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**ULTRASONIC REFLECTION
AND TRANSMISSION FACTORS
FOR MATERIALS WITH ATTENUATION**

March 1970

**AEC RESEARCH &
DEVELOPMENT REPORT**

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ULTRASONIC REFLECTION AND TRANSMISSION FACTORS
FOR MATERIALS WITH ATTENUATION

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ABSTRACT

A model for the propagation of ultrasonic plane waves which includes the effects of attenuation has been developed. This report represents our viscoelastic ultrasonic wave model and its solutions. The effects of attenuation on reflection and transmission coefficients have been investigated using this model. It was found that in most cases attenuation can be ignored for incident angles less than approximately two degrees below the longitudinal critical angle. For larger angles especially, the longitudinal, shear and Rayleigh critical angles material attenuation is an important factor.

Reflection and transmission coefficients for aluminum, copper, iron, iron 30%nickel, and nickel plates immersed in water were calculated and are listed in Appendix B. These calculations were accomplished for material grain sizes of 0.01, 0.005 and 0.001 in. at frequencies of 1, 10 and 20 MHz.

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1.0 INTRODUCTION

Future developments in nondestructive testing (NDT) would be greatly aided by a firm theoretical basis. This is true not only for ultrasonic testing but also for most other forms of NDT. This report is directed toward establishing a firm theoretical basis in the hope of promoting new NDT techniques through a more thorough understanding of ultrasonic wave propagation.

Ultrasonic waves incident at interfaces between dissimilar materials are affected by attenuation losses^(1,2) as well as by velocities, densities and incident angles. The effect of attenuation has been ignored in most existing theories. Where it has been included, the effect on reflection and transmission factors has not been evaluated. Because of this, the creditability of many existing theoretical studies can be questioned,

This report presents our viscoelastic wave model and its solutions. The effects of attenuation are evaluated by examples and sample applications. The theoretical portion of the report (Section 3) is completely general in nature, while the application section (Section 4) is limited to the case of liquid solid boundaries common to NDT. Graphs of reflection and transmission factors, over a range of frequencies and grain sizes, for a variety of metals in water are presented in Section 4 and Appendix B. These graphs are included to help the reader better understand the modes of propagation of ultrasound and the partition of energy at a liquid solid interface. Appendix A presents some examples of the effect of variables on reflection and transmission factors. Our conclusions are presented in Section 5.

2.0 SUMMARY

A model for the propagation of ultrasonic plane waves which includes the effects of attenuation has been developed. This was done in a manner which allows the equation relating stress and strain to be independent of frequency. The accuracy and applicability of this model is demonstrated by its ability to predict observed phenomena such as the Rayleigh Critical angle reflection minimum. This is not possible when attenuation is ignored.

The most significant contribution of this study is the explanation of attenuation effects on reflection and transmission factors at the Rayleigh, shear and longitudinal critical angles. We have also shown that the effects of attenuation on reflection and transmission factors can for most practical purposes be ignored if the angle of incidence is less than the longitudinal critical angle by at least two degrees.

We have also calculated the reflection and transmission coefficients at water-to-metal and metal-to-water interfaces for aluminum, copper, iron, iron 30% nickel, and nickel. This was accomplished for frequencies of 1, 10, and 20 MHz, also for the zero attenuation case. The attenuations used in these calculations were approximated from the grain sizes. Grain sizes of 0.01, 0.005 and 0.001 in. were chosen. The reflection and transmission factors for each test condition are presented in chart form. Sample applications of the charts are included to demonstrate their use. These charts can be used to additional advantage for the case of metal plates immersed in water; only the incident angle in water is needed to determine the coefficients for reflection, multiple reflections or transmission through plate.

3.0 THEORY

We develop the generalized linear viscoelastic wave model by adding the effects of attenuation to the classical case. The boundary value problem is then solved to obtain the reflection and transmission factors which include the effects of attenuation. This is done for the general case, all possible combinations of materials, incident wave types and directions. The special case of liquid solid boundaries is also solved. The generalized form of Snell's law is also developed. This was necessary to obtain the complex propagation constants in terms of measurable physical properties.

3.1 GENERALIZED LINEAR VISCOELASTIC WAVE MODEL

Models which require dissipation of the energy in a stress wave as it propagates have been discussed in detail. Thompson⁽¹⁾ discussed the relative merits of models accounting for the two inconsistencies of Hooke's law:

(A) Damping of vibrations in solids (damping)

(B) Hysteresis between stress and strain (hysteresis).

Earlier, Kimball and Lovell⁽²⁾ performed experiments on shafts forced to rotate in a strained position. Their experiments showed the hysteresis to be independent of frequency for a number of materials. Knopoff and McDonald⁽³⁾ discussed classical modifications of Hooke's law in which they attempt to explain the inconsistencies (A) and (B). They discussed experiments and observations pertaining to hysteresis between stress and strain for constituents of the earth's crust. They concurred with Kimball and Lovell that these materials have constant hysteresis (specific dissipation function) over a wide range of frequencies (10^{-2} to 10^7 Hz). They also concluded that a one-dimensional partial differential equation cannot be made to satisfy (A) and (B), with the latter (hysteresis) being independent of frequency. We propose that such an equation does indeed exist. This can be demonstrated algebraically if we examine the linear

relationship, Eq. (1) between stress $\sigma(\omega, t)$ and strain $\epsilon(\omega, t)$.

$$\sigma = \kappa_1 \epsilon + \frac{\kappa_2}{\omega} \frac{\partial \epsilon}{\partial t} \quad (1)$$

Here, ω is the angular frequency, κ_1 and κ_2 are constants and $\partial/\partial t$ is a linear differential operator with respect to time. Suppose $\epsilon = \sin \omega t$. Then $\sigma = \kappa_1 \sin \omega t + \kappa_2 \cos \omega t$. Eliminating time between the two equations yields

$$\sigma^2 + \left(\frac{\sigma - \kappa_1 \epsilon}{\kappa_2} \right)^2 = 1 \quad (2)$$

an ellipse independent of ω . From Eq. (2) we may conclude the Eq. (1) has properties which satisfy the requirements (A) and (B), with (B) independent of frequency, as Knopoff and MacDonald desired.

The analysis proceeds with a generalization of Eq. (1) to three dimensions

$$\sigma_{ij} = C_{ij}^{\kappa\ell} \epsilon_{\kappa\ell} + \frac{D_{ij}^{\kappa\ell}}{\omega} \epsilon_{\kappa\ell, t}$$

where $C_{ij}^{\kappa\ell}$ and $D_{ij}^{\kappa\ell}$ are arbitrary constants.

In Eq. (3) we used the Einstein convention for tensor notation. If we assume the medium to be isotropic, then Eq. (3) can be written with two pairs of constants where the unprimed constants of Eq. (4) are the Lamé constants and the primed constants follow in a manner parallel to that given in Love,⁽⁴⁾ Chapter III.

$$\sigma_{ij} = \lambda \delta_{ij} \epsilon_{\kappa\kappa} + 2\mu \epsilon_{ij} + \frac{\lambda'}{\omega} \delta_{ij} \epsilon_{\kappa\kappa, t} + \frac{2\mu'}{\omega} \epsilon_{ij, t} \quad (4)$$

Neglecting body forces, the dynamic equilibrium conditions per unit volume in terms of density ρ and the components of displacement U_i are given by

$$\sigma_{ij, j} = \rho U_{i, tt} \quad (5)$$

The strain-displacement equations are

$$\epsilon_{ij} = \frac{1}{2} (U_{i, j} + U_{j, i}) \quad (6)$$

We obtain the equation of particle motion [Eq. (7)] by eliminating the strains from Eqs. (4) and (6) and using the derivatives of the result to eliminate the stresses from Eq. (5).

$$\mu \nabla^2 \vec{U} + (\lambda + \mu) \nabla (\nabla \cdot \vec{U}) + \frac{\mu'}{\omega} \nabla^2 \vec{U}_{,t} + \frac{(\lambda' + \mu')}{\omega} \nabla (\nabla \cdot \vec{U}_{,t}) = \rho \vec{U}_{,tt} . \quad (7)$$

In Eq. (7) the notation $\vec{U}$ is the vector notation for the displacement components and ∇ is the vector gradient operator. Equation (7) is similar to a special case of the equation of particle motion, given by Newlands⁽⁵⁾ with the exception that Eq. (7) allows the angular frequency ω to take on complex values. In addition, Newlands states that the various wave velocities, compression, shear, and Rayleigh have complex values. We intend to reveal a physical interpretation for these complex valued velocities. The investigation of Eq. (7) continues by assuming, as in the lossless case, that the displacement vector ($\vec{U}$) is made up of the superposition of two independent displacements $\vec{L}$ and $\vec{S}$. Since Eq. (7) is linear, each component of $\vec{U}$ yields an independent equation. If $\vec{L} = \nabla \phi$, then

$$\nabla \left[(\lambda + 2\mu) \nabla^2 \phi + \frac{\lambda' + 2\mu'}{\omega} \nabla^2 \phi_{,t} - \rho \phi_{,tt} \right] = \vec{0} \quad (8)$$

If $\vec{S} = \nabla \times \vec{\psi}$ then

$$\nabla \times \left[\mu \nabla^2 \vec{\psi} + \frac{\mu'}{\omega} \nabla^2 \vec{\psi}_{,t} - \rho \vec{\psi}_{,tt} \right] = \vec{0} . \quad (9)$$

In Eq. (8) the quantity in brackets must be equivalent to a scalar constant which is equated to zero for convenience. Similarly, in Eq. (9) the constant vector field is chosen to be null. An equation of type (8) or (9) will yield specific dissipation functions independent of frequency, and may be represented by a single equation

$$\left(\nu + \frac{E}{\omega} \frac{\partial}{\partial t} \right) \nabla^2 f(\vec{R}, t | \omega) = \rho \frac{\partial^2}{\partial t^2} f(\vec{R}, t | \omega) , \quad (10)$$

where ν and E take on appropriate values.

Assume a solution be of the form

$$f(\vec{R}, t | \omega) = Z(\omega) e^{i\omega \left(\frac{\vec{N} \cdot \vec{R}}{c} - t \right)}, \quad (11)$$

where $\vec{N}$ is a unit vector and c is a real valued propagation constant. Newlands⁽⁵⁾ has shown that c must be complex for $E \neq 0$ in Eq. (10). In addition, we find examples of analyses where $\vec{N}$ takes on complex values in the expansion of a spherical wave potential function into a system of plane waves.

If we substitute Eq. (11) into Eq. (10), we obtain

$$\left(\frac{i\omega}{c} \right)^2 \vec{N} \cdot \vec{N} \left[v + \frac{E}{\omega} (-i\omega) \right] = \rho (-i\omega)^2, \quad ,$$

which imposes two generalizations:

$$\vec{N} \cdot \vec{N} = 1 \quad \text{for all complex valued } \vec{N} \quad , \quad (12)$$

and

$$c^2 = \frac{v - iE}{\rho} \quad . \quad (13)$$

At least two questions arise. First, how fast are wave crests moving? Second, what is the direction of motion of wave crests? These questions have been answered by Fitch and Richardson⁽⁶⁾. However, a more heuristic approach is to inspect directions of $\vec{R}$ in $f(\vec{R}, t | \omega)$, given by a generalization of $\vec{N}/c$ in Eq. (11). Again, it is customary to define a relation between $\vec{N} \cdot \vec{R}/c$ with t which makes the exponent vanish. This relation then yields the velocity of wave crests, commonly called the phase velocity. Here, we are required to choose real values of the position vector $\vec{R}$ and real values of time. Consequently, the relation must be real valued.

