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A Division of North American Aviation, Inc.

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ATOMICS INTERNATIONAL <small>A Division of North American Aviation, Inc.</small>		NAA-SR-9203 TDR NO		APPROVALS <div style="text-align: right; margin-top: 10px;"> <i>J. L. Anderson</i> <i>J. D. Sutherland</i> </div>	
TECHNICAL DATA RECORD		PAGE 1 OF 35			
AUTHOR C. C. Rampton		DEPT & GROUP NO. 728-824		DATE 10-14-62	
				GO NO. 7622	
TITLE Structural Analysis of SNAP 2 Reactor Vessel Top Head		S/A NO. 1310		TWR 52483	
		SECURITY CLASSIFICATION			
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STATEMENT OF PROBLEM Analyze the top head of the reactor vessel for the loads imposed by NaK outlet pipe and internal pressure.					
ABSTRACT <p>The top head of the SNAP-2 reactor vessel was analyzed and found to be adequate for the assumed conditions. Discontinuity stresses in the spherical shell at the junction of the NaK outlet pipe and at the perimeter of the shell during assumed launch vibration were calculated. A maximum stress of 29,700 psi was calculated at the NaK outlet junction and a maximum stress of 33,200 psi was calculated near the perimeter of the spherical shell. The maximum membrane stress in the head was found to be 7,900 psi for short duration loads and 5,250 psi for long duration loads. These stresses indicate that, at a temperature of 1250° F, the creep of the head will be less than 1% in 10,000 hours.</p> <p>Discontinuity loads on the reactor vessel were also calculated for the assumed launch vibration.</p>					

ATOMICS INTERNATIONAL

A Division of North American Aviation, Inc.

NO NAA-SR-9203
DATE 10-14-63
PAGE 2 OF 35

TABLE OF CONTENTS

	Page No.
Abstract	1
Summary	3
References	4
Discussion	5
Critical Loads on Head	6
Assumed Reactor Top Head Configuration	7
Ring Details	8
Head Material Properties	9
Stresses in Head Near Outlet	10
Discontinuity Forces in Head Near Perimeter	12
Discontinuity Forces in Head Near Perimeter Due to Pressure	29
Max Stress Near Perimeter of Head	34
Stresses in Ring	35

ATOMICS INTERNATIONAL

A Division of North American Aviation, Inc.

NO. NAA-SR-9203

DATE 10-14-63

PAGE 3 OF 35

SUMMARY

1. Max stresses during launch (vibrations)

2. During long term operation (30 psi)

3. Discontinuity load on Reactor Vessel from Assumed Launch Vibration.

ATOMICS INTERNATIONAL

A Division of North American Aviation, Inc.

NO. NAA-SR-9203

DATE 10-14-63

PAGE 4 OF 35

REFERENCES

1. Timoshenko, S. P. "Theory of Plates & Shells", second edition, McGraw Hill Book Co.
2. Bijlaard, P. D. "Computation of the Stresses from Local Loads in Spherical Pressure Vessels or Pressure Vessel Heads", Welding Research Council Bulletin Series Number 34, March 1957.
3. "Tentative Structural Design Basis for Reactor Pressure Vessels and Directly Associated Components", December 1, 1958 Revision United States Navy.
4. Rampton, C. C. NAA-SR-TDR #9043 "Analysis of Upper Portion of NaK Outlet Pipe".

ATOMICS INTERNATIONAL

A Division of North American Aviation, Inc.

NO. NAA-SR-9203

DATE 10-14-63

PAGE 5 OF 35

DISCUSSION:

The loads considered critical in the analysis of the head result from pipe vibrations and internal pressure at room temperature. Pipe vibration loads were obtained from reference 4, and internal pressures were specified by J. Sutherland. Stresses were calculated based on the "Hencky-von Mises Theory of Failure" and compared to the yield point stress of the material. Near the junction of the head and the NaK outlet pipe a stress of 29,200 psi was found. This resulted from "in plane" lateral pipe vibrations (Condition #1).

It was noted from "Bijlaard's Papers" (Reference #2) that maximum edge rotations and deflections of the head were more sensitive to an applied moment than to a direct force. Thus, pipe loads from "out of plane" lateral vibrations (Condition #2) were used to evaluate the edge rotations and deflections.

The following assumptions were made in the analysis of the head perimeter:

- 1) Deformation of the head, under the action of a uniformly applied edge moment and horizontal radial force, is the same as that caused by edge moments and radial forces that vary around the shell multiplied by an estimated factor.
- 2) The stress distribution given in reference #2 is not affected by the reinforcement at the head perimeter.
- 3) Continuity of rotation and deflection exists at the junction of the head and ring (point #1); ring and vessel lip (point #2); and vessel lip and vessel body (point #3).

Deflections and rotations of the head, ring, vessel lip, and vessel body were equated; discontinuity forces evaluated; and stresses calculated. The stresses in the ring were found to be low. The stress near the junction of the ring and the head was calculated at 33,200 psi.

The stresses in the head due to internal pressure at 1250° F were found to give a creep rate below 1% in 10,000 hrs.

