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CALCULATIONS FOR PIPE INTERSECTIONS
CONTAINING FISSILE SOLUTION

Deanne Dickinson

THE DOW CHEMICAL COMPANY
ROCKY FLATS DIVISION
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GOLDEN, COLORADO 80401

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CALCULATIONS FOR PIPE INTERSECTIONS CONTAINING FISSILE SOLUTION

Deanne Dickinson

Abstract. Calculations of the effective neutron multiplication factor (k_{eff}) for complicated pipe intersection problems have been performed using the O5R Monte Carlo neutron transport code. Results are compared with experimental data and with the predictions of an earlier empirical model, identified as the AI Model. Calculations for simple pipe intersections such as T's and crosses are also presented. A new empirical model is formulated and its predictions compared with experimental and calculational data and with other model calculations.

INTRODUCTION

Many experiments simulating intersections of pipes containing fissile solution have been performed at the Rocky Flats Plant critical mass laboratory. The results of these experiments, together with criteria for safe design of piping systems for fissile solutions, are contained in an earlier topical report.¹ The current report compares these experimental results with Monte Carlo calculations and describes calculations performed to evaluate various aspects of the AI Model presented in the earlier report.

An obvious difficulty in pipe-intersection problems is their geometrical complexity and variability. For all but the simplest problems, many parameters are required to describe a configuration, and it is of interest to examine the behavior of the system as the parameter values are varied singly and in combination. Such a study leads to the problem of choosing certain parameters whose values characterize the criticality safety aspect of a particular arrangement. The only type of pipe geometry treated here relates to a main pipe, referred to as the *central column*, which is intersected by smaller pipes, called *arms*. The arms are usually arranged in layers, where a *layer* is a group of arms whose axes intersect the column in a plane perpendicular to the axis of the column. Some of the parameters considered are column dimensions, arm diameter, number of arms, spacing between layers

of arms, and the angle between arm and column. The empirical model described in RFP-1197² uses as a critical parameter the *area of intersection* of the arms with the column. For brevity, this model hereafter will be referred to as the *AI Model*. (See Appendix A, Page 17.)

Details of calculational methods are given, followed by results of calculations concerning complicated, many-arm intersections similar to those examined experimentally. Finally, an empirical model for calculating safe intersections is described, with examples of the application of the model. The predictions and range of applicability of the current model are compared with those of previous empirical models developed by C. L. Schuske,^{3,4} and which are summarized in Appendix A.

DETAILS OF CALCULATIONAL METHODS

Computer Code: All calculations were performed using the O5R Monte Carlo code.⁵ Cross-section information was averaged using 64 subgroups per supergroup. Angular scattering used a P_0 approximation. Thermal neutrons were treated by the one-velocity option, using Hansen-Roach group sixteen data.⁶ Each neutron batch contained 200 neutrons, and generally 20 batches were run. However, most problems were limited to 15 minutes of time on a CDC-3800 computer, and thus some problems could not be finished in the allotted time. The effective neutron multiplication factor, k_{eff} , was computed for each batch as the ratio of the statistical weight of neutrons produced during a batch to the statistical weight of the neutrons present at the beginning of the batch. The first two batches were always discarded because of source effects.

² *Ibid.*³ *Op. cit.*⁴ C. L. Schuske. *An Empirical Method for Calculating Subcritical Pipe Intersections*. TID-5451. Rocky Flats Division, The Dow Chemical Company, Golden, Colorado. July 17, 1956.⁵ D. C. Irving, R. M. Freestone, Jr., and F. B. K. Kam. *O5R, A General-Purpose Monte Carlo Neutron Transport Code*. ORNL-3622. Union Carbide Corporation, Oak Ridge National Laboratory, Oak Ridge, Tennessee. February 1965.⁶ G. E. Hansen and W. H. Roach. *Six and Sixteen Group Cross Sections for Fast and Intermediate Assemblies*. LAMS-2543. Los Alamos Scientific Laboratory, Los Alamos, New Mexico. November 1961.¹ Bruce B. Ernst and C. L. Schuske. *Empirical Methods for Calculating Pipe Intersections Containing Fissile Solutions*. RFP-1197. Rocky Flats Division, The Dow Chemical Company, Golden, Colorado. September 9, 1968.

Statistics: Since little experimental or calculational data exist currently on pipe intersections, it was felt that a larger number of *rough* calculations would prove more informative than a smaller number of precisely determined values. Thus, values of k_{eff} are calculated to an accuracy of 1 to 2 percent. All calculated values of k_{eff} are given with an associated error estimate of *one* standard error (sigma, σ). This error pertains only to the statistical uncertainty inherent in a Monte Carlo calculation and does not include errors caused by uncertainties in dimensions, cross-section data, etc. In terms of confidence limits, one standard error (σ) corresponds to 67 percent confidence limits for an average based on 18 batches, and 90 percent confidence limits are equivalent to 1.73 standard errors.

Material Composition: The fissile solution was uranyl nitrate at a concentration of 451 grams of uranium per liter, enriched to 93.1 weight percent (wt %) uranium 235. In the experiments, the pipes were made of stainless steel. Since nickel and chromium cross-section information was not available in an appropriate form, the stainless steel was replaced by mild steel (iron) in calculations. Data by Paxton, *et al.*⁷ indicate that mild and stainless steels give nearly identical critical dimensions. Number densities are given in Table I.

Experimental and Calculational Geometries: In all experiments reported by Ernst and Schuske,⁸ the central column (the term used to describe the main pipe) had a square cross-section, 7 inches on a side, steel walls 0.125 inches thick, and a length of 96 inches. Square or cylindrical pipes, called arms,

⁷H. C. Paxton, J. T. Thomas, Dixon Callihan, and E. B. Johnson. *Critical Dimensions of Systems Containing U^{235} , Pu^{239} and U^{233}* . TID-7028. Oak Ridge National Laboratory, Oak Ridge, Tennessee and Los Alamos Scientific Laboratory, Los Alamos, New Mexico. June 1964. (See Figure 86, Page 121.)

⁸RFP-1197. *Loc. cit.*

TABLE I. Number Densities.

