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SINGLE PARTICLE EXCITATION THEORY
OF THE PHOTONUCLEAR GIANT RESONANCE

Livermore, California

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PHOTONUCLEAR GIANT RESONANCE

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July 30, 1963

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ABSTRACT

Calculations of the photonuclear giant resonance according to the Wilkinson model are presented in this report. In this model the giant resonance in the cross section for absorption of gamma rays by nuclei is ascribed to the excitation of large numbers of nucleons in closed shells. The model generally predicts correctly the integrated cross section for the giant resonance, but the resonance energy is too low. Results are presented in a series of tables and formulas so that the integrated cross section may be calculated for any given nucleus.

I. INTRODUCTION

Wilkinson¹ has described a theory of the photonuclear giant resonance which is quite different from the classical Goldhaber-Teller collective model.² In the latter model, the large cross section for absorption of radiation is due to the excitation of a collective mode of vibration in which all the protons oscillate together and out of phase with the neutrons. This oscillation has a large electric dipole moment which accounts for the strength of the giant resonance.

In contrast to this collective model, the Wilkinson model is based on the independent particle model (shell model) of the nucleus. If there are no interactions between particles, then the incoming radiation can only excite one particle. The large radiation absorption cross section is then due to the large numbers of particles in closed shells which may absorb radiation individually. The various resonance peaks due to the various transitions tend to cluster in energy, and together these peaks form the giant resonance.

The purpose of this report is to calculate the transition strengths quantitatively, so that a comparison with experiment is possible. The most useful application of these calculations would be in identification of the

individual transitions which are present when the giant resonance appears to be broken into several sharp resonances.

These calculations are divided into several sections. The basic theory is described in Section II. The required transition matrix elements are calculated in Sections III and IV. In Section V a recoil correction to the dipole moment is derived and the energy levels are calculated in Section VI. The results are collected in Section VII and in Section VIII an example, Pb^{208} , is worked out in detail.

The model assumed for the nucleus is the j-j coupling shell model, where the potential well is assumed to be an infinite square well, i. e., $V = 0$ for $r < r_0$ and $V = \infty$ for $r > r_0$. This means that both the ground state and the excited states of a particle are bound states, which seems peculiar if the object is to describe a process in which a nucleon is ejected. However, this is not such a serious defect as it seems. If we consider the free-state wave function for a single particle in the nuclear potential at an arbitrary energy, it will have oscillations both inside and outside the potential well. However, at certain energies the wave function will be very large inside the potential well, and, at these energies, the transition probability for absorption of radiation will be large because the electric dipole transition matrix element, $\int \psi_f^* r \psi_i dv$, will be large. Thus, at these resonances, the final-state wave function is large in the well and small outside, so the wave function may be approximated by a bound-state wave function. This point is discussed by Bauer.³

Finally, it might seem rather arbitrary to use such a crude form of potential as the square well, and not try calculations involving other wells. The main effect of changing the well shape would be to vary the resonance energies, but not to change the absorption strength of the resonance. However, it is well known that the resonance energies always come out too small by independent particle model calculations, and this defect must be remedied in more subtle ways, such as in the Brown-Bolsterli theory.⁴

II. ABSORPTION OF RADIATION FOR TRANSITIONS BETWEEN DISCRETE LEVELS

In this section the absorption of radiation by a single particle in a potential well will be calculated. Later this may be applied to the nuclear problem if it is assumed that the nucleus consists of independent particles in

shell model states, by merely summing the cross sections for the individual particles.