Consider first the generalization of medium velocity given by Eq. (13) and its effect on the wave potential Eq. (11). The parameters which determine c^2 in Eq. (13) are inconvenient, and so new parameters v and d are defined such that

$$\begin{aligned}
 c &= v/(1 + id) \quad \text{where } d, v \geq 0 \\
 \text{consequently, } d &= E/\left(v + \sqrt{v^2 + E^2}\right) \\
 \text{and } v &= \sqrt{2(v^2 + E^2)/\rho\left(v + \sqrt{v^2 + E^2}\right)}
 \end{aligned}
 \quad , \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad (14)$$

The substitution of Eq. (14) into Eq. (11) yields

$$f(\vec{R}, t | \omega) = Z(\omega) e^{-\frac{\omega d}{v} \vec{N} \cdot \vec{R}} e^{i\omega \left(\frac{\vec{N} \cdot \vec{R}}{v} - t\right)} . \quad (15)$$

From Eq. (15) we see immediately that generalizing c yields a plane wave which has the usual attenuation factor $\alpha_n = \omega d/v$ (in nepers per unit distance) in the direction of $\vec{N}$ (i.e., the direction of wave propagation). These choices of v and d impose physical limitations on the propagating wave which require that $\vec{N}$ attenuate with d nepers per radian as $\vec{N}$ propagates with velocity v . By taking limits as E goes to zero, d also goes to zero and v becomes the usual undamped plane wave velocity in the medium.

In addition to the previous generalization of c , the generalization of $\vec{N}$ to complex values is still to be accomplished. In a rectangular coordinate system with unit vectors $\vec{i}$ in the positive x direction and $\vec{j}$ in the positive y direction, we let

$$\vec{R} = \vec{i}x + \vec{j}y \quad (16)$$

The unit normal ($\vec{N}$) takes the form

$$\vec{N} = \vec{i} \cos \gamma + \vec{j} \sin \gamma \quad (17)$$

where γ is positive when measured counter-clockwise from the positive x axis. We generalize $\vec{N}$ by requiring

$$\gamma = A + ib \quad , \quad (18)$$

where $\vec{i} = \sqrt{-1}$ and is distinct from the unit vector $\vec{i}$ by the indicated notation.

Substituting Eq. (18) into Eq. (17), and expanding, we see that

$$\vec{N} = \vec{N}_r \cosh B + i \vec{N}_i \sinh B \quad , \quad (19)$$

where

$$\begin{aligned} \vec{r} &= \vec{i} \cos A + \vec{j} \sin A \quad , \\ \vec{N}_i &= -\vec{i} \sin A + \vec{j} \cos A \end{aligned}$$

Note that $\vec{N}$ obeys the usual requirements $\vec{N} \cdot \vec{N} = 1$ when the usual rules of vector dot product and the product of complex numbers are applied. Also, the unit vectors $\vec{N}_r$ and $\vec{N}_i$ form a right-handed coordinate system where the vector product $\vec{N}_r \times \vec{N}_i = \vec{k}$, $\vec{k}$ being a unit vector in the direction of the positive z axis. Proceeding with the generalization of the unit normal, we substitute Eq. (19) into Eq. (11) which yields

$$f(\vec{R}, t | \omega) = Z(\omega) e^{-\frac{\sinh B}{c} \vec{N}_i \cdot \vec{R}} e^{i\omega \left(\frac{\cosh B}{c} \vec{N}_r \cdot \vec{R} - t \right)} \quad . \quad (20)$$

Plane wave motion in Eq. (20) is in the direction of $\vec{N}_r$, whereas attenuation is perpendicular to $\vec{N}_r$. Consequently, the amplitude in the plane of the wave is not uniform, but varies exponentially, increasing or decreasing in the direction of $\vec{N}_i$ depending on the sign of B. Schoch⁽⁷⁾ and Brekhovskikh⁽⁸⁾ called these generalized plane waves "inhomogeneous" plane waves. Much confusion has resulted from overuse of the term homogeneous in analysis. It seems better to use the term "generalized" plane wave since it is more descriptive. Not only is the generalized plane wave nonuniform in amplitude, but it also moves in the direction of $\vec{N}_r$ at a velocity less than that of sound in the medium. The physical example of this phenomenon is given by the Rayleigh wave, and the Stonely waves. More study of the relationship between this generalized plane wave representation and the structure of boundary waves is anticipated.

Since each factor c and $\vec{N}$ of plane wave generalizations has been studied separately, the factors are now considered simultaneously. As occurs in each previous case, we expect the wave to move in the direction given by $\text{Re}(\vec{N}/c)$ and to be attenuated in the direction $\text{Im}(\vec{N}/c)$. However, $\text{Im}(\vec{N}/c)$ has a meaningless direction unless it is interpreted as before, by

examining components in the direction of motion and perpendicular to the direction of motion. We need to resolve $\text{Im}(\vec{N}/c)$ into components in the direction $\text{Re}(\vec{N}/c)$ and in the direction perpendicular to $\text{Re}(\vec{N}/c)$. To carry out the generalization, we need to rotate from the rectangular (x,y) coordinates to the rectangular (ζ,η) coordinates, and from the unit vectors $\vec{i}$ and $\vec{j}$ to the unit vectors $\vec{U}_N$ and $\vec{U}_C$. Here, $\vec{U}_N$ is a unit vector in the direction of $\text{Re}(\vec{N}/c)$ and $\vec{U}_C$ is such that $\vec{U}_N \times \vec{U}_C = \vec{k}$, where $\vec{k}$ is the unit vector in the positive z direction. The geometric parameters (γ, A, B) given in Eq. (18) and the medium parameters (c,v,d) given in Eq. (14) are used to provide values for $\vec{U}_N$ and $\vec{U}_C$, from which the physical character of the propagating generalized plane waves may be determined.

$$\frac{\vec{N}}{c} = \text{Re}\left(\frac{\vec{N}}{c}\right) + i \text{Im}\left(\frac{\vec{N}}{c}\right) = \left|\text{Re}\left(\frac{\vec{N}}{c}\right)\right| \vec{U}_N + i \left(\lambda \vec{U}_N + \mu \vec{U}_C\right) \quad (21)$$

Here, λ and μ are parameters to be evaluated.

$$\text{Re}\left(\frac{\vec{N}}{c}\right) = \left(\vec{N}_r \cosh B - \vec{N}_i d \sinh B\right) / v \quad (22)$$

$$\text{Im}\left(\frac{\vec{N}}{c}\right) = \left(\vec{N}_r d \cosh B + \vec{N}_i \sinh B\right) / v \quad (23)$$

$$\left|\text{Re}\left(\frac{\vec{N}}{c}\right)\right| = \sqrt{\cosh^2 B + d^2 \sinh^2 B} / v \quad (24)$$

$$\vec{U}_N = \left(\vec{N}_r \cosh B - \vec{N}_i d \sinh B\right) / \sqrt{\cosh^2 B + d^2 \sinh^2 B} \quad (25)$$

$$\vec{U}_C = \left(\vec{N}_i \cosh B + \vec{N}_r d \sinh B\right) / \sqrt{\cosh^2 B + d^2 \sinh^2 B} \quad (26)$$

The $\vec{U}_N$ and $\vec{U}_C$ given in Eqs. (25) and (26) fulfill the requirements for a right-handed coordinate system. The vector $\vec{R}$ is transformed from the (x,y) coordinate system to the (ζ,η) coordinate system where:

x is measured positive in the direction $\vec{i}$,
 y is measured positive in the direction $\vec{j}$,
 ζ is measured positive in the direction $\vec{U}_N$, and
 η is measured positive in the direction $\vec{U}_C$.

Thus,

$$\vec{R} = \zeta \vec{U}_N + \eta \vec{U}_C \quad . \quad (27)$$

The value of λ is found from the dot product of Eq. (23) with (25):

$$\lambda = d / \left(v \sqrt{\cosh^2 B + d^2 \sinh^2 B} \right) \quad . \quad (28)$$

The value of μ is found from the dot product of Eq. (23) with Eq. (26):

$$\mu = (1 + d^2) \sinh 2B / \left(2v \sqrt{\cosh^2 B + d^2 \sinh^2 B} \right) \quad (29)$$

With Eqs. (27) and (21) we are able to evaluate

$$\vec{R} \cdot \left(\frac{\vec{N}}{C} \right) = \left| \text{Re} \left(\frac{\vec{N}}{C} \right) \right| \zeta + i(\lambda \zeta + \mu \eta) \quad . \quad (30)$$

Substituting Eq. (30) into Eq. (11) we are able to interpret the character of the generalized plane wave:

$$f(\vec{R}, t | \omega) = Z(\omega) \exp(-\omega \lambda \zeta) \exp(-\omega \mu \eta) \exp \left[i \omega \left(\left| \text{Re} \left(\frac{\vec{N}}{C} \right) \right| \zeta - t \right) \right] \quad . \quad (31)$$

The phase velocity of the wave $f(\vec{R}, t | \omega)$ is found by setting

$$\zeta = t / \left| \text{Re} \left(\frac{\vec{N}}{C} \right) \right| \quad .$$

Thus, the phase velocity of the generalized plane wave is called v^* , and is equal to the coefficient of t .

$$v^* = 1 / \left| \text{Re} \left(\frac{\vec{N}}{C} \right) \right| = v / \sqrt{\cosh^2 B + d^2 \sinh^2 B} \quad (32)$$

Curiously, generalized plane waves do not travel at medium velocity v even when no damping ($d = 0$) occurs, but plane waves ($B = 0$) always travel at medium velocity v as given in Eq., (14). Note that the medium velocity v , for a viscoelastic medium, is not the same as the one derived from the lossless wave equation for an elastic medium. This result is consistent with the findings of Eqs. (15) and (20).

The attenuation of the generalized plane wave in the direction of motion is $\omega \lambda$, as shown in Eq. (31), and is called the normal attenuation α_n . The use of Eq. (32) in Eq. (28) simplifies α_n :

$$\alpha_n = \omega d v^* / v^2 \quad . \quad (33)$$

Crest attenuation of the generalized wave is $\omega\mu$, as shown in Eq. (31), and is called α_c . Similarly, the use of Eq. (32) in Eq. (29) simplifies α_c :

$$a_c = \omega(1 + d^2) (v^*/2v^2) \sinh 2B \quad (34)$$

From Eq. (33) it is obvious that a nonattenuating medium does not have normal attenuation α_n . But Eq. (34) shows that the nonattenuating medium will support plane waves with exponentially varying crest amplitude. These results were anticipated in Eqs. (15) and (20). We have now answered our first question of Section 3.2 concerning the phase velocity of waves in the medium.

The second question was "What is the direction of motion of wave crests?". To answer this, we must find the real direction of motion of the generalized plane waves with respect to the (x,y) coordinates; i.e., the direction of $\vec{U}_N$ in terms of the unit vectors $\vec{i}$ and $\vec{j}$. From Eqs. (26) and (32)

$$\vec{U}_N = (\vec{N}_r \cosh B - \vec{N}_i d \sinh B) v^*/v \quad (35)$$

Substituting the values of $\vec{N}_r$ and $\vec{N}_i$ given in Eq. (19) into Eq. (35) yields

$$\begin{aligned} \vec{U}_N = & \vec{i} [\cos A \cosh B + d \sin A \sinh B] (v^*/v) \\ & + \vec{j} [\sin A \cosh B - d \cos A \sinh B] (v^*/v) \end{aligned} \quad (36)$$

Hence, the plane waves move in the direction which makes a positive angle α with respect to the positive x axis:

$$\alpha = \tan^{-1} \left\{ \frac{-\tan A - d \tanh B}{1 + d \tan A \tanh B} \right\} . \quad (37)$$

The values of d and $\tanh B$ are expected to be small. The value of α is approximately A , with the possible exception of waves colliding with the plane $x = 0$, for which the value of $\tan A$ becomes quite small.