C. C. RAMPTON PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 6 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63	SNAP-2 REACTOR HEAD	MODEL NO.

CRITICAL LOADS ON HEAD

CONDITION #1 * (Vibration)

CONDITION #2 * (Vibration)

* FOR LOADS SEE TDR # 9043 - REF # 4

CONDITION #3 (max pressure)

MAX TEMP = 1250°F

max long time pressure = 30psi

These values are per J.D. Sutherland's verbal instructions.

PREPARED BY: CCR	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 7 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63	SNAP-2 REACTOR HEAD	MODEL NO.

ASSUMED REACTOR TOP HEAD CONFIGURATION

RING DETAIL

	AREA	DIST.	Ad	Ad ²	I _{cg}
1.	.044	.526	.02314	.01217	.00274
2.	.065	.562	.03653	.02053	.00069
3.	.023	.189	.00548	.00104	.00053
4.	.010*	.438	.00438	.00192	—
5.	.004	.460	.00184	.00085	—
6.	.011	.340	.00374	.00127	—
	<u>A_R = .163</u>		<u>.07538</u>	<u>.03778</u>	<u>.00396</u>

03778

I_{BASE} = 04174

03486

$$0.7538 \times 46.245 = 0.7538$$

.163

I_R = 00688

* small error (assumed .040 thick) - reinforcement not considered

PREPARED BY: CCR	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 9 OF 35
CHECKED BY:		REPORT NO. NAA-8R-9203
DATE: 10-14-63		MODEL NO.

HEAD MATERIAL PROPERTIES

I. TYPE 321 STAINLESS STEEL

REF: J.D. SUTHERLAND, IDL " MATERIALS
USED IN SNAP 2 REACTOR"
OCTOBER 1, 1963

II. NOTE THAT FOR LOADING CONDITIONS #1 & #2

THE ASSUMED AVERAGE HEAD TEMPT. IS 70°F.

WITH A YIELD STRESS OF 33,000 PSI

REF: "STEELS FOR ELEVATED TEMPERATURE
SERVICE" U.S. STEEL HANDBOOK

III. NOTE THAT FOR LOADING CONDITION #3

THE ASSUMED AVERAGE HEAD TEMPT.

IS 1250°F WITH A STRESS OF 7400 PSI

TO PRODUCE A CREEP RATE OF 1% IN 10,000 HRS.

REF: SEE PARAGRAPH II.

PREPARED BY: CCR	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 10 OF 35
CHECKED BY:		REPORT NO. NAA SR-9203
DATE: 10-14-63		MODEL NO.

STRESSES IN HEAD NEAR OUTLET

$$s = 1.82 \frac{r_o}{R} \sqrt{\frac{R}{t}} = 1.82 \times \frac{.75}{14} \sqrt{\frac{14}{.04}} = 1.82$$

MAX STRESS DUE TO MOMENT *

$$\sigma_M = 0.50 \frac{M}{t^2 R} \sqrt{\frac{R}{t}} \cos \theta = \frac{.5 \times M}{.04^2 \times 14} \sqrt{\frac{14}{.04}} = 417 M$$

MAX STRESS DUE TO DIRECT LOAD *

$$\sigma_p = \frac{0.30 P}{t^2} = \frac{0.30 \times P}{.04^2} = 188 P$$

MAX STRESS DUE TO TORQUE

$$\tau = \frac{T/r}{A} = \frac{T}{2\pi r_o^2 t} = \frac{T}{2\pi \times .75^2 \times .04} = 7T$$

MEMBRANE STRESSES DUE TO PRESSURE

$$\sigma_p = \frac{pR}{2t} = \frac{14}{2 \times .04} = 175 p$$

* SEE REF. # 3 (PAGE 66)

PREPARED BY: CCR	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 11 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

CONDITION #1 - STRESSES

$$\sigma_m = 417 \times 40 = 16,680 \text{ psi}$$

$$\sigma_p = 188 \times 44 = 8,270 \text{ psi}$$

$$\gamma = \text{small}$$

$$\sigma_p = 175 \times 30 = 5,250 \text{ psi}$$

$$\sigma = \sqrt{30,200^2 + 5250^2 - 30,200 \times 5250} = 29,700 \text{ psi } \text{OK}$$

CONDITION #2 - STRESSES

$$\sigma_m = 417 \times 48 = 20,000 \text{ psi}$$

$$\sigma_p = 0$$

$$\gamma = 7 \times 544 = 3,800 \text{ psi}$$

$$\sigma_p = 175 \times 30 = 5,250$$

$$\begin{aligned} \sigma &= \sqrt{25,250^2 + 5,250^2 + 3 \times 3800^2 - 25,250 \times 5250} \\ &= 24,000 \text{ psi } \text{OK} \end{aligned}$$

CONDITION #3 - STRESSES

$$\sigma_p = 175 \times 45 = 7,900 \text{ psi (short duration) } \text{OK}$$

$$\sigma_p = 175 \times 30 = 5,250 \text{ psi (long duration) } \text{OK}$$

PREPARED BY: C. C. R.	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 12 OF 35
CHECKED BY:		REPORT NO. RAA-SR-9203
DATE: 10-14-63		MODEL NO.

