	GMSC2268
	G0 T0 603	GMSC2269
600	D0 602 I=1,MWGP	GMSC2270
	M=NWGP(I)	GMSC2271
	D0 601 J=1,M	GMSC2272
	A(J)=EWGP(I,J)	GMSC2273
601	B(J)=WGP(I,J)	GMSC2274
602	CALL INTRPL(M,A,B,EN,WORK(I))	GMSC2275
	CALL ANGLE(0.0,YD,ZD,X,Y,Z,X,Y,Z,XSAV2,YSAV2,ZSAV2,THNG)	GMSC2276
	CALL DISTR(WORK,THNG,SG,MWGP,10)	GMSC2277
	DSIGGP=VSGP*SG/4.0/PI	GMSC2278
603	DELTG=DELTA(0.0,0.0,DNO,0.0,YD,ZD,X,Y,Z,RS,1)	GMSC2279
	ETAG=EXF(-SIGGT*DELTG)	GMSC2280
	YLDG=FLUX*DSIGGP*ETAG/DG/DG	GMSC2281
C		GMSC2282
C	UPDATE YLDSUM(ISCAT) AND FIX INDEX(ISCAT=1)	GMSC2283
C		GMSC2284
	YLDSUM(ISCAT)=YLDSUM(ISCAT)+YLDG	GMSC2285
	J=ISCAT-1	GMSC2286
	INDEX(J)=INDX	GMSC2287
C		GMSC2288
C	TEST FOR END OF MULTIPLE SCATTERING LOOP, RESET PARAMETERS FOR	GMSC2289
C	NEXT CYCLE IF REQUIRED	GMSC2290
C		GMSC2291
	ISCAT=ISCAT+1	GMSC2292
	IF(ISCAT=NSCAT) 610,610,700	GMSC2293
610	XSAV1=XSAV2	GMSC2294
	YSAV1=YSAV2	GMSC2295
	ZSAV1=ZSAV2	GMSC2296
	XSAV2=X	GMSC2297
	YSAV2=Y	GMSC2298
	ZSAV2=Z	GMSC2299


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      GO TO 300
C
CXXX END OF MULTIPLE SCATTERING LOOP
C
C      UPDATE HISTORIES AVERAGE, INTERMEDIATE OUTPUT IF SS1 UP, TEST
C      FOR COMPLETION OF REQUESTED NUMBER OF HISTORIES OR SS2 UP AND
C      PROCEED TO OUTPUT OR CONTINUE WITH HISTORY ACCORDING TO OUTCOME
C
700 DO 701 I=1,NSCAT
701 YLD(I)=YLDSUM(I)/FLOAT(IHIST)
      CALL SSWTCH(1,K1)
      IF(K1=2) 702,706,702
702 WRITE(5,703) IHIST
703 FORMAT(5I7)
      WRITE(5,703) (JNDX(J),J=1,NSCAT)
      IF(NSCAT=1) 704,705,704
704 I=NSCAT-1
      WRITE(5,703) (INDEX(J),J=1,I)
      WRITE(5,703) (NLEV(J),J=1,MNS)
705 WRITE(5,703) (NHIT(J),J=1,NSCAT)
      WRITE(5,22) (YLD(I),I=1,NSCAT)
706 CALL SSWTCH(2,K2)
      IF(K2=1) 707,800,707
707 IF(IHIST=NHIST) 708,800,800
708 IHIST=IHIST+1
      GO TO 100
C
C.....END OF HISTORY LOOP
C
C      FINAL OUTPUT-IF NSCAT.GT.1 RENORMALIZE YLD(I) SO YLD(1)=1 AND
C      COMPUTE ALFA
C
800 WRITE(5,801) IHIST
801 FORMAT(10HHISTORIES=.18)
      IF(NSCAT=1) 804,804,802
802 WRITE(5,803)
803 FORMAT(7HNLV(I))
      WRITE(5,703) (NLEV(I),I=1,MNS)
804 WRITE(5,805)
805 FORMAT(7HNNHIT(I))
      WRITE(5,703) (NHIT(I),I=1,NSCAT)
      WRITE(5,806)
806 FORMAT(6HYLD(I))
      WRITE(5,22) (YLD(I),I=1,NSCAT)
      IF(NSCAT=1) 807,1,807
807 FYLD=YLD(1)
      DO 808 I=1,NSCAT
808 YLD(I)=YLD(I)/FYLD
      ALFA=0.0
      DO 809 I=2,NSCAT
809 ALFA=ALFA+YLD(I)
      WRITE(5,810)
810 FORMAT(17HNNORMALIZED YLD(I))
      WRITE(5,22) (YLD(I),I=1,NSCAT)
      WRITE(5,811) ALFA
811 FORMAT(5HALFA=.E10.4)
C
      IF(K2=1) 1,812,1
812 PAUSE
      GO TO 1