The derivation of the basic equations for absorption of radiation will follow the argument given in Schiff,⁵ Ch. X, Sec. 35. It is shown that if radiation of frequency ω is incident on a system in the initial state ψ_i , the probability that it will be in state f at time t is given by:

$$|a_f(t)|^2 = \frac{4|H'_{fi}|^2}{\hbar^2} \sin^2 \left[\frac{1}{2} (\omega_{fi} - \omega)t \right] \cdot \frac{1}{(\omega_{fi} - \omega)^2}.$$

The transition probability per unit time is then

$$w = \lim_{t \rightarrow \infty} \frac{|a_f(t)|^2}{t}.$$

Finally, the cross section for the process is given by $\sigma = w/F$, where F = incident flux of photons. Furthermore, Schiff calculates the matrix element H'_{fi} in the dipole approximation:

$$\begin{aligned} H'_{fi} &= \frac{ie\hbar}{mc} \int \psi_f^*(\vec{r}) e^{i\vec{k} \cdot \vec{r}} \vec{A}_0 \cdot \vec{\text{grad}} \psi_i(\vec{r}) d^3r \\ &= -\frac{ie\hbar}{mc} A_0 \frac{m\omega}{\hbar} \int \psi_f^*(\vec{r}) r_A \psi_i(\vec{r}) d^3r \\ &= -\frac{ie\omega_{fi} A_0}{c} \langle f | r_A | i \rangle. \end{aligned}$$

Here r_A is the component of $\vec{r}$ in the direction $\vec{A}$. The average flux of electromagnetic energy is

$$\bar{S} = \frac{\omega^2}{2\pi c} A_0^2,$$

so the flux of photons is

$$F = \bar{S}/\hbar\omega = \frac{\omega}{2\pi\hbar c} A_0^2.$$

Also, a needed representation of the δ function is:

$$\lim_{t \rightarrow \infty} \frac{\sin^2\left(\frac{\omega t}{2}\right)}{\omega^2 t} = \frac{\pi}{2} \delta(\omega).$$

Combining these relations to calculate σ , we have

$$\sigma = \frac{4\pi^2 e^2}{\hbar c} \omega \langle f | r_A | i \rangle^2 \delta(\omega_{fi} - \omega).$$

Note that this is of the form of an infinitely narrow resonance function, if we consider σ as a function of ω . The quantity which would be measured in an experiment would actually be $\int_R \sigma dE$, where the range of integration R is a small range of energy centered at the resonance peak. Since $E = \hbar\omega$, we have

$$\int_R \sigma dE = \frac{4\pi^2 e^2}{\hbar c} \int \omega |\langle f | r_A | i \rangle|^2 \cdot \delta(\omega_{fi} - \omega) \hbar d\omega,$$

$$\int_R \sigma dE = \frac{4\pi^2 e^2}{\hbar c} \hbar \omega_{fi} |\langle f | r_A | i \rangle|^2. \quad (1)$$

The above calculation was for a transition between two discrete levels, which showed an infinitely sharp resonance. The resonance would be expected in reality to have a finite width, but the broadened resonance should have the same integrated cross section as given above. In later sections the subscript R will be omitted from the integral.

Evaluation of the Matrix Element

In order to evaluate the integrated cross section for an arbitrary transition, we must know the initial and final wave functions and also the transition energy, $\hbar\omega_{fi}$. The wave function for the initial and final states may be factored into a function of r and a second function which includes the angular dependence and spin functions. The quantity r_A is the component of r in the direction of A , which is the polarization direction for the incoming wave. This will be taken to be along the z axis. Thus, letting $d\Omega =$ element of solid angle:

$$r_A = r \cos \theta$$

$$\psi_i = R_i(r) y_{jm}(\theta, \phi, s)$$

$$\begin{aligned} \langle f | r_A | i \rangle &= \iiint R_f^* y_{j'm'}^*(r \cos \theta) R_i y_{jm} r^3 dr d\Omega \\ &= \int R_f^* R_i r^3 dr \iint y_{j'm'}^* \cos \theta y_{jm} d\Omega \end{aligned}$$

$$\langle f | r_A | i \rangle_\infty = I_r \cdot I_\theta \quad (2)$$

$$I_r = \int_0^\infty R_f^*(r) R_i(r) r^3 dr \quad (3)$$

$$I_\theta = \iint Y_{j'm'}^*(\theta, \phi) \cos \theta Y_{jm}(\theta, \phi) d\Omega. \quad (4)$$

The Y_{jm} represent eigenfunctions of the total angular momentum j and the z component of the total angular momentum m . These eigenfunctions are formed from orbital angular momentum wave functions (spherical harmonics $Y_{\ell m}$) combined with spin wave functions by using Clebsch Gordon coefficients.