Material	Element	Number Density (atoms per barn- centimeter)
Solution	Hydrogen	0.05505
	Nitrogen	0.002744
	Oxygen	0.037849
	Uranium 235	0.001076
	Uranium 238	0.000079
Mild Steel	Iron	0.08385

intersected the central column at an angle of 45 or 90 degrees. The square arms were identical in cross-section to the column and had a length of 54 inches. The cylindrical arms, also 54 inches long, had steel walls 0.116 inches thick and diameters of 5.35 and 6.4 inches. Information on the exact positioning of the arms along the column was not available. In calculations it was assumed, unless otherwise stated, that the solution in the column extended at least 40 inches above the top layer of arms and 40 inches below the bottom layer and that the column walls did not extend beyond the solution.

In all calculations, certain experimental details were neglected; e.g., the steel bottom on the central column, the steel on the arm ends not adjacent to the column, fill lines and dump valves containing fissile solution, and the supporting apparatus for the arms and column. Also omitted from the calculations is the large concrete room in which the experiments were performed, so that calculated critical values are for the *bare* case. Figures 1(a) and (b) show the geometrical details of the eight-arm intersection used for calculations of experimental cases. In the experiments, the arms were actually separate from the column, since the arms were closed on both ends and merely placed against the central column. The arrangement facilitates changes in the positions of the arms, but it may result in a small gap between the ends of the arms and the column wall, as well as two layers of steel at the arm-column interface.

For calculations not directly modeled after experimental cases, the arms were connected to the central column, and the extra steel at the interface was eliminated (as in an actual intersection). A typical geometry is shown in Figures 2(a) and (b).

CALCULATIONS FOR MANY-ARM INTERSECTIONS

The calculations presented in this section are directly related to the experimental data of RFP-1197 or to the Area of Intersection Model. (See Appendix A.) First, calculations for a variety of experimentally determined critical geometries are presented. These are followed by calculations which evaluate assumptions used in the formulation of the AI Model or study the dependence of k_{eff} on a parameter.

Experimental Cases: In order to judge the accuracy of the O5R code and cross-section data when applied to pipe-intersection problems, values of k_{eff} were

FIGURES 1(a) and (b). Experimental Geometry for Eight-Arm Intersections. Cross-Sectional Views: Top (a) and Side (b).

FIGURES 2(a) and (b). Geometry Used for Calculations of Eight-Arm Intersections: (a) Cross Section in Plane Perpendicular to Axis of Column and through Axes of Arms in a Layer; and (b) Cross Section in Plane through Axis of Column Parallel to Side of Column.

calculated for a variety of experimentally critical cases. A second purpose of the calculations was the evaluation of the effect of various idealizations and omissions arising in the conversion of experimental data to tractable calculational problems.

The calculated values of k_{eff} as given in Table II, are all slightly less than 1.0, as would be expected when room return is neglected. The possibility of a small gap between the arm end and the column was mentioned earlier. Since the actual gap width was not known, calculations were performed for the case of eight 6.4-inch arms with no gap and with a $\frac{1}{16}$ -inch gap, values which bracket the estimated experimental gap width. The difference in k_{eff} for the two cases was only 0.005, much smaller than the standard errors involved. Hence further calculations for experimental geometries were performed with no gap between the arm and the column.

Evaluation of the AI Model: Several assumptions were necessary in order to formulate the empirical AI Model from the data presented in RFP-1197. The nature of the assumptions prevented easy investigations

TABLE II. Calculations for Experimentally Critical Cases.

Description	Figure References	$k_{eff} \pm \sigma$	Number of Batches Averaged
Eight 6.4-inch arms, four arms per layer, 5.19-inch separation, $\frac{1}{16}$ -inch gap between arm and column.	1(a) and (b) Page 3	0.989 ± 0.015	17
Eight 6.4-inch arms, four arms per layer, 5.19-inch separation, no gap between arm and column.	1(a) and (b) Page 3	0.994 ± 0.018	16
Eight 5.35-inch arms, four arms per layer, 0.25-inch separation, no gap between arm and column.	1(a) and (b) Page 3	0.995 ± 0.017	23
Six 6.4-inch arms, three layers, two arms per layer, 90° angle between arms in layer, layers edge to edge.		0.991 ± 0.016	15
Two square arms, 7-inches on a side, at 90° to each other and at 45° to the column, solution in column 4.82 inches above top of arms.		0.984 ± 0.018	15

experimentally, and a calculational check was desirable. In some cases (e.g., the reflector-savings calculations), the evaluation of a hypothesis included the calculation of values of k_{eff} for intersections designed according to the AI Model. These results verified that the AI Model is conservative.

STEEL INTERFACE AND GAP CORRECTION -

In modifying the experimental data of RFP-1197 for application to actual intersections, the authors (Schuske, *et al.*) proposed a reduction of the arm diameter by 0.28 inches to compensate for the two layers of steel (total thickness 0.241 inches) at the arm-column interface and the possible gap between the arm and the column in the experiments [see Figures 1(a) and (b) and 2(a) and (b)]. The validity of this correction is especially important because the AI Model is based on the postulated dependence of criticality and safety on the properties of the intersection. The calculated k_{eff} for the reduced diameter arms (6.12 versus 6.4 inches for the experimental case) and the geometry of Figures 2(a) and (b) was 0.982 ± 0.018 compared to $k_{eff} = 0.989 \pm 0.015$ for the experimental geometry in Figures 2(a) and (b), with a $\frac{1}{16}$ -inch gap. The extremely good agreement between these values supports the diameter correction suggested in RFP-1197. As noted in Figure 3, removing the steel and not reducing the diameter gives $k_{eff} = 1.030 \pm 0.017$.

The magnitude of the diameter correction was calculated from experimental data for the 6.4-inch arms and assumed to apply to, or at least to be conservative for, other diameters. Since future experimental data may be expected to require such correction, a calculational and experimental study of the variation of the correction with arm diameter would be desirable. Only one other case was examined. The values of k_{eff} for 5.35-inch arms [as in Figures 1(a) and (b)] and 5.07-inch arms [Figures 2(a) and (b)] were found to be 0.995 ± 0.017 and 0.953 ± 0.013 , respectively.

