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MULTIPLE CHARGED PION AMPLITUDES

MASTER

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and the Department of Physics
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July, 1970

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OPERATORIAL CONSTRUCTION OF
MULTIPLE CHARGED PION AMPLITUDES*

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ABSTRACT

It is shown that taking the "chiral symmetry" result $\alpha_p(0) - \alpha_\pi(0) = 1/2$ as a hypothesis allows straightforward use of the oscillator formalism to construct 4π and 6π amplitudes with no tachyon ghosts, and full Regge behaviour. An attempt is made to isolate which features of such schemes to generalize the Lovelace amplitude lead to problematic results in the multi-pion amplitudes which are obtained.

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I. In this paper we discuss a generalization of the Lovelace amplitude¹ for multiple charged pion scattering in the framework of the dual resonance model², without introducing additional constructs such as quarks.

While it is possible that an entirely satisfactory derivation of the many pion amplitude will not occur without introducing quarks, the interesting features of multi-pion amplitudes at this stage hinge upon the fact the ρ^+ and πA_1 trajectories have different intercepts, whereas the quark model tells us the ρ and π particles are degenerate.

The present treatment differs from other recent works^{4,5} on this problem in that the terms in the amplitude are built operatorially from the outset.³ That is to say, the harmonic oscillator formalism is employed for "evaluating" the necessary graphs. The evaluations are therefore systematic, and relationships between classes of graphs are recognizable through use of the formal apparatus which has been set up by several authors.^{6,7}

The results obtained in this manner are very similar to those of Ref. 5 for six pions, and the present methods can be applied straightforwardly for generalizing to amplitudes with greater numbers of pions. The only novel feature in passing from six to eight pions is that the $2\pi \cdot \rho$ Reggeon vertex must be introduced.

However, the details of this generalization will be deferred to a later publication. Our principal concerns for the present are to illustrate the basis of the method with the

4 and 6 pion amplitudes; and to discuss those features which appear to be independent of the evaluational scheme used.

II. Unlike the scalar particle case, each planar tree "graph" which can be drawn has meaning in two distinct senses. The new meaning is that the graph specifies which negative mass ghosts are killed by factors α_{ij} , and which are killed by variables in the integrand U_{ij} .

In the Lovelace amplitude,

$$(1) \quad \frac{\Gamma(1-\alpha_{12}) \Gamma(1-\alpha_{23})}{\Gamma(1-\alpha_{12}-\alpha_{23})} \quad ; \quad \alpha_{ij} \equiv \alpha_{ij}(0) + (p_i + \dots + p_j)^2$$

both kinds of ghost killing occur; recall this is required to maintain full Regge behaviour. Thus, the four pion amplitude is defined⁴ as

$$-A_4 = \alpha_{12} \int_0^1 dx x^{-\alpha_{12}-1} (1-x)^{-\alpha_{23}} + \alpha_{23} \int_0^1 dx x^{-\alpha_{12}} (1-x)^{-\alpha_{23}-1}$$

in order to display symmetry in the way the ghosts are killed. The summands are equal in this case because an integration by parts takes us from one term to the other. But integration by parts corresponds to no operator natural to the theory, as the generators of the group of Moebius transformations are, so this property should not be expected to generalize.

Fig. 4 represents in this sense that the ρ ghost in the 56 channel is killed by α_{56} , and the ghost in the 45 channel by a factor U_{45} . In Fig. 5 the roles are reversed. The total amplitude may contain both these terms, as well as terms from cluster configurations such as that of Fig. 7, and all cyclic permutations.

The second meaning of the graphs is the usual one in a dual theory: the initial configuration in which the graph was evaluated can be distorted to exhibit poles in other channels, i.e., the integrands must be functions of the invariant quantities U_{ij} .