III. EVALUATION OF THE RADIAL INTEGRAL

In Section II, the radial integral was defined in the following manner:

$$I_r = \int_0^\infty R_f^*(r) R_i(r) r^3 dr.$$

Of course, R_i and R_f depend on the form of the potential chosen, and in this calculation only the infinite square well potential is used. The solutions for the radial part of the Schrödinger equation will be spherical Bessel functions $R(r) = N j_\ell(ar)$, where N is the normalization constant and ℓ is the orbital angular momentum of the state.

Since $V = 0$ for $r < r_0$ and $V = \infty$ for $r \geq r_0$, we must have $R(r) = 0$ for $r \geq r_0$. This means that ar_0 must be a zero of the Bessel function $j_\ell(x)$. If $\beta_{n\ell}$ is the n th zero of $j_\ell(x)$, then $ar_0 = \beta_{n\ell}$ enables one to calculate a . Here n is the radial quantum number and a is related to the energy eigenvalue as follows (Schiff, Ref. 5, p. 76):

$$a^2 = \frac{2mE}{\hbar^2}. \quad (5)$$

The calculation of I_r has been carried out by Wilkinson¹ and the values of I_r^2 for various transitions are reported in Table I. Several values of I_r^2 have been checked and there seem to be no errors. Note that the quantity listed in the table must be multiplied by r_0^2 in order to obtain I_r^2 .

IV. CALCULATION OF THE ANGULAR INTEGRAL I_θ

The angular integral defined in Section II is:

$$I_\theta = \iint Y_{j'm'}^* Y_{jm} \cos \theta d\Omega.$$

Table I. $D = I_r^2/r_0^2$ for various transitions $n, \ell \rightarrow n', \ell'$.

$n = 1, n' = 1$	ℓ	0	1	2	3	4	5	6
$\ell' = \ell + 1$	D	0.28	0.38	0.44	0.49	0.53	0.56	0.58
$n = 2, n' = 2$	ℓ	0	1	2	3			
$\ell' = \ell + 1$	D	0.23	0.28	0.33	0.37			
$n = 3, n' = 3$	ℓ	0	1					
$\ell' = \ell + 1$	D	0.22	0.25					
$n = 1, n' = 2$	ℓ	1	2	3	4	5		
$\ell' = \ell - 1$	D	0.092	0.065	0.050	0.039	0.036		
$n = 1, n' = 2$	ℓ	0	1	2	3			
$\ell' = \ell + 1$	D	0.001	0.002	0.002	0.003			
$n = 2, n' = 3$	ℓ	1	2					
$\ell' = \ell - 1$	D	0.12	0.09					

The functions y_{jm} are eigenfunctions of the total angular momentum for the single particle and the projection of the total angular momentum along the z axis, with quantum numbers j and m, respectively.

$$y_{jm} = \sum_{m_1 m_2} C_{\ell s}(j, m; m_1, m_2) Y_{\ell m_1}(\theta, \phi) \chi_{m_2}$$

Here $Y_{\ell m_1}$ is the orbital angular momentum eigenfunction and χ_{m_2} is the spin eigenfunction.

In applying the Wilkinson theory to an actual nucleus, we are involved mainly with transitions from closed shells, in which all values of m from j to -j are occupied. For each m value, the cross section is proportional to I_θ^2 , so that the total cross section for a filled shell will involve $I^2 = \sum_m I_\theta^2$. This latter quantity I^2 is easy to calculate using the very general methods developed by Racah and described by Edmonds⁶ and Rotenberg.⁷ The calculation is carried out in the appendix and the result is:

$$I^2 = \frac{1}{6} \frac{(2j' + 1)(2j + 1)(2\ell' + 1)(2\ell + 1)(\ell' + \ell + 1)}{(\ell' + \ell + 2)(\ell' + \ell)} \left\{ \begin{matrix} 1 & j' & j \\ \frac{1}{2} & \ell & \ell' \end{matrix} \right\}^2$$

Here the quantity in braces is the Wigner 6-j symbol, which is related to the Racah coefficient. The 6-j symbol may be evaluated by means of numerical tables or in algebraic form when two of the indices are small. In this report the algebraic forms given on p. 16 of Rotenberg⁷ were used. The final results are given in Table II.