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END
FUNCTION DELTA(X0,Y0,Z0,X1,Y1,Z1,X2,Y2,Z2,RS,INDEX)
FUNCTION TO DETERMINE PENETRATION DEPTH FOR R.C. CYLINDER.
INDEX=1,2 OR 3 IMPLIES CYLINDER AXIS PARALLEL TO X,Y OR Z-AXIS
RESPECTIVELY. (X0,Y0,Z0) IS CYLINDER CENTER, (X1,Y1,Z1) IS
EXTERIOR POINT, (X2,Y2,Z2) IS INTERIOR POINT. RS=CYLINDER
RADIUS

R=SQRT((X2-X1)*(X2-X1)+(Y2-Y1)*(Y2-Y1)+(Z2-Z1)*(Z2-Z1))
A=(X2-X1)/R
B=(Y2-Y1)/R
C=(Z2-Z1)/R
GO TO(1,2,3),INDEX
1 Y0W=Y0
Z0W=Z0
Y1W=Y1
Z1W=Z1
BW=B
CW=C
GO TO 4
2 Y0W=Z0
Z0W=X0
Y1W=Z1
Z1W=X1
BW=C
CW=A
GO TO 4
3 Y0W=X0
Z0W=Y0
Y1W=X1
Z1W=Y1
BW=A
CW=B
4 S=(BW*(Y0W-Y1W)+CW*(Z0W-Z1W)-SQRT(ABS((BW*BW+CW*CW)*RS*RS-(CW*(Y1W-
1-Y0W)-BW*(Z1W-Z0W))*(CW*(Y1W-Y0W)-BW*(Z1W-Z0W))))/(BW*BW+CW*CW)
DELTA=R-S
RETURN
END
SUBROUTINE KINAM(A1,A2,A3,Q,E1,TH3,E31,E32)

W1=931.478*A1
W2=931.478*A2
W3=931.478*A3
W4=W1+W2+W3-Q
EF=-Q*(1.0+(W1/W2)-(0.5*Q/W2))
EB=-Q*(1.0+(W1/(W2-W3))-(0.5*Q/(W2-W3)))
IF(E1=EF) 1,1,2
1 E31=0.0
11 E32=0.0
GO TO 6
2 C=COS(TH3)
A=2.0*(W3+W4+E1*Q)
B=2.0*E1*(W1-W4+Q)-(2.0*W4*Q+Q*Q)
D=E1*(E1+2.0*W1)*C*C
TERM=(B*B-2.0*W3*A*B+4.0*W3*W3*D)*E1*(E1+2.0*W1)
IF(TERM) 1,1,3
3 DEN=A*A-4.0*D
U=(4.0*W3*D+A*B)/DEN
V=2.0*C*SQRT(ABS(TERM))/DEN