A. How are the graphs to be calculated? The matrix element corresponding to Fig. 1 in the oscillator formalism is

$$(2) \quad \langle 0 | e^{k_1 x_0} T_2 \Delta T_3 e^{k_4 x_0} | 0 \rangle = \int_0^1 dx x^{-\alpha_{12}-1} (1-x)^{-\alpha_{23}}$$

when evaluated with the propagator

$$(3) \quad \Delta(x) = \int_0^1 \frac{dx}{x(1-x)} \left(\frac{x}{1-x} \right)^{-\alpha_p(0)} x^H;$$

$$H \equiv R - \frac{\pi_0^2}{2}; \quad R \equiv \sum_n n a_n^+ a_n,$$

and the standard vertices

$$(4) \quad T_i = : e^{\sum_n \frac{k_i a_n^+}{\sqrt{n}}} e^{\sum_n \frac{k_i a_n}{\sqrt{n}}} : ;$$

$$k_i^2 \equiv 2 p_i^2 = 2 m_\pi^2 = -2 \alpha_\pi(0).$$

Only a factor α_{12} is lacking to obtain Eq. (1). This can be inserted operatorially by defining the modified ρ propagator to be

$$(5) \quad \Delta_\rho(x) \equiv \left(\alpha_\rho(0) + \frac{\pi_0^2}{2} \right) \Delta(x)$$

When Eq. (5) is sandwiched between the boosted ground states (conserving momentum) $\langle 0 | e^{kx_0} \dots e^{-kx_0} | 0 \rangle$ there is a series of poles for $\alpha_p(s) = 1, 2, \dots$; thus Eq. (5) is satisfactory for propagation of particles on the degenerate $p-f$ trajectories.

The trick in identifying Eq. (2) with Eq. (1) is that the difference in intercepts is assumed in advance to be

$$(6) \quad \alpha_p(0) - \alpha_\pi(0) = 1/2,$$

assuring Adler zeroes¹¹ for the 4π amplitude at the outset.

Now, instead of choosing the π of momentum p_4 to be the oscillator ground state, as in Fig. 1, the π of momentum p_3 could have been taken for the ground state. It is readily verified that if

$$(7) \quad \tilde{\Delta}_p(x) = \int_0^1 \frac{dx}{(1-x)^2} \left(\frac{x}{1-x} \right)^{H - \alpha_p(0) - 1} (-1)^R \cdot \left(\alpha_p(0) + \frac{\pi_0^2}{2} \right),$$

then

$$\langle 0 | e^{k_1 x_0} T_2 \tilde{\Delta}_p T_4 e^{k_3 x_0} | 0 \rangle = \text{Eq. (1)}.$$

At this point, a discussion of the previously mentioned formal developments is useful, both because of the advantages they will offer, and because of the limitations to them that become apparent in the present circumstances.

First, note that the p propagators introduced so far are related. Using Eq. (3) of Ref. 6b, it is possible to write

$$(8) \quad \tilde{\Delta}_p = S^+ \Delta_p \Omega$$

with ⁷

$$(9) \quad S_\alpha = (1-\alpha) \frac{\pi_0^2}{2} + \sum_n n a_n^+ a_n - \sum_n \sqrt{n(n+1)} a_n^+ a_{n+1};$$

and ⁶

$$(10) \quad \Omega = e^{\pi_0 a_1^+ + \sum_n \sqrt{n(n+1)} a_{n+1}^+ a_n} (-1)^R;$$

$$\Omega |p_1 \dots p_n\rangle = \Omega T_n \Delta \dots T_2 e^{k_1 x_0} |0\rangle = |p_n \dots p_1\rangle$$

Being an operator equation, (8) is valid only when

its matrix elements are taken with respect to appropriate coherent states. Specifically, it is necessary that⁷

$$(11) \quad S_\alpha |p_i\rangle = |p_i'(\rho_i, \alpha)\rangle$$

In the case at hand, $|p_i\rangle$ is a 2π state, which may be denoted $|2\pi\rangle$, $|p_i\rangle = |2\pi\rangle = T e^{k x_0} |0\rangle$,

and Eq. (11) is valid. But, with a similar notation,

$$(12) \quad S_\beta |3\pi\rangle = (1-\beta)^{\alpha_\beta(0) - \alpha_\pi(0)} |3\pi\rangle$$

There is a residual dependence upon the parameter of the transformation which arises from the fact the state $|3\pi\rangle$ includes a β propagator. Consequently the intercept does not cancel with a (momentum)² as in the usual case. This is found to be true for any state with more than two pions.