V. CORRECTION FOR RECOIL OF THE RESIDUAL NUCLEUS

In the development of the theory in Section II it was assumed that a single particle was bound in a fixed potential well, and then the absorption of radiation associated with transitions from the ground state to an excited state was calculated. The cross section for this process was governed by a factor $e^2 |\langle f | r | i \rangle|^2$ which may be rewritten as $|\langle f | e r | i \rangle|^2$. In the latter form, it appears as the transition matrix element of the dipole moment operator, $e r$.

In the actual nucleus, the dipole moment is modified due to motion of the recoil residual nucleus. Now consider a nucleus with mass A and charge Z which undergoes a single proton transition. Let $\vec{r}$ represent the coordinate of the proton with respect to the recoil nucleus. Then $\vec{r}' = \vec{r} \cdot (A - 1)/A$ is the

Table II. I^2 for transitions $(j, \ell) \rightarrow (j', \ell')$.

$$j = \ell - \frac{1}{2} \qquad j' = \ell' - \frac{1}{2} \qquad (1)$$

$$I^2 = \frac{1}{6} \frac{(\ell + \ell' - 1)(\ell + \ell' + 1)}{\ell + \ell'}$$

$$j = \ell + \frac{1}{2} \qquad j' = \ell' + \frac{1}{2} \qquad (2)$$

$$I^2 = \frac{1}{6} \frac{(\ell + \ell' + 1)(\ell + \ell' + 3)}{(\ell + \ell' + 2)}$$

$$j = \ell - \frac{1}{2} \qquad j' = \ell' + \frac{1}{2} \qquad (3)$$

$$I^2 = \frac{1}{6} \frac{(\ell' - \ell + 2)(\ell - \ell' + 1)(\ell + \ell' + 1)}{(\ell + \ell' + 2)(\ell + \ell')}$$

$$j = \ell + \frac{1}{2} \qquad j' = \ell' - \frac{1}{2} \qquad (4)$$

$$I^2 = \frac{1}{6} \frac{(\ell - \ell' + 2)(\ell' - \ell + 1)(\ell + \ell' + 1)}{(\ell + \ell' + 2)(\ell + \ell')}$$

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coordinate of the proton with respect to the center of mass and $\vec{r}' = -\vec{r}l/A$ is the coordinate of the recoil nucleus with respect to the center of mass. Then let $\vec{d}$ = dipole moment with respect to the center of mass.

$$\begin{aligned}\vec{d}_p &= +e \cdot \left(\vec{r} \frac{A-1}{A}\right) - (Z-1)e \left(\vec{r} \frac{1}{A}\right), \\ \vec{d}_p &= e\vec{r} \cdot \left(\frac{A-Z}{A}\right).\end{aligned}$$

In a similar manner, if we have a neutron transition, the dipole moment is

$$\begin{aligned}\vec{d}_n &= 0 \cdot \vec{r} \left(\frac{A-1}{A}\right) - Ze \left(\vec{r} \frac{1}{A}\right), \\ \vec{d}_n &= -\frac{Z}{A} e\vec{r}.\end{aligned}$$

Thus, the dipole moment may be written as

$$\vec{d} = K \cdot e\vec{r}$$

where

$$K_n = -\frac{Z}{A}, \quad K_p = \frac{A-Z}{A}.$$

Then the cross section must be modified by replacing $e^2 \langle f|r_A|i \rangle^2$ by $K^2 e^2 \langle f|r_A|i \rangle^2$.

