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Energy Dependent Beta-Gamma Circular Polarization
and Nuclear Matrix Elements of Rb^{86} *

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ABSTRACT

The energy dependent beta-gamma circular polarization was measured in order to set better limits on the matrix element ratio $\Lambda = \int \vec{\alpha} / (\int i \vec{r} / \rho)$ for the 696 keV first forbidden beta transition of Rb^{86} . The results show that Λ is consistent with the Fujita-Eichler relation, so the contributions of the off-diagonal matrix elements of the coulomb hamiltonian to Λ are small for this transition. The nuclear matrix elements are in agreement with previous results, but the limits of error have been reduced.

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I. INTRODUCTION

The determination of nuclear matrix elements for first forbidden beta transitions provides a sensitive test for nuclear models. Since there can be a relatively large number of matrix elements which contribute to the beta transition, more can be learned about the details of nuclear structure from first forbidden transitions than from allowed transitions. Even though the large number of quantities which can be measured make the investigation interesting, the fact that there are a large number of unknowns also makes it difficult to extract the matrix elements from the experimental data.

A valuable aid to simplify the extraction of the matrix elements has been proposed by Fujita¹ and Eichler.² The CVC theory of beta decay can be used to predict the ratio Λ of two of the vector matrix elements.

$$\Lambda \equiv \frac{\int \vec{\alpha} \cdot d\vec{r}}{\int i\vec{r} / \rho}$$

If Λ can be used to remove one unknown from the problem, it is much easier to determine the matrix elements. The Fujita-Eichler method for predicting Λ is attractive because it does not depend on any details of nuclear structure. The assumption is made that the off-diagonal matrix elements of the coulomb hamiltonian H_C are so small that they can be neglected. The validity of this assumption was questioned by Damgaard and Winther.³ They used nuclear model calculations for Tl^{207} and Pb^{209} to propose that even if the off-diagonal matrix elements of H_C are small, it is possible that the vector matrix element ratio Λ will depend on details of nuclear structure.

The Fujita-Eichler expression for Λ is

$$\Lambda = \frac{\int \vec{\alpha} \cdot d\vec{r}}{\int i\vec{r} / \rho} = \pm 2.4 \frac{QZ}{2} + (W_0 \mp 2.5) \rho \quad \text{for } \beta^\mp \text{ decay} \quad (1)$$

Z is the charge of the daughter, $\alpha = 1/137$, and natural units ($m_e = \hbar = c = 1$) are used for the nuclear radius ρ and the endpoint energy W_0 . When the off diagonal matrix elements of H_C are included the following correction term⁵ must be added to Λ :

$$\sum_{f' \neq f} \langle f | H_C | f' \rangle \langle f' | i \vec{r} | i \rangle - \sum_{i' \neq i} \langle f | i \vec{r} | i' \rangle \langle i' | H_C | i \rangle \quad (2)$$

The final state $|f\rangle$ and the initial state $|i\rangle$ of the beta transition are members of the complete set of states $|f'\rangle$ and $|i'\rangle$ respectively. Damgaard and Winter proposed that the correction term be evaluated by using a realistic form for the coulomb hamiltonian. A more general result for Λ is then obtained.⁵

$$\Lambda = \pm \frac{\alpha Z}{2} (3 - \lambda) + (W_0 \mp 2.5) \rho \quad \text{for } \beta^\mp \text{ decay} \quad (3)$$

The parameter λ is the ratio of a higher order to a first order matrix element.

$$\lambda = \frac{\int r(r/\rho)^2}{\int r}$$

when $\lambda=0.6$ the Fujita-Eichler relation and the general formula give the same result for Λ .

It is evident from Eq. (3) that the theoretical value for Λ depends on the value for λ . The experimental limits which are set on Λ also depend upon the value of λ because the higher order matrix element $\int r(r/\rho)^2$ contributes to the beta transition.

We have shown^{4,5} in a previous experiment that the Fujita-Eichler relationship for Λ is not valid for the 2.2 Mev beta transition of La^{140} ($\lambda=2.45$). Nevertheless it is still interesting to measure Λ for other nuclei for two reasons. First, a measurement of Λ shows whether or not the off-diagonal matrix elements of the coulomb hamiltonian H_C are important for the transition. This is useful additional information for nuclear structure studies, and it can also be used to investigate isobaric analogue

states.⁵ Second, if it can be shown that the off-diagonal elements of H_C are not important for a nucleus, then the Fujita-Eichler relation can be useful in the analysis of similar nuclei.

