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RHIC PROJECT

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Analytic Studies of Decapole Correction Schemes

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Introduction

As part of the current design parameters of RHIC[1], the expected systematic decapole error in the arc dipoles is stated to be $|b_4'| \leq 0.7$, where b_4' is the decapole field harmonic in primed units, i.e., at the reference orbit of 2.5 cm multiplied by 10^4. However, for the most severe operating conditions at RHIC, i.e., ^{197}Au beams at 30 GeV/u, the decapole tolerance is calculated to be[2] $|b_4'| \leq 0.56$, or a factor of 1.25 less than the expected value.

The mismatch between expected decapole error and the required tolerance has prompted the need for a decapole corrector system at RHIC. This decapole corrector system has been discussed in two previous reports[3,4]. The corrector system previously proposed uses two families of correctors situated next to arc quadrupoles QF, QD. With this scheme a factor of about two reduction in the decapole error can be achieved.[4]

More recently, an alternative decapole corrector system was discussed by Neuffer.[5,6] He proposed using three correctors, two situated next to arc quadrupoles QF, QD, and one at the mid–point of the arc dipole, QC. With this three corrector arrangement, it has been claimed that it is possible to get a reduction in the decapole error by factors of 1000 or more[6].

In this report, the decapole corrector scheme proposed for RHIC is reviewed and the effectiveness of a two family scheme is compared with a three family scheme. Because the dipoles at RHIC have an unbroken 9.45 m length, it is not possible to implement directly the three corrector scheme discussed above. However, within the insertions, there is a region between Q8 – Q9 where the lattice functions exhibit similar behavior as in the arc

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[1] *RHIC Conceptual Design Manual*, May 1989. BNL 52195.

[2] F. Dell, H. Hahn, G. Parzen, M. Rhoades- Brown, A.G. Ruggiero, *"Tolerances on Systematic Field Errors in RHIC Magnets"*, RHIC Technical Note AD/RHIC-58.

[3] H. Hahn, Report AD/RHIC–22, (1987).

[4] F. Dell, H. Hahn, G. Parzen, A. Ruggiero, *"RHIC Decapole Correction Magnet Requirements"*, RHIC Technical Note AD/RHIC-28.

[5] D. Neuffer, *"A Novel Method for Correcting the SSC Multipole Problem"*, Report SSC-172 (1988).

[6] D. Neuffer, *"Multipole Correction in Synchrotrons,"* Proc. 1989 IEEE Accelerator Conference, Chicago, p.1432.

dipoles. It was pointed out by Claus that together with the original correctors at QF and QD, this region could be utilized to form a three family corrector scheme for decapole errors in RHIC. In fact, it will also be shown that by making the corrector strengths at QF, QD equal an adequate corrector scheme for RHIC can be introduced that uses only two families of power supplies.

The Tuneshift Due to Decapole Errors

Utilizing the formalism due to Jackson[7], the first order perturbative expression for the tune shift due to a decapole error in the dipole magnet is given by,

$$\frac{\Delta\nu_x^{(4)}}{b_4} = 2\delta^3\langle\beta_x X_p^3\rangle - \frac{3}{2}\delta\epsilon_T\big(2(1-F)\langle\beta_x\beta_y X_p\rangle - F\langle\beta_x^2 X_p\rangle\big) \tag{1}$$

$$\frac{\Delta\nu_y^{(4)}}{b_4} = -2\delta^3\langle\beta_y X_p^3\rangle + \frac{3}{2}\delta\epsilon_T\big((1-F)\langle\beta_y^2 X_p\rangle - 2F\langle\beta_x\beta_y X_p\rangle\big) \tag{2}$$

where b_4 is related to the systematic error in primed units, $b_4 = 10^{-4}b_4'(2.5\ \text{cm})^{-4}$, δ is the momentum deviation, and the total emittance is written as

$$\epsilon_T = \epsilon_x + \epsilon_y = \epsilon_T F + \epsilon_T(1-F) \tag{3}$$

with $0 \le \text{F} \le 1$.

