AD/RHIC/RD-30

RHIC PROJECT

Brookhaven National Laboratory

The Beam ν-Spread Due to the Random b_3 and the Random b_4 and its Correction

G. Parzen

November 1991

THE BEAM ν-SPREAD DUE TO THE RANDOM b_3 and THE RANDOM b_4 AND ITS CORRECTION

G. Parzen
Brookhaven National Laboratory
November 1991

1. Introduction

The random b_2, b_3, b_4 in the magnets may cause a large ν-spread[1] in the beam. In one worst case, a possible ν-spread of $\Delta\nu = 21 \times 10^{-3}$ was found. RHIC is required to operate within an area in ν-space, which is free of all resonances up to 10^{th} order and whose width is about 33×10^{-3}.

It appears desirable to correct the ν-spread due to the random b_2, b_3, b_4 down to something like $\Delta\nu \simeq 3 \times 10^{-3}$. The work in Ref. 1 computed the beam ν-spread due to the random b_k and a_k and showed it might be large. This note extends this work by studying and comparing two ways of correcting this beam ν-spread. One way has b_3, b_4 correction coils at QF and QD. The second way has b_3, b_4 correction coils effectively at the center of the dipoles, by placing them between Q8 and Q9.

Previously, a detailed study[8] was done of the ν-shift due to b_4 and its correction. This study was done for a pure b_4, no a_4 present, and used the analytical results to compute the ν-shift. It is my opinion that the analytical results cannot be used reliably for random field multipoles. They do not include the effects of the random skew multipoles, a_2, a_3, a_4, etc., nor the possible effects of the many harmonics generated by the random b_k, a_k that can excite nearby resonances. The results found in ref. 8 are nevertheless valuable, as the more general methods used in this study are more difficult to carry through. In particular, it is difficult to optimize the choice of the corrector strengths.

The present study does tracking runs and computes the ν-shift by fourier analyzing the particle motion. Random a_3 and a_4 were included, as well as random b_3 and b_4. The presence of the a_3 and a_4 couples the particle motion and leads to two ν-values being present in the particle motion. The ν-spread computed by adding the ν-spread found in two modes is usually larger than the ν-shift found for a pure b_3 or b_4 using the analytical results. This larger ν-spread found for the random b_3, a_3 and the random b_4, a_4, and the appreciable ν-spread found for the random b_2, a_2 can lead to a total ν-spread which may not be acceptable and needs to be corrected.

With the assumptions made in this paper for the size of the average b_3 and average b_4 in the dipole, the beam ν-spread was found to be large, and the correctors at Q8, Q9 are necessary to correct the beam ν-spread.

If the average b_3, b_4 turn out to be considerably larger than was assumed in this paper, then this study would have to be repeated for this case, and the correction of the ν-spread may require having correctors at Q8-Q9 and at QF, QD. The higher correcting ability that comes from having both Q8-Q9 and QF, QD correctors may also be desirable for correcting the larger ν-shift of large amplitude particles that may be contributing to the background.

The b_3, b_4 correctors at QF, QD or at Q8-Q9 may affect the dynamic aperture. Tracking studies for 1000 turns were done to see the effect on the dynamic aperture. No appreciable effect was seen in these studies.

To keep a proper perspective on this problem, one should keep in mind the following aspects of the problem:

1. A small fraction, about 25%, of accelerators will have random error distributions that cause large ν-spreads.
2. The ν-spread computed below is for the beam dimensions after 10 hours of growth due to intrabeam scattering for the case of Au at $\gamma = 30$.
3. Only particles with large x and small y exhibit the large ν-shifts that cause the large ν-spread. This again is some fraction of all the particles.
4. The $\Delta\nu$ due to random errors is not simply additive to the $\Delta\nu$ due to the beam-beam interaction, $\Delta\nu \simeq 25 \times 10^{-3}$. The beam-beam $\Delta\nu$ is smaller at large betatron amplitudes, where the $\Delta\nu$ due to b_k, a_k is largest.

2. Sources of ν-Spread

It appears now that the largest beam ν-spread due to magnet field errors for Au ions will probably occur at $\gamma = 30$ and is primarily due to the random b_2, b_3, b_4 in the dipoles.

