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September, 1963

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PRELIMINARY ANALYSIS
of
NEUTRON ELASTIC SCATTERING ANISOTROPY
in
TUNGSTEN
by

Harry Alter
H. J. Lorenzi

Applied Mathematics

This work was performed by the Atomics International Cross Section Analysis and Applications Project

ACKNOWLEDGMENT

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ABSTRACT

Based on the angular distribution data contained in the second edition of BNL-400, a preliminary analysis of Tungsten was performed. The results of this analysis appear as sets of coefficients (which describe the degree of anisotropy in the elastic scattering of neutrons by Tungsten) and a series of curves which help illustrate the analysis.

INTRODUCTION

The random variable associated with stochastic treatments of anisotropic neutron scattering must represent the cumulative distribution function

$$\xi = F(\mu, E) = \int_{-1}^{\mu} G(\mu', E) d\mu' \quad (1)$$

where

μ = cosine of deflection angle (here, in center of mass),

$G(\mu, E)$ = probability that scattering is into $d\mu$ at μ

$$= \frac{2\pi}{\sigma(E)} \frac{d\sigma(\mu, E)}{d\Omega} \quad (1a)$$

E = neutron initial energy (here merely a parameter).

Thus, $F(-1, E) = 0$ and $F(1, E) = 1$, and also $G(\mu, E) \geq 0$ so that $F(\mu, E)$ is a monotonic function of μ , all properties required of the random variable ξ . From this random variable the values of μ are to be determined by an inversion of the function $\xi(\mu)$.

The coefficients of such a polynomial representation are obtained by a least-squares analysis of an auxiliary function, $y(\xi)$, introduced to ensure proper behavior at $\mu = \pm 1$, and defined by the relations

$$\mu(\xi, E) = 2\xi - 1 + \xi(1-\xi)y(\xi, E) \quad (2)$$

or

$$y(\xi, E) = \frac{\mu - 2\xi + 1}{\xi(1-\xi)} \quad (3)$$

Then $y(\xi)$ is approximated by a polynomial

$$y(\xi) \approx A_0 + A_1 \xi + A_2 \xi^2 + \dots + A_n \xi^n \quad (4)$$

whose coefficients A_i can be obtained by least square.

Equations (2) and (4) imply for $\mu(\xi)$ the representation

$$\mu \approx B_0 + B_1 \xi + B_2 \xi^2 + \dots + B_{n+2} \xi^{n+2} \quad (5)$$

where

$$B_0 = -1$$

$$B_1 = A_0 + 2$$

$$B_2 = A_1 - A_0$$

$$B_3 = A_2 - A_1$$

$$B_{n+1} = A_n - A_{n-1}$$

and consequently

$$\sum_{i=1}^{n+2} B_i = 2$$

Although the function $\xi(\mu)$ may be rather like a polynomial-- arising, for example, from the nautral expansion of $G(\mu, E)$ in Legendre polynomials -- its inverse $\mu(\xi)$, and the auxiliary function $y(\xi)$, are often quite different from polynomials. The available leastsquares computing routines are for relatively low-degree polynomials. The difficulties of representing $\mu(\xi)$ adequately by a few terms of a power series have called for a good deal of effort in improving the fit. At present, most of the fits obtained are reasonably satisfactory.

RESULTS

The results of the analysis of the Tungsten anisotropic elastic scattering are presented in the following tables and curves.

The first curve (solid line), in each set of four, is the data from BNL-400 represented by equation (1a).

The circles represent the normalized cross section reconstituted by taking the reciprocal of the derivative of the power expansion in μ vs ξ , i.e.

$$\sigma(E, \mu) \sim \frac{1}{d\mu/d\xi}$$

The oscillations in the fit μ vs ξ are greatly magnified when the derivation is taken. In spite of the sometime oscillatory nature of the reconstituted cross section, the distribution μ vs ξ is considered valid providing the function satisfies the conditions expressed in the introduction.

The second curve represents the cumulative distribution function, equation (1). The solid line of the third curve is the inverse of ξ vs μ , i.e., μ vs ξ , and the circles represent the fit to this curve by the polynomial. This distribution satisfies equation (2). The fourth curve illustrates the inverse fitting function $y(\xi)$. The solid line represents equation (3) where μ and ξ are computed from equation (1). Although the majority of the fits obtained are satisfactory some are not. These are so marked. Methods of improving the fits are presently being investigated.