Rb^{86} is an interesting nucleus to study because earlier experimental work⁶ has shown that a measurement of the energy dependence of the beta-gamma circular polarization would greatly improve the experimental limits which could be set on Λ for Rb^{86} .

II. EXPERIMENTAL METHOD

The instrument that was used to measure the beta-gamma circular polarization P_Y has been described previously.⁴ The electronic system has the following major features: it can measure the energy of beta particles with reasonable accuracy at counting rates of 5×10^5 counts per second, the detectors are stabilized by using feedback systems to control the high voltage for the photomultipliers, and strong sources can be used because true and accidental coincidences are recorded simultaneously. The circular polarization P_Y of the gamma rays was measured by forward Compton scattering from magnetized iron. The stabilizing system was very effective. There was no appreciable change in the gain of the beta detector when the beta counting rate was changed from 10^4 to 5×10^5 counts per second. The difference in beta and gamma single counting rates as a function of the magnetic field was typically 0.02%. The average deviation of the single counting rates was typically 0.1%.

The efficiency of the circular polarization analyzing magnet was calculated with a computer program. The accuracy of the program was checked by measuring P_Y for Co^{60} . The calculated efficiency agreed to within 5% with the experimental value for Co^{60} . Since the degree of

polarization of the electrons in the iron was only known to 5%, the agreement was considered to be good.

The sources were prepared by vacuum evaporating the Rb onto thin (0.6 mg/cm^2) aluminum leaf mounts. This procedure is not difficult for Rb since evaporation occurs from Rb Cl at a temperature between 800°C and 1000°C . The procedure has the advantage that it produces a source with uniform surface density (typically 1 mg/cm^2 in this experiment).

The beta-gamma angular correlation function for a first forbidden beta transition can be expressed in the following manner:

$$N(W, \theta, S) = A_0(W) + SA_1(W)P_1(\theta) + A_2(W)P_2(\theta) + SA_3(W)P_3(\theta) \quad (4)$$

where W is the total energy of the beta particle, θ is the angle between the direction of emission of the beta and gamma rays, and the helicity factor S is $+1$ and -1 for right-hand and left-hand circular polarization, respectively. The coefficients A_ℓ are functions of the nuclear matrix elements, and P_ℓ are legendre polynomials. The beta-gamma circular polarization P_γ is defined as,

$$P_\gamma = \frac{N(+1) - N(-1)}{N(+1) + N(-1)} = \frac{A_1P_1 + A_3P_3}{A_0 + A_2P_2} \quad (5)$$

The experimental difference in coincidence counting rate δ can be related to P_γ by the efficiency ϵ of the analyzing magnet,⁷

$$\begin{aligned} \delta(W) &= \frac{C_+(W) - C_-(W)}{C_+(W) + C_-(W)} \\ &= \frac{\epsilon_1 A_1(W) + \epsilon_3 A_3(W)}{A_0(W) + \epsilon_2 A_2(W)} \end{aligned} \quad (6)$$

$C_\pm(W)$ is the beta-gamma true coincidence counting rate for the two directions of magnetic field in the analyzing magnet.

The parameters ϵ_j are obtained by a numerical integration over the geometry of the experiment. Solid angle corrections, average values for P_ℓ , and the circular polarization efficiency are all combined in ϵ_j . The following values were calculated for the efficiency factors:

$$\epsilon_1 = -0.0512 \quad \epsilon_2 = 0.770 \quad \epsilon_3 = -0.0328$$

In order to determine P_γ from δ , the relative size of A_1 to A_3 and A_0 to A_2 must be known. Even though these parameters are relatively well known from measurements of the angular dependence of $P_\gamma^{8,9}$ and the β - γ directional correlation,⁹ the most accurate procedure in extracting the matrix elements is to compare the theoretical calculations directly to δ using Eq. (6).

The average angle between the directions of emission of the beta and gamma rays was 161° . At this angle, the factors $A_1 P_1(\theta_{\beta\gamma})$ and $A_3 P_3(\theta_{\beta\gamma})$ are approximately equal in magnitude and have opposite signs so that P_γ is very sensitive to the nuclear matrix elements.⁶ Small changes in the matrix elements cause the sign of the circular polarization to change.