The foregoing analysis of generalized plane waves is an attempt to convince the reader of the validity of Eq. (3) as a linear approximation to the solution of the original questions concerning attenuation of stress waves. A more convincing example is given by seeking an explanation of

the anomaly given by Brekhovskikh^(8, p.33) concerning the dip in the magnitude of the reflected ultrasonic wave at the Rayleigh critical angle. The irregularities between observed and computed data are eliminated by using Eq. (3) in the solution of the problem of reflection of a plane wave at oblique incidence from a plane boundary between two dissimilar attenuating media. This subject will be further discussed in the application section.

3.2 REFLECTION AND TRANSMISSION FACTORS

In solving for the reflection and transmission factors we shall first solve the case where both media may be either liquid or solid. This is done first for an incident longitudinal wave in medium 1 and then for the general case where a shear or longitudinal wave is incident in either medium. The special case, where medium 1 is a liquid, is then solved.

3.2.1 General Case

The analysis leading to the reflection factor is given by Reference (6) with anticipated use of the digital computer. Next we will use the common procedures to compute a solution in an attempt to indicate the similarity of this solution to those in the existing literature. (See, for example, Cooper⁽⁹⁾ for the frequency dependent case.) Figure 1 reveals the geometric pattern of response waves anticipated from an incident longitudinal wave in medium 1. The other three possible incident wave conditions will be analyzed in this section.

The wave potentials ϕ_{jk} and ψ_{jk} have been described in Eqs. (8) and (9). The following index notation will be adopted:

- Incident medium parameters have subscript $j=1$,
- Refracting medium parameters have subscript $j=2$,
- Incident wave parameters have subscript $k=1$,
- Response wave parameters have subscript $k=2$.

The superposition of an incident longitudinal displacement potential ϕ_{11} , a reflected longitudinal displacement potential ϕ_{12} , and a reflected shear displacement potential ψ_{12} , make up the total displacement potential in

FIGURE 1. Geometry of Response Wave Potentials From an Incident Longitudinal Wave Potential

the incident medium. The superposition of a refracted longitudinal displacement potential ϕ_{22} and a refracted shear displacement potential ψ_{22} make up the total displacement potential in the refracting medium. At the plane boundary ($y = 0$) separating the two media, the potentials in medium 1 have the following relationships to the potentials in the second medium:

- (i) The two components of displacement are equal.
- (ii) The direct and shear components of stress normal to the plane are equal.

We have used superposition to resolve the displacement vector according to $\vec{U} = \nabla\phi + \nabla\times\vec{\psi}$, which is always possible for any vector field. In the sequel, it is natural to use cartesian coordinates (x,y,z) with respective unit vectors $\vec{i}, \vec{j}, \vec{k}$. The following analysis is not violated by assuming

z independence for the potentials $\vec{\psi} = \vec{k}\psi(x, y, t|\omega)$ and $\phi = \phi(x, y, t|\omega)$, where ϕ and ψ have the format of $f(\vec{R}, t|\omega)$ in Eq. (11). Continuity, (i) above, of the x and y components of the displacement vector $\vec{U}$ yields the respective equations

$$\phi_{11,x} + \phi_{12,x} + \psi_{12,y} = \phi_{22,x} + \psi_{22,y} \quad , \quad (38)$$

$$\phi_{11,y} + \phi_{12,y} - \psi_{12,x} = \phi_{22,y} - \psi_{22,x} \quad (39)$$

at the interface between the two media ($y = 0$), where the variable subscripts following the comma denote partial differentiation.

The generalized relation between stress and strain, Eq. (4), may be used to equate direct and shear stresses at the interface. Continuity, (ii) above, of ϕ_{YY} at the boundary ($y = 0$) is written in terms of the potentials ϕ and ψ in Eq. (4):

$$\left[\lambda_1 + 2\mu_1 + \frac{\lambda_1' + 2\mu_1'}{\omega} \frac{\partial}{\partial t} \right] \nabla^2 (\phi_{11} + \phi_{12}) - 2 \left[\mu_1 + \frac{\mu_1'}{\omega} \frac{\partial}{\partial t} \right] (\phi_{11,xx} + \phi_{12,xx} + \psi_{12,xy}) =$$

$$\left[\lambda_2 + 2\mu_2 + \frac{\lambda_2' + 2\mu_2'}{\omega} \frac{\partial}{\partial t} \right] \nabla^2 \phi_{22} - 2 \left[\mu_2 + \frac{\mu_2'}{\omega} \frac{\partial}{\partial t} \right] (\phi_{22,xx} + \psi_{22,xy}) \quad . \quad (40)$$

Continuity, (ii) above, of σ_{xy} at the boundary ($y = 0$) is written in terms of the potentials ϕ and ψ in Eq. (4):

$$\left[\mu_1 + \frac{\mu_1'}{\omega} \frac{\partial}{\partial t} \right] \left\{ \nabla^2 \psi_{12} + 2 [\phi_{11,xy} + \phi_{12,xy} - \psi_{12,xx}] \right\} =$$

$$\left[\mu_2 + \frac{\mu_2'}{\omega} \frac{\partial}{\partial t} \right] \left\{ \nabla^2 \psi_{22} + 2 [\phi_{22,xy} - \psi_{22,xx}] \right\} \quad . \quad (41)$$

Using the following subscript notation in Eq. (11),

$$Z_{ijk}(\omega) \exp \left[i\omega \left(\frac{\vec{N}_{ijk} \cdot \vec{R}}{c_{ij}} - t \right) \right] = \begin{cases} \phi_{jk}(\vec{R}, t|\omega) & \text{if } i = 1, \\ \psi_{jk}(\vec{R}, t|\omega) & \text{if } i = 2 \end{cases} ; j, k = 1, 2. \quad (42)$$

In the boundary conditions Eqs. (38) through (41), we carry out the indicated linear operations on the functions given by Eq. (42).

Consequently, setting $y = 0$, we find a function

$$\exp \left[i\omega \left(\frac{x \cos \gamma_{ijk}}{c_{ij}} - t \right) \right]$$

common to every term. Obedience of these potentials to the boundary condition demands that the coefficients of x be equivalent:

$$\frac{\cos \gamma_{ijk}}{c_{ij}} = \text{a complex constant; } i, j, k = 1, 2 \quad (43)$$

Equation (43) is the generalized form of Snell's law resulting from the generalization of the equation relating stress to strain. Here, we have used the definitions

$$\vec{N}_{ijk} = \vec{i} \cos \gamma_{ijk} + \vec{j} \sin \gamma_{ijk} \quad (44)$$

and

$$\vec{R} = \vec{i}x + \vec{j}y \quad . \quad (45)$$

The index notation is given in detail for clarification:

Longitudinal wave parameters have subscript $i = 1$,

Shear wave parameters have subscript $i = 2$,

Incident medium parameters have subscript $j = 1$,

Refracting medium parameters have subscript $j = 2$,

Incident wave parameters have index $k = 1$,

Response wave parameters have index $k = 2$.

It is important to adopt the policy that the angle of reflection is the negative of the angle of incidence when angles are measured positively in the counter-clockwise direction from the positive x -axis (even if they are complex). The respective boundary Eqs. (38) through (41) can be simplified by omitting the common function of time and the function of x (made common by the generalized Snell's law). In addition, we can delete the index k by writing all angles in terms of response angles ($k = 2$).

We then obtain

$$(Z_{111} + Z_{112}) \frac{\cos \gamma_{11}}{c_{11}} + Z_{212} \frac{\sin \gamma_{21}}{c_{21}} = Z_{122} \frac{\cos \gamma_{12}}{c_{12}} + Z_{222} \frac{\sin \gamma_{22}}{c_{22}}, \quad (46)$$

$$(-Z_{111} + Z_{112}) \frac{\sin \gamma_{11}}{c_{11}} - Z_{212} \frac{\cos \gamma_{21}}{c_{21}} = Z_{122} \frac{\sin \gamma_{12}}{c_{12}} - Z_{222} \frac{\cos \gamma_{22}}{c_{22}}, \quad (47)$$

$$\begin{aligned} [\lambda_1 + 2\mu_1 - i(\lambda'_1 + 2\mu'_1)] \left(\frac{Z_{111} + Z_{112}}{c_{11}^2} \right) - 2(\mu_1 - i\mu'_1) \left[(Z_{111} + Z_{112}) \frac{\cos^2 \gamma_{11}}{c_{11}^2} \right. \\ \left. + Z_{212} \frac{\sin \gamma_{21} \cos \gamma_{21}}{c_{21}^2} \right] = [\lambda_2 + 2\mu_2 - i(\lambda'_2 + 2\mu'_2)] \frac{Z_{122}}{c_{12}^2} \\ - 2(\mu_2 - i\mu'_2) \left(Z_{122} \frac{\cos^2 \gamma_{12}}{c_{12}^2} + Z_{222} \frac{\sin \gamma_{22} \cos \gamma_{22}}{c_{22}^2} \right), \end{aligned} \quad (48)$$

$$\begin{aligned} (\mu_1 - i\mu'_1) \left\{ \frac{Z_{212}}{c_{21}^2} + 2 \left[(-Z_{111} + Z_{112}) \frac{\sin \gamma_{11} \cos \gamma_{11}}{c_{11}^2} - Z_{212} \frac{\cos^2 \gamma_{21}}{c_{21}^2} \right] \right\} = \\ (\mu_2 - i\mu'_2) \left\{ \frac{Z_{222}}{c_{22}^2} + 2 \left[Z_{122} \frac{\sin \gamma_{12} \cos \gamma_{12}}{c_{12}^2} - Z_{222} \frac{\cos^2 \gamma_{22}}{c_{22}^2} \right] \right\}. \end{aligned} \quad (49)$$

The addition of the subscripts to Eq. (13) yields

$$\rho_j c_{ij}^2 = \begin{cases} \lambda_j + 2\mu_j - i(\lambda'_j + 2\mu'_j) & \text{if } i = 1 \\ \mu_j - i\mu'_j & \text{if } i = 2 \end{cases}, j = 1, 2. \quad (50)$$

In incorporating Eq. (50) into Eq. (46) through Eq. (49) and rearranging, we obtain the following simultaneous equations:

$$\begin{aligned} \frac{\cos \gamma_{11}}{c_{11}} Z_{112} - \frac{\sin \gamma_{22}}{c_{22}} Z_{222} - \frac{\cos \gamma_{12}}{c_{12}} Z_{122} + \frac{\sin \gamma_{21}}{c_{21}} Z_{212} = \\ - \frac{\cos \gamma_{11}}{c_{11}} Z_{111}, \end{aligned} \quad (51)$$

$$\frac{\sin \gamma_{11}}{c_{11}} Z_{112} + \frac{\cos \gamma_{22}}{c_{22}} Z_{222} - \frac{\sin \gamma_{12}}{c_{12}} Z_{122} - \frac{\cos \gamma_{21}}{c_{21}} Z_{212} =$$

$$\frac{\sin \gamma_{11}}{c_{11}} Z_{111} \quad , \quad (52)$$

$$\rho_1 \left(1 - 2 \frac{c_{21}^2}{c_{11}^2} \cos^2 \gamma_{11} \right) Z_{112} + \rho_2 \sin 2\gamma_{22} Z_{222} +$$

$$-\rho_2 \left(1 - 2 \frac{c_{22}^2}{c_{12}^2} \cos^2 \gamma_{12} \right) Z_{122} - \rho_1 \sin 2\gamma_{21} Z_{212} = -\rho_1 \left(1 - 2 \frac{c_{21}^2}{c_{11}^2} \cos^2 \gamma_{11} \right) Z_{111} \quad (53)$$

$$2\rho_1 \frac{c_{21}^2}{c_{11}^2} \sin \gamma_{11} \cos \gamma_{11} Z_{112} - \rho_2 \left(1 - 2 \cos^2 \gamma_{22} \right) Z_{222} +$$

$$-2\rho_2 \frac{c_{22}^2}{c_{12}^2} \sin \gamma_{12} \cos \gamma_{12} Z_{122} + \rho_1 \left(1 - 2 \cos^2 \gamma_{21} \right) Z_{212} =$$

$$2\rho_1 \frac{c_{21}^2}{c_{11}^2} \sin \gamma_{11} \cos \gamma_{11} Z_{111} \quad . \quad (54)$$

