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Y-1594
Engineering and Equipment

AUTOMATIC CENTERING INSPECTION MACHINE

Rasey R. Feezell

MASTER

UNION CARBIDE CORPORATION
NUCLEAR DIVISION
OAK RIDGE Y-12 PLANT

operated for the ATOMIC ENERGY COMMISSION under U. S. GOVERNMENT Contract W-7405 eng 26

UNION CARBIDE

OAK RIDGE Y-12 PLANT
P. O. Box Y
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Rasey R. Feezell

This report is based on a study by the author as partial fulfillment of requirements for the degree of Master of Science in Electrical Engineering from The University of Tennessee.

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ABSTRACT

The inspection of accurately machined parts which are surfaces of revolution require the part to be placed precisely in the center of a rotary inspection machine. This job is normally done manually, requiring considerable time and an experienced inspector.

A two-axis, sampling-type control system has been designed to perform this operation automatically without the use of skilled operators. The ultimate centering accuracy is limited by the mechanical performance of the inspection machine. Specifically, the accuracy is limited by the backlash and free play of the cross slides on this particular machine.

Two design-aiding computer programs are also reported.

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TABLE OF CONTENTS

CHAPTER PAGE

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The inspection of accurately machined parts which are surfaces of revolution usually involves a rotary table, some form of electronic displacement gage, and the services of an experienced machinist or mechanical inspector. A typical inspection machine is shown in Figure 1 with a hemispherical part shown in place ready for inspection. Usually a part of this type will be inspected for variations of the radius as the part is rotated one revolution about its center of rotation. The gage shown is a linear voltage differential transformer type, abbreviated LVDT, which is typically calibrated to read ± 0.0005 inch from its center position. By positioning the arm holding the LVDT to the desired radius, and moving the ZERO ADJUSTMENT till the spring-loaded plunger is in the center of the gage, the recorder output will read zero and inspection can proceed. As the part is rotated through 360°, the output meter or recorder will indicate the variation of the radius from ideal at this particular latitude. Usually the part is checked at several different latitudes and the resulting deviations recorded. From this information it is determined if the part conforms to specified dimensions within the required accuracy.

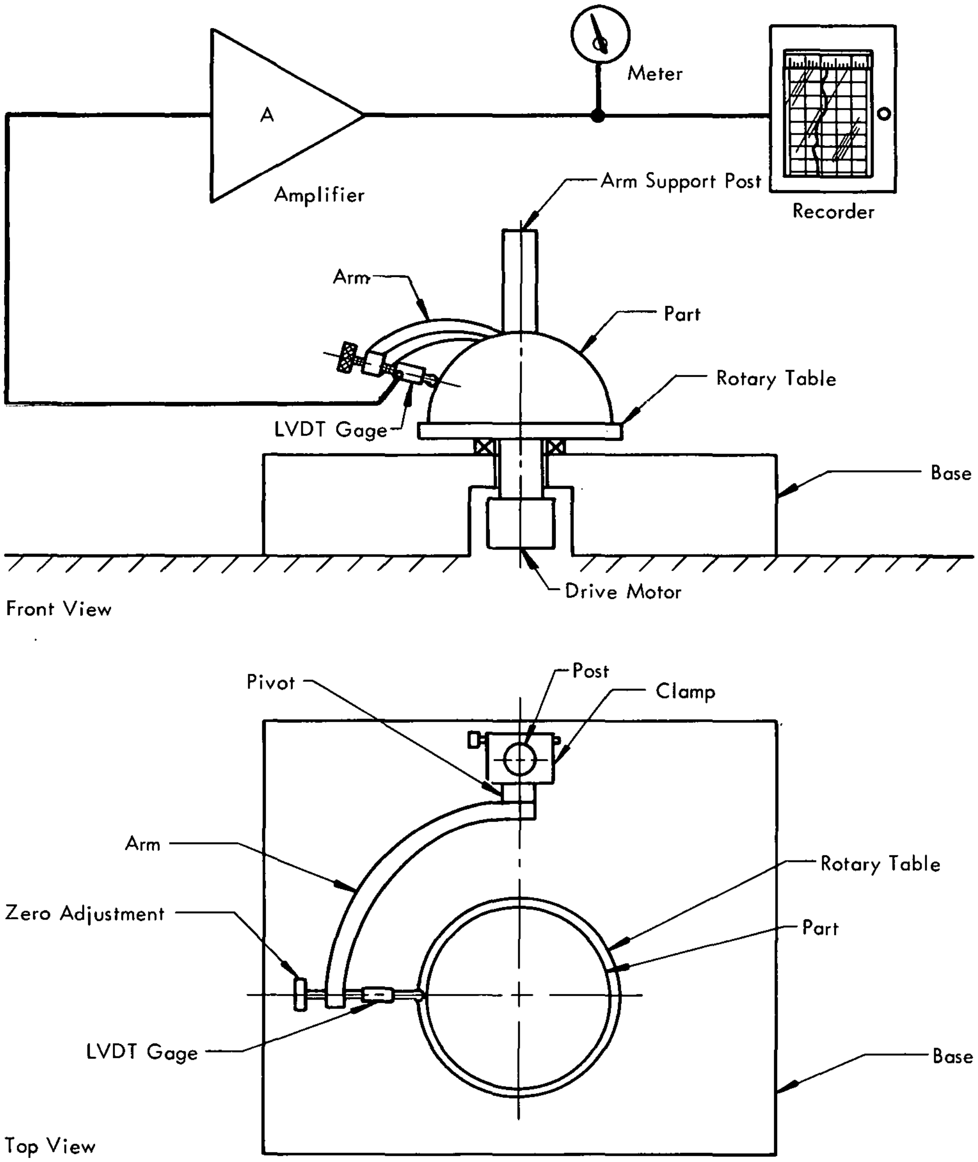


FIGURE 1

INSPECTION MACHINE ASSEMBLY VIEW

A not unusual requirement for so-called precision parts is that they deviate no more than ± 0.0005 inch from some specific radius. In order to measure this deviation correctly, the center of rotation of the part and the center of rotation of the table should coincide to within 0.0001 inch. This value assumes there are no other errors in the system. The usual method of centering the part on the rotary table requires the inspector to make two measurements of the part radius 180° apart, subtract the two measurements, divide their difference by two, and attempt to move the part on the table by this amount. After this is accomplished, the operator must pick two other points 180° apart and 90° from the first set and repeat the procedure. It is apparent that centering the part on the table to within 0.0001 inch in two directions requires much trial and error with a subsequent loss of time. The subject of this report is a machine to perform this operation automatically.

CHAPTER II

SYSTEM DESCRIPTION

An existing inspection machine has been modified to incorporate two servo-controlled orthogonal cross slides mounted on top of the rotary table with a flat platform mounted on top of the slides to receive the part to be inspected. The bottom slide provides movement in the "X" direction and the top slide provides movement in the "Y" direction. The mechanical arrangement is shown in Figure 2.

Development of Error Signal

Each slide is driven by an independent servo system which is identical for each axis. Since it is desirable to have the system entirely automatic, a method has been devised for automatically sampling a part off-center error at four points sequentially, storing this information, and then making proportional slide movements to reduce this error; then, again, taking a sample of the error and reducing this error with proportional slide movements. This process is continued until the part is centered to the desired accuracy without stopping the table rotation. When this condition is reached, a panel lamp signals the operator that the part is centered.