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E31=U+V	GMSCT420
IF(E1=EB) 4,4,5	GMSCT421
4 IF(TM3=1.5707963) 41,11,11	GMSCT422
41 E32=U-V	GMSCT423
G0 T0 6	GMSCT424
5 E32=E31	GMSCT425
6 RETURN	GMSCT426
END	GMSCT427
SUBROUTINE ANGLE(X1H,Y1H,Z1H,X1T,Y1T,Z1T,X2H,Y2H,Z2H,X2T,Y2T,Z2T,T)	GMSCT428
1H)	GMSCT429
V1=SQRT(ABS((X1H-X1T)*(X1H-X1T)+(Y1H-Y1T)*(Y1H-Y1T)+(Z1H-Z1T)*(Z1H-Z1T)))	GMSCT430
1-Z1T)))	GMSCT431
V2=SQRT(ABS((X2H-X2T)*(X2H-X2T)+(Y2H-Y2T)*(Y2H-Y2T)+(Z2H-Z2T)*(Z2H-Z2T)))	GMSCT432
1-Z2T)))	GMSCT433
D0T=(X1H-X1T)*(X2H-X2T)+(Y1H-Y1T)*(Y2H-Y2T)+(Z1H-Z1T)*(Z2H-Z2T)	GMSCT434
CTH=D0T/V1/V2	GMSCT435
TH=ARCCOS(CTH,2)	GMSCT436
RETURN	GMSCT437
END	GMSCT438
SUBROUTINE DISTR(W,TH,V,NW,NMAX)	GMSCT439
DIMENSION W(NMAX)	GMSCT440
V=1,0	GMSCT441
IF(NW.EQ.0) G0 T0 4	GMSCT442
D0 2 I=1,NW	GMSCT443
IF(W(I)) 21,20,21	GMSCT444
20 VADD=0,0	GMSCT445
G0 T0 22	GMSCT446
21 VADD=W(I)*POLYL(2,1,TH)	GMSCT447
22 V=V+VADD	GMSCT448
2 CONTINUE	GMSCT449
IF(V) 3,4,4	GMSCT450
3 V=0,0	GMSCT451
4 RETURN	GMSCT452
END	GMSCT453
FUNCTION POLYL(I0P,N,ANGLE)	GMSCT454
X = ANGLE	GMSCT455
G0 T0(10,11,12),I0P	GMSCT456
10 X = .017453293*X	GMSCT457
11 X = COS(X)	GMSCT458
12 NBIG = N-1	GMSCT459
IF(NBIG) 1,2,3	GMSCT460
1 POLYL = 1,0	GMSCT461
G0 T0 100	GMSCT462
2 POLYL = X	GMSCT463
G0 T0 100	GMSCT464
3 PL = X	GMSCT465
PLM1 = 1,0	GMSCT466
D0 4 L=1,NBIG	GMSCT467
POLYL = (FLOAT(2*L+1)*X*PL = FLOAT(L)*PLM1)/FLOAT(L+1)	GMSCT468
PLM1 = PL	GMSCT469
4 PL = POLYL	GMSCT470
100 RETURN	GMSCT471
END	GMSCT472
SUBROUTINE INTRPL(N,XT,YT,X,Y)	GMSCT473
DIMENSION XT(N),YT(N)	GMSCT474
IF(X=XT(1)) 1,3,4	GMSCT475
1 WRITE(1,2)	GMSCT476
2 FORMAT(8HRANG ERR)	GMSCT477
PAUSE	GMSCT478
3 Y=YT(1)	GMSCT479

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      G0 T0 24
4 IF(X=XT(N)) 7.5,1
5 Y=YT(N)
      G0 T0 24
7 I=0
  J=N
8 K=0.5*FLOAT(J-1)+0.1
  K=K+1
  IF(X=XT(K)) 9,10,11
9 J=K
  G0 T0 12
10 Y=YT(K)
  G0 T0 24
11 I=K
12 IF(J-I-1) 13,13,8
13 I=J
  J=I-1
  DEN=XT(J)-XT(I)
  C1=(XT(J)*YT(I)-XT(I)*YT(J))/DEN
  C2=(YT(J)-YT(I))/DEN
  Y=C1+C2*X
24 RETURN
END
FUNCTION ARCCOS(X,K)
  ARCCOS=1.5707963
  IF(ABS(X).GT..999999) X=.999999*X/ABS(X)
  IF(X*X.GT.1.0E-70) ARCCOS=ATAN(SQRT(ABS(1./X/X-1.)))
  IF(X.LT.0.) ARCCOS=3.1415926-ARCCOS
  G0 T0 (100,200),K
100 ARCCOS=ARCCOS+57.2957795
200 RETURN
END
FUNCTION EXP(Z)
  IF(Z) 1,1,3
1 IF(Z.LT.-70.0) Z=-70.0
  IF(Z.GT..1E-04) G0 T0 2
  EXP=EXP(Z)
  G0 T0 4
3 IF(Z.GT.70.0) Z=70.0
  IF(Z.LT..1E-04) G0 T0 2
  EXP=EXP(Z)
  G0 T0 4
2 EXP=1.0+Z
4 CONTINUE
RETURN
END
SUBROUTINE FINDI(Y,N,NDIM,Z,IZ)
  DIMENSION Y(NDIM)
  NMIN=1
  NMAX=N
36 INTER=0.5*FLOAT(NMAX-NMIN)+0.1
  NTEST=NMIN+INTER
  IF(Z=Y(NTEST)) 1,2,3
1 NMAX=NTEST
  G0 T0 4
2 IZ=NTEST
  G0 T0 999
3 NMIN=NTEST
4 IF(NMAX=NMIN-1) 5,5,36
5 IZ=NMAX-1

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GMSCT532
GMSCT533
GMSCT534
GMSCT535
GMSCT536
GMSCT537
GMSCT538
GMSCT539