Equations (51) through (54) yield the response amplitudes of waves rebounding from an interface between two dissimilar media for an incident longitudinal wave in medium one. The remaining incident wave conditions may be handled in a similar manner. The solutions for the general case are found by adding the incident potentials ϕ_{21} , ψ_{11} and ψ_{21} to Eqs. (38) through (41) as follows:

$$\phi_{1.,x} + \psi_{1.,y} = \phi_{2.,x} + \psi_{2.,y} \quad , \quad (55)$$

$$\phi_{1.,y} - \psi_{1.,x} = \phi_{2.,y} - \psi_{2.,x} \quad (56)$$

$$\left[\lambda_1 + 2\mu_1 + \frac{\lambda_1' + 2\mu_1'}{\omega} \frac{\partial}{\partial t} \right] \nabla^2 \phi_{1.} - 2 \left[\mu_1 + \frac{\mu_1'}{\omega} \frac{\partial}{\partial t} \right] (\phi_{1.,xx} + \psi_{1.,xy}) =$$

$$\left[\lambda_2 + 2\mu_2 + \frac{\lambda_2' + 2\mu_2'}{\omega} \frac{\partial}{\partial t} \right] \nabla^2 \phi_{2.} - 2 \left[\mu_2 + \frac{\mu_2'}{\omega} \frac{\partial}{\partial t} \right] (\phi_{2.,xx} + \psi_{2.,xy}) \quad (57)$$

$$\left[\mu_1 + \frac{\mu_1'}{\omega} \frac{\partial}{\partial t} \right] \left\{ \nabla^2 \psi_{1.} + 2 [\phi_{1.,xy} - \psi_{1.,xx}] \right\} =$$

$$\left[\mu_2 + \frac{\mu_2'}{\omega} \frac{\partial}{\partial t} \right] \left\{ \nabla^2 \psi_{2.} + 2 [\phi_{2.,xy} - \psi_{2.,xx}] \right\} \quad (58)$$

In the above, the second subscript is replaced by a period to indicate the sum of the incident and respondent potentials; i.e., $\phi_{1.,x} = \phi_{11,x} + \phi_{12,x}$.

The indicated operations are performed and the results evaluated at $y = 0$. The four boundary value equations are then simplified using Eq. (50) and trigonometric identities. Writing these in matrix form results in the following equation:

$$(W) \vec{X} = (P) \vec{Q} \quad (59)$$

$$(W) = \begin{bmatrix} \tan \gamma_{11} & 1 & -\tan \gamma_{12} & -1 \\ -\rho_1 \cos 2\gamma_{21} & \rho_2 \sin 2\gamma_{22} & \rho_2 \cos 2\gamma_{22} & -\rho_1 \sin 2\gamma_{21} \\ \rho_1 \frac{c_{21}^2}{c_{11}^2} \sin 2\gamma_{11} & \rho_2 \cos 2\gamma_{22} & -\rho_2 \frac{c_{22}^2}{c_{12}^2} 2\gamma_{12} & -\rho_1 \cos 2\gamma_{21} \\ 1 & -\tan \gamma_{22} & -1 & \tan \gamma_{21} \end{bmatrix} \quad (60)$$

$$\vec{X} = \begin{bmatrix} Z_{112} \\ Z_{222} \\ Z_{122} \\ Z_{212} \end{bmatrix} \quad , \quad \vec{Q} = \begin{bmatrix} Z_{111} \\ Z_{221} \\ Z_{121} \\ Z_{211} \end{bmatrix} \quad \begin{matrix} (61) \\ (62) \end{matrix} \text{ and}$$

$$(P) = \begin{bmatrix} \tan \gamma_{11} & -1 & -\tan \gamma_{12} & +1 \\ +\rho_1 \cos 2 \gamma_{21} & \rho_2 \sin 2 \gamma_{22} & -\rho_2 \cos 2 \gamma_{22} & -\rho_1 \sin 2 \gamma_{21} \\ +\rho_1 \frac{c_{21}^2}{c_{11}^2} \sin 2 \gamma_{11} & -\rho_2 \cos 2 \gamma_{22} & -\rho_2 \frac{c_{22}^2}{c_{12}^2} \sin 2 \gamma_{12} & +\rho_1 \cos 2 \gamma_{21} \\ -1 & -\tan \gamma_{22} & +1 & +\tan \gamma_{21} \end{bmatrix} \quad (63)$$

In these Eqs. (W) is the matrix of respondent coefficients while (P) is the matrix of incident coefficients. The column vector $\vec{X}$ represents the unknown or respondent displacement potential vector and $\vec{Q}$ represents the known or incident displacement potential vector. All but one element of $\vec{Q}$ is assumed to be zero which corresponds to the case of only one incident wave.

The rows of (W), (P), $\vec{X}$ and $\vec{Q}$ correspond to Eqs. (56), (57), (58), and (55), respectively.

Comparing elements of (W) with those of (P) yields the relation

$$P_{mn} = (-1)^{m+n} W_{mn} \quad . \quad (64)$$

Equation (59) can also be expressed as

$$\sum_{n=1}^4 W_{mn} X_n = \sum_{n=1}^4 (-1)^{m+n} W_{mn} Q_n, \quad m = 1, 2, 3, 4 \quad . \quad (65)$$

In general, to solve Eqs. (59) and (64) for the unknown elements in terms of the known elements, it is necessary to adopt new indices for the known element of $\vec{Q}$ which is non-zero. Let this element be Z_{pq1} , where p takes the place of i and q takes the place of j in the known element. The indices on the unknown elements of $\vec{X}$ remain unchanged.

Equation (59) then becomes

$$(W) \begin{bmatrix} Z_{112} \\ Z_{222} \\ Z_{122} \\ Z_{212} \end{bmatrix} = \begin{bmatrix} (-1)^{1+r} W_{1r} \\ (-1)^{2+r} W_{2r} \\ (-1)^{3+r} W_{3r} \\ (-1)^{4+r} W_{4r} \end{bmatrix} Z_{pq1} \quad (66)$$

where p, q = 1, 2 and $r = -4pq + 7p + 6q - 8$. The relation, r, is such that if p = q = 1 then r = 1; p = q = 2 then r = 2; p = 1 and q = 2 then r = 3; and p = 2 and q = 1 then r = 4. Thus, r selects the correct column of p, depending on the choice of p and q.

Solving Eq. (66) for Z_{ij2} , the quantity $s = -4ij + 7i + 6j - 8$ is introduced where the parameter, s , plays a role similar to r . Using Cramer's rule in Eq. (66), the solution takes the form of the ratio of determinants. The denominator is the determinant of W and the numerator is found by replacing column s of W with the righthand side of Eq. (66). As example of a solution to Eq. (66) for $i = j = 1$ is

$$Z_{112} = \frac{\begin{vmatrix} (-1)^{1+r} W_{1r} Z_{pq1}, & W_{12}, & W_{13}, & W_{14} \\ (-1)^{2+r} W_{2r} Z_{pq1}, & W_{22}, & W_{23}, & W_{24} \\ (-1)^{3+r} W_{3r} Z_{pq1}, & W_{32}, & W_{33}, & W_{34} \\ (-1)^{4+r} W_{4r} Z_{pq1}, & W_{42}, & W_{43}, & W_{44} \end{vmatrix}}{|W|} \quad (67)$$

or

$$\frac{Z_{112}}{Z_{pq1}} = \frac{\sum_{m=1}^4 (-1)^{m+r} (-1)^{m+1} W_{mr} M_{m1}}{|W|} \quad (68)$$

where M_{m1} is the minor of element W_{m1} in the matrix W .

The general solution is obtained by expanding the numerator determinant in minors M_{ms} of column s :

$$\frac{Z_{ij2}}{Z_{pq1}} = \frac{\sum_{m=1}^4 (-1)^{m+r} (-1)^{m+s} W_{mr} M_{ms}}{|W|} \quad (69)$$

This can be simplified by noting that

$$\begin{aligned} (-1)^{m+r} + m+s &= (-1)^{r+s} = (-1)^{-4(pq) - 4(ij)} \\ &\quad + 7p + 7i + 6j + 6q - 16 \end{aligned} \quad (70)$$

and the even powers of (-1) are eliminated since they do not change the sign. Equation (70) is then rewritten as

$$(-1)^{m+r} + m+s = (-1)^{7(p+i)} = (-1)^{p+i} \quad (71)$$

Using this result, Eq. (69) is expressed as

$$\frac{Z_{ij2}}{Z_{pq1}} = \frac{(-1)^{p+i} \sum_{m=1}^4 W_{mr} M_{ms}}{|W|} \quad (72)$$

The values of r and s for the various incident and respondent wave types and media are listed in Table 1.

TABLE 1. Table of Coefficients r and s

Incident			Respondent		
Wave Type	Medium	r	Wave Type	Medium	s
p	q		i	j	
1 - longitudinal	1	1	1	1	1
2 - shear	2	2	2	2	2
1 - longitudinal	2	3	1		3
2 - shear	1	4	2	1	4

Equation (72) is the general equation for the reflection and transmission factors at the interface between the incident medium $k = 1$, and the refraction medium, $k = 2$. It is to be noted that Eq. (72) gives the ratio of unknown to known displacement potentials. Usually, in experimental work, one wishes to know the reflection and transmission factors in terms of pressure ratios since measurements are made with pressure sensitive transducers. The ratios of pressures can be obtained from the displacement potentials by

$$\frac{R_{pq1}}{Z_{pq1}} = \frac{\omega^2 \rho_j Z_{ij2}}{\omega^2 \rho_q Z_{pq1}} = \frac{\rho_j Z_{ij2}}{\rho_q Z_{pq1}} \quad (73)$$

where R_{pq1} and R_{ij2} are the incident and respondent (either reflection or refraction, depending on choice of j) pressures, respectively.

It is also interesting to note that Eqs. (72) and (73) are complex relations involving real and imaginary parts. When using these expressions, it is necessary then to obtain the absolute value of the real and imaginary

parts by taking the square root of the sum of the squares. The phase shift at the interface is represented by the arctangent of the imaginary part divided by the real part of the quantities obtained.