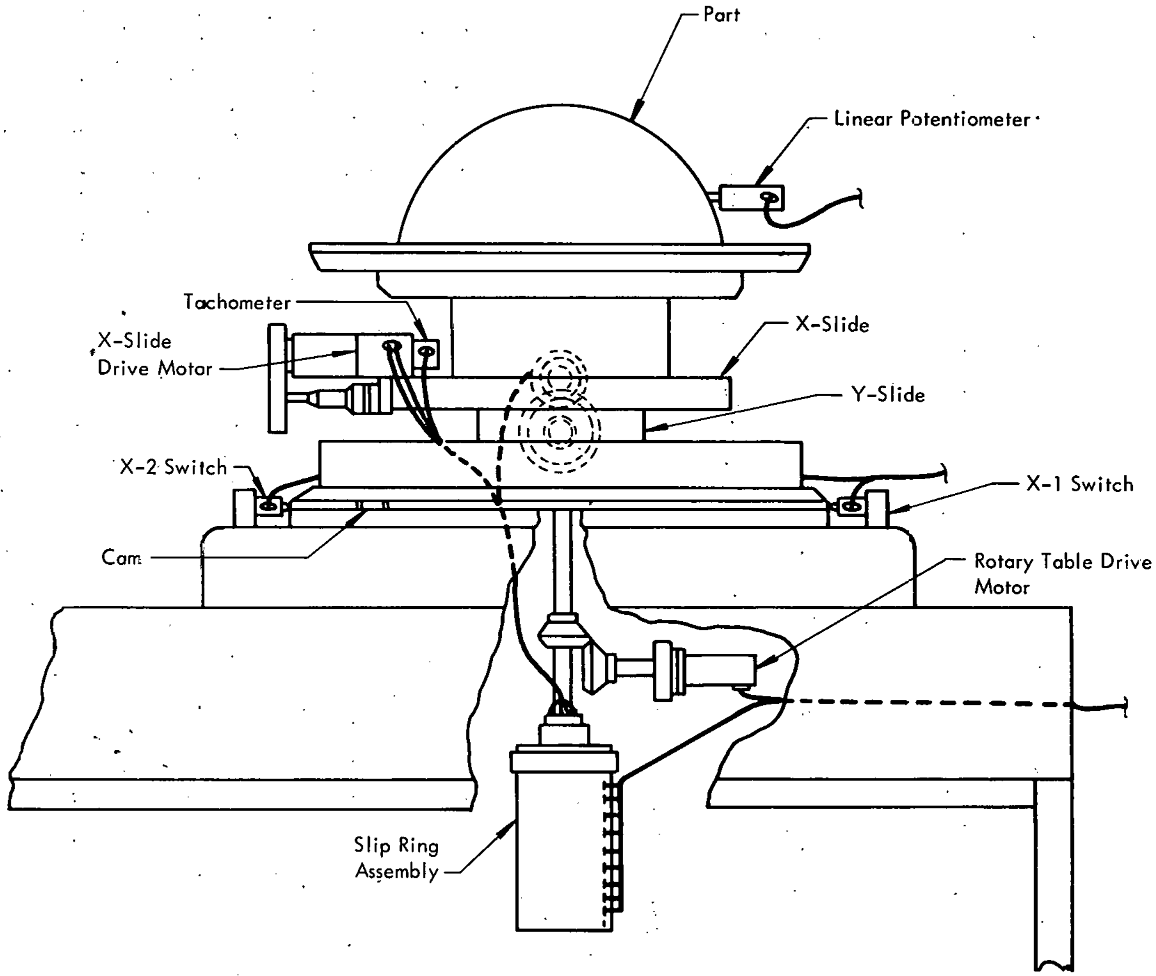


FIGURE 2

ROTARY TABLE

The base below the rotary table contains four orthogonally mounted switches, labeled S-1 through S-4, which are operated sequentially by a short actuating cam located on the rim of the rotary table (see Figure 2). The method for developing the off-center error signal can best be seen by observing Figures 3 and 4 simultaneously. Figure 3 shows the four quadrants labeled A, B, C, and D where the part position is sampled.

X' and Y' are the axes of the rotary table; X and Y are the axes of the part. Note from Figure 2 that the correction cross slides are mounted between the rotary table and the part. One correction slide is mounted coaxially with the X' axis and the other slide is mounted on top of the first, coaxially with the Y' axis.

The transducer shown in Figures 3 and 4 is a precision carbon-film, linear-motion potentiometer with its actuating plunger spring loaded to keep the plunger in contact with the part at all times. The bearings and mechanical design of the transducer plunger and wiper assembly have been carefully assembled to minimize backlash and free play. The transducer has a range of ± 0.500 inch and an applied voltage of ± 70 volts with respect to ground. Thus, the developed error signal will be 14 millivolts when the part is only 0.0001 inch off center.