VI. TRANSITION ENERGIES

For the infinite square well potential which has been assumed, the energy level of a given state was calculated in Section III, Eq. (5):

$$\begin{aligned}E_{nl} &= \frac{\hbar^2}{2m} a_{nl}^2, \\ a_{nl} &= \frac{1}{r_0} \beta_{nl}.\end{aligned}$$

Here β_{nl} is the n th zero of the spherical Bessel function $j_l(x)$, and the integer n is the principal quantum number. Thus, combining the above equations:

$$E_{nl} = E_0 \beta_{nl}^2, \tag{6}$$

$$E_0 = \frac{\hbar^2}{2m} \frac{1}{r_0^2}. \tag{7}$$

In practical units, where E is in MeV and r_0 is in Fermis, the following relation may be used:

$$E_0 \text{ (MeV)} = \frac{20.8}{r_0^2}, \quad r_0 \text{ in Fermis.} \quad (8)$$

For any transition between two levels, the difference between the initial and final energies is required.

$$E_{fi} = E_0 (\beta_{n'l'}^2 - \beta_{nl}^2)$$

For convenience, a table of the β_{nl} values is given in Table III. These are just zeros of Bessel functions and any values not in the table may be obtained from Bessel function tables.

VII. SUMMARY OF RESULTS

In this section the results of the previous calculations are collected and arranged in a manner which will simplify the application of the theory to any nucleus. For a single particle transition from an initial state i to a final state f , the basic equations for the integrated cross section are:

$$\int \sigma dE = K^2 \cdot \frac{4\pi e^2}{\hbar c} \cdot E_Y \langle f | r_A | i \rangle^2 = K^2 \frac{4\pi e^2}{\hbar c} E_Y I_r^2 I_\theta^2. \quad (9)$$

If we are dealing with transitions from a closed shell of total angular momentum j and z component m , all of the $2j + 1$ values of m will be occupied, so that the above result should be summed on m . The only quantity which depends on m is I_θ . If we define $I^2 = \sum_m I_\theta^2$, and define σ_c to be the cross section for the filled shell, then we have

$$\int \sigma_c dE = K^2 \cdot \frac{4\pi e^2}{\hbar c} E_Y I_r^2 I^2. \quad (10)$$

In practical units, with E_Y in MeV, σ in millibarns, I_r in Fermis, the result may be written

$$\int \sigma_c dE \text{ (MeV-mb)} = 2.88 K^2 E_Y I_r^2 I^2, \quad (11)$$

$$E_Y = E_0 (\beta_{n'l'}^2 - \beta_{nl}^2), \quad (12)$$

$$E_0 \text{ (MeV)} = \frac{20.8}{r_0^2} \quad (r_0 \text{ in Fermis}), \quad (13)$$

Table III. β_{nl} for various n, l values.

n, l	β_{nl}	β_{nl}^2
1,0 (1s)	3.1416	9.8696
1,1 (1p)	4.4934	20.1906
1,2 (1d)	5.7635	33.2179
1,3 (1f)	6.9879	48.8307
1,4 (1g)	8.1826	66.9549
1,5 (1h)	9.3558	87.5310
1,6 (1i)	10.5128	110.5190
1,7 (1j)	11.6573	135.8926
2,0 (2s)	6.2832	39.4786
2,1 (2p)	7.7253	59.6802
2,2 (2d)	9.0950	82.7190
2,3 (2f)	10.4171	108.5160
2,4 (2g)	11.7049	137.0047
3,0 (3s)	9.4248	88.8269
3,1 (3p)	10.9041	118.8994
3,2 (3d)	12.3232	151.8613

$$K^2 = \left(\frac{A - Z}{A} \right)^2 \text{ for proton transitions,} \quad (14)$$

$$K^2 = \frac{Z^2}{A^2} \text{ for neutron transitions,} \quad (15)$$

where n, j, l are the principal, total angular momentum and orbital angular momentum quantum numbers, for the initial state, and n', j', l' are the corresponding quantum numbers for the final state. The quantities I_r^2 , I^2 , and β_{nl}^2 may be found in Tables I, II, and III. The above result applies to the cross section for all transitions from a closed shell. If the cross section σ for one particle in a shell is desired, one must divide σ_c by $2j + 1$:

$$\int \sigma dE = \frac{1}{2j + 1} \int \sigma_c dE.$$

Equations (11) through (15) are the equations which are actually used in practical calculations, along with Tables I, II, and III.