All further calculations in this section assume *no steel* at the interface; i.e., an actual intersection.

EFFECT OF DIAMETER AND SPACING CHANGES -

The case of 8 arms on a square column, as shown in Figures 2(a) and (b), was used to examine the variation in k_{eff} caused by changing the arm diameter at a fixed edge-to-edge spacing of 5.19 inches or by varying the spacing for arms of a fixed diameter of 6.12 inches. The results of these calculations are shown in Figures 3 and 4. Comparison of the two graphs shows that the change in k_{eff} produced by a change in arm

FIGURE 3. Values of k_{eff} versus Arm Diameter for 5.19-Inch Separation.FIGURE 4. Value of k_{eff} versus Separation for Eight 6.12-Inch Diameter Arms.

diameter (d) is much greater than that caused by a change in separation (s). For example:

$$\text{At near critical: } \left| \frac{\Delta k_{\text{eff}}}{\Delta d} \right| \sim 0.17 \text{ inches}^{-1}$$

$$\left| \frac{\Delta k_{\text{eff}}}{\Delta s} \right| \sim 0.015 \text{ inches}^{-1}$$

$$\text{far from critical: } \left| \frac{\Delta k_{\text{eff}}}{\Delta d} \right| \sim 0.014 \text{ inches}^{-1}$$

$$\left| \frac{\Delta k_{\text{eff}}}{\Delta s} \right| \sim 0.005 \text{ inches}^{-1}$$

These data support an attitude common to all the empirical models, all of which place more stringent restrictions on diameter than on spacing. Thus, the AI Model allows a maximum diameter of 3.9 inches for each of two arms in a quadrant, regardless of whether they are in contact or 10 inches apart.

BUCKLING CORRECTION – The central column used in all of the experiments was square, and it was suggested in RFP-1197 that the simple buckling relation noted below could be used to convert data from a square column of a side of length a for use with a round column of a radius r :

$$\left(\frac{2.405}{r} \right)^2 = 2 \left(\frac{\pi}{a} \right)^2$$

In Table III, calculational results are given for two cases, evaluating the buckling correction for the column alone and for the column with 8 arms. In both cases, the square column was 7 inches on a side and the circular column had a diameter of 7.578 inches. (See Figure 5.) For the two cases calculated, the buckling correction is conservative, and the decrease in k_{eff} resulting from the replacement of the square column by the round one is larger for the column with arms, probably because of a decrease in the interaction of the arms with the column.

AREA OF INTERSECTION AS A CRITICAL PARAMETER – The AI Model is based on the observation that the *total area of intersection* of the arms with the column is closely related to the criticality of the system. For example, the arm diameter and the angle between the arm and the column may be jointly varied, keeping the intersection area constant, without

TABLE III. Buckling Correction.

Column Cross Section	Number of Arms	Figure References	$k_{\text{eff}} \pm \sigma$	Number of Batches Averaged
Square	0	—	0.840 ± 0.012	18
Circular	0	—	0.823 ± 0.014	18
Square	8	2 (a) (b) Page 3	0.953 ± 0.013	17
Circular	8	5 Page 6	0.927 ± 0.016	18

NOTE: Square column is 7 inches on a side.
Circular column has diameter of 7.578 inches.
Arms have diameters of 5.07 inches, with layers separated by 0.25 inches.

FIGURE 5. Cross-Sectional View in Plane of Arms for Round Column.

changing the k_{eff} of the system. This hypothesis implies that the interaction between arms and between arm and column is small compared to the arm-column interaction at the interface. Calculations were performed for two different geometries to examine the change in k_{eff} with arm angle.

First, a simple T intersection (a column with one arm) was studied. For θ (theta), the angle between the arm and the column, equal to 90° , both arm and column had a diameter of 7.7 inches. As the angle θ was varied, the arm diameter was altered according

Legend

Angle (θ)	$k_{eff} \pm \sigma$	Angle (θ)	$k_{eff} \pm \sigma$
0 =	0.812 ± 0.017	45 =	0.867 ± 0.019
10 =	0.868 ± 0.020	90 =	0.869 ± 0.013
30 =	0.872 ± 0.014		

FIGURE 6. The k_{eff} versus the Angle for T Intersection.

to the formula $d = d_0 \sqrt{\sin \theta}$, where $d_0 = 7.7$ inches; the column diameter was unchanged. For angles of 10, 30, 60, and 90 degrees, the values of k_{eff} varied by only 0.6 percent (see Figure 6). In the case where θ (theta) = 0, this corresponds to an arm with zero diameter, i.e., no arm, and the intersection area reduces to a line, so that the area is discontinuous at $\theta = 0$. As a second test of the area of intersection concept, k_{eff} was calculated for eight 5.15-inch diameter arms at 45° (corresponding to 6.12-inch arms at 90°) with a separation of 5.19 inches measured parallel to the column (Figure 7). The resulting k_{eff} was 0.961 ± 0.021 compared to $k_{eff} = 0.982 \pm 0.018$ for the 6.12-inch arms at 90° , a difference of about 2 percent in k_{eff} .

REPEATED INTERSECTIONS – The AI Model describes conditions for arm intersections which may be safely repeated at 18-inch intervals, and a series of calculations was performed to see how k_{eff} increased as layers of arms were added. According to the minimal reflection model, the maximum permissible column diameter is 6.5 inches and the maximum arm diameter is 5.5 inches. Since the exact placing of the arms on the column is not specified by the model, and four 5.5-inch arms will not fit in a plane layer around a 6.5-inch column, the staggered arrangement of arms

FIGURE 7. Side View of Eight Arms at an angle of 45° on a Square Column.

FIGURE 8. Arrangement of Staggered Arms on a Round Column (inch dimensions).

shown in Figure 8 was adopted. Calculations were done for the 1-, 2-, 3-, and 6-layer cases. In all examples, the column was 133.9 inches long and the layers of arms were centered along the column; except for the six-layer case, the column extended at least 40 inches beyond the last layer of arms. As shown in Figure 9, k_{eff} increases linearly from one to three layers, and then an additional three layers produces a much smaller change in k_{eff} as the curve appears to level off at $k_{eff} < 0.86$.