3.2.2 Liquid-Solid Interfaces

At liquid-solid and solid-liquid interfaces test results indicate the amount of energy going into shear waves in the liquid is small. Neglecting this small effect, by allowing slip to occur, we obtain a simplification of the analysis. The simplification is parallel to that given by Ewing et al.⁽¹⁰⁾ where the velocity of shear waves in the liquid approaches zero. Since the generalized Snell's law must always be true, when medium 1 is the liquid we have

$$\lim_{c_{2l} \rightarrow 0} \left(\frac{\cos \gamma_{2l}}{c_{2l}} \right) = \text{complex constant.}$$

Hence, $\gamma_{2l} \rightarrow \pm \frac{\pi}{2}$, $Z_{2l2} \rightarrow 0$, and Eq. (55) is no longer valid since the horizontal component of displacement is not continuous at $y = 0$, where slip is allowed. The second subscript, j , is replaced with a 2 when medium 2 is the liquid. This simplification eliminates a row and a column of (W). The ratio of displacement potentials can then be found by the same method as in the general case.

The solution for these two special cases are given below in terms of the response angles. The subscript j is replaced by a or s which corresponds to liquid or solid media, respectively.

Equations (74), (75) and (76) are for the liquid to solid interface, and Eqs. (77) through (83) are for the solid to liquid case:

$$\frac{Z_{1l2}}{Z_{1l1}} = \frac{\left(\frac{c_{2s}}{c_{1s}} \right)^2 \sin 2\gamma_{2s} \sin 2\gamma_{1s} + \cos^2 2\gamma_{2s} + \frac{\rho_l}{\rho_s} \frac{c_{1l}}{c_{1s}} \frac{\sin \gamma_{1s}}{\sin \gamma_{1l}}}{\Delta}, \quad (74)$$

$$\frac{Z_{1s2}}{Z_{1l1}} = \frac{\rho_l}{\rho_s} \frac{2 \cos 2\gamma_{2s}}{\Delta}, \quad (75)$$

$$\frac{Z_{2s2}}{Z_{1\ell 1}} = -2 \frac{\rho_{\ell}}{\rho_s} \frac{C_{2s}}{C_{1s}} \frac{\sin 2\gamma_{1s}}{\Delta} , \quad (76)$$

$$\frac{Z_{1s2}}{Z_{1s1}} = \frac{\left(\frac{C_{2s}}{C_{1s}}\right)^2 \sin 2\gamma_{2s} \sin 2\gamma_{1s} - \cos^2 2\gamma_{2s} - \frac{\rho_{\ell}}{\rho_s} \frac{C_{1\ell}}{C_{1s}} \frac{\sin \gamma_{1s}}{\sin \gamma_{1\ell}}}{\Delta} , \quad (77)$$

$$\frac{Z_{1\ell 2}}{Z_{1s1}} = \frac{2 \frac{C_{1\ell}}{C_{1s}} \frac{\sin \gamma_{1s}}{\sin \gamma_{1\ell}} \cos 2\gamma_{2s}}{\Delta} , \quad (78)$$

$$\frac{Z_{2s2}}{Z_{1s1}} = \frac{-2 \frac{C_{2s}}{C_{1s}} \cos 2\gamma_{2s} \sin 2\gamma_{1s}}{\Delta} , \quad (79)$$

$$\frac{Z_{2s2}}{Z_{2s1}} = \frac{\left(\frac{C_{2s}}{C_{1s}}\right)^2 \sin 2\gamma_{2s} \sin 2\gamma_{1s} - \cos^2 2\gamma_{2s} + \frac{\rho_{\ell}}{\rho_s} \frac{C_{1\ell}}{C_{1m}} \frac{\sin \gamma_{1s}}{\sin \gamma_{1\ell}}}{\Delta} , \quad (80)$$

$$\frac{Z_{1s2}}{Z_{2s1}} = -\frac{\sin 4\gamma_{2s}}{a} , \quad (81)$$

$$\frac{Z_{1\ell 2}}{Z_{1s1}} = -2 \frac{C_{1\ell}}{C_{1s}} \frac{\sin \gamma_{1s}}{\sin \gamma_{1\ell}} \frac{\sin 2\gamma_{2s}}{\Delta} , \quad (82)$$

$$\Delta = \left(\frac{C_{2s}}{C_{1s}}\right)^2 \sin 2\gamma_{2s} \sin 2\gamma_{1s} + \cos^2 2\gamma_{2s} - \frac{\rho_{\ell}}{\rho_s} \frac{C_{1\ell}}{C_{1s}} \frac{\sin \gamma_{1s}}{\sin \gamma_{1\ell}} . \quad (83)$$

The results of Eqs. (74) through (83) agree with those of Schoch⁽⁷⁾ (recognizing his special coordinates and unknowns) when losses are neglected. Evaluation of these factors requires the use of the generalized Snell's law, Eq. (43).

3.3 GENERALIZED SNELL'S LAW

In experimental work only the real part of the incident angle and medium velocities, v_i , can be measured. The normal attenuations, n , densities and frequency can also be measured. The remaining factors needed to evaluate Eqs. (72) and (74) through (83) can be found by using the generalized form of Snell's law, Eqs. (43) and (14) and (15). This concluding segment of our theoretical section gives the details of these calculations.

To evaluate the response angles, first we can find the parameter d from Eq. (15), where

$$\alpha_{nij} = \omega d_{ij} / v_{ij} \quad . \quad (84)$$

Remember that positive angles are measured counterclockwise from the positive x axis to the arrow end of the vector pointing in the direction of propagation as shown in Figure 1.

Next, c_{ij} is evaluated from Eq. (14),

$$c_{ij} = v_{ij} / (1 + id_{ij}) \quad (85)$$

and then the ratio $c_{ij} \cos \gamma_{11} / c_{11}$ is evaluated as a complex constant, which is used in the generalized Snell's law, Eq. (43):

$$\cos \gamma_{ij} = \frac{(1 + id_{11})}{(1 + id_{ij})} \frac{v_{ij}}{v_{11}} \cos \gamma_{11} = f_{ij} + ig_{ij} \quad (86)$$

We use Eq. (18) to evaluate $A + iB = \gamma$, where the subscripts have been dropped for convenience:

$$\cos (A + iB) = f + ig = h \quad (87)$$

Expand the cosine function

$$\cos A \cosh B - i \sin A \sinh B = f + ig$$

and equate real and imaginary parts:

$$\left\{ \begin{array}{l} \cos A \cosh B = f \\ \sin A \sinh B = -g \end{array} \right\} \quad . \quad (88)$$

Using the trigonometric identity $\cos^2 A + \sin^2 A = 1$, and the hyperbolic identity $\cosh^2 B - \sinh^2 B = 1$, we get

$$\left\{ \begin{array}{l} \frac{f^2}{\cosh^2 B} + \frac{g^2}{\sinh^2 B} = 1 \\ \frac{f^2}{\cos^2 A} - \frac{g^2}{\sin^2 A} = 1 \end{array} \right\} . \quad (89)$$

Solve Eq. (89) by eliminating $\sinh^2 B$, and by eliminating $\sin^2 A$:

$$\left\{ \begin{array}{l} \cosh B = \sqrt{\frac{1 + |h|^2}{2}} + \sqrt{\left(\frac{1 + |h|^2}{2}\right)^2 - f^2} \\ \cos A = \sqrt{\frac{1 + |h|^2}{2}} - \sqrt{\left(\frac{1 + |h|^2}{2}\right)^2 - f^2} \end{array} \right\} . \quad (90)$$

Use of the identities yields

$$\left\{ \begin{array}{l} \sinh B = \pm \sqrt{\frac{|h|^2 - 1}{2}} + \sqrt{\left(\frac{1 + |h|^2}{2}\right)^2 - f^2} \\ \sin A = \mp \sqrt{1 - \frac{|h|^2}{2}} + \sqrt{\left(\frac{1 + |h|^2}{2}\right)^2 - f^2} \end{array} \right\} , \quad (91)$$

The signs of A and B are determined from the geometry and the sign of g . First, $\cosh B \geq 1$ for all B . Second, every wave geometry can be chosen to make A an acute angle. Consequently, $\cos A > 0$. If A is measured counter-clockwise from the positive x axis, $A > 0$. It follows from Eq. (88) that B has a sign opposite the sign of g . Conversely, if A is measured clockwise from the positive x axis, $A < 0$ and Eq. (88) requires that B have the same sign as the sign of g .

Computation of Eqs. (90) and (91) is made more precise if we introduce parameters α and β such that

$$\begin{aligned} \cosh B &= \sqrt{\alpha} + \sqrt{\beta} \\ \cos A &= \sqrt{\alpha} - \sqrt{\beta} \end{aligned} \quad (92)$$

Equation (88) shows that $a - \beta = f$. Equation (90) is used to show

$$\alpha + \beta + 2\sqrt{\alpha\beta} = \frac{1 + |h|^2}{2} + \sqrt{\left(\frac{1 + |h|^2}{2}\right)^2 - f^2}$$

and

$$\alpha + \beta - 2\sqrt{\alpha\beta} = \frac{1 + |h|^2}{2} - \sqrt{\left(\frac{1 + |h|^2}{2}\right)^2 - f^2}.$$

Addition of these equations yields a result which together with the result from Eq. (88) leads to a solution for α and β :

$$\begin{cases} \alpha = \frac{1}{4} \left[(1 + f)^2 + g^2 \right] \\ \beta = \frac{1}{4} \left[(1 - f)^2 + g^2 \right] \end{cases}. \quad (93)$$

Again, introducing parameters γ and δ , we let

$$\begin{cases} \sin A = \pm (\sqrt{\gamma} + \sqrt{\delta}) \\ \sinh B = \pm i (\sqrt{\gamma} - \sqrt{\delta}) \end{cases}. \quad (94)$$

It follows from Eq. (88) that $\gamma - \delta = ig$. Equation (91) is used to show

$$\gamma + \delta + 2\sqrt{\gamma\delta} = \frac{1 - |h|^2}{2} + \sqrt{\left(\frac{1 + |h|^2}{2}\right)^2 - f^2}$$

and

$$-\gamma - \delta + 2\sqrt{\gamma\delta} = -\frac{1 - |h|^2}{2} + \sqrt{\left(\frac{1 + |h|^2}{2}\right)^2 - f^2}$$

Subtraction yields a result which together with $\gamma - \delta = ig$ leads to a solution for γ and δ :

$$\begin{cases} \gamma = \frac{1}{2} \left[\left(\frac{1 - |h|^2}{2} \right) + ig \right] \\ \delta = \frac{1}{2} \left[\left(\frac{1 - |h|^2}{2} \right) - ig \right] \end{cases},$$

However, γ and δ are not easy to compute. To aid in the computation, we introduce a parameter

$$\xi = \tan^{-1} \left[2g / (1 - |h|^2) \right]. \quad (95)$$

Then γ and δ become

$$\left\{ \begin{array}{l} \gamma = \frac{1}{2} \sqrt{\frac{1 - |h|^2}{2} + g^2} e^{i\xi} \\ \delta = \frac{1}{2} \sqrt{\frac{1 - |h|^2}{2} + g^2} e^{-i\xi} \end{array} \right\}$$

Noting that $\left(\frac{1 - |h|^2}{2}\right)^2 + g^2 = \left(\frac{1 + |h|^2}{2}\right)^2 - f^2 = 4\alpha\beta$, we are able to write Eq. (91) with α , β , and ξ :

$$\begin{aligned} \sin A &= \pm 2(\alpha\beta)^{1/4} \cos(\xi/2) \\ \sinh B &= \mp 2(\alpha\beta)^{1/4} \sin(\xi/2) \end{aligned} \quad (96)$$

The signs of A and B were discussed previously following Eq. (91).