VIII. CALCULATIONS FOR Pb^{208}

Pb^{208} has closed shells for both neutrons and protons. The shell structure used here is from Mayer and Jensen.⁸ In both cases the selection rules $\Delta l = \pm 1$, $\Delta j = 0, \pm 1$ operate, and it is seen from the radial integrals of Table I that transitions with $\Delta l = +1$ and $\Delta n = +1$ are very weak.

Proton Transitions in Pb^{208}

Wilkinson shows that the following transitions are strong, so only these will be calculated here — namely, $1g_{7/2} \rightarrow 1h_{9/2}$, $1h_{11/2} \rightarrow 1i_{13/2}$, $2d_{3/2} \rightarrow 2f_{5/2}$, $2d_{5/2} \rightarrow 2f_{5/2}$, and $2d_{5/2} \rightarrow 2f_{7/2}$.

Assuming $r_0 = 7$ Fermis, $E_0 = 0.425$ MeV, and $E_Y = E_0 (\beta_{n'l'}^2 - \beta_{nl}^2)$. From this we find: $E_Y = 8.75$ MeV for $1g \rightarrow 1h$, $E_Y = 9.75$ MeV for $1h \rightarrow 1i$, and $E_Y = 10.95$ MeV for $2d \rightarrow 2f$. The constant $K^2 = [(A - Z)/A]^2$ is 0.366. Then using the basic equation, $\int \sigma dE = K^2 (2.88) E_Y I_r^2 I^2$, we can calculate the various integrated cross sections.

Transition (Protons)	E_γ (MeV)	I_r^2 (Fermi ²)	I^2	$\int \sigma dE$ (MeV-mb)
$1g_{7/2} \rightarrow 1h_{9/2}$	8.75	26.0	1.48	354
$1h_{11/2} \rightarrow 1i_{13/2}$	9.75	27.4	2.15	603
$2d \rightarrow 2f$	10.95	16.2	2.0	375

The radius of the potential well, r_0 , has been chosen as 7 Fermis in this calculation. This is an approximate value, but it should be noticed that $\int \sigma dE$ is independent of r_0 since $\int \sigma dE$ is proportional to $E_\gamma - I_r^2$. E_γ varies as r_0^{-2} and I_r^2 varies as r_0^2 , so $\int \sigma dE$ is independent of r_0 .

Neutron Transitions in Pb²⁰⁸

The main neutron transitions should be $1h_{11/2} \rightarrow 1i_{11/2}$, $2f_{7/2} \rightarrow 2g_{9/2}$, $2f_{7/2} \rightarrow 2g_{7/2}$, $2f_{5/2} \rightarrow 2g_{7/2}$, $3p_{3/2} \rightarrow 3d_{5/2}$, $3p_{3/2} \rightarrow 3d_{3/2}$, $3p_{1/2} \rightarrow 3d_{3/2}$, and $1i_{13/2} \rightarrow 1j_{15/2}$, $1h_{9/2} \rightarrow 1i_{11/2}$.

The calculations are quite similar to those for the proton transitions except that here $K^2 = Z^2/A^2 = 0.1553$. Again $r_0 = 7$ Fermis is used.