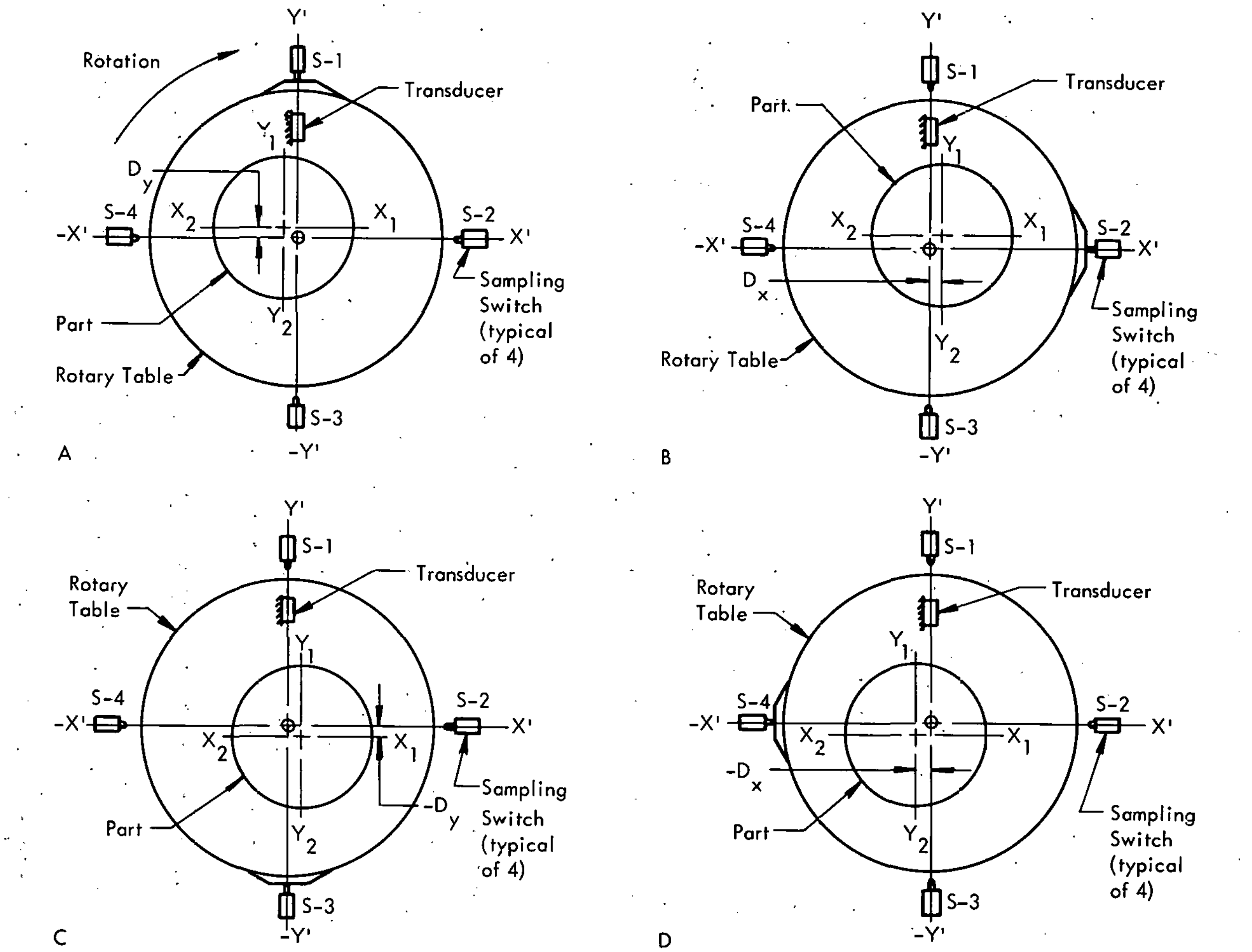


FIGURE 3

DIAGRAM OF AUTOMATIC CENTERING SEQUENCE

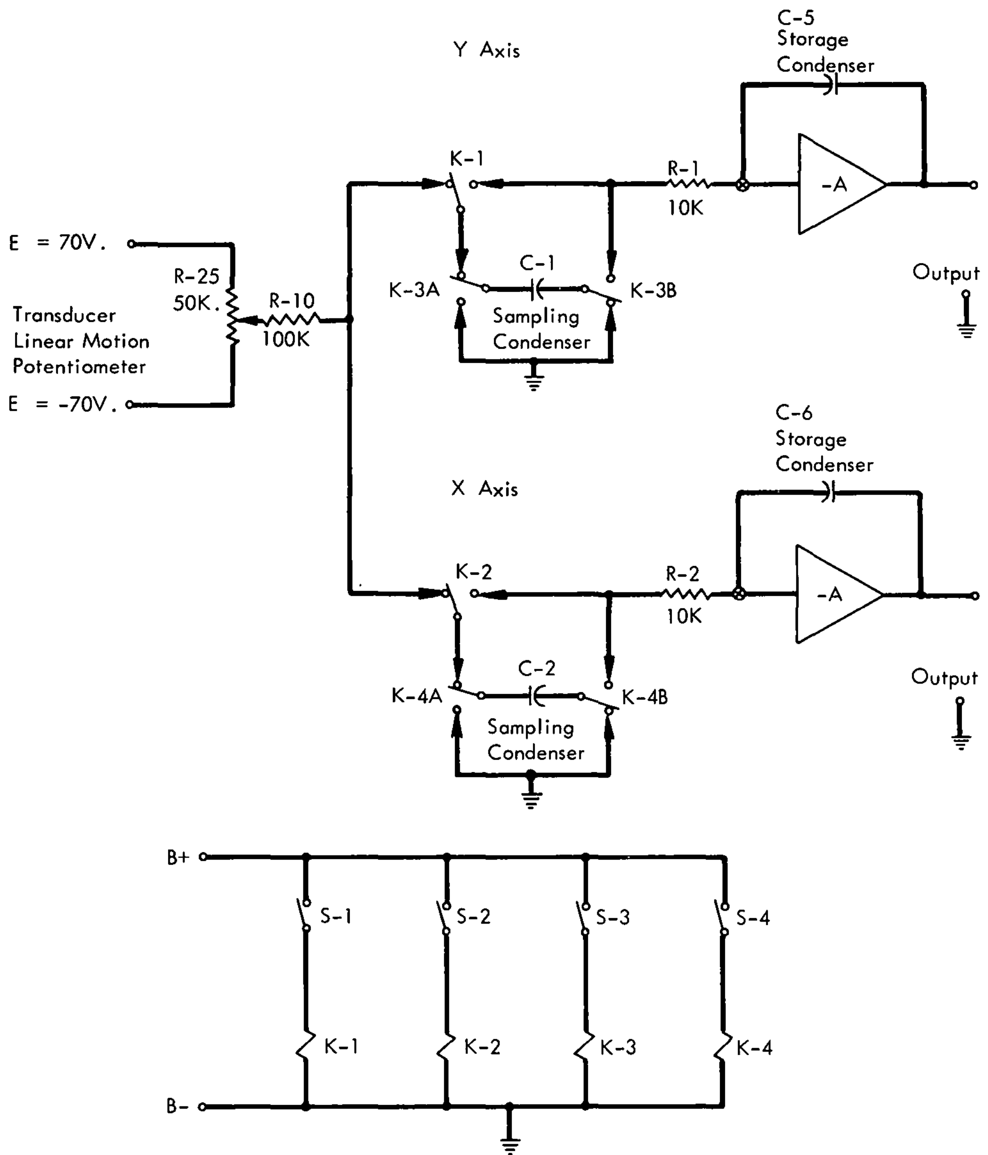


FIGURE 4

SIMPLIFIED SCHEMATIC OF SAMPLING CIRCUIT

Assuming the table rotation has just reached the point shown in Figure 3A, the voltage across C-1 is essentially equal to the transducer voltage since the R-10, C-1 time constant is small compared to the tangential part surface velocity. When S-1 is actuated, relay K-1 operates and C-1 discharges into C-5 through R-1. C-1 has time to completely discharge before S-1 opens and returns C-1 to the transducer. C-5 now has a stored voltage which is proportional to the amount of part off center in the Y_1 direction. As the table continues to rotate in a clockwise direction, the cam closes S-2 which operates relay K-2. C-2 has a voltage drop which is proportional to the amount of part off center in the X_1 direction. When K-2 operates, this voltage is transferred to C-6 through R-2. When the table reaches the point shown in Figure 3C, the voltage across C-1 (which is proportional to the amount of part off center in the Y_2 direction) is applied to C-5, but with reversed polarity since S-3 operates K-3 which reverses C-1 before discharging it into C-5 through R-1. Thus, C-5 now contains a voltage proportional to $Y_1 - Y_2 = D_Y$ which is the amount of part off center in the Y-axis direction. In a similar manner, S-4 operates K-4 to produce the same results in the X-axis direction. By using this method of sampling to develop the error signal, it does not matter how close to the part surface the transducer plunger is placed as long as it stays in its operating range of ± 0.5 inch. This is an important practical consideration which allows the operator to place the part on the surface anywhere within ± 0.5 inch of the center without having to adjust the position of the transducer. This method of sampling can be more clearly seen in Figure 5A and B.

Let ρ = transducer displacement from electrical zero in the direction of +E.

Let γ = transducer displacement from electrical zero in the direction of -E.

Then the voltage across the sampling condenser in Figure 5A will be proportional to ρE. This signal is applied directly to the storage condenser. The table is rotated 180° and a sample taken at Y_2, as shown in Figure 5B, which produces a voltage proportional to $-\gamma E$. This voltage is reversed before applying it to the storage condenser. This voltage is equal to $E(\rho + \gamma)$ which is proportional to the part off set in the Y-axis direction. If the transducer is always on the same side of electrical zero, the error signal will be proportional to either $E(\rho_1 - \rho_2)$ or $-E(\gamma_2 - \gamma_1) = E(\gamma_1 - \gamma_2)$, where the subscript 1 denotes the position fartherest away from electrical zero. In all cases the error signal will be positive if the part is off set in the Y_1 direction.

It can be easily seen that if the part is off center in the Y_2 direction, the error signal will be negative for all possible transducer positions.

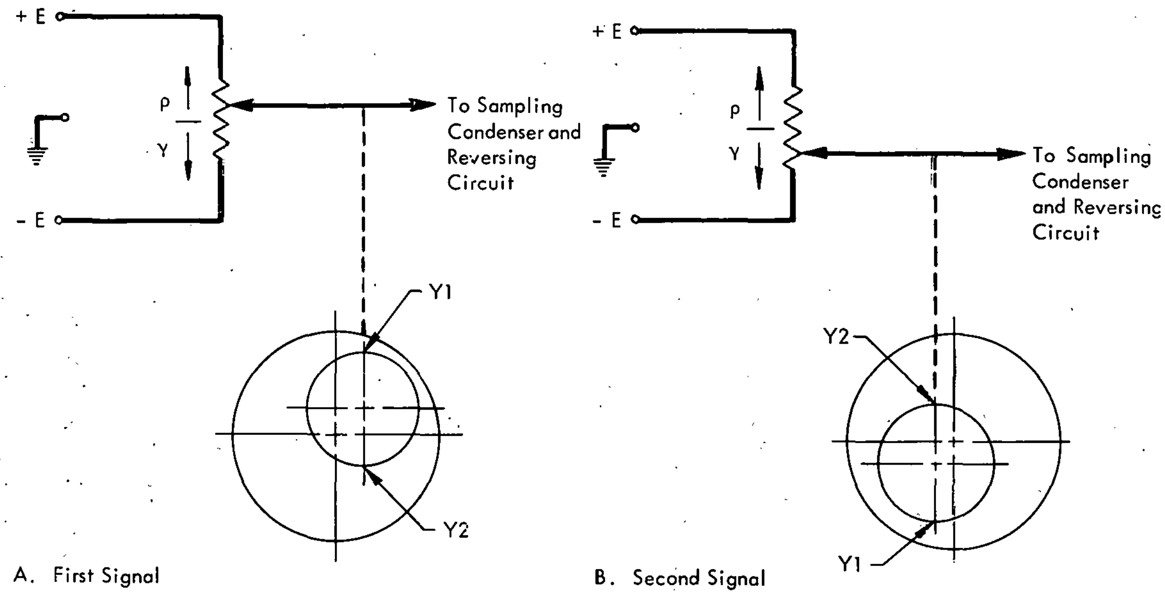


FIGURE 5

ERROR SIGNALS

Derivation of the Error Signal

The Y-axis circuit is shown in Figure 6 just after K-1 has operated. Let the voltage across C-1 equal e_{r1} and summing the currents at the junction,

$$i_1 - i_2 + i_3 = 0,$$

$$\frac{e_{Y1} - e_i}{R_1} - i_2 + C_5 \frac{d}{dt}(e_o - e_i) = 0. \qquad (1)$$

But,

$$e_i = -\frac{e_o}{A},$$

and

$$A \geq 10^7.$$

In this case,

$$e_i \ll e_{Y1},$$

and

$$e_i \ll e_o.$$

If $i_2 \ll i_1 \cong i_3$, then i_2 can be set equal to zero.[1]

The input current $i_2 = 10^{-10}$ amperes in the operational amplifier used in this circuit.