DISCONTINUITY FORCES IN HEAD NEAR THE PERIMETER (CONDITION #2 CHECKED)

IN THE ANALYSIS, TWO MAJOR ASSUMPTIONS
WERE MADE:

- 1- DEFORMATIONS OF THE SHELL UNDER THE ACTION OF A UNIFORM HORIZONTAL FORCE AND A UNIFORMLY APPLIED MOMENT AT THE OUTER EDGE ARE THE SAME AS THOSE CAUSED BY MOMENTS AND FORCES THAT DECREASE UNIFORMLY AROUND THE EDGE OF THE SHELL.
- 2- THE STRESS DISTRIBUTION GIVEN IN REF # 2 IS NOT AFFECTED BY THE REINFORCEMENT AT THE PERIMETER.

CONDITION #2 WAS CONSIDERED CRITICAL
BY EXAMINATION OF "w" CURVES IN
REF #2.

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 13 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

DETAILS

FORCES RESOLVED

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 14 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

OUTLINE OF SOLUTION OF DISCONTINUITY
STRESSES IN THE HEAD OF THE
REACTOR VESSEL.

REFERENCES #1 & #2

GENERAL EQUATIONS:

FOR POINT #1

EQ. I. $\delta_0 + \delta = \delta_1 + \delta_2$

EQ. II. $V_0 + V = V_2 + V_3$

FOR POINT #2

EQ. III. $\delta_1 + \delta_2 + V_2 d + V_3 d = \delta_4$

EQ. IV. $V_2 + V_3 = V_4$

FOR POINT #3

EQ. V. $\delta_5 = \delta_6$

EQ. VI. $V_5 = V_6$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 15 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

EXPLANATION OF NOTATION

a) $\delta_0 = w \cos \alpha$

SPHERE

where "w" is taken from reference # 2
page # 3

b) $\delta = \frac{2\lambda^2 \sin \alpha F_s M_\alpha}{Eh} - \frac{2R\lambda \sin^2 \alpha F_s H}{Eh}$

SPHERE

where $\lambda^4 = 3(1-\nu^2) \left(\frac{R}{t}\right)^2$

c) $\delta_1 = \frac{H_0 r_R^2 F_{\delta_1}}{EA_R}$

RING

where $H_0 = N_x \cos \alpha$
and N_x is from ref. # 2 page # 4

d) $\delta_2 = \frac{r_R^2 F_{\delta_2} (H + Q_0)}{EA_R}$

RING

e) $V_0 = \frac{12(\nu M_Y - M_X)}{Eh}$

SPHERE

This equation was obtained from
those given in ref. # 1 page 432

M_Y & M_X are taken from ref. # 2 page # 4

f) $V = \frac{4\lambda^3 F_v M_\alpha}{EtR} + \frac{2\lambda^2 \sin \alpha F_v H}{Et}$

SPHERE

where $\lambda^4 = 3(1-\nu^2) \left(\frac{R}{t}\right)^2$

g) $V_2 = \frac{r_R^2 F_{v_2} M_\alpha}{EI_R}$

RING

h) $V_3 = \frac{r_R^2 F_{v_3} (M_0 + d Q_0)}{EI_R}$

RING

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 16 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

$$i) \delta_4 = \left(\frac{C_3 Q_0}{2D\beta^3} + \frac{C_4 M_0}{2D\beta^2} + \frac{C_3' Q_1}{2D\beta^3} + \frac{C_4' M_1}{2D\beta^2} \right) F_e$$

SHORT CYL.

$$\text{where } \beta^4 = \frac{3(1-\nu^2)}{R_c^2 t_c^2}$$

$$D = \frac{E t_c^3}{12(1-\nu^2)}$$

and C_3 & C_4 are taken from
ref. # 3 page # 89

$$j) V_4 = \left(\frac{C_4 Q_0}{2D\beta^2} + \frac{C_6 M_0}{D\beta} + \frac{C_4' Q_1}{2D\beta^2} + \frac{C_6' M_1}{D\beta} \right) F_e$$

SHORT CYL.

where C_4 & C_6 are taken from
ref. # 3 page # 89

$$k) \delta_5 = \left(-\frac{C_3' Q_0}{2D\beta^3} - \frac{C_4' M_0}{2D\beta^2} - \frac{C_3 Q_1}{2D\beta^3} - \frac{C_4 M_1}{2D\beta^2} \right) F_e$$

SHORT CYL.

where C_3 & C_4 are taken
from ref. # 3 page # 89

$$l) \delta_c = - \frac{1}{2\beta_c^3 D_c} (\beta_c M_1 + Q_1) F_e$$

LONG CYL.

$$\text{where } \beta_c^4 = \frac{3(1-\nu^2)}{a^2 h^2}$$

$$D_c = \frac{E h^3}{12(1-\nu^2)}$$

$$m) V_5 = \left(\frac{C_4' Q_0}{2D\beta^2} + \frac{C_6' M_0}{D\beta} + \frac{C_4 Q_1}{2D\beta^2} + \frac{C_6 M_1}{D\beta} \right) F_e$$