REFLECTOR SAVINGS CORRECTION – All experiments were performed under conditions of *minimal* reflection, which include effects from vessel walls, the supporting apparatus, and concrete walls at least ten feet away. In order to formulate a model for use under other conditions (e.g., a small room, a nearby wall, or water flooding) without obtaining more experimental data, a reflector savings correction was used in RFP-1197, based on a reflector savings curve for a single, infinite cylinder. In order to see how the correction applied to intersection geometries, two sets of calculations were performed, one for a cross

FIGURE 9. Values of k_{eff} versus Number of Layers for Minimal Reflection Case of the AI Model.

intersection and the second for the limiting cases of the AI Model.

A cross intersection [i.e., a column with two arms at 90° to the column and 180° to each other, as noted in Figure 10(a)] was used to test the reflector savings correction on a nearly critical intersection. Values of k_{eff} were calculated for a bare system and for systems reflected by $\frac{1}{2}$ and by 4 inches of water around the arms and column. In the first case, the arm and column radii were reduced by a $\frac{1}{2}$ inch, and in the second case by 1.6 inches. As shown in Table IV, the reflector savings correction is conservative; for the reflected cases, k_{eff} is lower than for the bare case.

A second test of the reflector savings correction was made on pipe intersections designed according to the three cases designated in RFP-1197 as minimal, nominal, and full reflection. Values of k_{eff} were calculated for the case of one arm per quadrant (staggered as in Figure 8) using the maximum permissible arm and column diameters. Again, Table IV

shows the correction to be conservative. Refer also to Figures 10(a) and (b). (The diameter corrections for the case of full reflection were based on the ratio of diameters of fully reflected and bare infinite cylinders).⁹

Two Simple Intersections:

In order to provide critical data (for cases where no experimental information exists) against which the predictions of empirical models can be compared, critical diameters were calculated for a simple cross intersection and for a six-layer T intersection [see Figures 10(a) and (b)]. Values of k_{eff} were found for one subcritical and one supercritical point as shown in Table V, and the critical diameter was found by linear interpolation. The critical diameter for the cross was found to be 8.4 inches ± 0.1 , and for the six-layer T, 8.6 inches ± 0.1 .

⁹C. L. Schuske. Private communication. 1969.

FIGURES 10(a) and (b). Simple Cross Intersection(a) and Six-Layer T Intersection (b).

TABLE IV. Reflector Savings.

Description	Number of Arms	Column Diameter (inches)	Arm Diameter (inches)	Water Reflector Thickness (inches)	Figure References	$k_{eff} \pm \sigma$	Number of Batches Averaged
Cross	2	8.35	8.35	0.0	10(a) Page 9	0.987 ± 0.017	18
Cross	2	7.35	7.35	0.5	—	0.976 ± 0.019	18
Cross	2	5.15	5.15	4.0	—	0.866 ± 0.008	16
Minimal Reflection Model of RFP-1197	4	6.5	5.5	0.0	8 Page 8	0.807 ± 0.013	18
Nominal Reflection Model of RFP-1197	4	5.5	4.5	0.5	—	0.692 ± 0.014	18
Full Reflection Model of RFP-1197	4	4.1	3.5	4.0	—	0.773 ± 0.017	9

NOTE: All steel thicknesses are 0.125 inches, except for the arm walls in the last three cases, where the arm wall thickness is 0.116 inches.

TABLE V. Computational Results for Cross- and Six-Layer T Intersections.

Type of Intersection	*Diameter (inches)	$k_{eff} \pm \sigma$	Number of Batches Averaged
Cross	8.35	0.987 ± 0.017	18
Cross	8.5	1.056 ± 0.014	18
Six-Layer T	8.5	0.982 ± 0.014	18
Six-Layer T	8.8	1.056 ± 0.017	18

*Arm and column have same diameter and same wall thickness (1/8 inches).

cylinder of height, H_E , and diameter, D_E , is subcritical, the intersection is safe. (The safety factor is included in the definitions of H_E and D_E .)

Definition of the Generalized Equivalent Cylinder Model: The model is defined for the case of one or more identical planar layers of arms around a central column. Extensions to other cases are made by using conservative approximations. The following terms and symbols are used:

Sector = any 18-inch length of the central column

a = inner diameter of the central column

b_i = inner diameter of i^{th} arm in layer:
 $i \leq 4, b_i \leq a$

θ_i = acute angle between column and i^{th} arm ($0 < \theta_i \leq 90^\circ$)

n = number of arms per layer:
 $1 \leq n \leq 4$

N = number of layers in sector:
 $1 \leq N \leq 4$

$r = \begin{cases} 0 & \text{if all arms are in one sector, and} \\ 1 & \text{if more than one sector contains arms} \end{cases}$
(the repeating case)

$$D_E = \left(1 + \frac{Nr}{10n}\right) \sqrt{a^2 + \sum_{i=1}^n \frac{b_i^2}{\sin \theta_i}}$$

$$H_E = \left(\frac{10N}{n+2}\right)a$$

FORMULATION OF THE GENERALIZED EQUIVALENT CYLINDER MODEL

The model for designing safe pipe intersections for fissile solutions presented herein is an extension of the model described earlier,¹⁰ referred to as the Equivalent Cylinder Model, or EC Model. The new model, the Generalized Equivalent Cylinder (GEC) Model, seeks to modify the EC Model, which is restrictively conservative for many-arm intersections, incorporating features of the AI Model,¹¹ which is too conservative for few-arm intersections. The basic idea of the EC and GEC Models is that of the *equivalent cylinder* for a particular intersection; the diameter (D_E) and height (H_E) of the equivalent cylinder are defined in terms of parameters of the intersection such as column diameter, arm diameters and positions, and number of arms. The values of H_E and D_E are then compared with an appropriate *critical H* versus a *D* curve, and if the equivalent

¹⁰TID-5451. *Loc. cit.*

¹¹RFP-1197. *Loc. cit.*

Rules for Application of Model:

1. The quantities D_E and H_E must be calculated for each sector, and any 18-inch length of the central column (i.e., any possible choice for a sector) must satisfy the safety condition (see Rule 3).
2. The particular H versus D curve to be used for a problem is the *critical* H versus D curve for a cylinder containing the *same fissile solution* under the *same conditions of reflection* as the intersection in question. (If the appropriate H versus D curve is not available, it may be possible to make use of other data, together with conservative approximations. For example, if H versus D data were available for a bare cylinder, one could calculate the allowable pipe diameters for a bare intersection and use a reflector savings correction to calculate a reflected intersection.)
3. The intersection is safe if the point (D_E , H_E) lies below the critical H versus D curve. (See Figure 11, where the safe region is indicated.) For the examples which have thus far been considered by the GEC Model, the *safe* dimensions predicted by the model are at least 14 percent less than the critical diameters.