With the above simplifications, Equation 1 reduces to:

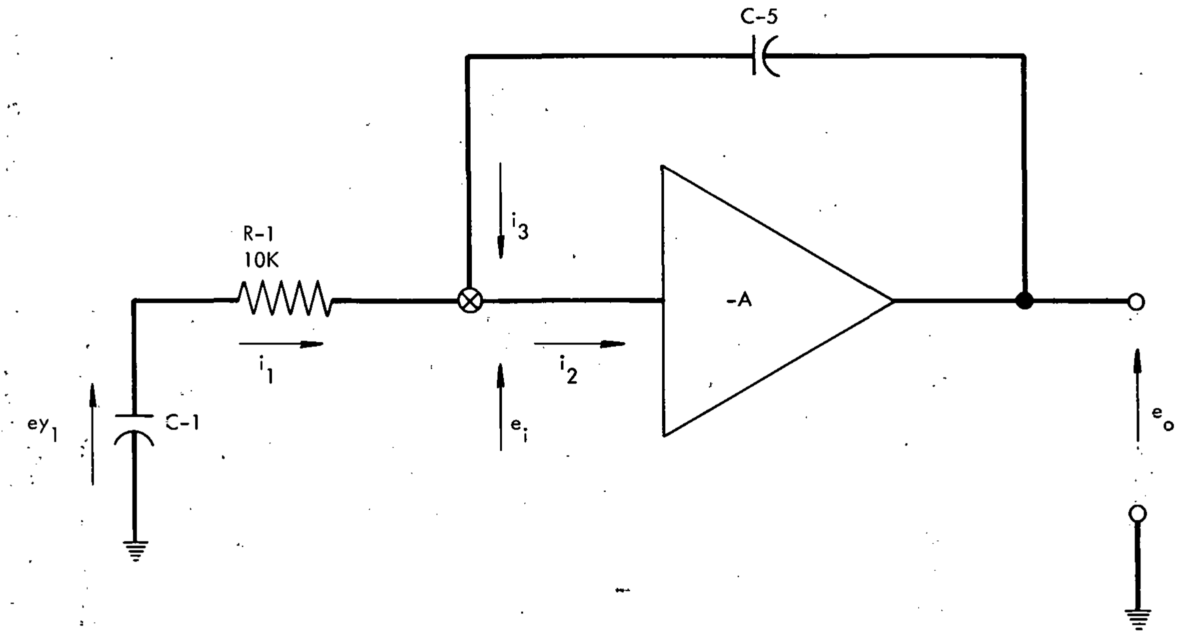


FIGURE 6

ERROR SIGNAL STORAGE CIRCUIT

$$\frac{e_{Y1}}{R_1} + C_5 \frac{d e_o}{dt} = 0. \tag{2}$$

Since $e_i \cong 0$, the summing junction is virtually at ground. Therefore, the instantaneous voltage, e_{Y1}, across C_1 is:

$$e_{Y1} = E_{Y1}\, e^{-\frac{t}{R_1 C_1}}, \tag{3}$$

where E_{Y1} is the initial value of e_{Y1}. Substituting Equation 3 into Equation 2 and solving for e_o,

$$\frac{de_o}{dt} = -\frac{E_{Y1}}{R_1 C_5}\, e^{-\frac{t}{R_1 C_1}}, \text{ or}$$

$$e_o = -\frac{E_{Y1}}{R_1 C_5} \int_o^t e^{-\frac{t}{R_1 C_1}} dt,$$

$$e_o = \frac{C_1 E_{Y1}}{C_5} \left[e^{-\frac{t}{R_1 C_1}} \right]_o^t,$$

$$e_o = -\frac{C_1 E_{Y1}}{C_5} \left[1 - e^{-\frac{t}{R_1 C_1}} \right]. \tag{4}$$

Since the length of time S-1 is kept closed by the actuating cam is "t" in the above equation, it is easily seen that if $t \gg R_1 C_1$, the output voltage will be:

$$e_o = - \frac{C_1 E_{Y1}}{C_5} = - \frac{0.5}{0.25} E_{Y1} = -.2 E_{Y1}.$$

This signal must be held until the table has rotated 180° and then a new sample equal to $- 2E_{Y2}$ is stored in C_1. The final output voltage is,

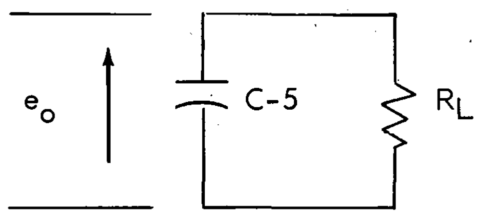
$$e_o = - 2 (E_{Y1} - E_{Y2}) = - E_Y.$$

During one revolution of the table, a similar error signal is developed for the X axis such that its output voltage is:

$$e_o = - 2 (E_{X1} - E_{X2}) = - E_X.$$

Storage Capacitor Requirements

Since the rotary table is designed to have a minimum speed of one revolution per minute, and the storage condenser is required to hold the signal for a maximum of five table revolutions, as will be shown later, the leakage resistance of the storage capacitors to produce no more than one per cent loss of signal is:



$$0.99 e_o = e_o E^{-\frac{t}{R_L C_5}},$$

$$\ln 0.99 = - \frac{t}{R_L C_5} = - \frac{300}{(2.5)\, 10^{-7}\, R_L},$$

$$\ln 1.0101 = \frac{(1.2)\, 10^{9}}{R_L},$$

$$R_L = 1.2 \times 10^{11} \text{ ohms.}$$

This efficiency can be achieved by the use of high-quality condensers.

Error-Correction Technique

An iterative technique is used on both axes to progressively reduce the part off-center error to the desired value. Figure 7 shows a simplified circuit of one axis. The voltage E_Y is equal to the part off-center error in the Y direction. At the completion of all four samples (at the end of one table revolution), relay circuits inhibit further sampling and close S-A which applies the error voltage E_Y to the input of the servo amplifier and, in turn, to the servo motor which moves the Y-axis slide in a direction to reduce the error. The motor also turns a DC tachometer producing a voltage $e_T = K_T \omega$, where:

K_T = tachometer constant, and

ω = tachometer angular velocity.

The speed of the servo motor is proportional to the error voltage, e_e.

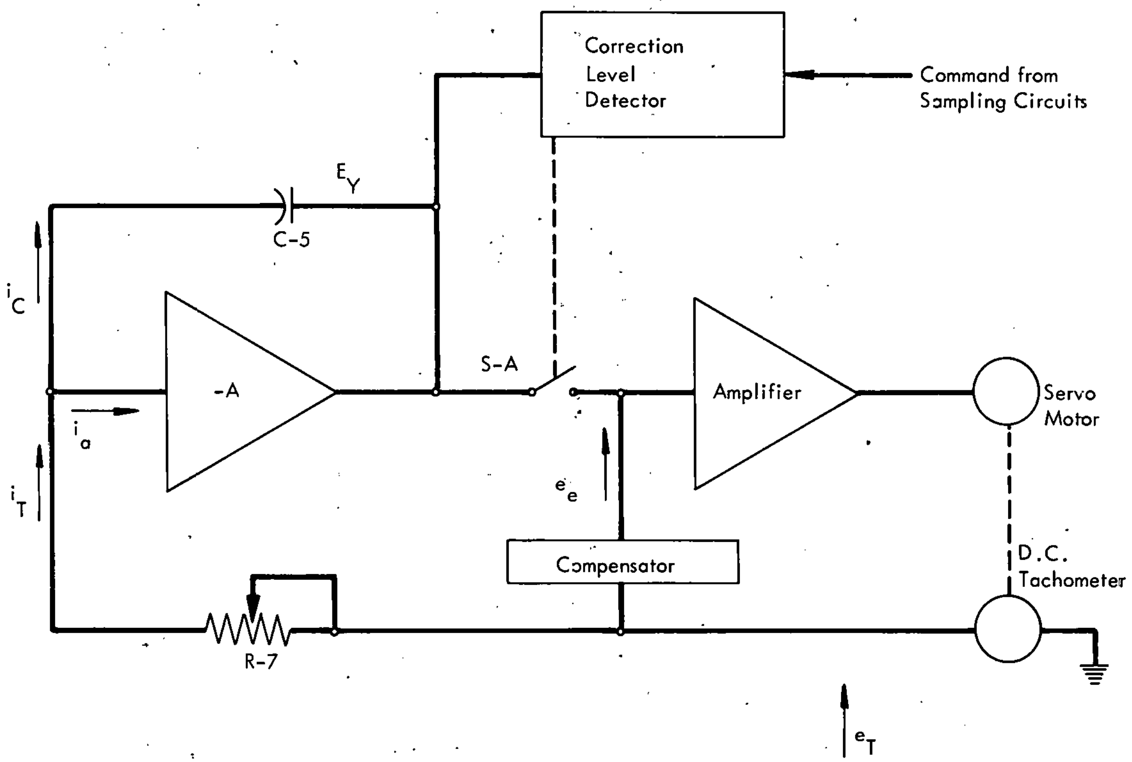


FIGURE 7

SYSTEM DIAGRAM AFTER SAMPLING

If $i_a \ll i_T$ then,

$$i_c = i_T = \frac{e_T}{R_7}.$$

The tachometer polarity is such that i_c is in a direction to discharge the storage condenser:

$$e_e = E_Y - \frac{1}{C_5}\int_o^t i_c\, dt = E_Y - \frac{1}{R_7C_5}\int_o^t e_T\, dt,$$

$$e_e = E_Y - \frac{K_T}{R_7C_5}\int_o^t \omega_m dt = E_Y - \frac{K_T}{R_7C_5}\int_o^{\theta_m} d\theta, \text{ or}$$

$$e_e = E_Y - \frac{K_T}{R_7C_5}\,\theta_m. \qquad (5)$$

But,

$$\theta_m = N\theta_L, \qquad (6)$$

and

$$\theta_L = \frac{Y}{P}, \qquad (7)$$

where:

θ_m = motor shaft angle,

θ_L = load shaft angle producing a proportional slide movement, N = gear reduction between the motor and slide, and

P = pitch of the lead screw.