SHORT CYL.

where C_4 & C_6 are taken
from ref. # 3 page 89

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 17 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

$$n) \quad V_c = \frac{1}{2\beta_c^2 D_c} (2\beta_c M_1 + Q_1) F_e$$

LONG CYL

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 18 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

EXPLANATION OF "F" FACTORS

"F" FACTORS IN THE ABOVE EQUATIONS
ARE TO ACCOUNT FOR THE NON UNIFORM
LOADING AND WILL BE ESTIMATED

1) $F_\delta = 1$ THIS GIVES THE SOLUTION FOR A
UNIFORM DISTRIBUTION AND IS
CONSERVATIVE

2) $F_{\delta_1} = \frac{1}{2}$ SEE FOLLOWING

3) $F_{\delta_2} = \frac{1}{2}$ SEE ABOVE

7) $F_c = \frac{1}{2}$

4) $F_v = 1$ "

5) $F_{v_2} = \frac{1}{2}$ "

6) $F_{v_3} = \frac{1}{2}$ "

EVALUATION OF $M_x, M_y, \text{ and } M_z$ (from Bijlaards paper) REF. #2

$$U = 1.82 \frac{r_0}{R} \sqrt{\frac{R}{t}} \cos \theta = \frac{1.82 \times .75}{14} \sqrt{\frac{14}{.04}} = 1.82$$

$$S = \frac{1.82 r}{R} \sqrt{\frac{R}{t}} \cos \theta = \frac{1.82 \times 4.09}{14} \sqrt{\frac{14}{.04}} = 9.94$$

$$C_3 = - \frac{2(1+\nu)[3(1-\nu^2)]^{1/2}(2\ker'U - U\ker U)}{\pi U \{ (1+\nu)[U^3(\ker^2 U + \ker'^2 U) + 4(\ker'^2 U + \ker'^2 U) + 4U(\ker'U\ker U - \ker U\ker'U)] - U^3(\ker U\ker'U + \ker'U\ker U) \}} \times \left[\frac{MR}{Et^2 l} \right]$$

$$\ker U = -0.17947$$

$$\ker'U = -1.59208$$

$$\nu = 0.3$$

$$\ker U = -2.44439$$

$$\ker'U = +2.47361$$

$$C_3 = - \frac{22.66595}{838.36745} \times \frac{MR}{Et^2 l} = -0.02703 \frac{MR}{Et^2 l}$$

$$C_4 = -C_3 \left(\frac{2\ker'U + U\ker U}{2\ker'U - U\ker U} \right) = -0.04465 \frac{MR}{Et^2 l}$$

$$C_{12} = \frac{[3(1-\nu^2)]^{1/2}}{\pi} \times \frac{MR}{Et^2 l} = +0.52616 \frac{MR}{Et^2 l}$$

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PAGE NO. 20 OF 35

CHECKED BY:

REPORT NO. A-66-9203

DATE: 10-14-63

MODEL NO.

$$\text{Ker } s = +1.49019$$

$$\text{Kei } s = -3.15760$$

$$\text{Ker } s' = -3.36259$$

$$\text{Kei } s' = +1.33391$$

$$s = 9.94$$

$$v = 0.3$$

$$M_x = \frac{Et^2}{R[12(1-v^2)]^{1/2}} \left\{ C_3 \left[\frac{(1-v)}{s} \text{Kei } s + 2 \frac{(1-v)}{s^2} \text{Ker } s' - \text{Kei } s' \right] + C_4 \left[-\frac{(1-v)}{s} \text{Ker } s + 2 \frac{(1-v)}{s^2} \text{Kei } s' + \text{Ker } s' \right] \right\} \cos \theta$$

$$= \frac{1}{1.82} (0.04335 + 0.15398) \frac{M}{R} \sqrt{\frac{R}{t}} = 0.11 \frac{M}{R} \sqrt{\frac{R}{t}}$$

$$M_y = \frac{Et^2}{R[12(1-v^2)]^{1/2}} \left\{ -C_3 \left[\frac{(1-v)}{s} \text{Kei } s + 2 \frac{(1-v)}{s^2} \text{Ker } s' + v \text{Kei } s' \right] + C_4 \left[\frac{(1-v)}{s} \text{Ker } s - 2 \frac{(1-v)}{s^2} \text{Kei } s' + v \text{Ker } s' \right] \right\} \cos \theta$$

$$= \frac{1}{1.82} (+0.00352 + 0.04120) = .02 \frac{M}{R} \sqrt{\frac{R}{t}}$$

$$w = (C_3 \text{Ker } s' + C_4 \text{Kei } s') \cos \theta = .03 \times \frac{M}{Et^2} \sqrt{\frac{R}{t}} \cos \theta$$

$$N_x = \frac{Et}{Rs} \left[C_3 \left(\text{Ker } s - \frac{2}{s} \text{Kei } s' \right) + C_4 \left(\text{Kei } s + \frac{2}{s} \text{Ker } s' \right) - 2 \frac{C_{12}}{s^2} \right] \cos \theta$$

$$= .02 \frac{M}{Rt} \sqrt{\frac{R}{t}} \cos \theta$$

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CHECKED BY:		REPORT NO. NAA-SR-2203
DATE: 10-14-63		MODEL NO.