In equations (1) – (2), the average over the lattice functions $\langle\beta_x^l\beta_y^m X_p^n\rangle$ is defined by

$$\langle\beta_x^l\beta_y^m X_p^n\rangle = \frac{N}{2\pi\rho}\int_{\frac{-L}{2}}^{\frac{L}{2}} \beta_x^l(s)\beta_y^m(s)X_p^n(s)\mathrm{d}s \tag{4}$$

where L represents the length of one arc dipole, and N represents the total number of magnet elements.

It is advantagous to rewrite equations (1)–(2) in terms of a single dimensionless variable defined by $A_\delta = \eta A_\beta$, where $A_\delta = \delta X_{p_o}$ and $A_\beta = \sqrt{\epsilon_T\beta_o}$. Both X_{p_o} and β_o are defined at the focussing quadrupole. In terms of the independent variables η and F,

[7] A. Jackson, " *Tune Shifts and Compensation from Systematic Field Components*", Report SSC–107 (1987).

$$\frac{\Delta\nu_x^{(4)}}{b_4\delta\epsilon_T} = 2\eta^2\langle\beta_x X_p^3\rangle\frac{\beta_o}{X_{p_o}^2} - \frac{3}{2}\Big(2(1-F)\langle\beta_x\beta_y X_p\rangle - F\langle\beta_x^2 X_p\rangle\Big) \tag{5}$$

$$\frac{\Delta\nu_y^{(4)}}{b_4\delta\epsilon_T} = -2\eta^2\langle\beta_y X_p^3\rangle\frac{\beta_o}{X_{p_o}^2} + \frac{3}{2}\Big((1-F)\langle\beta_y^2 X_p\rangle - 2F\langle\beta_x\beta_y X_p\rangle\Big) \tag{6}$$

When there are correctors at QF, QD and the mid–point of the arc dipole QC, equations (5) and (6) also have contributions from the lattice functions at the positions of the correctors,

$$\begin{aligned}\frac{\Delta\nu_x^{(4)}}{\delta\epsilon_T} &= \frac{2\beta_o}{X_{p_o}^2}\eta^2\left(\langle\beta_x X_p^3\rangle_{\mathrm{DIP}}b_4 + \sum_{i=QF,QD,QC}\langle\beta_x X_p^3\rangle_i\ b_i\frac{l}{L}\right)\\ &-\frac{3}{2}\left(2(1-F)\left(\langle\beta_x\beta_y X_p\rangle_{\mathrm{DIP}}b_4 + \sum_{i=QF,QD,QC}\langle\beta_x\beta_y X_p\rangle_i\ b_i\frac{l}{L}\right)\right.\\ &\left.-F\left(\langle\beta_x^2 X_p\rangle_{\mathrm{DIP}}b_4 + \sum_{i=QF,QD,QC}\langle\beta_x^2 X_p\rangle_i\ b_i\frac{l}{L}\right)\right)\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\Delta\nu_y^{(4)}}{\delta\epsilon_T} &= -2\frac{\beta_o}{X_{p_o}^2}\eta^2\left(\langle\beta_y X_p^3\rangle_{\mathrm{DIP}}b_4 + \sum_{i=QF,QD,QC}\langle\beta_y X_p^3\rangle_i\ b_i\frac{l}{L}\right)\\ &+\frac{3}{2}\left((1-F)\left(\langle\beta_y^2 X_p\rangle_{\mathrm{DIP}}b_4 + \sum_{i=QF,QD,QC}\langle\beta_y^2 X_p\rangle_i\ b_i\frac{l}{L}\right)\right.\\ &\left.-2F\left(\langle\beta_x\beta_y X_p\rangle_{\mathrm{DIP}}b_4 + \sum_{i=QF,QD,QC}\langle\beta_x\beta_y X_p\rangle_i\ b_i\frac{l}{L}\right)\right)\end{aligned} \tag{8}$$

where l is the effective length of the correctors, and we have scaled the contribution from the corrector matrix elements in terms of the ratio l/L. In Table I, the matrix elements for the arc dipoles and the three correctors at QF, QD and the mid–point of the arc dipole QC are tabulated.