4. In practical situations, one must impose a maximum pipe diameter to insure that any single pipe is safely subcritical, and this maximum will vary with the environment. For example, if there is danger of flooding, one would require that a fully-reflected pipe be subcritical. Suggested maximum pipe diameters for various conditions of reflection are given in RFP-1197.

The application of the above rules will be illustrated by several detailed examples. For these, the fissile solution is uranyl nitrate at a concentration of 451 grams per liter of uranium enriched to 93.1 percent by weight ^{235}U , and the only reflection is assumed to be from $\frac{1}{8}$ -inch steel walls. This case is used because an approximate equation is known for the critical H versus D curve¹² shown in Figure 11:

$$(D - 9.248)(H - 5.488) = 16.182 \text{ square inches}$$

FIGURE 11. Critical Height versus Diameter for a Cylinder.

EXAMPLE 1 – The problem is to determine whether the intersection shown in Figure 12 is safe. Note that all four arms (1, 2, 3, 4) in Figure 12 cannot be placed in one sector, and that Arms 3 and 4 must be put in the same sector (see Rule 1). For Sector I: $n = 2$, $N = 1$, $a = 6.0$ inches, $b = 5.5$ inches, and $r = 1$ (since there is another sector to be considered). From these values one computes:

$$D_E = 10.3 \text{ inches}, H_E = 15 \text{ inches}$$

This point lies in the safe region of the curve of Figure 11. For Sector II: a , b , and r as for Sector I, $n = 1$, $N = 2$, and;

$$D_E = 9.8 \text{ inches}, H_E = 40 \text{ inches}$$

This point lies outside the safe region of the graph, and hence the intersection is *unsafe*.

EXAMPLE 2 – Find the maximum safe diameter for Arms 3 and 4 of Figure 12. (Maximum problems such as these require an analytic expression for the H versus D curve.) Thus, $H_E = 40$ inches, as before, $D_E = 1.2 \sqrt{6^2 + b^2}$, and b is determined by the equation:

$$(1.2 \sqrt{6^2 + b^2} - 9.248)(40 - 5.488) = 16.182$$

This gives $b = 5.4$ inches. For the same example, the AI Model gives a maximum diameter of 3.9 inches for Arms 3 and 4. For Arms 1 and 2, the GEC model allows a maximum diameter of 6.0 inches, compared to a 5.5-inch maximum for the AI Model.

¹²Reactor Physics Constants. Second Edition. ANL-5800. Argonne National Laboratory, Argonne, Illinois. July 1963.

Legend

Arm Diameters (1, 2, 3, 4) = 5.5 inches.

Column Diameter = 6.0 inches.

FIGURE 12. Example Used to Illustrate Application of Generalized Equivalent Cylinder Model.

Comparison of the Models: It is difficult to give a general, point-by-point comparison of the EC, AI, and GEC models. A few differences and similarities are noted, as well as a number of examples. For all examples which have thus far been considered, all three models give conservative results.

The most significant difference between the EC Model and the GEC Model is in the formula for the equivalent cylinder diameter. In the EC Model, the cross-sectional area of the equivalent cylinder is the sum of the cross-sectional areas of the column and the arms. However in the GEC Model (at least for $r = 0$), the cross-sectional area of the equivalent cylinder equals the cross-sectional area of the column plus the total area of intersection of a layer of arms with the column. For both models, the formula for D_E involves both arm and column diameters, but in the AI Model the maximum arm diameter is not related to the column size. For

FIGURE 13. Cross Intersection with Arms at Angle of 45° to Column.

example in the case of minimal reflection, the largest arm diameter is 5.5 inches whether the column diameter is 5.5 or 6.5 inches.

An important practical difference between the EC and GEC Models and the AI Model concerns the type of experimental data required to employ the models. The EC and GEC Models need critical data for cylinders only, whereas the AI Model requires data on intersections, and the latter type of data is more difficult and expensive to obtain.

A unique feature of the GEC Model is the distinction between repeated and nonrepeated intersections, permitting larger diameter arms if only a single intersection were required.

EXAMPLE 3 – Illustrates further the application of the GEC Model and the considerations involved in comparing the three models.

Find the maximum safe diameter for two arms intersecting a 6.5-inch column at 45° as shown in Figure 13. The fissile solution is again uranyl nitrate and Figure 11 is used.

The maximum arm diameter is calculated using each of the three models. First, for the EC Model, the maximum diameter is 5 inches, and the intersection may be repeated every 46 inches. For the AI Model, the maximum diameter is 4.6 inches, and the intersection may be repeated every 18 inches. Using the GEC Model with $r = 1$, the repeating distance is again 18 inches and the maximum arm diameter is 4.7 inches. For the case of a single intersection, the GEC Model permits a maximum arm diameter of 5.09 inches.

In order to compare the results obtained from the different models, one must distinguish the case where only a single intersection is required from the case where in actuality the intersection is to be repeated at some interval along the column. In the latter, one must consider the length of the repetition interval as well as the maximum arm diameter. The AI and GEC Models give nearly identical results for the repeating situation, whereas the EC Model allows a diameter which is approximately 9 percent greater, but requires a 250 percent increase in the interval length. Thus, in terms of the amount of fissile solution which can be placed in a given volume, the EC Model is more conservative than the other two. If only a single intersection is desired, then the EC and GEC Models give about the same result, while the AI Model is more conservative.