Conventionally, Snell's law relates the wave direction-cosines to the velocities of sound in the media. An examination of the generalized Snell's law Eq. (43) reveals a modification of the convention. The generalized Snell's law can be rewritten by introducing the parameters σ and τ equal to the real and imaginary parts of the complex constant:

$$\frac{\cos \gamma}{c} = \sigma + i\tau \quad (97)$$

Expanding with $\gamma = A + iB$ and $c = v/(1 + id)$,

$$\begin{aligned} \frac{\cos A \cosh B + d \sin A \sinh B}{v} &= \sigma, \\ \frac{d \cos A \cosh B - \sin A \sinh B}{v} &= \tau \end{aligned}$$

Suppose we compare the result Eq. (98) with some important direction cosines obtained from $\vec{N}/c$ of Eq. (21). First, we denote the angle between the vectors $\vec{i}$ and $\text{Re}(\vec{N}/c)$ as α_1 . The cosine of that angle can be written

$$\cos \alpha_1 = \vec{i} \cdot \vec{U}_N = \frac{\cos A \cosh B + d \sin A \sinh B}{\sqrt{\cosh^2 B + d^2 \sinh^2 B}}$$

Noting that v^* is the generalized plane wave velocity, we may write the first equation in Eq. (98) as

$$\frac{\cos \alpha_1}{v} = \sigma \quad (99)$$

for all waves. Next we denote the angle between $\vec{n}$ and $\text{Im}(\vec{N}/c)$ by α_2 , and obtain its cosine from

$$\cos \alpha_2 = \frac{\vec{n} \cdot \text{Im}(\vec{N}/c)}{|\text{Im}(\vec{N}/c)|} = \frac{d \cos A \cosh B - \sin A \sinh B}{\sqrt{d^2 \cosh^2 B + \sinh^2 B}}$$

If we define a velocity v' as

$$v' = v / \sqrt{d^2 \cosh^2 B + \sinh^2 B} \quad (100)$$

then

$$\frac{\cos \alpha_2}{v'} = \tau \quad \text{for all waves.} \quad (101)$$

Thus, there are two physical requirements imposed by the generalization of Snell's law. The first relates generalized plane-wave direction-cosines to generalized wave-velocities. The second relates the direction cosines of generalized plane wave attenuation to a type of velocity v' which usually exceeds medium velocity v . Perhaps this mechanism accounts for the diffusion associated with acoustical losses.

In this section we have added attenuation to the classical wave model and studied its effects on the propagation direction and velocity. These two effects were then considered in the solution of the boundary value problem to obtain the reflection and transmission factors. The generalized form of Snell's law was then developed so that the complex propagation constants could be solved for in terms of measured quantities. In the following section we shall demonstrate the application of the model and the effects of attenuation at liquid solid boundaries.

4.0 APPLICATION

Application of the reflection and transmission factors at liquid solid interfaces with attenuation will be demonstrated by the use of illustrative examples. Nickel was chosen for this demonstration. Reflection and transmission factors were also calculated for Al, Cu, Fe and Fe-30 Ni over a range of grain sizes and frequencies. These factors are listed in Appendix B. Appendix B should serve as a convenient guide to reflection and transmission effects as well as attenuation effects. Before proceeding with this demonstration we shall discuss the material constants required for calculating the factors.

4.1 MATERIAL CONSTANTS

The influence of attenuation on reflection and transmission factors can now be evaluated. To do this we must first determine the material constants, V , ρ , and d or α_n . Velocities and densities are easily found by standard methods. The attenuations are, however, much more difficult to obtain. Experimental data for both types of waves are limited. Of these limited data, none have apparently been corrected for shear wave diffraction effects. The correction for longitudinal wave diffraction effects has been the subject of many studies in the literature and calculating techniques are available. Similar methods are not available for shear wave corrections, although an experimental method has been devised.⁽¹¹⁾ Utilization of this new method is in progress but results are as yet limited to one material.

A method⁽¹²⁾ has been developed to estimate the losses due to scattering which is the principal loss mechanism in granular material at ultrasonic frequencies. The method has been shown to predict adequately⁽¹³⁾ the frequency versus attenuation curve in the Rayleigh scattering region. The Rayleigh region is defined by the inequality $\lambda > 2\pi\bar{D}$ where $\bar{D}$ is the average grain diameter. We will now use this method to calculate the attenuations needed to solve for the reflection and transmission factors. The method is as follows:

1. The Rayleigh and stochastic frequencies are calculated from the relationships

$$f_R = \frac{1}{10} \frac{2\pi\bar{D}}{V} \quad , \quad f_S = 10 \frac{2\pi\bar{D}}{V}$$

where V is the velocity of either shear or longitudinal waves.

2. Attenuation at these frequencies is given by

$$\alpha_R = T f_R^4 S \quad , \quad \alpha_S = \bar{D} f_S^2 \sum$$

where T is the average grain volume, R and $\sum$ are the Rayleigh and stochastic scattering factors, respectively, which are given in Reference 13.

3. The Rayleigh and stochastic points are plotted on log paper. Vertical lines are then drawn at the stochastic frequency.
4. The arc of a circle is found such that it passes through the Rayleigh point with a slope of 4 and intersects the stochastic frequency line with a slope of 2. This arc is then used to represent the frequency versus attenuation curve.

For the purpose of demonstration we studied five materials, Al, Cu, Fe, Ni, and Fe-30 Ni and attenuation factors for three hypothetical grain sizes of each were obtained. The necessary parameters and constants were obtained from Reference 14 for the first four materials and from Reference 13 for Fe-30 Ni.

The above information was used to calculate the frequency versus attenuation curves for three grain sizes of the five materials under consideration. Rayleigh and stochastic points for $\bar{D}$ s of 0.001, 0.005, 0.010 are listed in Table 2 for each material. Frequency versus attenuation curves were then constructed as previously prescribed. Examples of three such curves are given in Figure 2. This figure represents the three grain sizes of Ni. Attenuations at 1, 10, and 20 Mtiz for each material, obtained from these curves, are listed in Table 3. These three frequencies were chosen because they encompass the general area of NDT interest. The last column of Table 3 cross references to figures which are discussed

FIGURE 2. Calculated Frequency Versus Attenuation Curves for Nickel. Three Grain Sizes (0.001 in., 0.004 in. and 0.01 in. are Shown.

TABLE 2. Attenuation at Rayleigh and Stochastic Frequencies

	$\bar{D}$, in.	$\alpha_{R\ell}$, (db/ft) $\times 10^{-3}$	$f_{R\ell}$, MHz	α_{Rs} , (db/ft) $\times 10^{-3}$	f_{Rs} , MHz	$\alpha_{R\ell}$, (db/ft) $\times 10^3$	$f_{R\ell}$, MHz	α_{Ss} , (db/ft) $\times 10^3$	f_{Ss} , MHz
Aluminum (Al)	0.001	2.75	4.02	0.985	1.908	0.7376	402.1	9.22	190.8
	0.005	0.55	0.8043	0.197	0.3817	0.1475	80.43	1.844	38.17
	0.010	0.275	0.40214	0.0985	0.198	0.0736	40.21	0.922	19.084
Copper (Cu)	0.001	77.1	3.133	26.6	1.426	16.64	313.3	248.8	142.6
	0.005	15.4	0.6266	5.328	0.2852	3.328	62.66	49.77	28.53
	0.010	7.7	0.3133	2.66	0.1426	1.664	31.33	24.88	14.26
Iron (Fe)	0.001	35.54	3.733	14.14	2.018	18.66	373.3	132.4	201.8
	0.005	7.108	0.7465	2.83	0.4036	3.732	74.65	26.48	40.36
	0.010	3.554	0.3733	1.414	0.2018	1.866	37.33	13.24	20.18
Iron-30 Nickel (Fe-30 Ni)	0.001	71.9	2.843	33.0	1.732	65.07	284.3	297.3	173.2
	0.005	14.4	0.5685	6.59	0.3464	13.01	56.85	59.45	34.64
	0.010	7.19	0.2843	3.3	0.1732	6.507	28.43	29.73	17.32
Nickel (Ni)	0.001	48.0	3.78	17.7	1.87	16.2	378.0	166.0	187.0
	0.005	9.6	0.757	3.54	0.375	3.24	75.7	33.3	37.5
	0.010	4.8	0.378	1.77	0.187	1.62	37.8	16.6	18.7

TABLE 3. Attenuation at 1, 10 and 20 MHz

Material	Grain Size, in.	$\alpha=0$, Fig. No.	1 MHz			10 MHz			20 MHz		
			α_l db/ft	α_s db/ft	Fig. No.	α_l db/ft	α_s db/ft	Fig. No.	α_l db/ft	α_s db/ft	Fig. No.
Al	0.001	B-1	$1. \times 10^{-6}$	$6. \times 10^{-5}$		0.072	0.235		1.9	4.1	
	0.005		0.00125	0.008		1.4	3.1		7.2	14.5	
	0.010		0.007	0.023		3.7	6.6		18.	28.	B-2
Cu	0.001	B-3	0.00025	0.005		4.7	14.		102.	260.	B-4
	0.005		0.009	0.38		64.	130		310	570.	B-5
	0.010		0.43	1.5	B-6	176.	350.	B-7	800.	1400.	B-8
Fe	0.001	B-9	3.4×10^{-5}	$6. \times 10^{-4}$		1.2	2.95		10.	18.8	
	0.005		0.00235	0.00		21.	35.		113.	170.	B-10
	0.010		0.11	0.295		53.	85.	B-11	255.	370.	B-12
Fe-30 Ni	0.001	B-13	$04. \times 10^{-4}$	$3. \times 10^{-3}$		5.7	10.6		41.	67.	B-14
	0.005		0.11	0.255		77.	110.		380.	510.	B-15
	0.010		0.54	0.83	B-16	193.	237.	B-17	840.	970.	B-18
Ni	0.001	10	4.6×10^{-5}	$1. \times 10^{-3}$		1.6	48.		13.	31.	B-19
	0.005		0.027	0.11		26.5	51.		140.	240.	B-20
	0.010		0.15	0.44	B-21	76.	130.	B-22	330.	550.	4a,b,c

later in the report. Velocities and densities necessary for these calculations are also listed in Table 4. This information was obtained from References 13 and 14. Poisson's ratio is included here for reference only.

TABLE 4. Velocities and Densities

<u>Material</u>	<u>Longitudinal Velocity x 10⁵ in./sec</u>	<u>Shear Velocity x 10⁵ in./sec</u>	<u>Density lb/in.³</u>	<u>Poisson's Ratio</u>
Al	2.527	1.2	0.0975	0.355
Cu	1.969	0.8962	0.324	0.366
Fe	2.345	1.268	0.285	0.2955
Fe-30 Ni	1.79	1.09	0.3945	0.2055
Ni	2.377	1.178	0.322	0.337
H ₂ O	0.587	---	0.03613	---

4.2 CALCULATION OF REFLECTION AND TRANSMISSION FACTORS

Transmission and reflection factors which include attenuation may now be calculated using the above information. A program has been written in Fortran IV for the Hanford Univac 1108 computer for this purpose. This program is actually the solution of the four by four matrix, Eq. (72). This is the general case and is capable of handling all possible combinations of liquid and solid interfaces. The solutions obtained as stated in the section on theory are the ratio of displacement potentials and should not be confused with pressure or energy ratios.

For the special case under study (metal plates immersed in water) B_{ij} in water is set equal to zero. This assumes that there is a source of plane waves in the water. Waves refracted into the attenuating metal would then have B_{ij} values dependent on their incident angle and the attenuation in both media.