The coefficients describing the neutron elastic scattering anisotropy for Tungsten are defined through equation (5), and appear in the following tables.

REFERENCES

(1)

M. D. Goldberg, V. M. ~~My~~ and J. R. Stehn, "Angular Distributions in Neutron Induced Reactions, Vol I and II", BNL-400 Second Edition, October, 1962.

EXPANSION COEFFICIENTS FOR THE COSINE OF THE SCATTERING ANGLE FOR TUNGSTEN

CENTER OF MASS SYSTEM

ENERGY (MEV)

COEFFICIENTS	0.350	0.400	0.475	0.490	0.590
B0	-0.100000E 01	-0.100000E 01	-0.100000E 01	-0.100000E 01	-0.100000E 01
a1	0.300152E 01	0.274715E 01	0.300872E 01	0.343608E 01	0.278992E 01
B2	-0.309406E-00	-0.176034E-00	-0.451334E-00	0.125646E-00	-0.303076E 01
83	0.924536E 01	0.320588E 02	0.371891E 02	0.275486E 02	0.999621E 02
84	-0.452117E 02	-0.166990E 03	-0.199496E 03	-0.176549E 03	-0.494311E 03
85	0.812128E 02	0.353505E 03	0.431419E 03	0.412527E 03	0.105728E 04
Et	-0.734400E 02	-0.384748E 03	-0.475563E 03	-0.0477915E 03	-0.116870E 04
87	0.338911E 02	0.213704E 03	0.265877E 03	0.277207E 03	0.657124E 03
B8	-0.638961E 01	-0.481002E 02	-0.599833E 02	-0.643805E 02	-0.149110E 03
	0.640	0.690	0.775	0.790	0.950
B0	-0.100000E 01	-0.100000E 01	-0.100000E 01	-0.100000E 01	-0.100000E 01
81	0.295007E 01	0.348258E 01	0.324007E 01	0.329145E 01	0.345238E 01
e2	-0.400053E 01	0.957503E 00	0.119899E 02	-0.285344E 01	0.198567E 02
B3	0.131604E 03	0.928448E 02	0.364713E 02	0.141369E 03	-0.216678E 02
84	-0.676486E 03	-0.585448E 03	-0.490509E 03	-0.779142E 03	-0.321268E 03
B5	0.149941E 04	0.141428E 04	0.140055E 04	0.178975E 04	0.114394E 04
B6	-0.170822E 04	-0.169765E 04	-0.182530E 04	-0.208630E 04	-0.161218E 04
B7	0.985076E 03	0.101554E 04	0.114962E 04	0.322279E 04	0.105832E 04
B8	-0.228331E 03	-0.242008E 03	-0.284068E 03	-0.286907E 03	-0.268463E 03

EXPANSION COEFFICIENTS FOR THE COSINE OF THE SCATTERING ANGLE FOR TUNGSTEN CONT.

CENTER OF PASS SYSTEM

ENERGY (MEV)

COEFFICIENTS	1.000	1.100	1.250	4.100	5.000
B0	-0.100000E 01	-0.100000E 01	-0.100000E 01	-0.100000E 01	-0.100000E 01
B1	0.338125E 01	0.537019E 01	0.719260E 01	0.203669E 02	0.228255E 02
B2	-0.884660E 01	0.793059E 01	0.602115E 01	-0.848874E 02	-0.225988E 03
B3	0.974766E 02	-0.680170E 01	-0.502796E 02	0.122899E 03	0.136667E 04
B4	-0.340893E 03	-0.292430E 03	-0.438038E 02	0.156109E 03	-0.453619E 04
B5	0.534251E 03	0.974633E 03	0.448517E 03	-0.788403E 03	0.846531E 04
B6	-0.416045E 03	-0.135241E 04	-0.772051E 03	0.110326E 04	-0.889846E 04
B7	0.150433E 03	0.881561E 03	0.554489E 03	-0.697673E 03	0.492468E 04
B8	-0.177566E 02	-0.222582E 03	-0.148085E 03	0.170320E 03	-0.111684E 04

TUNGSTEN

BNL 400

ENERGY IS .35000 MEV

XI (NORMALIZED DISTRIBUTION)

Figure 1.(a)

Figure 1 (b)

Figure 1 (c)

TUNGSTEN

BNL 400

ENERGY IS .35000 MEV

Y (INVERSE FITTING FUNCTION)