Table I. Lattice Dependent Components of the Decapole Tune Shift

Component	Dipole $\times\, b_4$	Corrector @ QF $\times\, b_{QF} l/L$	Corrector @ QD $\times\, b_{QD} l/L$	Corrector @ QC $\times\, b_{QC} l/L$
$\langle \beta_x X_p^3 \rangle$, m^4	34.8	78.13	1.87	24.92
$\langle \beta_x \beta_y X_p \rangle$, m^3	489.84	310.66	155.31	466.4
$\langle \beta_x^2 X_p \rangle$, m^3	670	1.61×10^3	7.25×10^3	466.4
$\langle \beta_y X_p^3 \rangle$, m^4	26.05	15.14	10.25	24.92
$\langle \beta_x \beta_y X_p \rangle$, m^3	489.84	310.66	155.31	466.4
$\langle \beta_y^2 X_p \rangle$, m^3	556.25	54.38	851.56	466.4

Dipole Length $L = 9.45$ m, Corrector Length $l = 0.5$ m

Two Corrector Scheme

With a corrector near each of the quadrupoles QF and QD, the corrector strength is chosen such that the tune shifts due to the betatron motion in equations (7) and (8) is independent of F. This optimal condition corresponds to $\partial\nu_x^{(4)}/\partial F = 0$, $\partial\nu_y^{(4)}/\partial F = 0$, and results in the conditions for the corrector strength,

$$\sum_{i=QF,QD} \left(2\langle\beta_x\beta_y X_p\rangle_i + \langle\beta_x^2 X_p\rangle_i\right) b_i l = -\left(2\langle\beta_x\beta_y X_p\rangle_{\rm DIP} + \langle\beta_x^2 X_p\rangle_{\rm DIP}\right) b_4 L \quad (9)$$

$$\sum_{i=QF,QD} \left(2\langle\beta_x\beta_y X_p\rangle_i + \langle\beta_y^2 X_p\rangle_i\right) b_i l = -\left(2\langle\beta_x\beta_y X_p\rangle_{\rm DIP} + \langle\beta_y^2 X_p\rangle_{\rm DIP}\right) b_4 L. \quad (10)$$

Note this condition is independent of β_o and X_{p_o}. Using the results of Table I, one finds,

$$b'_{\rm QF} l = -0.59 \times b'_4 L$$
$$(11)$$
$$b'_{\rm QD} l = -0.98 \times b'_4 L$$

where these results are within 3% of those calculated in an earlier report[4]

In Fig. 1, the quantity $\nu^{(4)}/b'_4\eta$ is plotted as a function of η. At RHIC, the values $\eta \approx 1$ are the most relevant. The tune shift shown corresponds to ^{197}Au beams at 30 GeV/u. For this case we use the values $\epsilon_T = 1.71 \times 10^{-6}$ mrad, and $X_{p_o}^2/\beta_o = 0.051$m at quadrupole QF. With the corrector values of (11), the dipole tune shift is reduced by a factor of two.