Let:

$$K = \frac{K_T N}{R_7 C_5 P}, \tag{8}$$

then:

$$e_e = E_Y - KY. \tag{9}$$

Notice that K is adjustable by changing the value of R_7.

Iterative Correction Method

To prevent over correcting and provide a rapid method of correcting convergence, K is adjusted such that the error voltage, e_e, will become essentially zero and stop the servo motor before all of the part off-center error has been removed.

Let,

$$D_{Y1}, D_{Y2}, D_{Y3} \text{ --- } D_{Yn}$$

be the successive off-center errors.

But,

$$D_{Y2} = (1 - K)D_{Y1},$$

and

$$D_{Y3} = (1 - K)D_{Y2} = (1 - K)^2 D_{Y1}$$

$$\vdots$$

$$D_{Yn} = (1 - K)^{n-1} D_{Y1}. \tag{10}$$

For example, assume $D_{Y1} = 0.250$ inch, $K = 0.95$, and the desired final part off-center error, $D_{Yn} = 0.0001$ inch.

Then:

$$\frac{0.0001}{0.250} = (1 - 0.95)^{n-1}, \text{ or}$$

$$n = 3.6.$$

In other words, with four sample and correction cycles, an initial off set of 0.250 inch will be reduced to 0.0001 inch without danger of over correction.

Correction Level Detector

The correction level detector shown in Figure 7 (Page 17) consists of a high-gain differential amplifier with one input connected to the inner loop error signal and the other input connected to an adjustable bias voltage.

The purpose of the correction level detector is to stop the sampling and correction cycles when the part has been centered to the desired level. It alsokeeps the servos disconnected during the sampling period. Relay commands from the sampling circuit keep S-A open until all four sampling switches have operated, storing a voltage equivalent to the part off set in the respective storage capacitors. If this voltage is greater than the bias setting, the level detector will close S-A to make the necessary corrections; if the error voltage on both axes is below the bias voltage, S-A does not close and a pilot light signals the operator that the part is centered to the desired accuracy.

CHAPTER III

MOTOR AND LOAD ANALYSIS

Each slide assembly is completely supported on roller bearings running against hardened steel surfaces. The slide movements are controlled by recirculating, double-nut, ball-bearing lead screws. This system provides a very low coefficient of static friction with no backlash. Since the weight of a slide assembly is small compared to the weight of the part, both servo systems have essentially the same load even though one is carrying two slides plus the part. The equivalent load diagram is shown in Figure 8.

Here:

M = mass of the load,

B_L = viscous friction coefficient of load,

y = horizontal distance,

F_L = horizontal reaction force,

F_T = tangential force at the mean thread diameter,

P = lead screw pitch (inches horizontal travel/revolution),

r = lead screw radius (0.5 inch),

τ_L = load torque,

τ_m = motor torque,

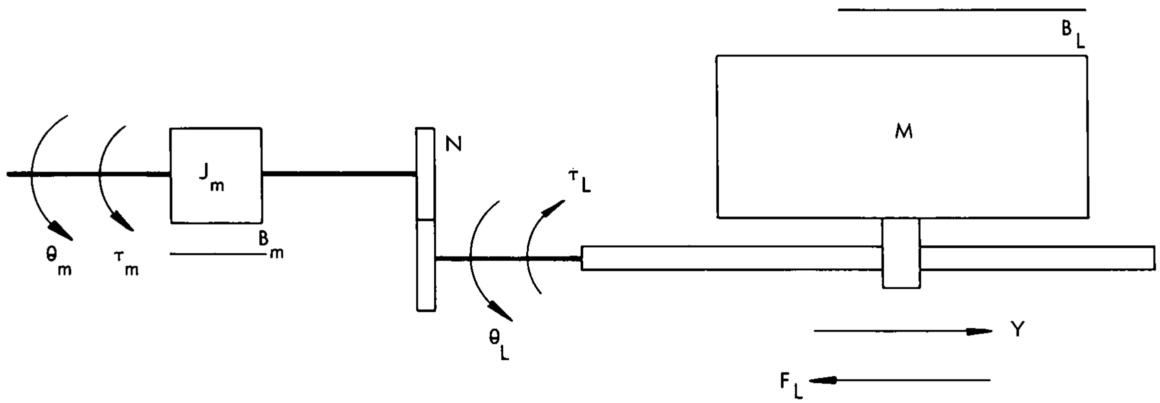


FIGURE 8

MOTOR AND LOAD DIAGRAM

N = gear reduction ratio,

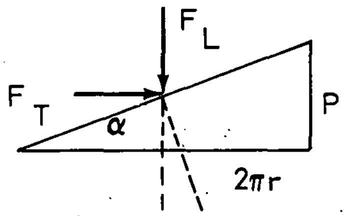
θ_L = lead screw angle,

θ_m = motor shaft angle, and

J_m = inertia of motor and equivalent gear box inertia combined.

Motor Torque Equation

The conversion of F_L to F_T through the threads of the lead screw can be found from examining one turn as an inclined plane problem with no friction:



F_T is the force necessary to move F_L up the plane and α is the angle of the threads.

Then:

$$\tan\ \alpha = \frac{F_T}{F_L} = \frac{P}{2\pi r}\ . \tag{11}$$

The load torque is:

$$\tau_L = r\,F_T, \tag{12}$$

and

$$\theta_L = \frac{2\pi}{P}\ Y. \tag{13}$$

The differential equation for the load in LaPlace notation is:

$$F_L(S) = MS^2Y(S) + B_L SY(S). \tag{14}$$

Substituting Equations 11, 12, and 13 into Equation 14, the load torque is:

$$\tau_L(S) = m\left(\frac{P}{2\pi}\right)^2 S^2\theta_L(S) + B_L\left(\frac{P}{2\pi}\right)^2 S\theta_L(S). \tag{15}$$

$$\theta_L = \frac{\theta_m}{N}, \text{ and} \tag{16}$$

$$\tau_m(S) = J_m S^2\theta_m(S) + B_m S\theta_m(S) + \frac{\tau_L(S)}{N}. \tag{17}$$

Combining Equations 15, 16, and 17:

$$\tau_m(S) = \left[J_m + \frac{m}{N^2}\left(\frac{P}{2\pi}\right)^2\right] S^2\theta_m(S) + \left[B_m + \frac{B_L}{N^2}\left(\frac{P}{2\pi}\right)^2\right] S\theta_m(S). \tag{18}$$

From mechanical considerations, the pitch of the lead screw was selected to be 0.0167 ft./rev. The mass is:

$$m = \frac{300 \text{ lb.-sec.}^2}{32 \text{ ft.}} = 9.4\ \frac{\text{lb.-sec.}^2}{\text{ft.}}.$$

The viscous friction coefficient was found by running a slide with typical lubrication at constant speed and measuring the torque. It was found to be 1,000 lb.-ft./sec. Substituting these values into Equation 18:

$$\tau_m(S) = \left(J_m + \frac{0.000066}{N^2}\right) S^2\theta_m(S) + \left(B_m + \frac{0.007}{N^2}\right) S\theta_m(S). \tag{19}$$

The largest available two-phase servo motor and gear reducer that would physically fit into the available space was selected in order to provide maximum acceleration. For this motor and gear reducer:

$$J_m = 3 \times 10^{-6} \text{ lb.-ft./sec.}^2 \text{ and } B_m = 1.7 \times 10^{-4} \text{ lb.-ft./sec.}$$

Therefore,

$$\tau_m(S) = 10^{-6} \left[\left(3 + \frac{66}{N^2}\right) S^2 \theta_m(S) + \left(170 + \frac{7{,}000}{N^2}\right) S\theta_m(S)\right]. \tag{20}$$

Selecting the Gear Ratio

It has been shown that for a given motor torque the maximum acceleration is greatest when the load inertia referred to the motor is equal to the motor inertia.[2]

Therefore:

$$J_m = \frac{J_L}{N^2},$$

and

$$N = \sqrt{\frac{J_L}{J_m}} = 5.74.$$

However, a larger gear reduction will reduce the motor's mechanical time constant. But, the main reason for a larger gear reduction is to allow a larger angular rotation of the motor shaft during the last correction period. A compromise must be made when selecting the gear ratio between the total time required to center the part and the difficulty in controlling small shaft angles.