Figure 1 (d)

Figure 2 (a)

078 000

TUNGSTEN

BNL 400

ENERGY IS ,40000 MEV

XI (NORMALIZED DISTRIBUTION)

Figure - 2

Figure 2 (c)

Figure 2 (d)

Figure 2 (a)

Figure 3(b)

TUNGSTEN

BNL 400

ENERGY IS .47500 MEV

MU (COSINE OF THE SCATTERING ANGLE)

Figure 3. (c)

TUNGSTEN

BNL 400

ENERGY IS .47500 MEV

084 000

Figure 3 (d)

Figure 4(a)

Figure 4 (b)

TUNOSTEN

BNL 400

ENERGY IS .49000 MEV

Figure 4(c)

TUNGSTEN

BNL 400

ENERGY IS .49000 MEV

000 000

Figure 4(d)

Figure 5(a)

TUNGSTEN

BNL 400

ENERGY IS .59000 MEV

090 000

Figure 5(b)

TUNGSTEN

BNL 400

ENERGY IS .59000 MEV

Figure 5(c)

TUNGSTEN

BNL 400

ENERGY IS .59000 MEV

092 - 000

Figure 5(d)

Figure 6(a)

TUNGSTEN

BNL 400

ENERGY IS .64000 MEV

XI (NORMALIZED DISTRIBUTION)

Figure 6(b)

Figure 6(c)

TUNOSTEN

BNL 400

ENERGY IS .64000 MEV

e⁻
o
o@@ #)

Figure 6(d)

Figure 7(a)

TUNGSTEN

BNL 400

ENERGY IS .69000 MEV

Figure 7(b)

TUNGSTEN

BNL 400

ENERGY IS .69000 MEV

Figure 7(c)

TUNGSTEN

BNL 400

ENERGY I§ .69000 MEV

INVERSE FITTING FUNCTION

Figure 7(d)

Figure 8(a)

TUNGSTEN

BNL 400

ENERGY IS .77500 MEV

Figure 8(b)

A

TUNGSTEN

BNL 400

ENERGY IS ,77500 MEV

103 000

MU (COSINE OF THE SCATTERING ANGLE)

Figure 8(c)

TUNOSTEN

BNL 400

ENERGY IS .77500 MEV

 $^{+0}_{-000}$

Figure 8(d)

TUNOSTEN

BNL 400

ENERGY IS ,79000 MEV

106 - 000

Figure 9(b)

TUNOSTEN

BNL 400

ENERGY IS ,79000 MEV

107 000

Figure 9(c)

TUNGSTEN

BNL 400

ENERGY IS ,99000 MEV

Figure 9(d)

Figure 10(a)

Figure 10(b)

TUNGSTEN

BNL 400

ENERGY IS .95000 MEV

111 000

Figure 10(c)

TUNGSTEN

BNL 400

ENERGY IS .95000 MEV

Y (INVERSE FITTING FUNCTION)

Figure 10(d)

Figure 11(a)

TUNGSTEN

BNL 400

ENERGY IS 1,000 MEV

Figure 11(b)

TUNOSTEN

BNL 400

ENERGY IS 1.0000 MEV

Figure 11(c)

TUNGSTEN

BNL 400

ENERGY IS 1.0000 MEV

116 000

Figure 11(d)

Figure 12(a)

TUNGSTEN

BNL 400

ENERGY IS 1.1000 MEV

118 000

Figure 12(b)

Figure 12(c)

6

TUNGSTEN

BNL 400

ENERGY IS 1.1000 MEV

120 000

Figure 12(d)

TUNOSTEN

BNL 400

ENERGY IS 1.2500 MEV

122 - 000

Figure 13(b)

Figure 13(c)

TUNOSTEN

BNL 400

ENERGY IS 1.2500 MEV

000

Figure 13(d)

Figure 14b

TUNGSTEN

BNL 400

ENERGY IS 4.1000 MEV

e⁺

Figure 14c

TUNGSTEN

BNL 400

ENERGY IS 4.1000 MEV

020 000

Figure 14d

Figure 15(a)

TUNOSTEN

BNL 400

ENERGY IS 3,000 MEV

Figure 15(b)

Figure 15(c)

TUNGSTEN

BNL 400

ENERGY IS 5.0000 MEV

132 000

Figure 15(d)