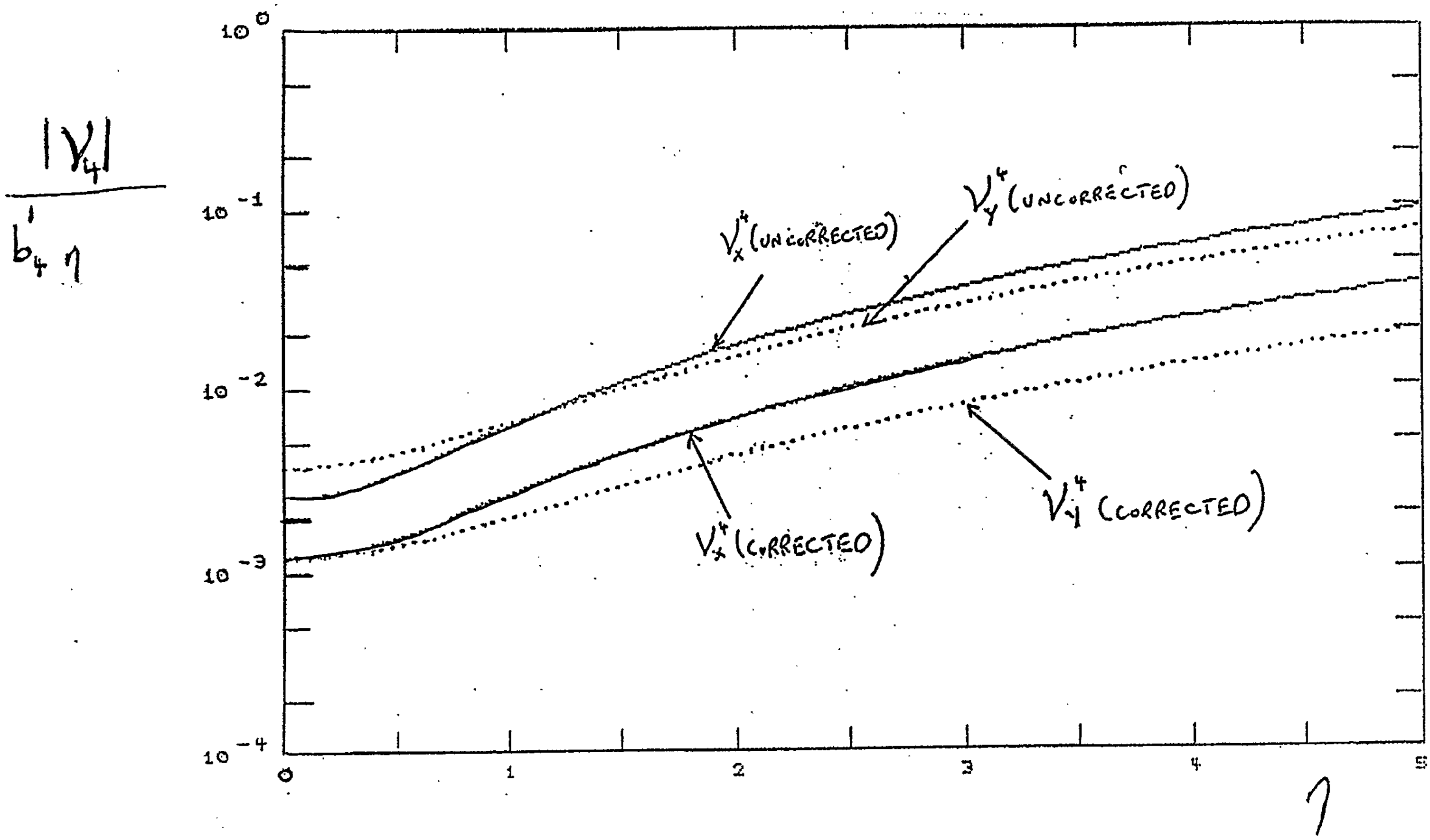

Fig. 1 Plot of the x,y component of $\nu^{(4)}/b_4'\eta$ as a function of η. The corrected values are shown for a two corrector scheme.

Three Corrector Scheme

In this section we study the effectiveness of placing a corrector at the mid– point of the dipole, but recognize that such a scheme is only of theoretical interest at RHIC. With three correctors, we determine the optimal corrector strength by requiring the constraints $\nu_x^{(4)} = \nu_y^{(4)}$ for all η and $\nu_x^{(4)}$, $\nu_y^{(4)}$ be independent of F. Simply equating (7) and (8) gives the following equation for the coefficients of η^2

$$\sum_{i=QF,QD,QC} \left(\langle \beta_x X_p^3 \rangle_i + \langle \beta_y X_p^3 \rangle_i \right) b_i l = - \left(\langle \beta_x X_p^3 \rangle_{\mathrm{DIP}} + \langle \beta_y X_p^3 \rangle_{\mathrm{DIP}} \right) b_4 L \tag{12}$$

.