Assume, temporarily, that the gear reduction ratio, N, in Equation 20 is large, such that:

$$170 \gg \frac{7{,}000}{N^2},$$

then the motor is effectively operating with no load. As a first choice, find N such that the acceleration time will consume one fifth of the total shaft angle necessary to make the last correction. The final allowable off-center error is:

$$D_{Yn} = 0.0001 \text{ inch.}$$

But:

$$D_{Yn} = 0.05\, D_{Yn-1}.$$

Then:

$$D_{Yn-1} = 0.002 \text{ inch,}$$

and the lead screw pitch is 0.2 inch per revolution:

$$\theta_L = 0.02\pi.$$

Solving Equation 20 with the assumption that $170 \gg \dfrac{7{,}000}{N^2}$,

$$\tau_m(S) = (3)\,10^{-6}\,S\,(S + 57)\,\theta_m(S).$$

If the torque-speed relationship for the motor characteristic curves shown in Figure 9 are assumed to be linear and the motor-control voltage is 100 per cent of the rated voltage times a unit step, then:

$$\tau_m(S) = \frac{0.0684}{S} - 0.000175\,\theta_m(S).$$

Combining these two equations gives:

$$\theta_m(S) = \frac{22800}{S^2\,(S + 114)}.$$

Transforming to the time domain:

$$\theta_m(t) = 1.75\,(e^{-114t} + 114t - 1). \tag{21}$$

But:

$$\theta_m(t) = N\,\theta_L\,(t).$$

Therefore:

$$\theta_L(t) = \frac{1.75}{N}\,(e^{-114t} + 114t - 1), \tag{22}$$

and the shaft speed is

$$\omega_L(t) = \frac{200}{N}\,(1 - e^{-114t}).$$

The acceleration will be complete and the shaft running at essentially constant speed when:

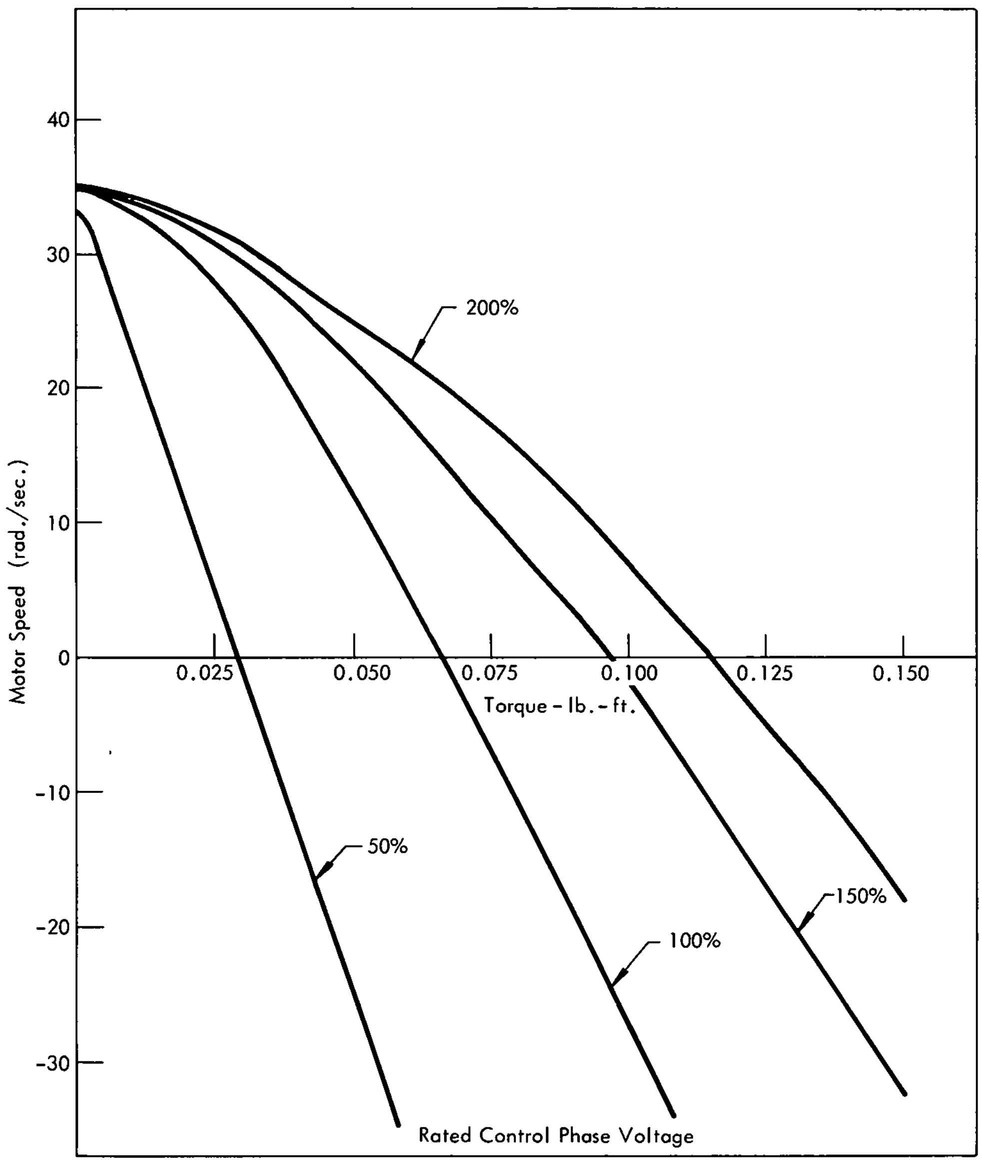


FIGURE 9

SERVOMOTOR CHARACTERISTICS

$$e^{-114t} = 0.05 \text{ and}$$

$$t = 0.0263 \text{ sec.}$$

Let the total time for the last correction be $t_1 = 5(0.0263) = 0.132$ second. Then, substituting the required angle and t_1 into Equation 22:

$$0.02\pi = \frac{1.75}{N}\left[e^{-114(0.132)} + 114(0.132) - 1\right], \text{ or}$$

$$N = 420.$$

The nearest commercially available ratio is 400:1 which was selected. Thus:

$$\theta_m = N\theta_L = 400(0.02\pi) = 8\pi,$$

or four revolutions for the last correction cycle.

Correction Time

A part which is 0.250 inch off center will require four sample-correction cycles. If the rotary table is rotating at its slowest speed of one revolution per minute, four revolutions or four minutes will be used in taking samples and the remaining time utilized for correction. The acceleration and deceleration time will be small compared to the total.

From Equation 13:

$$\theta_L = \frac{0.250(2\pi)}{0.2} = 7.86 \text{ rad.}$$

for the total correction. From Equation 22 the required time is:

$$7.86 = \frac{1.75}{400}(114)t, \text{ or}$$

$$t = 15.8 \text{ sec.}$$

Even with the rotary table running at its maximum speed of 25 rpm, the sampling time is 9.6 seconds; therefore, the above correction time is not excessive and a gear reducer of 400:1 is a good compromise. The gear reducer has been specially designed to produce zero backlash, but still operate at 90 per cent efficiency. The final equation for the motor torque is:

$$\tau_m(S) = (3)\,10^{-6}\,(S + 57)\,S\theta_m(S). \tag{23}$$

CHAPTER IV

SYSTEM TRANSFER FUNCTIONS

By observing the system diagram shown in Figure 7 (Page 17), the transfer function block diagram can be drawn, as shown in Figure 10. When the transfer functions are given in LaPlace notation:

$G_m(S)$ = motor,

$G_A(S)$ = servo amplifier,

$G_T(S)$ = tachometer,

$G_C(S)$ = compensator,

$G_I(S)$ = integrator,

$\theta_m(S)$ = motor shaft angle, and

$R(S)$ = step input of magnitude E_Y.

After the four samples have been taken, a voltage E_Y for the Y axis is stored in the integrating capacitor. The sampling relays operating through the level detection circuit closes the switch S-A which starts the correction cycle.