Higher multipoles and other magnets do not usually contribute much to the ν-spread. One possible exception is the iron saturation b_5 in the high β quadrupoles. The iron saturation b_4 in the dipoles which first stimulated the study of the beam ν-spread in RHIC (H. Hahn[2]), now appears small enough, $b_4 \simeq 2 \times 10^{-4}$ (R. Gupta) not to be of great concern. $(\Delta\nu \simeq 2 \times 10^{-3})$

It has been found that the ν spread due to b_3 and b_4 is generated by the average b_3, b_4 in the dipoles, $b_{3,av}$ and $b_{4,av}$ (A. Ruggiero[3]). If b_3 and b_4 were truly random then $b_{3,av}$, $b_{4,av}$ would obey

$$\left(b_{k,av}\right)_{rms} = \frac{1}{\sqrt{N}} b_{k,rms} \ , \qquad k = 3, 4$$

N is the number of dipoles (N=144 for RHIC).

I have assumed that the $b_{k,av}$ satisfy

$$b'_{3,av} \leq 0.42 \times 10^{-4}$$

$$b'_{4,av} \leq 0.70 \times 10^{-4}$$

This is a factor 2 larger than the result predicted by the $\sqrt{N}$ rule.

For the worse case studied, the beam $\Delta\nu$ has the following breakdown

$$\underline{\text{Total}}\ \Delta\nu = 21 \times 10^{-3}$$

$$\Delta\nu \text{ due to } b_2 = 6 \times 10^{-3}$$

$$\Delta\nu \text{ due to } b_3 = 10 \times 10^{-3}$$

$$\Delta\nu \text{ due to } b_4 = 7 \times 10^{-3}$$

One may note that an appreciable part of the ν-spread can come from the random b_2. This is not studied in this paper. The $\Delta\nu$ are computed from tracking runs which is necessary because of the presence of the random a_2, a_3, a_4.

3. Correction of Beam ν-Spread

The ν-shift is a function of $\delta = \Delta p/p$, ϵ_x, ϵ_y. The ν-shift due to a pure b_3, no a_3, is given by

$$\Delta\nu_x \left(\delta, \epsilon_x, \epsilon_y\right) = \frac{3}{4\pi\rho} \int ds\ b_3 \left(\beta_x \left(X_p \delta\right)^2 + \frac{1}{4}\beta_x^2 \epsilon_x - \frac{1}{2}\beta_x \beta_y \epsilon_y \right) .$$

Similar integrals give the ν-shift due to b_4. Assuming the dipoles dominate this shows that $\Delta\nu$ depends on $b_{3,av}$ and $b_{4,av}$ in the dipoles.

Local correction at each dipole can be done by writing $\Delta\nu_x$ due to each dipole as

$$\Delta\nu_x = \sum_k c_k\ b_3\left(s_k\right) f\left(s_k, \delta, \epsilon_x, \epsilon_y\right) L_D$$

where c_k are the weights of an integration algorithm and the s_k are several locations within the dipole. $\Delta\nu_x$ can be corrected by placing correction coils at the s_k with the weights c_k. Examples are

1. 2 correction coils at ends, weights $= \frac{1}{2}$
2. 1 correction coil at center, weight $= 1$
3. 3 correction coils at center and ends, weights $= 1/6,\ 4/6,\ 1/6$ (Simpsons Rule).

This approach is similar to that proposed by D. Neuffer[4].

The correction coil at the center of the dipole can be approximated by putting it between Q8 and Q9 (J. Claus[5]).

Table 1: Comparison of correction results for the two methods of correction for the 5 worse error distributions. $\Delta\nu$ is the beam ν-spread.

	$\Delta\nu/10^{-3}$, b_3, a_3 only		
Error Distribution No.	Uncorrected	Corrector at Q8,Q9	Corrector at QF,QD
5	10	1	5
12	6	1	2
14	4	4	7
18	1	1	2
20	10	2	4

	$\Delta\nu/10^{-3}$, b_4, a_4 only		
Error Distribution No.	Uncorrected	Corrector at Q8,Q9	Corrector at QF,QD
5	7	3	6
12	5	1	4
14	16	4	11
18	3	0	3
20	7	1	4

Assumptions in Table 1

1. $\gamma = 30$, $\beta^* = 6$ lattice, Au after 10 hours, $\sigma_x = 3.1$ mm, $\epsilon_t = 1.92$, for 95% of beams; $\Delta p/p = \pm 0.005$.
2. ν computed for $\epsilon_x = \epsilon_t$, $\epsilon_y = 0$ only.
3. Correction coil weights not optimized, set at $\frac{1}{2}$ for QF,QD correctors and at 1 for center Q8,Q9 corrector.

Similar computer results were also found by G.F. Dell.[6] Detailed studies for a pure b_4 were done in references 7,8.