Therefore although (see Fig. 3)

$$(13a) \quad \langle 0 | e^{k_1 x_0} T_2 \tilde{\Delta}_\beta T_4 e^{k_3 x_0} | 0 \rangle$$

$$= \langle 0 | e^{k_1 x_0} T_2 S^\dagger \Delta_\beta \Omega T_4 e^{k_3 x_0} | 0 \rangle$$

$$= \langle 0 | e^{k_1 x_0} T_2 \Delta_\beta T_3 e^{k_4 x_0} | 0 \rangle,$$

and

$$(13b) \quad \langle 2\pi | S^\dagger \Delta_p | 2\pi \rangle \\ = \langle 2\pi | \Delta_p | 2\pi \rangle \text{ (reverse order)} \rangle$$

nevertheless

$$(13c) \quad \langle 2\pi | S^\dagger \Delta_p S | 2\pi \rangle \neq \\ \langle 2\pi \text{ (r.o.)} | \Delta_p | 2\pi \rangle.$$

The utility of these relations will be clarified in Section III, where $\tilde{\Delta}_p$ is used in evaluating certain semiperipheral graphs.

B. The normal π propagator can be taken to be

$$(14) \quad \Delta_\pi(x) = \int_0^1 \frac{dx}{x(1-x)} x^{H - \alpha_\pi(0)} (1-x)^{\alpha_p(0)}$$

This expression has poles at $\alpha_\pi(s) = 0, 1, 2, \dots$, and so is satisfactory for propagation of particles on the π and A_1 trajectories. The peculiar appearance of $\alpha_p(0)$ in the π propagator is due to the presence of two-body channels dual to any three-body channel in an amplitude for more than four particles; this factor does not affect the poles of the propagator, only the position of the zeroes. As will be seen below, there is a serious difficulty arising from this manifestation of duality. (Such a factor was not required for the normal rho propagator because the π is the external particle, hence its intercept effectively appears as any (momentum)².)

Before actually calculating any graphs, however, note that Fig. 5, a semiperipheral graph, is not required to equal

Fig. 4 except at $\alpha_{13} = 0$, where each must factor into a pair of Lovelace terms. What is going to make the graphs different?

In the present view, a semiperipheral graph is to be evaluated using a "twisted" propagator which must be introduced separately from the regular propagator, unless, as was discussed for the ρ , some relation between the two is forced for reasons of consistency. In that case, independence of choice of ground state led to Eq. (7). In the case of the π , no such reason exists, and one is free to choose any object with the proper pole structure that leads to a dual, Regge behaved amplitude.

A reasonable ansatz is that $\tilde{\Delta}_{\pi}$ has a structure similar to what would have been obtained had the operations with S actually been valid. Thus introduce

$$(15) \quad \tilde{\Delta}_{\pi} = \int_0^1 dy \left(\frac{y}{1-y} \right)^{H - \alpha_{\pi}(0) - 1} (-1)^R$$

The relationships between graphs suggested by Eq. (8) will not be true precisely because the operations with S are not valid.⁸

These comments are better explained by turning to concrete illustrations.

III. Using Eq. (14) the graph of Fig. 4 can be evaluated:

$$(16) \quad A_6 = \int dV \pi u_{ij}^{-\alpha_{ij}-1} [\alpha_{12} \alpha_{56} u_{23} u_{25} u_{45} (1-u_{13})]$$

This turns out to be Eq. (4) of Ref. 5.

The corresponding expression from Ref. 4, on the other hand, is

$$(17) \quad A_6 = \int dV \pi u_{ij}^{-\alpha_{ij}-1} d_{12} d_{56} u_{23} u_{45} u_{45} (1-u_{13}) [u_{24} \cdot u_{35} u_{25}]$$

Note that the extra U_{1j} introduced by Olive and Zakrzewski are unnecessary for ghost-killing purposes - U_{24} and U_{25} are 3-body channels, and the single U_{25} required is already present. Of course, in the term $(1-U_{13})$ there are also 3-channel variables and 2-channel variables which kill more than the lowest ρ trajectory resonance. But whereas Eq. (16) is fully Regge-behaved for any two or three subenergies growing, Eq. (17) is not.

This is not intended as a criticism, since these expressions are not cyclically symmetric. Once the cyclic permutations are added, the amplitude of Ref. 4 does fully Reggeize. What is curious is that the oscillator formalism has led us to a tachyon-free amplitude so economically that the term-by-term Regge behaviour has not been lost.