Also, when equating (7) and (8), the F dependent components reduce to expressions that are identical to the requirement $\partial \nu_x^{(4)} / \partial F = 0$ and $\partial \nu_y^{(4)} / \partial F = 0$. From these expressions, we can write

$$\sum_{i=QF,QD,QC} \left(2 \langle \beta_x \beta_y X_p \rangle_i + \langle \beta_x^2 X_p \rangle_i \right) b_i l = - \left(2 \langle \beta_x \beta_y X_p \rangle_{\mathrm{DIP}} + \langle \beta_x^2 X_p \rangle_{\mathrm{DIP}} \right) b_4 L \tag{13}$$

$$\sum_{i=QF,QD,QC} \left(2 \langle \beta_x \beta_y X_p \rangle_i + \langle \beta_y^2 X_p \rangle_i \right) b_i l = - \left(2 \langle \beta_x \beta_y X_p \rangle_{\mathrm{DIP}} + \langle \beta_y^2 X_p \rangle_{\mathrm{DIP}} \right) b_4 L \tag{14}$$

Utilizing the values in Table I, we find the solution

$$\begin{aligned} b'_{QF} l &= -0.1511 \times b'_4 L \\ b'_{QD} l &= -0.1336 \times b'_4 L \\ b'_{QC} l &= -0.9047 \times b'_4 L \end{aligned} \tag{15}$$

In Fig. 2, the quantity $\nu^{(4)} / b'_4 \eta$ is plotted for ^{197}Au beams at 30 GeV/u. Using the corrector strengths in equation (15), the tune shift can be reduced up to a factor of 800. Figure 2 also shows the effect of a $\pm 1\%$ change in the optimal values of (15). This realistic variation in the corrector strength emphasizes the sensitivity of the three corrector scheme.