The computer program which was used to set limits on the matrix elements and the vector matrix element ratio has been described previously.^{5,6} The program uses exact electron wave functions, corrects for screening by atomic electrons, and considers the effect of higher order matrix elements. Previous experimental results for the energy dependence of the β - γ directional correlation,⁹ the shape correction factor,¹⁰ the angular dependence of $P_\gamma^{8,9}$ and the present measurement of the energy dependence of P_γ were used to determine the matrix elements.

III. RESULTS

Figure 1 shows plots of Λ (theoretical) and limits on Λ (experimental) as a function of λ . The theoretical value of Λ lies within the experimental limits as long as λ is within the following range:

$$-0.6 \leq \lambda \leq +0.8$$

The experimental values of the circular polarization effect δ are plotted in Fig. 2 and tabulated in Table I. In the calibration measurement, the average value for δ divided by v/c for Co^{60} was $+0.0170 \pm 0.0006$. The solid curve in Fig. 2 shows the theoretical values of δ which were calculated with a typical set of acceptable matrix elements. It is clear from Eq. (5) and (6) that additional uncertainty is introduced when P_Y is obtained from δ because the relative size of A_1 and A_3 are not known accurately. However, a good indication of the size and energy dependence of P_Y can be obtained from Fig. 3 where the theoretical value of $P_Y(\theta_{BY} = 161^\circ)$ is plotted for the same set of matrix elements used in Fig. 2. The matrix element parameters obtained with the new data and analysis are given in Table II. The limits of error are valid for all acceptable values of λ . The matrix element parameters which were obtained from the previous analysis⁶ are also given for comparison. The matrix elements which are associated with the parameters are also given in Table II. For x and u the matrix element is equal to the matrix element parameter. For the other parameters the relationship to the matrix element is not exact because of higher order matrix elements. The complete expressions for the matrix element parameters are given in Reference 5. The matrix elements are normalized such that their maximum physical size is $\sqrt{2}$.

IV. DISCUSSION

The contributions of the off-diagonal matrix elements of the coulomb hamiltonian H_C cannot be as large in Rb^{86} as they are⁵ in La^{140} ($\lambda=2.45$). Furthermore the results for the vector matrix element ratio Λ are consistent with a zero contribution ($\lambda=0.6$)--that is, the Fujita-Eichler relation may be valid for this transition. A calculation was performed to see how the value of λ would be affected if the present experimental result for δ were used and the limit of errors were reduced by a factor of 3. In that case, λ would be 0.5 ± 0.3 . Therefore it is likely that the contribution of the off-diagonal elements of H_C to Λ are small for this transition.

It is evident from Eq. (2) that the correction term in the expression for Λ will be small if the off-diagonal matrix elements of H_C ($\langle f|H_C|f' \rangle, \langle i'|H_C|i \rangle$) or the additional radial matrix elements ($\langle f'|i\vec{r}|i \rangle, \langle f|i\vec{r}|i' \rangle$) are small. The existence of relatively pure isobaric analogue states is one of the strong indications that $\langle f|H_C|f' \rangle$ and $\langle i'|H_C|i \rangle$ are small. The explanation that has been offered⁵ for the large correction in La^{140} is that the additional matrix elements $\langle f'|i\vec{r}|i \rangle$ and $\langle f|i\vec{r}|i' \rangle$ are large compared to the transition matrix element $\langle f|i\vec{r}|i \rangle$ which was shown to be reduced by an order of magnitude from its maximum physical size. However that same possibility exists for Rb^{86} because $\langle f|i\vec{r}|i \rangle$ (called $\int \frac{i\vec{r}}{p}$ in Table II) is also reduced by an order of magnitude from its maximum physical size ($\sqrt{2}$). Therefore there is no obvious reason why the correction term is large for La^{140} ($\lambda=2.45$) and apparently small ($\lambda \approx 0.6$) for Rb^{86} .

Until a clear reason can be given for the fact that the simple Fujita-Eichler relation is a good approximation for Rb^{86} , it will not be possible to use the relation with confidence to simplify matrix element

extractions in other nuclei. However the general expression (Eq. (3)) can be useful in matrix element extractions.