EVALUATION OF TERMS FOR GENERAL EQS.

$$\delta_0 = W \cos \alpha$$

from Bijlaard

$$W = .03 \times \frac{M}{Et^2} \sqrt{\frac{R}{t}} \cos \theta$$

$$EW = \frac{.03 \times 48}{.04^2} \sqrt{\frac{14}{.04}} = 16,830$$

$$E\delta_0 = 16,830 \times \cos 17^\circ = -16,094$$

$$\delta = \frac{2\lambda^2 \sin \alpha M_a}{Eh} + \frac{2\alpha \lambda \sin^2 \alpha H}{Eh}$$

$$\lambda^4 = 3(1-\nu^2) \left(\frac{a}{h}\right)^2 = 3(1-0.3^2) \left(\frac{14}{.04}\right)^2$$

$$\lambda^4 = 334,425 \quad \lambda^2 = 578 \quad \lambda = 24 \quad \lambda^3 = 13,872$$

$$E\delta = \frac{2 \times 578 \times .292 M_a}{.04} + \frac{2 \times 14 \times 24 \times .085 H}{.04}$$

$$= 8,439 M_a + 1,428 H$$

$$\delta_1 = \frac{H_0 r_0^2}{EA} \times \left(\frac{1}{2}\right)$$

$$N_x = .02 \times \frac{M}{Rt} \sqrt{\frac{R}{t}} = \frac{.02 \times 48}{14 \times .04} \sqrt{\frac{14}{.04}} = 32$$

$$H_0 = 32 \times \cos 17^\circ = 31$$

$$E\delta_1 = \frac{32 \times 4.09^2}{.163} \left(\frac{1}{2}\right) = +1,642$$

$$\delta_2 = \frac{r_0^2}{EA} \left(\frac{1}{2}\right) (H+Q_0) = \frac{4.09^2}{.163 E} \left(\frac{1}{2}\right) (H+Q_0)$$

$$\delta_2 E = (-51H - 51Q_0)$$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 22 OF 35
CHECKED BY:		REPORT NO. NAA-OR-9203
DATE: 10-14-63		MODEL NO.

$$V_0 = \frac{12(\sqrt{M_y - M_x})}{Eh^3}$$

from Bijlaard

$$M_x = 0.11 \frac{M}{R} \sqrt{\frac{R}{t}} = 0.11 \times \frac{48}{14} \sqrt{\frac{14}{0.04}} = 7.05$$

$$M_y = 0.02 \frac{M}{R} \sqrt{\frac{R}{t}} = 1.28$$

$$EV_0 = \frac{12(0.3 \times 1.28 - 7.05)}{0.04^3} = -1,250,000$$

$$V = \frac{4\lambda^3(1)M_a}{Eha} + \frac{2\lambda^2(1)\sin\alpha H}{Eh}$$

$$EV = \frac{4 \times 13,872 \times M_a}{0.04 \times 14} + \frac{2 \times 578 \times 292 H}{0.04}$$

$$= -9,208 M_a - 8,439 H$$

$$V_2 = \frac{r_0^2}{EI} \left(\frac{1}{2} \right) M_a$$

$$EV_2 = \frac{4.09^2 M_a}{0.0688 \times 2} + 1,216 M_a$$

$$V_3 = (M_0 + dQ_0) \frac{r_0^2}{EI} \left(\frac{1}{2} \right)$$

$$EV_3 = \frac{4.09^2 (M_0 + .498 Q_0)}{0.0688 \times 2}$$

$$EV_3 = -1216 M_0 - 605 Q_0$$

PREPARED BY: CHECKED BY: DATE: 10-14-63	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 23 OF 35 REPORT NO. NAA-68-9203 MODEL NO.
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$$\delta_4 = \left(\frac{C_3 Q_0}{2D\beta^3} + \frac{C_4 M_0}{2D\beta^2} + \frac{C'_3 Q_1}{2D\beta^3} - \frac{C'_4 M_1}{2D\beta^2} \right) \left(\frac{1}{2} \right)$$

$$\beta^{-1} = .778 \sqrt{R_c t_c} = .778 \sqrt{4.5 \times .125}$$

$$\beta^{-1} = .5835 \quad \beta^{-2} = .3405 \quad \beta^{-3} = .1987$$

$$D = 0.0916 E t_c^3 = .0916 E \times .125^3$$

$$= 0.000018 E$$

$$\beta L = 1.25 / .5835 = 2.14$$

then from ref# page#

$C_3 = 1.10$	$C'_3 = 0.36$
$C_4 = 1.20$	$C'_4 = 0.41$
$C_6 = 1.03$	$C'_6 = 0.27$

$$E \delta_4 = \frac{1.10 \times .1987 Q_0}{2 \times 0.000018 \times 2} + \frac{1.20 \times .3405 M_0}{2 \times 0.000018 \times 2} + \frac{0.36 \times .1987 Q_1}{2 \times 0.000018 \times 2}$$