However, a non-trivial objection to Eq. (16) (and to the corresponding term in the prescription of Ref. 4 as well) is that the residue at the triple pole $\alpha_{12} = \alpha_{34} = \alpha_{56} = 1$ is non-vanishing. This residue has the structure of a $(J^{PC} =) 1^{--}, 1^{--}, 1^{-+}$ coupling, and does not disappear after adding properly the cyclic and allowed non-cyclic graphs of the same type. Since three ρ^0 's cannot couple because of C-invariance, Eq. (16) predicts a vector meson of abnormal charge conjugation degenerate with the ρ meson.

It might be that the cluster diagrams (Fig. 7) could be fixed up to cancel these undesirable particles. However, Olive and Zakrzewski⁴ have already discussed in detail how such diagrams lead to negative metric ghosts (in the eight π

case) on the leading trajectory starting with spin 2. If matters were modified to enforce specific cancellations, it seems reasonable to anticipate even greater difficulties of that kind.

At the outset all possible classes of diagrams had to be considered, because there was no reason for excluding any. The above arguments give reasons for excluding two classes, the multiperipheral and the cluster. The reasons are a posteriori, and we maintain no pretense that they have been "derived". The statement is simply that if the diagrams are allowed, very undesirable predictions result. What is important is that these results appear to be independent of the calculational details.

There remains one class of graphs. From Fig. 5 we obtain

$$\begin{aligned}
 (18) \quad & \langle 0 | e^{k_1 x_0} T_2 \Delta_p T_3 \tilde{\Delta}_\pi T_6 \Delta_p T_5 e^{k_4 x_0} | 0 \rangle \\
 &= \int_0^1 dx \int_0^1 d\bar{z} \int_0^1 dz \, x^{\alpha_{12}-1} (-\bar{z})^{\alpha_{13}-1} \\
 & \quad z^{\alpha_{45}-1} (1-x)^{\alpha_{23}} (1-\bar{z})^{-2p_3 p_6-2} (1-z)^{-\alpha_{56}} \\
 & \quad (1-\bar{z}x)^{-2p_2 p_6} (1-\bar{z}z)^{-2p_3 p_5} (1-x\bar{z}z)^{-2p_2 p_5} \\
 &= \int dV \pi U_{ij}^{-\alpha_{ij}-1} [\alpha_{12} \alpha_{45} U_{23} U_{45} (1-U_{13})(1-U_{45})].
 \end{aligned}$$

In this evaluation, the following identities have been used:

$$\begin{aligned}
 & \left(\bar{z} = \frac{y}{y-1} \right); \quad Q_n = k_3 + k_2 x^n; \quad P_n = k_6 + k_5 z^n \\
 & \sum_{n=1}^{\infty} \frac{\bar{z}^n}{n} Q_n P_n = \sum_{n=1}^{\infty} \frac{(-1)^n Q_n P_n}{n!} \sum_{m=n}^{\infty} \frac{y^m}{m} \cdot m(m-1) \dots \\
 & \dots (m-n+1) = Q_0 P_0 \ln(1-y) + \sum_{n=1}^{\infty} \frac{y^n}{n} \{ k_2 k_6 \cdot \\
 & \quad \sum_m \binom{n}{m} (-x)^m + \dots \}
 \end{aligned}$$

$$= -h_2 h_6 \ln(1-y(1-x)) - h_3 h_5 \ln(1-y(1-z)) \\ - h_2 h_5 \ln(1-y(1-xz)) + (h_2 + h_3) \cdot (h_5 + h_6) \ln(1-y)$$

Eq. (18) is readily seen to be free of tachyon ghosts.

Full Regge behaviour is obtained in only some limits, e.g.,

$$\left. \begin{matrix} \alpha_{12} \\ \alpha_{25} \\ \alpha_{56} \end{matrix} \right\} \rightarrow \infty ; \quad \left. \begin{matrix} \alpha_{23} \\ \alpha_{34} \\ \alpha_{45} \end{matrix} \right\} \rightarrow \infty ; \quad \left. \begin{matrix} \alpha_{12} \\ \alpha_{34} \end{matrix} \right\} \rightarrow \infty ; \text{ etc.}$$

However, with the permutations of this term which preserve cyclic ordering, the complete amplitude exhibits Regge behaviour in all channels.

Triple poles at spin one on the leading trajectory are forbidden in Eq. (18) and its permutations. This result is not due to the specific choice of π propagator beyond the minimal requirement that the factor $(1-U_{13})$ be present to kill the ghosts in two body channels dual to 13.