Given a value of $\int \vec{\alpha}$ and λ , the values of $\int i\vec{r}$ and $\int i\vec{r}(r/\rho)^2$ are fixed by Eq. (3) and the definition of λ . Since $\int \vec{\alpha} \vec{r}$ and $\int i\vec{r}$ have the same radial dependence, it also is reasonable to use the following approximation:

$$\int \vec{\alpha} \vec{r} \left(\frac{r}{\rho}\right)^2 \approx \lambda \int \vec{\alpha} \vec{r}$$

The matrix elements $\int i\vec{r}(r/\rho)^2$ and $\int \vec{\alpha} \vec{r}(r/\rho)^2$ are the most important higher order matrix elements because they enter the expression for the transition probability with relatively large coefficients. Therefore the general expression for Λ can be used to include important high order terms in the matrix element extraction without making the extraction more difficult.

$\int \vec{\alpha}$ and λ can be treated as unknowns rather than $\int \vec{\alpha}$ and $\int i\vec{r}$.

A brief analysis of the matrix elements using the shell model was presented in our earlier work on this transition.⁶ A detailed study of Rb^{86} has been made by S. Wahlborn.¹¹ The better limits of error obtained in this work would not alter the descriptions provided in either of these earlier papers. The present results confirm our earlier observation⁶ that the experimental limits of error used by Wahlborn for the matrix element parameter w are too restrictive. The limits placed on λ provide new nuclear structure information. The higher order matrix element $\int i\vec{r}(r/\rho)^2$ must be less than or equal in magnitude to $\int i\vec{r}$, and the two matrix elements may have equal or opposite sign.

The authors would like to thank Dr. H. A. Smith for his assistance in the matrix element extraction.

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* Work supported by U. S. Atomic Energy Commission under Contract AT(11-1)1746.

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FIGURE CAPTIONS

Fig. 1. Variation of the theoretical value and experimental limits of the matrix element ratio $\Lambda = \sqrt{\alpha}/(\int \vec{r}/\rho)$ as a function of the parameter λ .

Fig. 2. Observed circular polarization effect δ as a function of beta particle energy. The solid line shows the theoretical values for δ which were calculated with a typical set of acceptable matrix elements.

Fig. 3. Theoretical value of P_γ as a function of beta particle energy for an angle of 161° between the beta and gamma rays. The calculation used the same set of matrix elements which were used to calculate the theoretical values of δ shown in Fig. 2.

Table I. Value of the circular polarization effect parameter δ as a function of the total energy of the beta particle W at $\langle \theta_{\beta\gamma} \rangle = 161$.

W	$\delta \times 10^{+3}$
1.30	$- 0.2 \pm 1.8$
1.41	$- 2.2 \pm 1.9$
1.51	$+ 0.4 \pm 2.1$
1.62	$+ 0.2 \pm 2.3$
1.73	$- 3.8 \pm 2.6$
1.83	$- 3.3 \pm 3.0$
1.94	$+ 0.9 \pm 3.7$
2.04	$- 1.7 \pm 5.0$
2.14	$+ 5.8 \pm 7.7$

Table II. Results for matrix element parameters

$$D = 0.149 \quad d = 0.205 \quad a = -0.032 \quad \rho = 0.0135$$

Matrix Element	Matrix Element Parameters	Result of Present Work	Previous Results (Reference 6)
$C_A \gamma_5 + D \quad C_A \int i \vec{\sigma} \cdot \vec{r} / \rho$	$\approx \eta_{DV}$	$= -0.0057 \pm 0.0004$	-0.006 ± 0.0015
$-C_A \int i \vec{\sigma} \cdot \vec{r} / \rho$	$\approx \eta_{w_0}$	0 ± 1.0	0 ± 1.0
$C_V \sqrt{\alpha} - D (C_V \int i \vec{r} / \rho + C_A \int \vec{\sigma} \times \vec{r} / \rho)$	$\approx \eta_{DY}$	-0.00063 ± 0.0007	-0.001 ± 0.001
$C_V \sqrt{\alpha}$	$\approx \eta_{D'y_0}$	-0.054 ± 0.01	-0.05 ± 0.015
$C_V \int i \vec{r} / \rho$	$\approx \eta_x$	-0.14 ± 0.03	-0.13 ± 0.05
$C_A \int \vec{\sigma} \times \vec{r} / \rho$	$\approx \eta_u$	-0.22 ± 0.048	-0.23 ± 0.05
$-C_A \int i B_{ij} / \rho$	$\approx \eta_{z_0}$	$+ .77 \pm .08$	$+0.78 \pm 0.15$