$$- \frac{0.41 \times .3405 M_1}{2 \times 0.000018 \times 2}$$

$$= +304 Q_0 + 568 M_0 + 99 Q_1 - 194 M_1$$

$$V_4 = \left(\frac{C_4 Q_0}{2D\beta^2} + \frac{C_6 M_0}{D\beta} + \frac{C'_4 Q_1}{2D\beta^2} + \frac{C'_6 M_1}{D\beta} \right) \left(\frac{1}{2} \right)$$

$$E V_4 = \frac{1.20 \times .3405 Q_0}{2 \times 0.000018 \times 2} + \frac{1.03 \times .5835 M_0}{0.000018 \times 2} + \frac{0.41 \times .3405 Q_1}{2 \times 0.000018 \times 2}$$

$$- \frac{0.27 \times .5835 M_1}{0.000018 \times 2}$$

$$= 568 Q_0 + 1669 M_0 + 194 Q_1 - 438 M_1$$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 24 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

$$\delta_5 = \left(-\frac{C_3' Q_0}{2D\beta^3} - \frac{C_4' M_0}{2D\beta^2} - \frac{C_3 Q_1}{2D\beta^3} - \frac{C_4 M_1}{2D\beta^2} \right) \left(\frac{1}{2} \right)$$

$$E\delta_5 = -\frac{0.36 \times 1987 Q_0}{2 \times 0.000018 \times 2} - \frac{0.41 \times 3405 M_0}{2 \times 0.000018 \times 2} - \frac{1.10 \times 1987 Q_1}{2 \times 0.000018 \times 2} + \frac{1.20 \times 3405 M_1}{2 \times 0.000018 \times 2}$$

$$= -99 Q_0 - 194 M_0 - 304 Q_1 + 568 M_1$$

$$\delta_c = \left(\frac{M_1}{2\beta_c^2 D_c} + \frac{Q_1}{2\beta_c^3 D_c} \right) \left(\frac{1}{2} \right)$$

$$\beta_c^{-1} = 7.78 \sqrt{ah} = 7.78 \sqrt{4.46 \times 0.4}$$

$$\beta_c^{-1} = .3283 \quad \beta_c^{-2} = 1078 \quad \beta_c^{-3} = 0354$$

$$D_c = 0.0916 E h^3 = 0.0916 E \times 0.4^3$$

$$= .000006 E$$

$$E\delta_c = +\frac{.1078 M_1}{2 \times 0.000006 \times 2} + \frac{0.0354 Q_1}{2 \times 0.000006 \times 2}$$

$$= +449 M_1 + 148 Q_1$$

$$V_5 = \left(\frac{C_4' Q_0}{2D\beta^2} + \frac{C_6' M_0}{D\beta} + \frac{C_4 Q_1}{2D\beta^2} - \frac{C_6 M_1}{D\beta} \right) \left(\frac{1}{2} \right)$$

$$EV_5 = \frac{0.41 \times 3405 Q_0}{2 \times 0.000018 \times 2} + \frac{0.27 \times 5835 M_0}{0.000018 \times 2} + \frac{1.20 \times 3405 Q_1}{2 \times 0.000018 \times 2} - \frac{1.03 \times 5835 M_1}{0.000018 \times 2}$$

$$= 194 Q_0 + 438 M_0 + 568 Q_1 - 1669 M_1$$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 25 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

$$V_c = \left(\frac{M_1}{2 D_c \beta_c} + \frac{Q_1}{2 D_c \beta_c^2} \right) \left(\frac{1}{2} \right)$$

$$EV_c = \frac{.3283 M_1}{2 \times 000006} + \frac{1078 Q_1}{2 \times 000006 \times 2}$$

$$= +2,736 M_1 + 449 Q_1$$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 26 OF 35
CHECKED BY:		REPORT NO. NAA-JR-9203
DATE: 10-14-63		MODEL NO.

SOLUTION OF GENERAL EQUATIONS:

EQ I $\delta_0 + \delta = \delta_1 + \delta_2$

$$-16,094 + 8439 M_a + 1428 H = 1642 - 51 H - 51 Q_0$$

$$+ 8439 M_a + 1479 H + 51 Q_0 = 17,736$$

EQ II $V_0 + V = V_2 + V_3$

$$-1,250,000 - 9908 M_a - 8439 H = +1216 M_a - 1216 M_0 - 605 Q_0$$

$$-11,124 M_a - 8439 H + 1216 M_0 + 605 Q_0 = 1,250,000$$

EQ III $\delta_1 + \delta_2 + V_2 d + V_3 d = \delta_4$

$$+ 1642 - 51 H - 51 Q_0 + 1216 M_a (498) - 1216 M_0 (498) - 605 Q_0 (498) =$$

$$+ 304 Q_0 + 568 M_0 + 99 Q_1 - 194 M_1$$

$$+ 605 M_a - 51 H - 1174 M_0 - 656 Q_0 + 194 M_1 - 99 Q_1 = -1642$$

EQ IV $V_2 + V_3 = V_4$

$$+ 1216 M_a - 1216 M_0 - 605 Q_0 = 568 Q_0 + 1669 M_0 + 194 Q_1 - 438 M_1$$

$$+ 1216 M_a - 2885 M_0 - 1173 Q_0 + 438 M_1 - 194 Q_1 = 0$$

EQ V $\delta_5 = \delta_c$

$$-99 Q_0 - 194 M_0 - 304 Q_1 + 568 M_1 = + 449 M_1 + 148 Q_1$$

$$-194 M_0 - 99 Q_0 + 119 M_1 - 452 Q_1 = 0$$

EQ VI $V_5 = V_c$

$$+ 194 Q_0 + 438 M_0 + 568 Q_1 - 1669 M_1 = + 2736 M_1 + 449 Q_1$$

$$+ 438 M_0 + 194 Q_0 - 4405 M_1 + 119 Q_1 = 0$$

SOLUTIONS (BY COMPUTER)

$$M_a = +35.037$$

$$H = -190.91$$

$$M_0 = -19.5046$$

$$Q_0 = +86.5719$$

$$M_1 = +1.59859$$

$$Q_1 = -10.1692$$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 27 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

DISCONTINUITY FORCES IN HEAD

NEAR PERIMETER

(ASSUME EDGE OF HEAD IS "FIXED")

OUTLINE OF SOLUTION OF DISCONTINUITY STRESSES IN HEAD OF THE REACTOR VESSEL

GENERAL EQS

$$\left. \begin{array}{l} \text{EQ I. } \delta_0 + \delta = 0 \\ \text{EQ II. } V_0 + V = 0 \end{array} \right\} \text{FIXED}$$

FOR EVALUATION & EXPLANATION OF TERMS
SEE PRECEEDING CASE.

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 28 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

SOLUTION OF GENERAL EQUATIONS

EQ. I $S_0 + S = 0$

$$-16.094 + 8.439 M_a + 1428 H = 0$$

$$8439 M_a + 1428 H = 16.094$$

EQ. II $V_0 + V = 0$

$$-1,250.000 - 9908 M_a - 8439 H = 0$$

$$9908 M_a + 8439 H = -1,250.000$$

SOLUTION OF EQS.

$$H = -192 \text{ #/11}$$

$$M_a = +36 \text{ #/11}$$

NOTE : BY COMPARING THE ABOVE RESULTS
 WITH THOSE CALCULATED PREVIOUSLY,
 THE VESSEL AND RING ARE VERY
 STIFF COMPARED WITH THE HEAD.

LOADS

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 29 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

DISCONTINUITY FORCES IN HEAD DUE TO PRESSURE

ASSUME THE EDGE OF HEAD IS "FIXED"

deflections $\leftarrow$ are + (δ)
rotations $\curvearrowright$ are + (V)

OUTLINE OF SOLUTION OF DISCONTINUITY STRESSES IN HEAD OF THE REACTOR VESSEL

GENERAL EQS.

EQ I. $\delta_0 + \delta = 0$

EQ II. $V_0 + V = 0$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 30 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

EVALUATION OF TERMS

$$\delta_o = \frac{r_o}{Eh} (N_\theta - \nu N_\phi)$$

$$N_\phi = N_\theta = \frac{14}{2} p = 7p$$

$$Eh \delta_o = 4.09 \times 7p \times (1 - 0.3) = 20.041 p$$

$$\delta = \frac{2\lambda^2 \sin \alpha}{Eh} M_\alpha + \frac{2\alpha \lambda \sin^2 \alpha}{Eh} H$$

$$\lambda^2 = 578 \quad \lambda = 24$$

$$\begin{aligned} Eh \delta &= 2 \times 578 \times .292 M_\alpha + 2 \times 14 \times 24 \times .085 H \\ &= 337 M_\alpha + 57 H \end{aligned}$$

$$V_o = \frac{1}{Eh} [(N_\phi - N_\theta)(1 - \nu)(\phi + \psi - \theta)]$$

$$Eh V_o = 0$$

$$V = - \frac{4\lambda^3}{Eh\alpha} M_\alpha - \frac{2\lambda^2 \sin \alpha}{Eh} H$$

$$\lambda^3 = 13,872 \quad \lambda^2 = 578$$

$$\begin{aligned} Eh V &= - \frac{4 \times 13,872}{14} M_\alpha - 2 \times 578 \times .292 H \\ &= - 3963 M_\alpha - 337 H \end{aligned}$$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 31 OF 35
CHECKED BY:		REPORT NO. NAA-SR-9203
DATE: 10-14-63		MODEL NO.