$$\int \sigma dE = 2.88 K^2 E_\gamma I_r^2 I^2.$$

Neutron Transitions

Transition	E_γ (MeV)	I_r^2 (Fermi ²)	I^2	$\int \sigma dE$ (MeV-mb)
$1h_{11/2} \rightarrow 1i_{11/2}$	9.75	27.4	0.028	3.3
$1h_{9/2} \rightarrow 1i_{11/2}$	9.75	27.4	1.817	217.0
$2f_{7/2} \rightarrow 2g_{9/2}$	12.11	18.1	1.481	145.0
$2f_{7/2} \rightarrow 2g_{7/2}$	12.11	18.1	0.0423	4.1
$2f_{5/2} \rightarrow 2g_{7/2}$	12.11	18.1	1.142	112.0
$3p_{3/2} \rightarrow 3d_{5/2}$	14.00	12.2	0.800	61.3
$3p_{3/2} \rightarrow 3d_{3/2}$	14.00	12.2	0.0890	6.8
$3p_{1/2} \rightarrow 3d_{3/2}$	14.00	12.2	0.444	34.0
$1i_{13/2} \rightarrow 1j_{15/2}$	10.78	28.4	2.94	341.0

Summary of Transitions in Pb²⁰⁸

Transition		$\int \sigma dE$ (MeV-mb)	E_γ (MeV)
(neutron)	1h $\rightarrow$ 1i	220	9.7
"	1i $\rightarrow$ 1j	341	10.8
"	2f $\rightarrow$ 2g	261	12.1
"	3p $\rightarrow$ 3d	102	14.0
(proton)	1g $\rightarrow$ 1h	354	8.7
"	1h $\rightarrow$ 1i	603	9.7
"	2d $\rightarrow$ 2f	375	11.0

These transitions add up to $\int \sigma dE = 2256$ MeV-mb, compared to 2980 MeV-mb predicted by the sum rule. The difference must be due to many weak transitions which have been ignored in this calculation.

IX. DISCUSSION

The calculations in this paper carry out an analysis of photonuclear reactions assuming that these reactions may be explained by assuming the shell model description of the nucleus, and assuming that the giant resonance is merely a superposition of many single particle excitations. The model gives the integrated cross section of the giant resonance, but the peak of the giant resonance is at too low an energy. The low resonance energy is known to be characteristic of the independent particle model, so there has been no attempt in this paper to change the radius of the potential well or to vary the form of the potential in order to correct this result.

The most promising approach to resolving this error in the resonance energy is that of Brown and Bolsterli,⁴ who consider the interaction between the particle which has been excited from a closed shell and the hole which is left behind. The schematic model of Brown and Bolsterli has been extended to specific cases in more recent calculations in Refs. 9, 10, and 11.

ACKNOWLEDGMENT

The author would like to thank Dr. S. C. Fultz and the Linac Group at the Lawrence Radiation Laboratory, Livermore, for the hospitality during the summer of 1963 which made this work possible.

This work was performed under the auspices of the U. S. Atomic Energy Commission.

APPENDIX

EVALUATION OF THE ANGULAR MATRIX ELEMENT

The basic methods used here are due to Racah, but more recent versions of the theory are to be found in Edmonds⁶ and Rotenberg.⁷ In this calculation, a reference such as [3.18] will refer to Rotenberg's numbered equations. Also, Dirac's notation will be used here for matrix elements.

Let $|j_1 j_2 j m\rangle$ be the notation for a wave function formed from angular momenta j_1 and j_2 combined to give total angular momentum j and z component m . Then we have [1.15]:

$$|j_1 j_2 j m\rangle = (-1)^{j_2 - j_1 - m} \sum_{m_1 m_2} (2j + 1)^{\frac{1}{2}} \begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & -m \end{pmatrix} |j_1 m_1 j_2 m_2\rangle, \quad (A1)$$

here

$$|j_1 m_1 j_2 m_2\rangle = |j_1 m_1\rangle \cdot |j_2 m_2\rangle, \quad (A2)$$

and the symbol in large parentheses is the Wigner 3-j coefficient which replaces the Clebsch-Gordon coefficient. For our calculation, $j_1 = \ell$ and $j_2 = 1/2$, representing orbital and spin angular momenta for a single particle. Also we may write

$$\cos \theta = \sqrt{\frac{4\pi}{3}} Y_{10}(\theta\phi).$$

The quantity we need is $\langle f | \cos \theta | i \rangle$ which may be written explicitly as:

$$\langle f | \cos \theta | i \rangle = \sqrt{\frac{4\pi}{3}} \langle j_1' j_2' j' m' | Y_{10}(\theta\phi) | j_1 j_2 j m \rangle. \quad (A3)$$