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999 RETURN
END

S

GMSCT540
GMSCT541
GMSCT542

REFERENCES

1. Donald L. Smith, Report ANL/NDM-12, Argonne National Laboratory (1975).
2. S. A. Cox and P. R. Hanley, IEE Trans. Nucl. Sci. 18, 108 (1971).
3. R. B. Day, Phys. Rev. 102, 767 (1956); Progress in Fast Neutron Physics, edited by G. C. Phillips, J. B. Marion and J. R. Risser (University of Chicago Press, Chicago, 1963).
4. K. Nishimura, K. Okano and S. Kikuchi, Nucl. Phys. 70, 421 (1965).
5. J. B. Parker, J. H. Towle, D. Sams and P. G. Jones, Nucl. Instr. 14, 1 (1961).
6. S. A. Cox, Nucl. Instr. 56, 245 (1967).
7. O. Aspelund and B. Gustafsson, Nucl. Instr. 57, 197 (1967).
8. P. Kuijper, J. C. Veefkind and C. C. Jonker, Nucl. Instr. 77, 55 (1970).
9. W. E. Kinney, Nucl. Instr. 83, 15 (1970).
10. J. K. Dickens, Nucl. Instr. 98, 451 (1972).
11. S. A. Elbaker, I. J. Van Heerden, W. K. Dawson, W. J. McDonald and G. C. Neilson, Nucl. Instr. 97, 283 (1971).
12. B. Minetti and A. Pasquarelli, Nucl. Instr. 120, 509 (1974).
13. E. D. Cashwell, C. J. Everett and O. W. Richards, "A Practical Manual on the Monte-Carlo Method for Random Walk Problems," Report LA-2120, Los Alamos Scientific Laboratory (1957).
14. Evaluated Neutron Data File, ENDF/B-IV, National Neutron Cross Section Center, Brookhaven National Laboratory.
15. E. Storm and H. I. Israel, Nucl. Data Tables A7, 565 (1970).

Table I

Nominal Conditions Considered in Sample
Geometry and Absorption Calculations

Geometry:

$$R_S = 1.9 \text{ cm}, H = 3.8 \text{ cm}, D_n = 12.8 \text{ cm},$$

$$D_\gamma = 130.0 \text{ cm}, a = 0.5 \text{ cm}, \text{ various } \theta_{\text{DET}}$$

Mesh size:

$$n_T = 1, n_H = 5, n_R = 5, n_\phi = 3$$

Sample material and gamma ray:

$$\text{Natural iron, } E_\gamma = 0.846 \text{ MeV}$$

Neutron source:

$$\text{Isotropic yield, } Q = 0, A_1 = 1, A_2 = 7.$$

$$\text{Incident energy selected so } E_{n,\text{max}} = 1 \text{ MeV.}$$

Cross sections^a:

$$\Sigma_{nT} = 0.27, \quad \Sigma_{\gamma T} = 0.5$$

$$(d\sigma/d\Omega)_\gamma = 0.038 \text{ b/sr (isotropic)}$$

Gamma-detector efficiency:

Arbitrary constant value

^aThese are nominal values which are not necessarily equal to
ENDF/B-IV values for iron.

Table II

Computed Yield Ratios Which
Demonstrate the Effects of
Geometry and Radiation Absorption

	"Front" - to "back" ratio ^a	"Near" - to "far" ratio ^a
i) Neutron and gamma-ray absorption	2.11	2.11
ii) Gamma-ray absorption only	1.19	2.19
iii) Neutron absorption only	1.78	1.03
iv) No absorption	1.16	1.02

^a Ratios are defined in Section 3.2.

Table III

Dependence of Gamma-Ray
Yield on the Sample Radius

R_S (cm)	$\bar{Y}_0^a$	$R_S \bar{Y}_0^a$
0.635	1	1
0.95	0.84	1.25
1.27	0.70	1.41
1.59	0.60	1.50
1.905	0.52	1.55
2.54	0.39	1.57

^a $\bar{Y}_0$ computed for $\theta_{DET} = 90^\circ$. Values are relative to corresponding values for $R_S = 0.635$ cm.