Motor

The motor transfer function can be derived by assuming the characteristic curves shown in Figure 9 (Page 28) are linear. The torque is a function of both speed and motor control voltage, e_c. The motor torque, τ_m, is: [3]

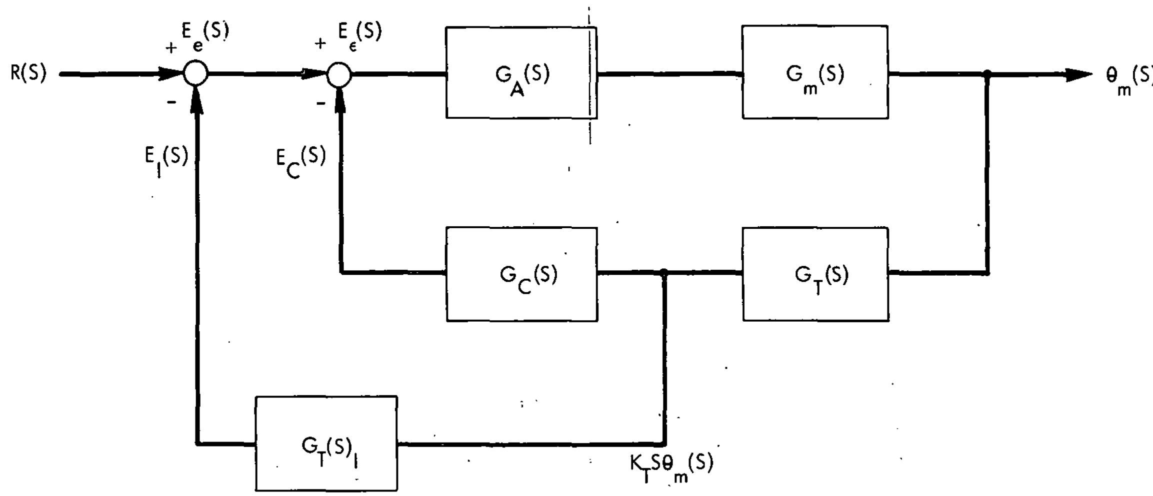


FIGURE 10

TRANSFER FUNCTION DIAGRAM

$$\frac{\partial \tau_m}{\partial e_c} e_m + \frac{\partial \tau_m}{\partial \omega_m} \omega_m = \tau_m(e_m, \omega_m),$$

$$\frac{\partial \tau_m}{\partial e_c} = 0.00045 \text{ lb.-ft./volt, and}$$

$$\frac{\partial \tau_m}{\partial \omega_m} = -0.00016 \text{ lb.-ft.-sec./rad.}$$

Then:

$$\tau_m(S) = 0.00045\, E_m(S) - 0.00016\, S\theta_m(S).$$

The load torque and motor torque must be equal, but with N = 400, the load torque is small compared to the motor. Since the data for Figure 9 (Page 28) were taken with the motor running at constant speed, the torque to overcome viscous friction is already taken into account. Therefore, $\tau_m = J_m S^2 \theta_m(S)$, the torque required to accelerate the motor's inertia. Combining these two equations gives the motor transfer function:

$$G_m(S) = \frac{\theta_m(S)}{E_m(S)} = \frac{150}{S(S + 53)}. \qquad (24)$$

The reciprocal of the electrical time constant is large compared with the value of 53 and can thus be neglected initially.

Amplifier and Tachometer

The amplifier has a differential input, a synchronized chopper, a high-gain AC amplifier, and has sufficient output voltage and current to supply the two-phase two-pole servomotor. The internal time constants of the amplifier are smaller than the mechanical time constant of the load. The transfer function is, therefore, just its gain "A" which is adjustable from zero to 5,000:

$$G_A(S) = A. \tag{25}$$

The lowest amplifier time constant will be considered later.

The internal inductance of the tachometer is low enough to provide a tachometer time constant well below the system mechanical time constant. Therefore,

$$G_T(S) = K_T S,$$

$$K_T = 0.062 \text{ volt/rad./sec., and}$$

$$G_T(S) = 0.062S. \tag{26}$$

Integrator

From Figures 7 (Page 17) and 10 (Page 32) plus Equation 7 (Page 18), the integrator transfer function $G_I(S)$ can be derived:

$$G_1(S) = \frac{E_1(S)}{K_T S\theta_m(S)}, \text{ and}$$

$$R(S) - E_1(S) = E_e(S) = \frac{E_Y}{S} - \frac{K_T}{R_7C_5}\theta_m(S).$$

Let:

$$R(S) = \frac{E_Y}{S}, \text{ and}$$

$$E_1(S) = \frac{K_T}{R_7C_5}\theta_m(S),$$

then:

$$G_1(S) = \frac{1}{R_7C_5S}, \text{ and} \tag{27}$$

$$C_5 = 0.25 \times 10^{-6} \text{ farads.}$$

R_7 is adjustable in order to set the value of the correction ratio near its design value of 0.95. In practice it will only vary about ± 10 per cent to take care of tolerances in the tachometer constant and C_5. R_7 can be treated as a constant. Its value can be found in the following way: The output from the transducer is:

$$\frac{70 \text{ volts}}{0.5 \text{ inch}} = 140 \text{ volts/inch.} \tag{28}$$

From Equation 5 (Page 18) it can be seen that the voltage stored in the integrating capacitor E_Y is twice the transducer voltage. When the part is off center 0.250 inch:

$$E_Y = 2(140)(0.250) = 70 \text{ volts.} \qquad (29)$$

From Equations 7 (Page 18), 8, and 9 (Page 19),

$$e_e = E_Y - \frac{K_T N}{R_7 C_5 P} y. \qquad (30)$$

Substituting the known values:

$$e_e = 70 - \frac{(3.1)\,10^9}{R_7}\, y. \qquad (31)$$

The command voltage to the servo amplifier, e_e, must be essentially zero when the slide has moved 95 per cent of the part off-center value.

Therefore:

$$0 = 70 - \frac{(3.1)\,10^9\,(0.250)(0.95)}{R_7}, \text{ and}$$

$$R_7 = (10.5)\,10^6 \text{ ohms.}$$

The final integrator transfer function is:

$$G_I(S) = \frac{1}{2.63S}, \text{ and}$$

$$E_e(S) = \frac{E_Y}{S} - 295\,Y(S). \qquad (32)$$

Compensator

Feedback compensation was chosen over cascade compensation because the input to the servo amplifier operates under highly saturated conditions a large portion of the time. For instance, the initial input voltage to the amplifier can be as high as 70 volts, but the servo amplifier will produce full output with less than 0.1 volt input. This factor places severe restrictions on the size of the storage elements used in a cascade compensator. Also, any mechanical system has some static friction which causes surging and erratic operation. Cascade compensation tends to increase these difficulties. In this particular application, a tachometer is necessary for other reasons and the same tachometer can be used for feedback compensation. The frequency shaping network for the compensator will be selected later.

Inner Closed Loop

The closed-loop transfer function for the inside loop is:

$$G_L = \frac{\theta_m(S)}{E_e(S)} = \frac{G_A\, G_m(S)}{1 + G_T(S)\, G_C(S)\, G_A\, G_m(S)},$$

$$G_L = \frac{\dfrac{150A}{S\,(S+53)}}{1 + \dfrac{0.062S\ G_C(S)\ 150A}{S\,(S+53)}}.$$

$$G_L(S) = \frac{150A}{S\,[S + 53 + 9.3A\ G_C(S)]}. \tag{33}$$

Outer Closed Loop

The outer closed-loop transfer function can be found from Figure 10 (Page 32) and the inner loop transfer function, thus:

$$R(S) - E_I(S) = E_e(S),$$

$$R(S) - G_I(S)\, G_T(S)\, \theta_m(S) = \frac{\theta_m(S)}{G_L(S)},$$

$$R(S) = \left[\frac{1}{G_L(S)} + G_I(S)\, G_T(S)\right] \theta_m(S), \text{ and}$$

$$G(S) = \frac{\theta_m(S)}{R(S)} = \frac{G_L(S)}{1 + G_I(S)\, G_T(S)\, G_L(S)},$$

$$G(S) = \frac{\dfrac{150A}{S\,(S + 53 + 9.3A\; G_C(S))}}{1 + \dfrac{0.062S\; 150A}{2.63S^2\,(S + 53 + 9.3A\; G_C(S))}}.$$

Or:

$$G(S) = \frac{\theta_m(S)}{R(S)} = \frac{150A}{S\,(S + 53 + 9.3A\; G_C(S)) + 3.54A}. \qquad (34)$$

CHAPTER V

ANALYSIS WITH UNCOMPENSATED VELOCITY FEEDBACK

Performance Criteria

The ideal objective is to automatically center the part to zero error in the shortest possible time. A final error of 0.0001 inch corresponding to a value of $e_e = 0.014$ volt has been previously decided as satisfactory. At this value of e_e, the error level detector opens the outside loop which makes $e_e = 0$ and stops the motor.

It is important that the motor is running at this point even though it is running very slowly. It would be possible to set the error level detector at zero and let the motor stop due to friction; however, the friction coefficients vary with environmental conditions, producing an indefinite stopping point. Therefore, the important criteria are minimum steady-state position error and minimum total correction time. As a starting point, these criteria will be examined with respect to the compensator in the minor feedback loop.

Critically Damped System

The circuit shown in Figure 10 (Page 32) has been modified in order to replace the motor shaft angle with slide displacement by the relation:

$$\theta_m = 4{,}000\pi y.$$

The new value for

$$G_m(S) = \frac{0.0119A}{S\,(S + 53)},$$

and the new value for

$$G_T(S) = 780S.$$

For this analysis, $G_C(s)$ is an adjustable constant K_1. The new circuit is shown in Figure 11.