Another aspect of the GEC Model illustrated by Example 3 concerns the angle ϕ (phi) between the two arms. In Figure 13 this angle is 180° . Obviously a decrease in ϕ would increase the k_{eff} of the intersection, but since insufficient information was available concerning the variation of k_{eff} with ϕ , the ϕ dependence was not included in the model. (In the AI Model, ϕ must be at least 90° , since it was assumed the arms were in different quadrants.) The experimental results in RFP-1197 were limited to $\phi = 90^\circ$ or $\phi = 180^\circ$ because of the square column, but with a circular column ϕ may vary continuously from ϕ_{min} to 180° , where ϕ_{min} (the minimum value which ϕ may assume) is determined by the geometrical restriction that the arms must not overlap. The formula for ϕ_{min} , as derived in Appendix B (Page 19), is:

$$\phi_{\text{min}} = \sin^{-1}\left(\frac{R_1}{R_c}\right) + \sin^{-1}\left(\frac{R_2}{R_c}\right)$$

where R_1 and R_2 are the *outer* radii of the two arms and R_c is the *outer* radius of the column. Assuming

$\frac{1}{8}$ -inch walls for the arms and column in Example 3, one finds $\phi_{\text{min}} \approx 102^\circ$ for the 5-inch arms and $\phi_{\text{min}} \approx 92^\circ$ for the 4.6-inch arms. According to RFP-1197, two 7-inch square arms on a 7-inch square column with $\theta = 45^\circ$ and $\phi = 90^\circ$ are critical with 4.82 inches of solution in the column above the arms; for the same arms at $\phi = 180^\circ$, the intersection is subcritical with the column full. The buckling assumption that a 7-inch square arm is equivalent to a 7.58-inch circular arm and subtracting 0.28 inches for the steel at the interface implies that (for the geometry of Example 3) two 7.3-inch arms on a 7.58-inch column are critical. Thus the predicted diameter of 5.09 inches is about 30 percent less than the critical diameter, even for $\phi = \phi_{\text{min}}$.

EXAMPLE 4 – Find the maximum diameter for two arms intersecting a 6.5-inch column, with $\theta = \phi = 90^\circ$. If the arms are required to lie in the same plane, the maximum diameter is $6.5/\sqrt{2} \approx 4.6$ inches by the purely geometrical considerations in Appendix B, Page 19. Now suppose the arms are not required to lie in a plane. The AI Model then allows each arm to be 5.5 inches in diameter, and the arms must be staggered by 3.2 inches, using the formula for h_{min} from Appendix B. The GEC Model does not apply directly to this example, but one can make the conservative approximation that the desired intersection is no more reactive than two arms with $\phi = 0$ as for Sector II in Figure 12. Under these conditions ($n = 1$, $N = 2$), the maximum arm diameter permitted by the GEC Model is 7.2 inches for the $r = 0$ case, but this value must be reduced to 6.5 inches, since that is the column diameter. For a repeated intersection ($r = 1$), the maximum diameter is 4.77 inches. (Note that it is *not conservative* to apply the GEC Model with $n = 2$ and $N = 1$ and then decrease ϕ to 90° by staggering the arms.)

The remaining examples are summarized in tabular form. Table VI gives predictions of the EC and GEC Models for some intersections containing uranyl fluoride (UO_2F_2) and compares the results with experimental data. Table VII compares the two models for some simple intersections containing uranyl nitrate at a concentration of 451 grams per liter of uranium enriched to 93.1 percent by weight ^{235}U in steel cylinders with minimal reflection and uses experimental data from RFP-1197 and calculations from this report to judge the amount of conservatism of the models. Finally, Table VIII extends Table VII to include the AI Model and more complicated intersections.

TABLE VI. Comparison of Two Models for 90-Degree Cross and 60-Degree Y Intersections Containing Uranyl Fluoride Solution in Aluminum Pipes.

Description	Equivalent Cylinder Model			Generalized Equivalent Cylinder Model			Critical Data (TID 7028, Page 69)
	^a Maximum Diameter (inches)	Percent Less Than Critical Diameter	Repeating Distance (inches)	$r \equiv 0$		$r = 1$	
				Maximum Diameter (inches)	Percent Less Than Critical Diameter	Maximum Diameter (inches)	
^b Cross, water reflected, (H/X = 44.3)	3.71	25.8	18.55	4.23	15.4	4.07	A 5-inch cross is critical with 5.75 inches of solution ^d above inter- section of axes.
Cross, water reflected, (H/X = 73.4)	3.73	25.4	18.67	4.25	15.	4.09	A 5-inch cross is critical with 7.8 inches of solution above inter- section of axes.
^c Y, water reflected, (H/X = 73.4)	3.66	26.8	36.6	4.30	14.	3.96	A 5-inch Y is critical with 15.6 inches of solution above inter- section of axes.
Y, unreflected	6.30	—	36.4	6.34	—	5.82	A 5-inch Y is subcritical.
Cross, unreflected	5.26	>29.9	26.3	5.72	>23.7	5.49	A 7.5-inch cross is subcritical.

^aAll pipes have same diameter.^bH/X = ratio of number of hydrogen atoms per unit volume to number of fissile atoms per unit volume.

The Height-Diameter Curves used were:

Reflected fluoride, H/X = 44.3 (ANL-5800, Page 185) $(D - 5.38)(H - 1.36) = 17.98$ square inchesReflected fluoride, H/X = 73.4 (ANL-5800, Page 185) $(D - 1.37)(H - 1.37) = 17.98$ square inchesUnreflected fluoride, (C. L. Schuske, Internal Communication, 1969) $(D - 8.48)(H - 4.724) = 13.7$ square inches.^cThe Y intersection is approximated by a column with one arm at 60°.^dUranyl Fluoride (UO_2F_4).

TABLE VII. Comparison of Two Models for Minimally Reflected Intersections Containing Uranyl Nitrate Solution in Steel Pipes.