Table IV

Nominal Conditions Considered in
Sample Multiple Scattering Calculations

Geometry:

$$R_S = 1.9 \text{ cm}, H = 3.8 \text{ cm}, D_n = 11.4 \text{ cm},$$

$$D_\gamma = 130.0 \text{ cm}, \text{ Various } \theta_{\text{DET}}$$

Sample material and gamma ray:

$$\text{Natural iron, } E_\gamma = 0.846 \text{ MeV}$$

Neutron source:

$$\text{Isotropic yield, } Q = 0, A_1 = 1, A_2 = \text{very large.}$$

$$\text{Neutron energy } E_n = 2 \text{ MeV.}$$

Cross sections^a:

$$\Sigma_{nT} = 0.17$$

$$\text{Elastic scattering, } Q = 0, A_1 = 1, A_2 = 55.85,$$

$$\Sigma_{EL} = 0.17 \text{ (isotropic)}$$

$$\text{Inelastic scattering, } Q = -0.85 \text{ MeV, } A_1 = 1, A_2 = 55.85,$$

$$\Sigma_{IN} = 0.05$$

$$(d\sigma/d\Omega)_\gamma = 0.038 \text{ b/sr (isotropic)}$$

^a These are nominal values which are not necessarily equal to ENDF/B-IV values for iron.

Table V

Effect of Coherent Photon Scattering

Sample Material	E_γ (MeV)	Ratio $Y_U/(Y_U + Y_S)^a$
Li (Z=3)	0.1	1.00
Al (Z=13)	0.1	0.96
	0.3	0.99
	0.5	1.00
Ti (Z=22)	0.1	0.95
	0.3	0.98
	0.5	0.99
	0.8	1.00
Fe (Z=26)	0.1	0.98
	0.3	0.98
	0.5	0.99
	0.8	1.00
Zn (Z=30)	0.1	0.99
	0.3	0.97
	0.5	0.99
	0.8	0.99
	1.0	1.00
Mo (Z=42)	0.1	1.00
	0.3	0.97
	0.5	0.98
	0.8	0.99
	1.0	0.99
	2.0	1.00

^a Y_U and Y_S are computed using Eqn. (81) and (82). Values of the ratio for larger E_γ are ~ 1.00 if not given in the table.

FIGURE CAPTIONS

- Fig. 1. The total cross section for natural iron in the energy range 0.1-2 MeV. The solid curve represents ENDF/B-IV values [14]. The dashed curve represents the same information after smoothing with a 0.1-MeV resolution function. The smoothed excitation function can be approximated by connecting the large dots with straight line segments. This simulates linear interpolation of a lookup table which is stored in the memory of a digital computer. (ANL Neg. No. 116-75-91).
- Fig. 2. Schematic diagrams to illustrate geometry applicable to computation of the yield of gamma-rays from $(n, X\gamma)$ reactions produced by unscattered neutrons. (ANL Neg. No. 116-75-89).
- Fig. 3. Distortion of an isotropic gamma-ray production angular distribution by radiation absorption. (ANL Neg. No. 116-75-85).
- Figs. 4 Sample midplane relative-yield profiles for the following re-
thru
Fig. 7 spective conditions: i) neutron and gamma-ray absorption, ii) gamma-ray absorption only, iii) neutron absorption only, and iv) no absorption. (ANL Neg. Nos. 116-75-84, 116-75-88, 116-75-86, 116-75-82).
- Fig. 8. Computed angular resolution functions for several values of θ_{DET} and Table I parameters. (ANL Neg. Nos. 116-75-80).
- Fig. 9. Demonstration of the factorization rule. The solid lines represent assumed differential cross section functions while the solid circles represent values of $\bar{Y}_0/(1 - \Delta \cos \theta_{DET})$ for various θ_{DET} but plotted at the corresponding angles $\langle \theta_{NY} \rangle$. All results are normalized to unity at $\theta_{DET} = 90^\circ$. (ANL Neg. No. 116-75-87).
- Fig. 10. Geometry appropriate to multiple-scattering calculations for the first two scattering orders. (ANL Neg. No. 116-75-92).
- Fig. 11. Plot of relative values for Y_0 , Y_1 , Y_2 and Y_3 computed using the parameters in Table IV. (ANL Neg. No. 116-75-90).
- Fig. 12. Plots of α_1 , α_2 , α_3 and α_{TOT} for various sample sizes ($H = 2 R_S$). (ANL Neg. No. 116-75-83).
- Fig. 13. The relative yield of 0.846-MeV gamma rays per atom for various natural iron samples. Comparison is made between the experimental results and the results of four sets of computations described in Section 5. (ANL Neg. No. 166-75-81).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

NEUTRON AND GAMMA-RAY ABSORPTION

Fig. 5

Fig. 6

Fig. 7

NO ABSORPTION

Fig. 8

Fig. 9

Fig. 10

γ_0 - UNSCATTERED NEUTRONS

γ_1 - ONCE-SCATTERED NEUTRONS

Fig. 11

Fig. 12

Fig. 13