Some remarks on feedback and feedforward employed to reduce beam induced voltages

(Mini-Workshop on RHIC RF Systems)

July 11-15, 1988
Collider Center

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7/14/88
\[ V_c = [\dot{z}_b + \dot{z}_g] z_c = [\dot{z}_b - AV_c] z_c \]

\[ i_g = -AV_c \]

\[ V_c = \frac{-\dot{i}_b z_c}{1 + AZ_c} = \frac{-\dot{i}_b 2\sigma s R}{s^2 + 2\sigma s + \omega_r^2 + 2\sigma A \omega_c e^{-s_2}} \]

And one needs the roots of \((s^2 + 2\sigma s + \omega_r^2 + 2\sigma A \omega_c e^{-s_2})\)

\[ \sigma = \frac{\omega_r}{2Q} \]

\[ \omega_c \] is loop delay, \(AR = G\) (gain)

Put \(s = \alpha + j\omega\) and solve for roots with \(\alpha = 0\)

\[-\omega^2 + \omega_r^2 + 2\sigma \omega G \sin \omega \alpha + 2\sigma j \omega [1 + G \cos \omega \alpha] = 0\]

\[ G \cos \omega \alpha = -1 \]

\[ G \sin \omega \alpha = \frac{\omega^2 - \omega_r^2}{2\sigma \omega} \]

\[ \tan \omega \alpha = \frac{\omega_r^2 - \omega^2}{2\sigma \omega} \quad \text{or} \quad \omega \alpha = \arctan\left(\frac{\omega_r^2 - \omega^2}{2\sigma \omega}\right) \]

\[ F(x) = \omega_r x \]

With \(G^2 - 1 = [x^2 - 1]^2 Q^2\) put \(x = 1 + \delta x\) to obtain

\[ G^2 - 1 = \left(\frac{x^2}{2\delta x + \delta x^2}\right) Q^2 \]

Which for \(G \gg 1, \delta x \ll 1\) gives

\[ G \approx 2 \delta x Q \]

From Graph we have \(\omega_r \gg \delta x \approx \pi / 2\)

So

\[ G \approx \frac{2 \pi \omega_c}{2 \omega_r} = \frac{\pi \omega_c}{\omega_r} \]
Thus for \( \frac{G}{2} = \frac{\pi}{4} \frac{\omega}{\omega_r} = \frac{\pi}{4} \frac{2Q}{\omega_r} \frac{1}{\xi} = \frac{2Q}{\pi} \sqrt{\frac{Q}{\xi}} \),

or \( \frac{1}{A} = R_{\text{min}} = \frac{2}{\pi} \frac{\omega_r R}{\xi} \).

Now in order to obtain the the roots for \( G < G_{\text{max}} \), one must plot the curves

\[
\frac{2S + G}{s^2 + 2\sigma s + \omega_r^2} = -e^{j\varphi}
\]

for different values of \( G \) and \( 180^\circ \leq \varphi \leq 90^\circ \) and find the intersection at the desired value of \( G \) with the line \( e^{j\varphi} \) such that \( \omega_r + \varphi = 180^\circ \). There will generally be singularity with \( \varphi < 0 \) in addition to conjugate roots with \( \omega < \omega_r \) and \( |\xi| > 0 \).
Also that will be conjugate roots at \( \omega_n \), the \( \omega_n \) where \( \omega_n = 2\pi / \tau \) for \( \tau > 1 / \gamma \) with \( K1 \gg \sigma \) and \( \gamma = 1/2 \)-

(thi assumes that \( G \) does not vary with \( \omega \) of course)

Next let us include a feedforward signal to the configuration. We then will have \( i_g = -AV_c + ic \)

where \( ic \) is a current \( \approx 0 \) so that one has

\[
V_c = \frac{(i_c - i_b) Z_c}{1 + AV_c e^{-s\tau}}
\]

Thus the stability condition are not altered and \( V_c \) can be made to approach zero even more closely for the same \( A \).

Finally let us redraw our circuit and include the drift signals required for the storage mode (ignoring phase and tuning loops)

\[
A = K(V_{res} - \epsilon V_c)
\]

During acceleration the amplitude control loop is open and \( A = A_{max} \& V_c \) is small (1-2 K\( \nu \)). At transfer the \( \epsilon \) feedback is switched off the amplitude loop is closed and \( V_c \) with the proper phase and amplitude is applied. \( \epsilon_c \) which contains the beam current at \( h = 2052 \).
Several rotation sidebands remain unchanged since it is required to compensate for the periodic transient beam loading due to the missing bunches.

The tuning and phase (average of all bunches) loops are also activated at this time.

We assume that during acceleration \( \omega_r = \omega_0 \)

i.e. that the cavities are triangular programmed to

track the \( \sim 480 \text{ Kc} \) frequency change that would

occur for gold at top energy.

The gain-bandwidth of the amplitude loop can be considerably less here than that needed for the \( \frac{1}{8} \) feedback loop.

It is evident now why we did not choose to add a compensation signal \( \frac{1}{14} \) at the \( V_s \) input

(it must be independent of the gain control loop during storage!)
Feedforward With Amplitude Loop

\[ V_{in} \rightarrow A \rightarrow G(s) \rightarrow V_c \]

\[ V_{res} \rightarrow K \rightarrow \epsilon |V_c| \rightarrow A \rightarrow -V_b \]

\[ A_0 V_{in} = V_o = V_b \]

\[ V_c = G(s) (A V_{in} - V_b) = G(s) V_o - K e^{s \zeta} \frac{V_c}{V_{in}} \]

or \[ |V_c| = \frac{G(s) [V_o - V_b]}{1 + K e^{s \zeta} G(s) e^{-s \zeta}} \]

Hence \[ |V_c| \sim \frac{G(s) [V_o - V_b]}{1 + K e^{s \zeta} G(s) e^{-s \zeta}} \]

Put \[ \epsilon = \frac{V_b}{V_{res}} \] and assume cavity is tuned so that \[ G(s) = \frac{1}{1 + s \zeta} \] i.e. the transfer function for Amplitude modulation of the carrier. Then one has

\[ |V_c| = \frac{(V_o - V_b)}{1 + s \zeta + K \epsilon e^{-s \zeta}} \]

Again put \[ s = j \omega \] with \( \alpha = 0 \) and solve

\[ K \tan \omega \zeta = -1 \quad K \sin \omega \zeta = \omega \zeta \]
and
\[ \tan \omega \gamma = - \omega \kappa \gamma \]
\[ K' = 1 + \omega^2 \kappa^2 \]

Now we know that \( G(s) \) can at most put in a \( \pi/2 \) phase shift in the loop so that the stability limit will occur when \( \omega \gamma \approx \pi/2 \). Since \( \gamma \gg \gamma \),
\[ \omega^2 \gamma^2 = \left(\frac{\pi}{2}\right)^2 \left(\frac{\gamma}{\kappa}\right)^2 \]
so that
\[ K_{max} = \frac{\pi \gamma}{2 \kappa} \]

which is the same result as for \( \kappa \) feedback except that \( \gamma \) is not required to be a multiple of the \( \kappa \) period.

In principal \( V_{in} \) can contain the components of \( i_0 \) at \( k = 2052 \pm m \) where \( m \leq 10 \) as well as \( \pi/2 \) of \( i_0 \). Again one would want the cavity always in resonance with 2052 so and when transferring the beam from the 26.7 MHz system excitation of the cavity would be as outlined above. Of course, the amplitude loop shown here would have to be disabled at the time. We note here that the gain bandwidth of the loop would in general be much greater than that required to control the storage voltage levels.