However, we have merely postponed meeting the 1^{-+} particles, perhaps as daughters, until higher in energy. There does not appear to be a way of avoiding a three vector-meson vertex completely, even if we start with no 3ρ Reggeon coupling⁴. Eq. (18) at least allows the attitude that if the ρ' is discovered, its opposite charge-conjugation brother may also be found.

Finally, the identities for $\Delta_{\mathcal{P}}$ discussed in Section I can be used to demonstrate that both semiperipheral graphs Fig. 5 and Fig. 6 are needed. (One is clockwise ordered, the other counterclockwise.) That is to say, the fact that $S|4\pi\rangle \neq |4\pi\rangle$ means Fig. (5) and Fig. (6) are not equal. This feature has an analog in the calculational scheme of Olive and Zakrzewski, in which the configuration $1\ 2\ 3 \rightarrow 4\ 5\ 6$ contributes differently from $6\ 1\ 2 \rightarrow 3\ 4\ 5$, the first having a factor $(1-U_{13})$ in the integrand, the latter a factor $(1-U_{35})$. In the present scheme, these same factors appear, but the reason for their appearance can be understood in terms of the allowed invariances under choice of ground state⁹.

The 6π amplitude calculated from Figs. 5 and 6 is, then,

$$(19) \quad A_6 = \alpha_{12} \alpha_{45} \int dV \pi u_{ij}^{-\alpha_{ij}-1} u_{23} u_{24} u_{25} u_{34} u_{56} \cdot [2 - u_{13} - u_{35}] + (\text{cycl. perm.})$$

IV. A. There are two further properties of Eq. (19) which we wish to discuss here. First, do the Adler zeroes built into the four π amplitude survive in the generalization? The term Eq. (18) will be studied first; of course, for the entire amplitude to have zeroes, the other five terms must have the same zeroes found for this single term. For a zero to occur, it is found necessary to go to a pole $\alpha_{13} = n^{10}$.

The pole comes from the endpoint singularity of the integrand near $\beta = 0$; then, since for $|\beta| > 1$ it is not legitimate to expand the integrand in powers of β , it has been suggested⁸ that expressions such as Eq. (18) be written

in the form

$$(20) \int dx dz \phi(x, z) \left[\int_{-\epsilon}^0 + \int_{-\infty}^{-\epsilon} \right] d\beta (-\beta)^{-\alpha_{13}-1} F(\beta, x, z)$$

This gives the impression that there is a regular term in addition to the poles in the amplitude.

Actually this procedure is unnecessary. As before, express the amplitude in terms of $y = \frac{z}{z-1}$ so that Eq. (18) reads

$$\alpha_{12} \alpha_{45} \int_0^1 dx dy dz x^{-\alpha_{12}-1} y^{-\alpha_{13}-1} z^{-\alpha_{45}-1} (1-x)^{-\alpha_{23}} (1-y)^{-\alpha_{24}+1} (1-z)^{-\alpha_{56}} (1-y(1-x))^{-2p_2 p_6} (1-y(1-z))^{-2p_3 p_5} (1-y(1-xz))^{-2p_2 p_5}$$

Since $0 \leq x, y, z \leq 1$ it is perfectly satisfactory to expand the integrand in powers of y .

A typical residue at the pole $\alpha_{13} = n$ is

$$(21) \quad \alpha_{12} \alpha_{45} P(d_{ij} \neq \alpha_{13}) B(m_1 - \alpha_{12}, 1 - \alpha_{23}) \cdot B(m_2 - \alpha_{45}, 1 - \alpha_{56})$$

$P = \text{Polynomial in the } s_{ij}.$

where m_1 and m_2 take the maximum value n . There will be a zero in the amplitude from the denominator of the B function if, e.g.,

$$(22) \quad m_1 - \alpha_{12} + 1 - \alpha_{23} = -k \quad (k \geq 0).$$

Suppose that $p_1 = 0$. Then because $S_{13} = n - \alpha_{13}(0) \rightarrow S_{23}$, we have $m_1 - \alpha_{12} + 1 - \alpha_{23} \rightarrow m_1 - n$.

Since $m_1 = n - l$, $l \geq 0$, there is a zero.

The same argument applies if $p_3 = 0$; and these remarks are easily repeated for the other side of the diagram.