Description	Equivalent Cylinder Model			Generalized Equivalent Cylinder Model			^b Critical Data
	^a Maximum Diameter (inches)	Percent Less than Critical Diameter	Repeating Distance (inches)	$r = 0$		$r = 1$	
				Maximum Diameter (inches)	Percent Less than Critical Diameter	Maximum Diameter (inches)	
One arm at 90°, T intersection	6.93	19	34.9	7.16	>17	6.57	An 8.6-inch, six-layer T is critical. For a 7.7-inch T, $k_{eff} = 0.87$ (see Page 9).
One arm at 45°	6.8	—	48.1	6.58	—	6.05	A 6.47-inch arm on a 7.7-inch column has $k_{eff} = 0.87$ (see Figure 6, Page 7).
One arm at 30°	6.72	—	∞	5.98	—	5.51	A 5.44-inch arm on a 7.7-inch column has $k_{eff} = 0.87$ (see Figure 6).
Two arms at 90°, cross intersection	5.74	31	28.7	6.25	25	6.01	An 8.35-inch cross is nearly critical (see Table IV, Page 10).
Planar array of three arms edge-to-edge: $\theta = 90^\circ, \phi = 0^\circ$	5.04	>18	25.2	6.72	—	5.21	An infinite array of 6.12-inch arms on a 7.58-inch column is subcritical (refer to

^aArms and column have same diameter.^bUranyl Nitrate Solution [$UO_2(NO_3)_2$].

TABLE VIII. Comparison of Maximum Arm Diameters Permitted by the Three Models for Minimal-Reflected Intersections Containing Uranyl Nitrate Solution in a 6.5-Inch Diameter Column.

Description	EC Model		AI Model	GEC Model		^b Critical Data
	Maximum Arm Diameter (inches)	Repeating Distance (inches)	Maximum Arm Diameter (inches)	Maximum Arm Diameter (inches) $r = 0$	Maximum Arm Diameter (inches) $r = 1$	
One arm at 90°	^a 6.5	32.5	5.5	^a 6.5	^a 6.5	See Table VII. Page 14).
One arm at 45°	^a 6.5	46.0	4.62	^a 6.5	5.61	See Table VII.
One arm at 30°	^a 6.5	∞	3.88	5.60	4.71	See Table VII.
One arm at 10°	Not permitted	—	1.15	3.30	2.78	A 3.21-inch arm on a 7.7-inch column has $k_{eff} = 0.868$ (see Figure 6).
Two arms at 90°	5.23	32.5	5.5	6.06	5.59	See Table VII.
Two arms at 45°	5.04	46.0	4.62	5.09	4.70	Two 7-inch square arms on a 7-inch square column are critical for $\phi = 90^\circ$, subcritical for $\phi = 180^\circ$.
Four arms at $\theta = 90^\circ$	3.70	32.5	5.5	5.2	5.02	Four 5.5-inch arms on a 6.5-inch column has $k_{eff} = 0.807$ (see Figure 9, Page 8).
Sixteen arms, four layers of four arms per layer, $\theta = 90^\circ$	1.85	32.5	2.75	3.45	2.80	Sixteen 4.06-inch arms edge-to-edge on a 7-inch square column are slightly subcritical.
Six arms, three layers of two arms per layer	3.02	32.5	3.17	5.01	3.72	A 7-inch square column with six 6.12-inch arms edge-to-edge is critical.

^aModel actually allows a larger diameter, but since the column diameter is fixed at 6.5 inches, no arm can be greater than 6.5 inches.

^bExperimental critical data from RFP-1197; experimental diameters have been decreased by 0.28 inches for the steel at the interface. (Uranyl nitrate at a concentration of 451 grams per liter of uranium enriched to 93 percent by weight ²³⁵U.)

CONCLUSIONS

Values of k_{eff} calculated for experimentally critical systems indicated that the O5R code gave reasonably accurate results for the geometries and materials considered. It should be noted that the versatility of the O5R geometry is necessary to describe complicated pipe intersections.

The calculational results given show that the hypotheses used in the formulation of the empirical model of RFP-1197 are conservative, but not overly so. Some general results for intersections are:

The value of k_{eff} depends much more strongly on pipe diameter than on the separation of two pipes, and hence an empirical model which gives sufficient

restrictions on pipe diameter need not restrict separation as severely.

2. The area of intersection of the arms with the column is a valid parameter to consider in nuclear safety problems.

The Generalized Equivalent Cylinder (GEC) Model provides restrictions which are reasonably, but not excessively, conservative for few- and many-arm intersections. The GEC Model (and also the EC and AI Models) has been found to be conservative in all the examples to which the model has been applied, with the maximum diameters predicted by the GEC Model ranging from 14 percent to more than 30 percent below the critical diameters.

Appendix A.

APPENDIX A. Description and Discussion of Earlier Models.

Equivalent Cylinder Model AI (EC):

The symbols a , b , and θ , used in the EC Model^{A-1} are defined as in the Generalized Equivalent Cylinder (GEC) Model, and in the number of arms in the intersection, and:

$$D_E = \sqrt{a^2 + nb^2}$$

$$H_E = \begin{cases} \frac{5a}{\sin \theta} & \theta > 30^\circ \\ \infty & \theta \leq 30^\circ \end{cases}$$

If a cylinder of diameter D_E and height H_E is subcritical, the intersection is *safe*. Intersections may be repeated at a distance H_E along the column.

^{A-1} C. L. Schuske. *An Empirical Method for Calculating Subcritical Pipe Intersections*. TID-5451. Rocky Flats Division, The Dow Chemical Company, Golden, Colorado. July 17, 1956.

FIGURE 1-A. Quadrant Definition.
(Shaded portion is one quadrant.)

Area of Intersection Model (AI):

A quadrant of the central column is defined as one-fourth of a sector (Figure 1-A) and a sector is any 18-inch length of the column:

The *area of intersection* of an arm is defined as the area of intersection of the arm with the tangent plane of the cylinder at the point where the axis of the arm intersects the column. If the arm and column axes lie in a plane, then:

$$\text{Area of intersection} = \frac{\pi d^2}{4 \sin \theta},$$

where

d is the arm diameter and θ is the angle between the axes.