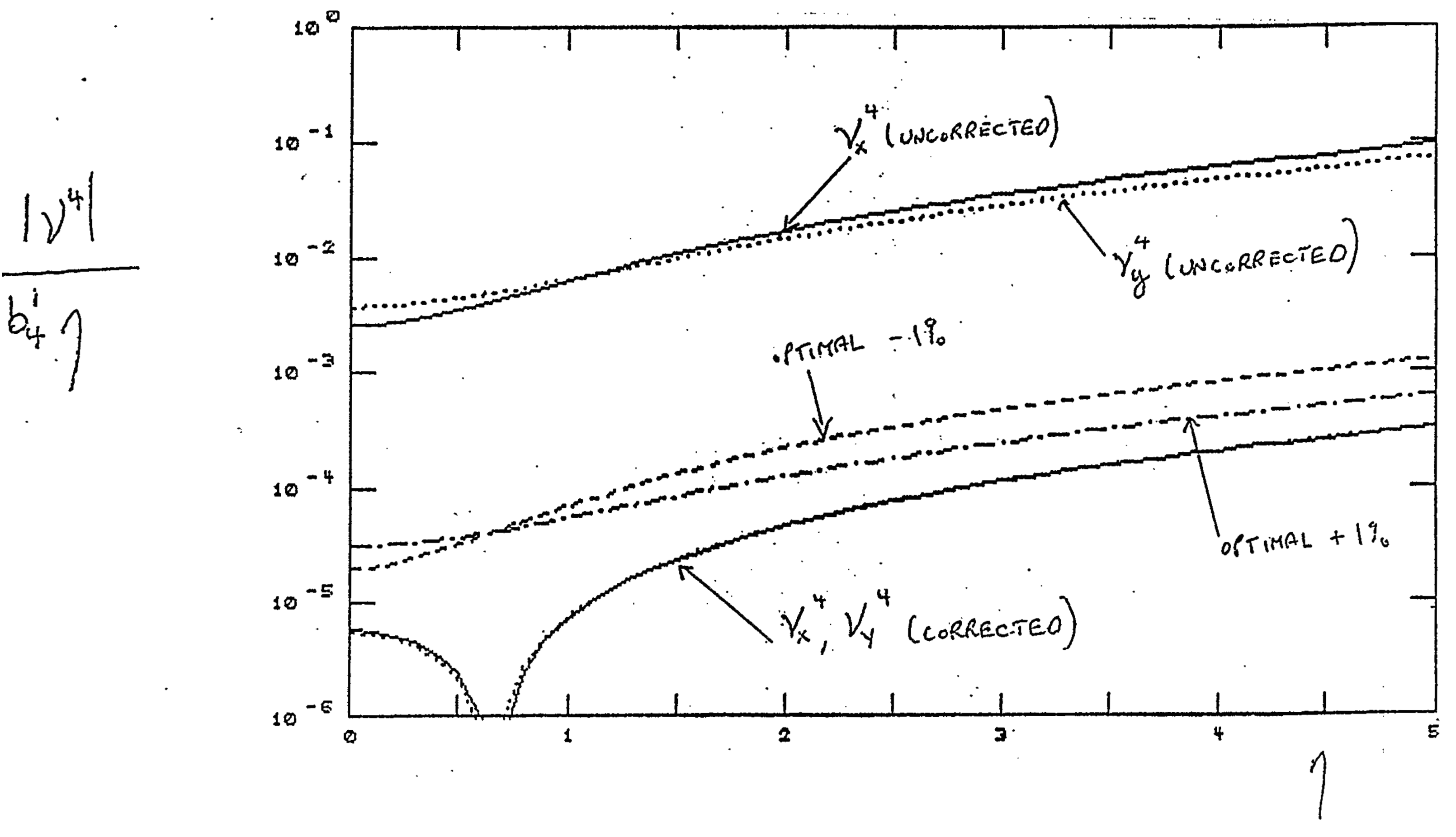

Fig. 2 Plot of the x,y component of $\nu^{(4)}/b_4'\eta$ as a function of η. The corrected values are shown for the optimal three corrector scheme, and for ± 1% of the optimal values.

Modified Three Corrector Scheme

A simplified, yet adequate, corrector scheme may be readily found by equating b_{QF} and b_{QD} in equations (12)–(14). For this case, the values in Table I now give the solution

$$b'_{QF}l = b'_{QD}l = -0.1708 \times b'_4 L$$

$$(16)$$

$$b'_{QC}l = -0.858 \times b'_4 L$$

In Fig. 3, the quantity $\nu^{(4)}/\mathrm{b}'_4\eta$ is plotted for ^{197}Au beams at 30 GeV/u. With the corrector strengths given by (16), the decapole tune shift is reduced by a factor of 80. The corrected tune shift is also less sensitive to ±1% variation in the corrector strengths than the full three corrector scheme. Although this simplified corrector scheme is not as effective as the full three corrector scheme, the reduction in tune shift calculated here would be more than adequate for any foreseeable error, and has the virtue of only requiring two families of power supplies.

Placing the Third Corrector in the Insertions at RHIC

Up to now we have focussed on the three corrector scheme, with correctors at the mid–point of the arc dipole, and the quadrupoles QF, QD. At RHIC, the arc dipole is an unbroken 9.45 m long, so this scheme is not appropriate.

At RHIC, however, there is adequate room in the insertion region to place a third corrector. Between Q8–Q9 the lattice functions have similar properties to an arc dipole. The ideal location for the third corrector would be between Q8–Q9, where $\beta_x = \beta_y$. In Table II., values of the integral as defined by equation (4) are calculated using lattice functions that correspond to the optimal location in the insertion.

Table II. Values of Decapole Tune Shift Components in the Insertion Region

Component	Corrector @ QC $\times\ b_{QC}\ l/L$
$\langle\beta_x X_p^3\rangle$ m^4	33.05
$\langle\beta_x\beta_y X_p\rangle$ m^3	724.6
$\langle\beta_x^2 X_p\rangle$ m^3	724.6
$\langle\beta_y X_p^3\rangle$ m^4	33.05
$\langle\beta_x\beta_y X_p\rangle$ m^3	724.6
$\langle\beta_y^2 X_p\rangle$ m^3	724.6

With these values, the modified three–corrector scheme ($b'_{QF} = b'_{QD}$) yields the following corrector strengths

$$b'_{QF}l = b'_{QD}l = -0.1708 \times b'_4 L$$

$$b'_{QC}l = -0.6474 \times b'_4 L \tag{17}$$

It is important to note that each corrector in the insertion would compensate for the decapole error in twelve dipoles. Thus the strength $b'_{QC}l$ quoted in equation (17) should be multiplied by a factor of 12 to achieve the required correction for RHIC.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the important contribution by Dr. J. Claus who suggested the use of decapole correctors in the insertions.