Before studying the cases $p_2, p_5 = 0$, consider some other term in the total amplitude. An interesting term is

$$(23) \quad \alpha_{25} \alpha_{34} \int dV \pi u_{ij}^{-\alpha_{ij}-1} [u_{12} u_{56} (1-u_{34})(1-u_{35})]$$

In this case, the left-hand residue for $\alpha_{13} = n$ is

$$(24) \quad B(m_1 - \alpha_{12} + 1, 2 - \alpha_{23}), \quad m_1 \leq n - 2,$$

because of the exponent of $u_{13} = y$ in the integrand.

Clearly the argument for $p_1 = 0$ and $p_3 = 0$ goes through once again, so this term also has zeroes for vanishing of these momenta. Similar considerations apply to the remaining terms in the amplitude. Therefore, at a pole $\alpha_{13} = n$, the entire amplitude vanishes for p_1, p_3, p_4 , or $p_6 = 0$ (separately).

It may now be questioned whether these are genuinely Adler zeroes of the six pion amplitude: as matters stand, they are zeroes in the subgraphs which refer to $\pi^+ \pi^- \rightarrow \pi^+ A^-$ scattering, where A^- is any particle on the π or A_1 trajectories, and its daughters. Indeed, graphs which can be separated into two allowed graphs by cutting a single line are excluded from consideration at the outset in Ref. 11. But realistically a resonance

"pole" should have a width, so there will be no zero in the denominator of the amplitude at $\text{Re } \alpha_{13} = n$. Then a zero in the numerator should still lead to an over-all zero of the amplitude. It is not obvious, however, that once such a unitarization of the amplitude has been made, the zero in the numerator we have found will survive. The question has to remain open.

For p_2 or $p_5 \rightarrow 0$, the method used above no longer applies. Eq. (22) becomes $m_1 - \alpha_{12} + 1 - \alpha_{23} \rightarrow m_1$, so there is no zero for $p_2 \rightarrow 0$. Fortunately the residue at $\alpha_{13} = n$ vanishes if p_2 or $p_5 \rightarrow 0$ in the case of Eq. (18), but that does not happen for the other terms in the amplitude unless $\alpha_{13} = 0$. This violation of the Adler consistency condition occurs in an interesting fashion, since the pion with momentum p_2 (e.g.) carries the same charge as the particle in the 13 channel. If the divergence of the axial current were dominated not only by the low-lying pion, but in addition by "excited" pions, a zero would not be necessary as the low-lying pion went soft. This matter requires more careful consideration than it can be given here, but we mention it because it is reminiscent of arguments that vector meson dominance predictions should be noticeably modified by the presence of higher lying vector mesons.

In any case, the point is that the zeroes based upon the intercept condition which were built into the four π amplitude have a great deal to do with the zeroes obtained in the six pion generalization, since the mechanism for their existence is the same. This is why we have no zeroes whatever away from $\alpha_{13} = n$. The difficulties we have noted may reflect that the mechanism is, after all, inappropriate.

B. Finally, although each term in the amplitude Eq. (19) factorizes according to the well-known scheme by construction, the sum factors in a different manner.

At $\alpha_{12} = 1$, for example, Eq. (18) contributes the normally expected ρ meson. In addition, there are terms

$$(25a) \quad \alpha_{23} \alpha_{56} \int dy dz F_1(yz) y^{-\alpha_{13}-1};$$

$$(25b) \quad \alpha_{34} \alpha_{25} \int dy dz F_2(yz) y^{-\alpha_{13}+1}.$$

Eq. (25a) contributes at the pole $\alpha_{13} = 0$ and so is present in $\pi\pi \rightarrow \pi\pi$ scattering. Eq. (25b) does not, since its first pole in the 13 channel is at $\alpha_{13} = 2$.

These distinguishable contributions have been noted in Ref. 4 and 5, and interpreted as a doubling of the ρ^{12} . They are present because if we take a many-pion amplitude and factor it into four-pion pieces, each must be a Lovelace amplitude. Hence there must be as many α_{1j} prefacing the integral as there are internal ρ lines in the graph. It is these factors which introduce the correlations between right and left of the pole that must be interpreted as new ρ 's.