The model provides restrictions on the maximum column diameter and maximum intersection area and number of arms per quadrant for three conditions of reflection. These values are summarized below:

Reflection	Maximum Column Diameter (inches)	Maximum Intersection Area per Quadrant (inches ²)	Maximum Number of Arms per Quadrant
Minimal (vessel walls only)	6.5	23.75	*—
Nominal (½ inch of water)	5.5	16.0	4
Full (4 inches of water)	4.1	9.6	4

* In an earlier report, ^{A-2} RFP-1197, the limit of four arms per quadrant is given for the case of minimal reflection also, but this restriction can be removed. ^{A-3}

^{A-2} Bruce B. Ernst and C. L. Schuske. *Empirical Methods for Calculating Pipe Intersections Containing Fissile Solutions*. RFP-1197. Rocky Flats Division, The Dow Chemical Company, Golden, Colorado. September 9, 1968.

^{A-3} C. L. Schuske. Internal Private Communication. 1969.

Appendix B.

APPENDIX B. Geometrical Results for a Circular Column.

Calculations relating to an earlier report,^{B-1} RFP-1197, were performed for both square and round columns. In the case of a square column, as used in the experiments, the only restriction on arm size is that the arm diameter be less than the column width. In particular, there is no relation between the sizes of arms in a planar layer around the central column. This is no longer true in the case of a circular column, and since it is the circular case which occurs in practice, the following two simple formulae are given, which may be thought of as *geometrical* restrictions of the Generalized Equivalent Cylinder (GEC) Model.

1. Minimum Angle Between Two Arms in a Plane:

Consider a column of outer radius R_c and two arms of outer radii R_1 and R_2 (both $\leq R_c$). The problem is to find $\phi_{\min}$, the minimum angle between the arms. Figure 1-B shows the geometrical situation.

FIGURE 1-B. Diagram of Plane of Intersection.

From the figure one can see the relations:

$$\sin \beta_1 = \frac{R_1}{R_c} \quad \sin \beta_2 = \frac{R_2}{R_c}$$

$$\phi_{\min} = \beta_1 + \beta_2$$

Thus $\phi_{\min}$ can be expressed in terms of the given radii as:

$$\phi_{\min} = \sin^{-1}\left(\frac{R_1}{R_c}\right) + \sin^{-1}\left(\frac{R_2}{R_c}\right)$$

EXAMPLE 1: $R_c = R_1 = R_2 = R$.

$$\phi_{\min} = \sin^{-1}\left(\frac{R}{R}\right) + \sin^{-1}\left(\frac{R}{R}\right) = 2 \sin^{-1}(1) = \pi$$

Thus for two arms equal in size to the central column, the angle between the arms must be 180° .

EXAMPLE 2: With R_c given, find the maximum radius of two arms which have a 90° angle between them.

$$\phi_{\min} = \frac{\pi}{2} = \sin^{-1}\left(\frac{R}{R_c}\right) + \sin^{-1}\left(\frac{R}{R_c}\right) = 2 \sin^{-1}\left(\frac{R}{R_c}\right)$$

^{B-1} Bruce B. Ernst and C. L. Schuske. *Empirical Methods for Isolating Pipe Intersections Containing Fissile Solutions*. RFP-1197. Rocky Flats Division, The Dow Chemical Company, Golden, Colorado. September 9, 1968.

Therefore,

$$\sin^{-1} \frac{R}{R_c} = \frac{\pi}{4} \quad \text{and} \quad \frac{R}{R_c} = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad \text{and}$$

$$R = \frac{R_c}{\sqrt{2}}$$

EXAMPLE 3: Find $\phi_{\min}$ for two 5.5-inch diameter arms intersecting a 6.5-inch diameter column.
Assume arm and column walls are 0.125 inches thick.

$$R_o = \frac{6.5}{2} + 0.125 = 3.375$$

$$R_i = R_o = \frac{5.5}{2} + 0.125 = 2.875$$

$$\phi_{\min} = 2 \sin^{-1} \frac{2.875}{3.375} = 2 \sin^{-1} (0.8519)$$

$$\phi_{\min} = 117^\circ$$

2. Minimum Vertical Staggering for Two Arms at 90° ($\phi = 90^\circ$):

If the formula for $\phi_{\min}$ shows that a desired intersection is impossible (i.e., would require $\phi < \phi_{\min}$), the two alternatives are to increase ϕ or to move one of the arms vertically along the column. Here the case is considered where $\phi = 90^\circ$ is fixed, $\phi_{\min} > 90^\circ$, and it is necessary to find the minimum vertical distance which separates the axes of the two arms. The notation used and the geometry is shown in Figure 2-B.

The arms are defined by the equations:

$$y^2 + z^2 = R^2$$

$$x^2 + (z - h)^2 = R^2$$

On the curve where the two arms intersect, both equations hold. To find $h_{\min}$, it is required that all points of intersection occur *within* the central column; i.e., for all points (ζ, η) which satisfy the set of equations of the curve:

$$\zeta^2 + \eta^2 > R_c^2$$

Define $u = \zeta^2 + \eta^2$; u is the square of the horizontal distance from the axis of the central column to a

point on the curve. Then, u can be expressed in terms of z , using the equations of the curve, as

$$u = [R^2 - (z - h)^2] + [R^2 - z^2]$$

Elementary calculus shows that u is a maximum for $z = h/2$, and hence the condition for all points of the curve to lie inside the column is:

$$u\left(\frac{h}{2}\right) \leq R_c^2$$

This condition can be rewritten as:

$$h \geq \sqrt{2(2R^2 - R_c^2)}$$

Hence,

$$h_{\min} = \sqrt{2(2R^2 - R_c^2)}$$

Note that this formula makes sense for $R \geq R_c / \sqrt{2}$, which is exactly the condition for $\phi_{\min} = \pi/2$. For arms of radii R_1 and R_2 , the condition is

$$h_{\min} = \sqrt{2(R_1^2 + R_2^2 - R_c^2)}$$

FIGURE 2-B. Staggered Arms with Angle, $\phi = 90$ Degrees.

Legend

R_c = Column outer radius.

R = Arm outer radius.

$h_{\min}$ = Minimum stagger distance.