EXPLANATION OF TERMS

$$a) \quad \delta_0 = \frac{r_0}{Eh} (N_\theta - v N_\phi)$$

$$\text{where } N_\phi = N_\theta = \frac{pr}{2}$$

$$b) \quad \delta = \frac{2\lambda^2 \sin \alpha}{Eh} M_\alpha + \frac{2\alpha \lambda \sin^2 \alpha}{Eh} H$$

$$\text{where } \lambda^4 = 3(1-v^2) \left(\frac{a}{h}\right)^2$$

$$c) \quad V_0 = \frac{1}{Eh} \left[(N_\phi - N_\theta)(1+v) \cot \phi - \frac{d}{d\phi} (N_\theta - v N_\phi) \right]_{\phi=\alpha}$$

$$\text{where } \frac{d}{d\phi} (N_\theta - v N_\phi) = 0$$

$$d) \quad V = - \frac{4\lambda^3}{Eh\alpha} M_\alpha - \frac{2\lambda^2 \sin \alpha}{Eh} H$$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 32 OF 35
CHECKED BY:		REPORT NO. NAA-OR-9203
DATE: 10-14-63		MODEL NO.

SOLUTION OF GENERAL EQS.

EQ. I $\delta_0 + \delta = 0$

$$20p + 337M_a + 57H = 0$$

EQ. II $V_0 + V = 0$

$$0 - 3963M_a - 337H = 0$$

SOLUTION:

$$\begin{aligned} 337M_a + 57H &= -20p \\ -3963M_a - 337H &= 0 \end{aligned}$$

$$H = -11.75964 M_a$$

$$337M_a + 57(-11.75964 M_a) = -20p$$

$$-333M_a = -20p$$

$$M_a = +.06p$$

$$H = -.70p$$

IF $p = 30 \text{ psi}$

$$M_a = 2 \#/\text{in}$$

$$H = -21 \#/\text{in}$$

actual directions

IF $p = 45 \text{ psi}$

$$M_a = 3 \#/\text{in}$$

$$H = -32 \#/\text{in}$$

STRESSES ARE SMALL
AND NOT CHECKED
ABT $\frac{1}{10}$ OF COND #1

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 33 OF 35
CHECKED BY:		REPORT NO. NAM-OR-9203
DATE: 10-14-63		MODEL NO.

MAX. MOMENT DUE TO DISCONTINUITY

EQ OF MOMENT DISTRIBUTION IN A SPHERE

$$M_\phi = - \frac{D}{a} \frac{dV}{d\phi}$$

$$= M_a e^{-\lambda\psi} (\sin \lambda\psi + \cos \lambda\psi) + H \frac{a}{\lambda} e^{-\lambda\psi} \sin \lambda\psi \sinh \lambda$$

NOTE THIS EQ. REACHES MAX AT

$$\tan \lambda\psi = \frac{H a \sinh \lambda}{H a \sinh \lambda + 2 \lambda M_a}$$

WHEN $\lambda = 24$ $H a \sinh \lambda = -867$ $2 \lambda M_a = -2064$

$\tan 24\psi = .2958$ $24\psi = 16.5^\circ$ $\psi = .70^\circ$

$\lambda\psi = .3 \text{ radians}$ $e^{-\lambda\psi} = .75$ $\sin \lambda\psi = .285$

$\cos \lambda\psi = .958$ $\sinh \lambda = .292$

MAX MOMENT = $-43 \times .75 (.285 + .958) - 212 \times \frac{14}{24} \times .75 \times .285 \times .292$

$= -40 - 7 = -47 \text{ #"}$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 34 OF 35
CHECKED BY:		REPORT NO. NA-6X-9203
DATE: 10-14-63		MODEL NO.

MAX. STRESS NEAR PERIMETER OF HEAD

DIRECT $\frac{212}{\cos 17^\circ} = 222 \#/\text{in.}$

$$\sigma_{D_x} = - \frac{222}{.094 \times 1} = -2,360 \text{ psi}$$

BENDING

$$\sigma = \frac{47 \times 6}{B_x \cdot .094^2 \times 1} = \pm 31,900 \text{ psi}$$

MEMBRANE

$$\sigma_m = \frac{p r}{2 t} = \frac{30 \times 14}{2 \times .094} = +2,240 \text{ psi}$$

$$\sigma_{m_y} = +2,240 \text{ psi}$$

TORQUE (SHEAR) (.094 thick)

$$\gamma = \frac{T/r}{A} = \frac{T}{2\pi \times 4.00^2 \times .094} = 60 \text{ psi}$$

HENCKY-VON MISES THEORY OF FAILURE

$$\sigma = \sqrt{32,020^2 + (32,020 \times 2,240) + 2,240^2 + 3 \times 60^2}$$

$$= 33,200 \text{ psi} \quad \text{OK}$$

PREPARED BY:	ATOMICS INTERNATIONAL A DIVISION OF NORTH AMERICAN AVIATION, INC.	PAGE NO. 35 OF 35
CHECKED BY:		REPORT NO. NAA-SK-9203
DATE: 10-14-63		MODEL NO.

STRESSES IN RING

$$\text{SECT MOD TOP} = \frac{00688}{.498} = .014$$

BENDING (ASSUME UNIFORM MOMENT)

$$\sigma_b = \frac{(35-24) 4.3}{.014} = 3400 \text{ PSI}$$

DIRECT (ASSUME FLEXIBLE)

$$\sigma_D = \frac{(190-87) \times 4.3}{.163} = 3000 \text{ PSI}$$

STRESSES ARE LOW