One would normally expect the difference in couplings to lead to a splitting of the masses of the initially degenerate mesons. This has to be regarded as a problem with the construction schemes. Once again, the details are different depending on how the graphs are calculated, but the general difficulties are the same.

These problems also arise at poles in a π line.

The residue of Eq. (19) at such a pole can be written

$$\sum_i L_i^{(n)} R_i^{(n)},$$

the effect of the sum of terms being to raise the degeneracy in the conventional manner. Still, the question of whether these degenerate particles couple in the same way to any number of external particles (always respecting G-parity, of course) cannot be answered without discussing the 8π amplitude.

V. In summary, it is possible to employ the oscillator formalism to construct 4 and 6 pion amplitudes embodying requirements of duality, Regge behaviour, and tachyon killing. This is intrinsically interesting because the scheme as outlined in the paper works by virtue of Eq. (6): if the intercepts are not carried except in the propagators, the terms we fail to pick up to obtain the Bardacki-Ruegg form are just the ones needed to kill the tachyons, yet preserve duality and Regge behaviour. Matters could certainly have turned out differently, and it is remarkable that this works in just the needed way.

Beyond this "cute" aspect, the oscillator formalism offers the possibility of generalizing to higher pion amplitudes in a "constructive" manner, i.e., in such a way that the connection to lower amplitudes is always clear. This hope is based on the fact each term in the amplitude factors in the standard way, and the new degeneracies arise entirely from writing the amplitude as a sum of such terms. In addition, there remains the possibility the oscillator formalism has some connection to field theory¹³.

Unfortunately, the formalism has been mutilated to some extent in the process of applying it. Certainly the "propagators" used have no connection with propagators in field theory, and the fact unrelated objects such as $\Delta\pi$ and $\tilde{\Delta}\pi$ are not only possible but necessary is unsettling, to say the least.

Where difficulties (1^- particles, doubling of the $\mathcal{P}$, no Adler zeroes, etc.) arise, it has been stressed that they seem independent of how one calculates the graphs. The afflictions really stem from what the calculational schemes have in common, namely, generalizing the Lovelace amplitude by adding together different graphs.¹⁴ This, in turn, is because the proposed calculational schemes (Ref. 4, 5, and the present scheme) regard the ghost-killing factors as accompanying the $\mathcal{P}$ to which they have been assigned wherever it goes. (Hence the double meaning of a graph.) These α_{ij} take on the appearance of kinematical factors. Perhaps what is required is a more "dynamical" ghost killing.

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8. Contrary to what was said before about the necessity of the ρ intercept in the π propagator, Eq. (17) does not have such a factor. This is a peculiarity of the semiperipheral case in general, as can be seen in the paper of Donini and Sciuto, (Torino preprint, 1969) and is not part of our assumptions. The intercept of the 2 body trajectory appears with that of the 3 body trajectory in the combination $2(\alpha_\rho - \alpha_\pi) = 1$.

References and Footnotes (continued)

9. Direct, though tedious, calculations verify the results of the formal operations in each case. The upshot of this discussion is that only graphs 5 and 6 and their cyclic permutations need to be considered, and graphs related to these only by a change of the ground state can be disregarded. Thus χ_ρ is superfluous.
10. Whether, e.g., the $\rho\pi \rightarrow \rho\pi$ amplitude derived from the $3\pi \rightarrow 3\pi'$ amplitude has Adler zeroes is a separate question.
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14. Note there may be wrong charge conjugation particles on the leading trajectory itself above spin 1. This trouble is present in each term of the amplitude, and not just in the sum.

Figure Captions

(Please note order.)

Fig. 1 Four pion graph, with particles 1 and 4 as ground states.

Fig. 2 Same, with 1 and 3 as ground states.

Fig. 3 Schematic representation of Eq. (13a.) The intermediate line in (a) is assigned $\tilde{\Delta}_p$. This is represented by (b), which also shows the order in which the vertices are written down. $\tilde{\Delta}_p = S^+ \Delta \mathcal{R}$ then leads to (c).

Fig. 4 Multiperipheral six pion graph.

Fig. 5 Semi-multiperipheral six pion graph.

Fig. 7 Cluster graph with 3 p Reggeon vertex.

Fig. 6 Semi-multiperipheral graph with ordering opposite that of Fig. 5.

Fig.1

Fig.2

Fig.3

Fig.4

Fig.5

Fig.7

Fig.6