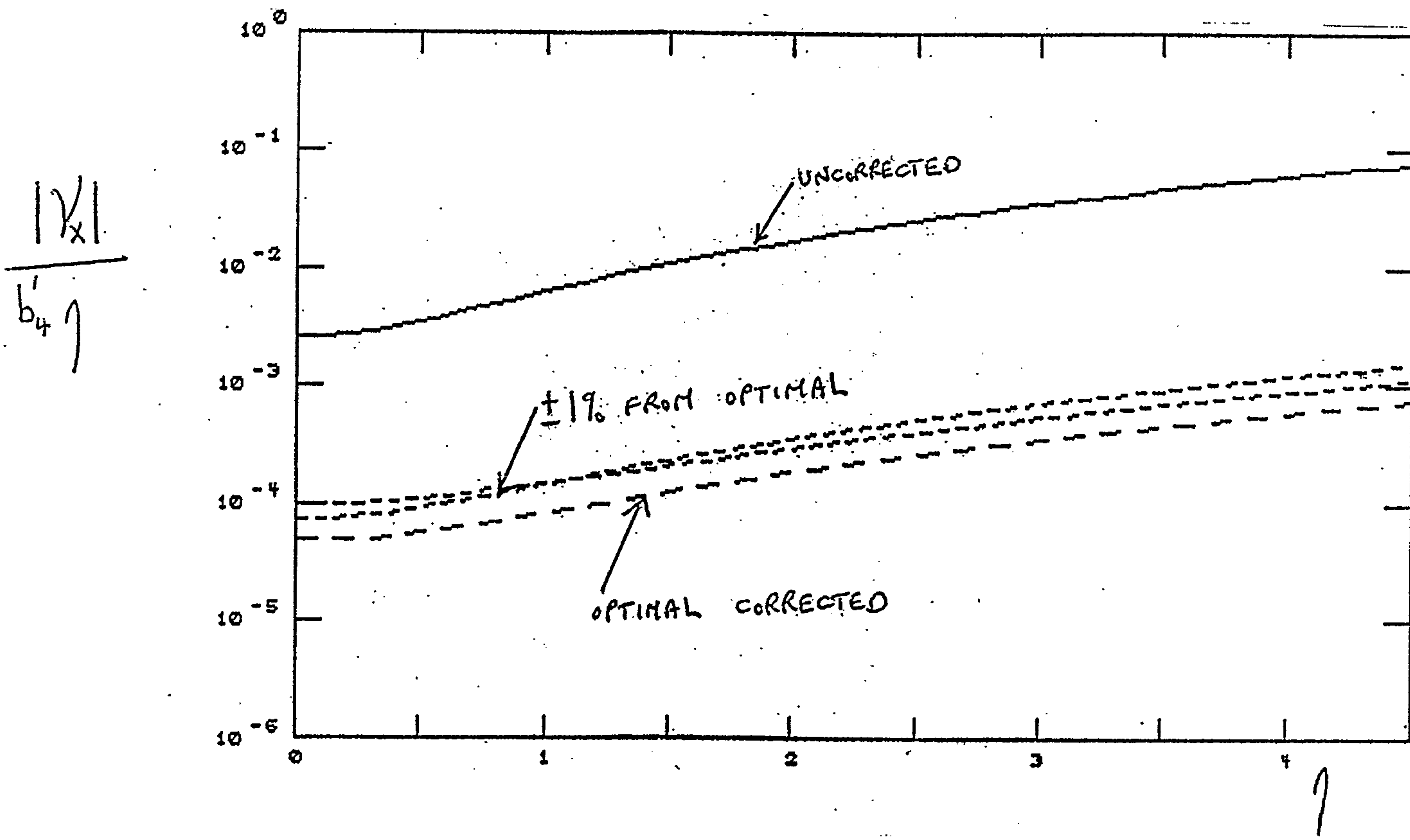

Fig. 3 Plot of the x,y component of $\nu^{(4)}/b'_4\eta$ as a function of η. The corrected values are shown for the modified corrector scheme when $b'_{QF} = b'_{QD}$.