In these calculations when an incident wave originates in a solid, B_{ij} of the respondent wave in water is again set equal to zero. B_{ij} factors in the solid are found as in Section 3.2. This assumes that there are generalized plane waves in the metal which become plane waves upon being transmitted into the water. The reason for this is that in most practical NDT applications the wave first starts as plane, then becomes a generalized plane wave in the metal and, upon being transmitted again into the water, returns to a plane wave.

Solutions for reflection and transmission factors generated by the program are in terms of the angle θ_w , where θ_w is the angle of propagation in water, as shown by Figure 3. With this arrangement, the refraction angle in the metal need not be of primary concern to determine reflection and transmission factors. The advantage of this system is that one can trace a beam through several reflections and mode conversions using only the angle θ_w . Phase velocities and refraction angles in the metal are also calculated by the program but are omitted from this report. If one

FIGURE 3. Test Conditions and Coordinate System

wants to find the refraction angles, the ordinary form of Snell's law may be used with certain limitations. The velocity v^* is approximately equal to the normal velocity below the critical angle (see Appendix A). Calculating v^* for various conditions, it can be shown that for $\alpha_n \leq 2,000$ db/ft and angles at least 1° less than critical, an error of less than 0.2% could be expected in using the usual Snell's law. Conditions beyond these limits require the use of the complex form.

4.3 HIGH ATTENUATION EXAMPLE

Reflection and transmission factors for many of the cases listed in Table 2 have been calculated and are graphically presented in Appendix B. Representative examples are also included in the body of this report to demonstrate their significant details and applications. The first metal examined was Ni with average grain diameter of 0.01 in. A frequency of 20 MHz was used and the three incident wave conditions: longitudinal in water, shear and longitudinal in the metal, are shown in Figures 4a, b, and c, respectively (see Figures 4 through 7 at the end of this section). The three respondent waves for each are labeled R , R_s , T , and T_s to indicate reflected longitudinal and shear waves and transmitted longitudinal and shear waves, respectively.

In Figure 4a the incident wave is longitudinal at the water-metal interface. The respondent waves R , T , and T_s are shown. The following significant details should be noted:

1. For $\theta_w = 0^\circ$, $T_s = 0$ as there is no mode conversion to shear waves at normal incidence.
2. The longitudinal critical angle occurs at 14.3° which is evidenced by the maximum in R_ℓ and T , and a minimum in T_s .
3. At this critical angle, α_c (for longitudinal waves) increases rapidly thus resulting in a severely damped generalized plane wave traveling near the surface (see Appendix A).
4. T approaches zero at 20.5° . T_s has a refraction angle of 45° . This particular condition is a useful tool in NDT and will be discussed further in a later section of this report.

5. The maximum in R , at 29.8° corresponds to the shear critical angle. Conditions similar to item three apply here also.
6. T , and T_s reach a maximum of 3 and 4.7, respectively, and R is a minimum at 32.2° . This corresponds to the Rayleigh critical angle.
7. R approaches one, while T_ℓ and T_s approach zero for angles greater than 32.2° .

At first it may appear as if the laws of conservation of energy are violated at the Rayleigh critical angle, but this is not the case if the following arguments are considered:

- The ratio of displacement potentials is calculated at the $y = 0$ interface.
- The crest damping factor α_c is very large at this angle even for materials of low attenuation (see Appendix A). The displacement, and therefore the energy will be much greater at the surface. Intuitively, one can see that while the energy density may be larger at the surface, the integral over the entire beam will be smaller than the total incident energy.
- T , and T_s are 90° out of phase and can trade energy back and forth or resonate. This results in elliptic particle motions where the direction of the rotation depends on the distance from the boundary. (15)

For the same conditions as above, reflection and transmission factors are calculated for incident shear waves at a metal-to-water interface. These are shown in Figure 4b. Remember here that the horizontal axis, θ_w , is the refraction angle in water, not the incident angle. The following significant details should be noted:

1. Shear waves are totally reflected at normal incidence.
2. The only minimum in R_s occurs at 13° . For materials with Poisson's ratio less than 0.26, such as Fe-30 Ni, this minimum reaches zero. (16)
3. For $\theta_w = 14.3$, R is a maximum and T , a minimum. This corresponds to the longitudinal critical angle.
4. R equals zero and R_s approaches unity at 20.5° .

5. The shear critical angle is indicated by the minimums in R_ℓ and T_ℓ at 29.8".
6. The sharp increase in the factors near and above the Rayleigh critical angle may be explained in the same manner as the previous case.

Incident longitudinal waves for this same condition behave in a similar manner and are shown in Figure 4c. Here both T_ℓ and R_S are minimums at 14.3" and 20.5". R_ℓ is also a minimum at 13° similar to item 2 above.

It was not possible in Figures 4b and 4c to show the behavior of the solutions past the Rayleigh critical angle. For this purpose Figures 5 and 6 present this information on a compressed scale. When considering these large values one should, as stated previously, consider the high degree of crest damping α_1 .

4.4 ZERO ATTENUATION CASE

As stated previously, the solutions of Eqs. (74) through (82) are the same as the classical case, which ignores attenuation, if α_n is set equal to zero. Figures 7a, b, and c represent the zero attenuation solutions for Ni of arbitrary grain size and frequency. In contrast to the classical case, these solutions are valid beyond the critical angles.

Comparing Figures 4 and 7 we can ascertain the principal differences and possible errors which arise when attenuation is neglected. The differences are as follows:

1. Water to metal interface (Figures 4a and 7a).
 - a. The minimum in T_S at 14.3" is much sharper and reaches zero.
 - b. Maximums and inflection points are much sharper for the zero attenuation case at the shear and longitudinal critical angles.
 - c. The minimum in R_ℓ at the Rayleigh critical angle does not occur for the nonattenuative case.
2. Metal to water interface for incident shear waves (Figures 4b and 7b).
 - a. The minimum in Figure 4b of T_ℓ and R_ℓ are sharper and reach zero.
 - b. R_S reaches unity at 14.3° and is flat from 16" to 20" for nonattenuative case.

- c. The maximum value of R_ℓ is slightly larger for the nonattenuative case 3.34 versus 3.1.
- 3. Metal to water interface for incident longitudinal waves (Figures 4c and 7c).
 - a. Minimums in T_ℓ and R_S are sharper and reach zero.
 - b. R_ℓ is equal to unity from 14.3° to 29.6° for the nonattenuative case.

Figures 7a, b and c are comparable to those given in Reference 4 if conditions stated previously concerning energy and existence of complex solutions are taken into account.

Failure to predict the reflection minimum at the Rayleigh critical angle, which has been observed by many researchers, is recognized as a failure of the classical model.⁽⁷⁾ As part of another study^(17,18) we obtained correlation between the measured values and that obtained by Eq. (74), in the area of this minimum. The conclusions of that study are listed below:

1. Depth of the minimum depends on attenuation.
2. The depth increases with increasing attenuation up to some critical value, then decreases.
3. The critical attenuation value is different for different materials and/or differing conditions for the same material.
4. Position of the minimum depends on the Rayleigh velocity, which in turn is dependent on the longitudinal and shear velocities.

4.5 SAMPLE APPLICATIONS

Applications of these reflection and transmission factors will be demonstrated through three examples. The hypothetical test configuration is the same as Figure 3. Mediums 1 and 3 are water, while medium 2 is Ni with an average grain diameter of 0.01 in. For a test frequency of 20 MHz, the applicable factors are obtained from Figure 4.

Example 1

The incident angle, θ_w , is 10° and the path is from medium 1 and 2 to 3 for longitudinal waves in all cases. From Figure 4a, T_ℓ at the 1-2 interface is 0.23 for $\theta_w = 10^\circ$. The angle of refraction θ_w at the 2-3 interface is also 10° . From Figure 4c, T_ℓ is then equal to 0.35. Since at each surface the incident wave was normalized to unity, the total insertion loss is the product of the factors, $0.23 \times 0.35 = 0.0805$. Attenuation and diffraction corrections could now be applied to calculate the relative amplitude of the received signal in medium 3.

Example 2

This example examines the other possible modes for the conditions of Example 1. For shear waves in medium 2 along the path 1-2-3, the factors T_s and T_ℓ are found from Figures 4a and 4b, respectively: $T_s \times T_\ell = 0.07 \times 0.3 = 0.021$.

For the path 1-2-1 four different modes are possible. The possible combinations are listed below:

1. $T_s \times R_s \times T_\ell$ from Figure 4a, c, and c is $0.23 \times 0.51 \times 0.35 = 0.0411$,
2. $T_\ell \times R_s \times T_\ell$ from Figure 4a, b, and b is $0.07 \times 0.57 \times 0.3 = 0.01197$,
3. $T_\ell \times R_s \times T_\ell$ from Figure 4a, c, and b is $0.225 \times 0.49 \times 0.31 = 0.0342$,
4. $T_s \times R_\ell \times T_\ell$ from Figure 4a, b, and c is $0.07 \times 1.32 \times 0.36 = 0.0333$.

It is interesting to note that, as expected, items (3) and (4) are equal within the accuracy of the graph. In fact, if the digital printout from the computer is used, the two are the same to seven significant figures.

Example 3

An interesting phenomenon occurs for an incident angle of 20.5° along the path 1-2-1. This is greater than the longitudinal critical angle. The four possible factors are shown below:

1. $T_\ell \times R_\ell \times T_\ell$ from Figure 4a, b, and b is $0.22 \times 0.92 \times 0.73 = 0.148$,
2. $T_\ell \times R_\ell \times T_\ell$ from Figure 4a, c, and c is $0.0 \times 1 \times 0.02 = 0$,
3. $T_\ell \times R_S \times T_\ell$ from Figure 4a, c, and b is $0.0 \times 0.02 \times 0.73 = 0$,
4. $T_S \times R_\ell \times T_\ell$ from Figure 4a, b, and c is $0.22 \times 0 \times 0.02 = 0$.

From items (2), (3) and (4) above we can see that not only is the total factor zero but that there are no longitudinal waves of any type in the metal. The above conditions are the basis of the 45" shear wave test. As the name implies, the refracted shear wave makes an angle of 45" with the vertical. The total loss for this test is found by taking the product $(T_S)(n)(R_S)(T_\ell)$ in the same manner as item (1), when n is the total number of reflections at the metal to water interface.

FIGURE 4a. Reflection and Transmission Factors for Nickel with Average Grain Diameter of 0.01 in. at a Frequency of 20 MHz. (The Incident Wave is Longitudinal at the Water to Metal Interface.)

FIGURE 4b Same Conditions as Figure 4a for Incident Shear Waves at the Nickel to Water Interface

FIGURE 4c. Same Conditions as Figure 4a for Incident Longitudinal Waves at the Nickel to Water Interface

FIGURE 5. Same as Figure 4b Except for Scale Changes

FIGURE 6 Same as Figure 4c Except for Scale Changes

FIGURE 7a. Reflection and Transmission Factors for Nickel Ignoring Attenuation. (The Incident Wave is Longitudinal at the Water to Metal Interface.)

FIGURE 7b. Same as Figure 7a Except for Incident Shear Waves at the Nickel to Water Interface.

FIGURE 7c. Same as Figure 7a Except for Incident Longitudinal Waves at the Nickel to Water Interface

5.0 CONCLUSIONS

The most significant effect of attenuation on the reflection and refraction of ultrasonic waves occurs at the Rayleigh critical angle. The accuracy with which this minimum is predicted by the model^(17,18) provides considerable support to its validity. The model also describes the propagation of ultrasound at angles above critical, which is not possible with most available models.