The results in Table 1 indicate that the correctors at Q8-Q9 are more effective than the correctors at QF,QD, and a satisfactory correction of the ν-spread requires having the correctors at Q8-Q9. It may be desirable to keep the QF,QD correctors as well as the Q8-Q9 correctors to obtain a better ability to correct the ν spread. This better ability to correct may be desirable if the $b_{3,av}$ or $b_{4,av}$ proves to be larger than assumed here, or if one wants to correct the ν-shift of large amplitude particles which may be getting lost and increasing the background.

4. Strength Requirements for the b_3, b_4 Correctors

Assuming that the b_3, b_4 errors in the dipoles satisfy

$$b'_{3,av} \leq 0.42 \times 10^{-4}$$

$$b'_{4,av} \leq 0.7 \times 10^{-4}$$

then the required corrector strengths; $\int ds\ B_3$ and $\int ds\ B_4$ are given in Table 2 for the two methods of correction

Table 2: Required correction strengths.

	Corrector at Q8-Q9	Corrector at QF,QD
$\int B_3\ ds$	1046 T/m^2	87 T/m^2
$\int B_4\ ds$	72 kT/m^3	6 kT/m^3

The results in Table 2 assume that each correction is correcting the ν-spread as well as it can. These strength requirements could be reduced some what by permitting a less favorable correction at $\gamma = 100$ where the beam is smaller and the ν-spread is smaller. Finding these lower strength requirements would require that this study be repeated at $\gamma = 100$ including the b_4 due to iron saturation.

5. Dynamic Aperture

The dynamic aperture effect of the b_3, b_4 correctors needs to be studied; particularly because of the large lumped Q8,Q9 corrector, and because the b_3 corrector at QF,QD are well placed to drive the 1/4 resonance (Ohnuma). Tracking studies do not indicate an appreciable loss in dynamic aperture.

Table 3: Dynamic aperture results for the two methods of correction

	A_{SL}(mm)			
Error Distribution No.	Uncorrected	Corrector at Q8,Q9	Corrector at QF,QD	$\Delta\nu/10^{-3}$ Corrected
5	16.5	15.5	16.5	b_3, 10
14	16.5	16.5	16.5	b_4, 16
20	16.5	16.5	16.5	b_3, 10

Tracking was done for 1000 turns. $\beta^* = 6$ lattice. Random b_k, a_k present up to $k = 10$. These tracking results do not include the long term effects of perturbations like tune modulation.

6. Comments on the Simulation Study

The ν-shifts is roughly a function of 3 parameters $\delta = \Delta p/p$ and ϵ_x, ϵ_y. To determine the correction ability of a certain set of correctors one should, in principle, study the remaining tune shift over the entire range of δ, ϵ_x, ϵ_y. In this study the ν-shift was studied only for the largest amplitude particles within the beam, and for $\epsilon_y = 0$ where the ν shift is believed to be largest. After correction, the largest remaining ν-shift may be at smaller amplitudes and for $\epsilon_y \neq 0$. A further complication is the choice of excitation of the correctors in order to optimize the correction, also the choice of the location of the Q8-Q9 corrector between Q8 and Q9.

Thus finding the correcting ability of a set of correctors is difficult. Based on the studies done so far, and some spot checks done at smaller amplitudes and for $\epsilon_y \neq 0$, I would hazard the guess that the Q8-Q9 correctors may reduce the ν-spread by about a factor which is probably somewhat smaller then that found in this study, and the QF,QD correctors may be about a factor 2 less able to reduce the ν-spread. If this is so, then it appears desirable that the magnets come close to the tolerances used in this study

$$b'_{3,av} \leq 0.42 \times 10^{-4}$$

$$b'_{4,av} \leq 0.7 \times 10^{-4} \quad .$$

References

1. G. Parzen, BNL AD/RHIC-61 (1990).
2. H. Hahn, BNL AD/RHIC-AP-37 (1986).
3. A.G. Ruggiero, BNL AD/RHIC-AP-86 (1989).
4. D. Neuffer, Proc. Second Advanced ICFA Beam Dynamics Workshop, p. 179 (1988).
5. J. Claus, private communication.
6. G.F. Dell, private communication.
7. M.J. Rhoades-Brown and H. Hahn, BNL AD/RHIC-66 (1990).
8. J. Claus, G.F. Dell, H. Hahn, G. Parzen, M.J. Rhoades-Brown, and A.G. Ruggiero, Proc. EPAC90, p. 1467 (1990).