Note that the Y_{10} function does not operate on the spin variables $j_2 m_2 j_2' m_2'$. Another needed relation is [1.19]:

$$\langle \ell' m' | Y_{10} | \ell m \rangle = (-1)^{m'} \left[\frac{(2\ell' + 1)(2\ell + 1)3}{4\pi} \right]^{\frac{1}{2}} \begin{pmatrix} \ell' & 1 & \ell \\ -m' & 0 & m \end{pmatrix} \begin{pmatrix} \ell' & 1 & \ell \\ 0 & 0 & 0 \end{pmatrix}. \quad (A4)$$

Note that $\langle j_2' m_2' | j_2 m_2 \rangle = \delta_{j_2 j_2'} \delta_{m_2 m_2'}$. Now substitute (A1) and (A2) into (A3), and also use the relation (A4). The result is:

$$\begin{aligned}
\langle j_1' j_2' j_1' m' | Y_{10} | j_1 j_2 j_1 m \rangle &= \sqrt{\frac{3}{4\pi}} \begin{pmatrix} j_1' & 1 & j_1 \\ 0 & 0 & 0 \end{pmatrix} \delta_{j_2' j_2} (-1)^{-j_2' + j_2 + j_1' - j_1} \\
&\times (-1)^{+m' - m} [(2j' + 1)(2j + 1)(2j_1' + 1)(2j_1 + 1)]^{\frac{1}{2}} \sum_{m_2' m_1' m_1} (-1)^{m_1'} \begin{pmatrix} j_1' & j_2 & j_1' \\ m_1' & m_2' & -m' \end{pmatrix} \\
&\times \begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & -m \end{pmatrix} \begin{pmatrix} j_1' & 1 & j_1 \\ -m_1' & 0 & m_1 \end{pmatrix} .
\end{aligned}$$

This last summation may be evaluated in terms of a Wigner 6-j symbol which is related to the Racah coefficients. Relations [1.7] and [2.20] from Rotenberg are required. The result is:

$$\begin{aligned}
\langle f | Y_{10} | i \rangle^2 &= \frac{3}{4\pi} \delta_{j_2' j_2} [(2j' + 1)(2j + 1)(2j_1' + 1)(2j_1 + 1)] \\
&\times \begin{pmatrix} j_1' & 1 & j_1 \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{pmatrix} j' & 1 & j \\ -m' & 0 & m \end{pmatrix}^2 \left\{ \begin{matrix} j' & 1 & j \\ j_1 & j_2 & j_1' \end{matrix} \right\}^2 . \quad (A5)
\end{aligned}$$

The index m labels the z component of j for the final state. If we sum on m to sum over all particles in a closed shell, we may use the relation [1.14]:

$$\sum_{m, m'} \begin{pmatrix} j' & 1 & j \\ -m' & 0 & m \end{pmatrix}^2 = \frac{1}{3} . \quad (A6)$$

Note that this last summation is not really a double sum since the 3-j symbol is zero unless the sum of the m indices is zero.

Now we may write the final result, which is obtained from (A5) with the aid of (A3) and (A6).

$$\begin{aligned}
\sum_m |\langle f | \cos \theta | i \rangle|^2 &= \frac{1}{3} \delta_{j_2' j_2} [(2j' + 1)(2j + 1)(2j_1' + 1) \\
&\times (2j_1 + 1)] \cdot \begin{pmatrix} j_1' & 1 & j_1 \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{pmatrix} j' & 1 & j \\ j_1 & j_2 & j_1' \end{pmatrix}^2 . \quad (A7)
\end{aligned}$$

Equations [2.12] to [2.16] and [1.53] in Rotenberg may be used to evaluate the 6-j and 3-j symbols, giving the results of Section IV.

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