The influence of attenuation on reflection and transmission factors has been ignored or dismissed without proof by most researchers. Thus, the validity of a large amount of work can be questioned. We have shown through our calculations and examples that these are valid assumptions only when the angle of incidence is well below critical. For example, at normal incidence there is no effect at all regardless of the magnitude of attenuation. At angles other than normal, the factors are influenced by attenuation. The effect depends on the magnitude of the attenuation but is significant only between approximately 2° below the longitudinal critical angle up to grazing incidence.

Further studies and verifications of the wave model are continuing. The method will be further expanded to include experimentally measured attenuation data on typical industrial materials, as this information becomes available. Considerable effort is also being applied to the theory and application of this model at the Rayleigh critical angle.

APPENDIX A

THE EFFECT OF VARIABLES ON REFLECTION AND TRANSMISSION FACTORS

APPENDIX A

THE EFFECT OF VARIABLES ON REFLECTION AND TRANSMISSION FACTORS

The concept of generalized or inhomogeneous plane waves, which is described in Section 3 of this report, is not discussed in any detail in the existing literature. To obtain a better idea of the magnitude of these effects we shall analyze a few practical examples. In this analysis we shall examine the effects of normal attenuation, α_n , and incident angle, θ , on the parameter B , which in turn determines the phase velocity v^* and the crest attenuation α_c . B is the imaginary part of the angle from Eq. (18). For the purpose of demonstration, 304L stainless steel is chosen. The necessary parameters are obtained from Reference 6 and are as follows:

1. Longitudinal velocity $v_{12} = 2.264 \times 10^5$ in./sec.
2. Shear velocity $v_{22} = 1.237 \times 10^5$ in./sec.
3. α_n at 0.5 MHz is 4.5×10^{-3} db/ft for longitudinal waves, and 9.5×10^{-3} db/ft for shear waves.
4. α_n at 15 MHz is 350 db/ft for longitudinal waves, and 1630 db/ft for shear waves.
5. The incident medium is water.

This is a very limited range of parameters but should serve to demonstrate some of the effects of attenuation on plane waves.

Figure A-1 is the parameter B_{22} , (shear waves in medium 2), plotted against the incident angle, e , for both 0.5 and 15 MHz. For the low attenuation case, B_{22} is equal to zero up to the shear critical angle then increases rapidly. The high attenuation case increases gradually from zero at normal incidence approaching the same value as the previous case above the critical angle. Longitudinal wave parameters B_{12} behave in a similar manner.

FIGURE A-1. B_{22} Versus the Incident Angle θ for a Water to Metal Interface. (The Incident Wave is Longitudinal and B_{22} Refers to the Refracted Shear Wave. The Metal is 304 Stainless Steel. Results for Both 0.5 and 15 MHz are Shown. Note Scale Change at $B_{22} = 0.4$.)

The phase velocities v_{12}^* and v_{22}^* in the metal can be found from Eq. (2) when B is known. Figure A-2 shows both of these phase velocities for a frequency of 15 MHz. The curve for 0.5 MHz is identical to Figure A-2 except for a slight increase in the sharpness of the transition at the critical angles. The phase velocities v^* , for all practical purposes, remains constant and equal to the measured velocity v up to the critical angle. It then decreases sharply and asymptotically approaches the longitudinal velocity in medium one, in this case water.

Snell's law in the general form as given by Eq. (43) can be reduced to:

$$\sin \theta_2 = \frac{v_2^*}{v_1} \sin \theta_1 \quad (\text{A-1})$$

for the case in which the incident wave is homogeneous, i.e., $B_1 = 0$. The identities

$$\theta_1 = \pi/2 + \gamma_1 \quad (\text{A-2})$$

and

$$\theta_2 = \pi/2 + \gamma_2 \quad (\text{A-3})$$

were used to obtain Eq. (A-1) in this form; where θ is the usual positive angle measured from the vertical as in Figure 3. From Figure A-2 it is apparent that as θ_1 increases past what is normally considered the critical angle, v_2 decreases, thus preventing $\sin \theta_2$ from reaching unity. This intuitive argument also corresponds with more rigorous calculations. These calculations indicate that the greater the attenuation, the smaller the upper limit of θ_2 .

Crest attenuation α_c can now be calculated using Eqs. (41) and (42) and the above information. Figure A-3 depicts the effects of α_c on a plane wave at an interface. Figure A-4 graphs the calculated value of α_c versus incident angle for longitudinal and shear waves at both 0.5 and 15 MHz; α_c is shown on the vertical scale in nepers per inch, one neper equals 8.686 db. The lower two curves are for the 0.5 MHz case while the upper two represent the 15 MHz case. Wave fronts for the low attenuation case are

FIGURE A-2. Longitudinal and Shear Wave Phase Velocities in 304 Stainless Steel at the Water to Metal Interface

FIGURE A-3. Homogeneous and Inhomogeneous Plane Waves

FIGURE A-4. α_c Versus Incident Angle θ for Same Conditions as Figure A-2. (Both Longitudinal and Shear Waves are Shown.)

approximately homogeneous up to their respective critical angle; α_c then increases rapidly approaching an asymptotic value at 90". For the higher attenuation case the wave fronts are inhomogeneous for all except normal incidence.

It is easy to see from Figure A-4 that above the critical angle all wave front crests are highly damped. For refracted waves traveling near the surface, this damping precludes significant penetration of energy below the surface.

It is more interesting to note that at 30.8° (the Rayleigh critical angle) the ratio between a_r for 0.5 and 15 MHz for both longitudinal and shear waves is 30, which is also the ratio of the wave lengths. This would indicate, at least for these examples, that the depth of penetration depends principally upon the wave length, not upon attenuation.

Since this discussion covers only a small range of material parameters, further investigations over a broad range of materials and frequencies are needed to better establish the character and magnitude of these effects.

APPENDIX B

REFLECTION AND TRANSMISSION FACTORS FOR
Al, Cu, Fe, Fe-30 Ni and Ni

APPENDIX B

REFLECTION AND TRANSMISSION FACTORS FOR
Al, Cu, Fe, Fe-30 Ni, and Ni

Reflection and transmission factors for each material and test condition listed in Table 2, (Al, Cu, Fe, Fe-30 Ni and Ni with grain sizes of 0.01, 0.004, and 0.001 in. at frequencies of 1, 10 and 20 MHz), were calculated. Frequencies and grain sizes which exhibit significant differences are shown in this Appendix. Zero attenuation cases were also calculated and are included here for comparison.

The factors are presented in the same form as Figures 4 through 7. The horizontal axis θ_w is the angle in water (see Figure 3) for all incident conditions. Reflection and transmission factors are depicted on the vertical axis. Each of the three conditions are shown on one page. Headings for each page indicate the material, grain size and frequency. Other necessary parameters may be obtained from Tables 3 and 4, which are reproduced for this discussion.

Reflection and transmission factors less than 1.2 and for angles from zero to 45° of θ_w are displayed in the graphs contained in this Appendix. This compromise was chosen to display the greatest amount of usable information in a limited area and still retain the fine structure detail. Important details which are eliminated are noted on the figure. The general shape of these omitted sections are similar to those shown in Figures 4 through 7.

The analysis of critical angle and inflection points shown by these factors follows the same pattern as that given in Section 4.3; the only exception being Fe-30 Ni. Two additional zeros occur in R_s for incident shear waves and in R_ϕ for incident longitudinal waves at the metal to water interface. This will be the case for all materials with a Poisson's ratio less than 0.26. (16)

TABLE 3. Attenuation at 1, 10, and 20 MHz

<u>Material</u>	<u>Grain Size, in.</u>	<u>$\alpha=0$, Fig. No.</u>	<u>1 MHz</u>			<u>10 MHz</u>			<u>20 MHz</u>		
			<u>α_l db/ft</u>	<u>α_s db/ft</u>	<u>Fig. No.</u>	<u>α_l db/ft</u>	<u>α_s db/ft</u>	<u>Fig. No.</u>	<u>α_l db/ft</u>	<u>α_s db/ft</u>	<u>Fig. No.</u>
Al	0.001	B-1	$1. \times 10^{-6}$	$6. \times 10^{-5}$		0.072	0.235		1.9	4.1	
	0.005		0.00125	0.006		1.4	3.1		7.2	14.5	
	0.010		0.007	0.023		3.7	6.6		18.	28.	B-2
Cu	0.001	B-3	0.00025	0.005		4.7	14.		102.	260.	B-4
	0.005		0.009	0.38		64.	130		310	570.	B-5
	0.010		0.43	1.5	B-6	176.	350.	B-7	800.	1400.	B-8
Fe	0.001	B-9	3.4×10^{-5}	$6. \times 10^{-4}$		1.2	2.95		10.	18.8	
	0.005		0.00235	0.007		21.	35.		113.	170.	B-10
	0.010		0.11	0.295		53.	85.	B-11	255.	370.	B-12
Fe-30 Ni	0.001	B-13	$04. \times 10^{-4}$	$3. \times 10^{-3}$		5.7	10.6		41.	67.	B-14
	0.005		0.11	0.255		77.	110.		380.	510.	B-15
	0.010		0.54	0.33	B-16	193.	237.	B-17	840.	970.	B-18
Ni	0.001	10	4.6×10^{-5}	$1. \times 10^{-3}$		1.6	48.		13.	31.	B-19
	0.005		0.027	0.11		26.5	51.		140.	240.	B-20
	0.010		0.15	0.04	B-21	76.	130.	B-22	330.	550.	4a,b,c

TABLE 4. Velocities and Densities

<u>Material</u>	<u>Longitudinal Velocity</u> <u>x 10⁵ in./sec</u>	<u>Shear Velocity</u> <u>x 10⁵ in./sec</u>	<u>Density</u> <u>lb/in.³</u>	<u>Poisson's</u> <u>Ratio</u>
Al	2.527	1.2	0.0975	0.355
Cu	1.969	0.8962	0.324	0.366
Fe	2.345	1.268	0.285	0.2955
Fe-30 Ni	1.79	1.09	0.3945	0.2055
Ni	2.377	1.178	0.322	0.337
H ₂ O	0.587	---	0.03613	---

FIGURE B-1. A1, Zero Attenuation

Reflection and Transmission Factors

FIGURE B-2. A1, 0.01 in., 20 MHz

FIGURE B-3. Cu, Zero Attenuation

FIGURE B-4. Cu, 0.001 in., 20 MHz

FIGURE B-5. Cu, 0.005 in., 20 MHz

FIGURE B-6. Cu, 0.01 in., 1 MHz

FIGURE B-7. Cu, 0.01 in., 10 MHz

FIGURE B-8. Cu, 0.01 in., 20 MHz

FIGURE B-9. Fe, Zero Attenuation

Reflection and Transmission Factors

FIGURE B-10. Fe, 0.005 in., 20 MHz

Reflection and Transmission Factors

FIGURE B-11. Fe, 0.01 in., 10 MHz

FIGURE B-12. Fe, 0.01 in., 20 MHz

FIGURE B-13. Fe-30 Ni, Zero Attenuation

FIGURE B-14. Fe-30, 0.001 in., 20 MHz

FIGURE B-15. Fe-30 Ni, 0.005 in., 20 MHz

Reflection and Transmission Factors

FIGURE B-16. Fe-30 Ni, 0.01 in., 1 MHz

FIGURE B-17. Fe-30 Ni, 0.01 in., 10 MHz

FIGURE B-18. Fe-30 Ni, 0.01 in., 20 MHz

FIGURE B-19. Ni, 0.001 in., 20 MHz

FIGURE B-20. Ni, 0.005 in., 20 MHz

FIGURE B-21. Ni, 0.01 in., 1 MHz

FIGURE B-22. Ni, 0.01 in., 10 MHz

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