The inner closed loop transfer function is:

$$\frac{Y(S)}{E_e(S)} = \frac{G_A(S)\,G_m(S)}{1 + G_C(S)\,G_T(S)\,G_A(S)\,G_m(S)}, \text{ or}$$

$$\frac{Y(S)}{E_e(S)} = \frac{0.0119A}{S\,(S + 53 + 9.3K_1A)}. \tag{35}$$

The outer closed loop transfer function is:

$$\frac{Y(S)}{R(S)} = \frac{0.0119A}{S\,(S + 53 + 9.3\,K_1A) + 3.5A}. \tag{36}$$

As a first step in the analysis, K_1 and A were chosen to allow critically damped operation. This equation is of the form:

$$\frac{Y(S)}{R(S)} = \frac{0.0119A}{S^2 + 2\zeta\omega_n S + \omega_n^{\,2}},$$

where:

$$\zeta = 1.0,$$

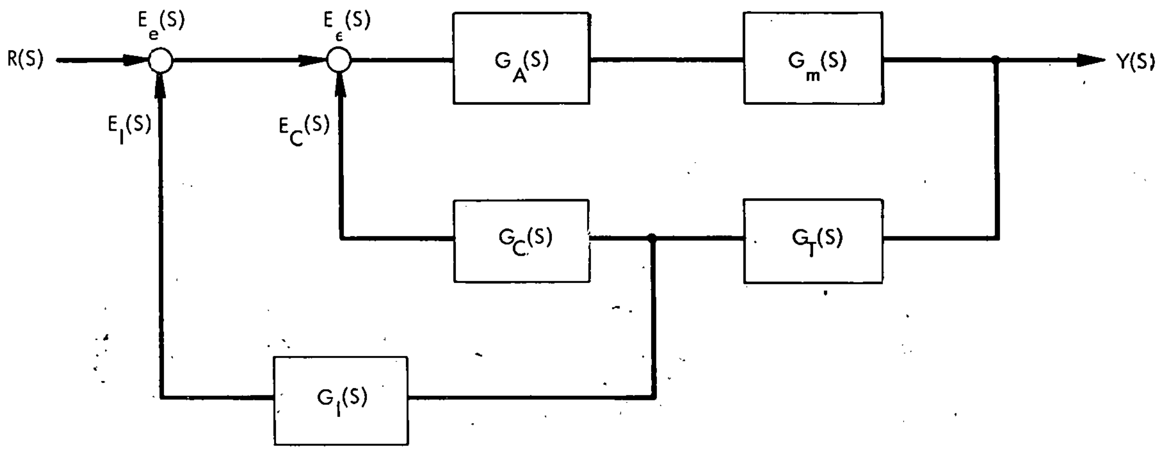


FIGURE 11

TRANSFER FUNCTION WITH VELOCITY FEEDBACK

$$2\zeta\omega_n = 53 + 9.3K_1A, \text{ and}$$

$$\omega_n = \sqrt{3.5A}.$$

Therefore,

$$2\sqrt{3.5A} = 53 + 9.3\,K_1A,$$

which reduces to:

$$A^2 + \left(\frac{11.4}{K_1} - \frac{0.164}{K_1^2}\right)A + \frac{32.5}{K_1^2} = 0, \text{ or}$$

$$A = -\frac{1}{K_1^2}\,(11.4K_1 - 0.164) \pm \frac{1}{K_1^2}\sqrt{0.027 - 1.87K_1}.$$

To insure positive values for A:

$$K_1 \leq 0.0144.$$

Let

$$K_1 = 0.01,$$

then:

$$A = 700.$$

First Correction Cycle

With a maximum part off-center value of 0.250 inch, the input to the amplifier is given by Equations 29 and 32 (Page 36) as:

$$e_e(t) = 70 - 295y(t). \qquad (37)$$

Initially, $e_e(t) = 70$ volts which will saturate the amplifier and the outside loop is effectively open. Under these conditions let:

$$e_e(t) = E_Y',$$

where E_Y' is the value of the input voltage applied to the closed inner loop which will just make the motor run at its full rated speed. Using the values of K_1 and A previously calculated, find E_Y' to produce the maximum output speed of 365 rad./second, or a maximum slide velocity of 0.029 inch/second.

From Equation 35,

$$\frac{Y(S)}{E_e(S)} = \frac{0.0119\ (700)}{S\ [S + 53 + 9.3\ (0.01)(700)]},$$

$$\frac{V(S)}{E_e(S)} = \frac{8.33}{S + 118}, \text{ and}$$

$$V(S) = \frac{8.33\ E_Y'}{S\ (S + 118)}.$$

From the final value theorem:

$$v(t)_{SS} = \lim_{S \to 0} SV(S) = \lim_{S \to 0} \frac{8.33\ E_Y'}{S + 118} = 0.029, \text{ or}$$

$$E_Y' = 0.412 \text{ volt.}$$

Using this value of E_Y', the output under saturated conditions is given by:

$$Y(S) = \frac{3.42}{S^2\ (S + 118)}. \tag{38}$$

The value of y(t) when the amplifier comes out of saturation can be found from Equation 37 as:

$$0.412 = 70 - 295\, y(t), \text{ or}$$

$$y(t) = 0.236 \text{ inch.} \tag{39}$$

Solving for the output:

$$y(t) = (2.46)\, 10^{-4}\, (e^{-118t} + 118t - 1), \quad y(t) \leq 0.236. \tag{40}$$

The maximum value of time in which this relationship is valid is

$$0.236 = (2.46)\, 10^{-4}\, (e^{-118t} + 118t - 1), \text{ or}$$

$$t = 8.06 \text{ seconds.}$$

Differentiating to find the velocity:

$$v(t) = 0.029\, (1 - e^{-118t}), \quad t \leq 8.06. \tag{41}$$

When y(t) reaches 0.236 inch, the amplifier is operating in its linear region with the outer loop closed. The transfer function with all initial conditions zero is given by Equation 36. Substituting $K_1 = 0.01$ and $A = 700$:

$$\frac{Y(S)}{R(S)} = \frac{8.33}{S\,(S + 118) + 2{,}460}. \tag{42}$$

Since the initial conditions are not zero, it will be necessary to convert back into the time domain, add the initial conditions, and retransform into the frequency domain:

$$S^2Y(S) - Sy(0) - v(0) + 118SY(S) - 118y(0) + 2{,}460Y(S) = \frac{8.33\,E_Y}{S}, \text{ or}$$

$$Y(S) = \frac{y(0)\left[S^2 + \left(\frac{v(0)}{y(0)} + 118\right) S + 8.33\,E_Y\right] e^{-8.06S}}{S\,(S^2 + 118S + 2{,}460)}\,.$$

Factoring the denominator:

$$Y(S) = \left[\frac{8.33\,E_Y}{S\,(S+27)(S+91)} + \frac{y(0)(S+118)}{(S+27)(S+91)} + \frac{v(0)}{(S+27)(S+91)}\right] e^{-8.06S}.$$

Solving for $y(t')$, where $t' = t - 8.06$:

$$y(t') = \frac{8.33\,E_Y}{2{,}450}\left[1 - \frac{(91e^{-27t'} - 27e^{-91t'})}{64}\right]$$

$$+\, y(0)\left[\frac{91e^{-27t'} - 27e^{-91t'}}{64}\right]$$

$$+\, v(0)\left[\frac{e^{-27t'} - e^{-91t'}}{64}\right].$$

Substituting the known values, and noting that $E_Y = 70$ volts, the initial voltage due to a 0.250 inch off-center value:

$$y(t') = 0.238 - 0.00239e^{-27t'} + 0.000393e^{-91t'}. \tag{44}$$

Differentiating to find the velocity:

$$v(t') = 0.0645e^{-27t'} - 0.0327e^{-91t'}. \tag{45}$$

It must be remembered that Equation 44 gives the total output y(t) as $e_e(t)$ goes from $E_Y = 70$ volts to essentially zero even though most of the time the outer loop is effectively open. Equations 40, 41, 44, and 45 are used to plot the output as a function of time for the first correction cycle under critically damped conditions with a 0.250 inch part off center. This information is plotted in Figure 12.

Second Correction Cycle

During the second sample, the part off-center value is 0.250 (1 - 0.95) = 0.0125 inch, which stores a voltage in the integrator condenser of:

$$E_Y' = 70\ (1.0 - 0.95) = 3.5 \text{ volts.}$$

Again, the amplifier will be saturated for a portion of the correction cycle and,

$$E_Y' = 0.412.$$

Using Equation 37 to find the value of y(t) which reduces the input voltage to the linear region:

$$e(t) = E_Y - 295y(t), \text{ or}$$

$$0.412 = 3.5 - 295y(t), \text{ or}$$

$$y(t) = 0.0105 \text{ inch.}$$

The output under saturated conditions is the same as before. Using Equation 40 to find the time when the outer loop becomes effective:

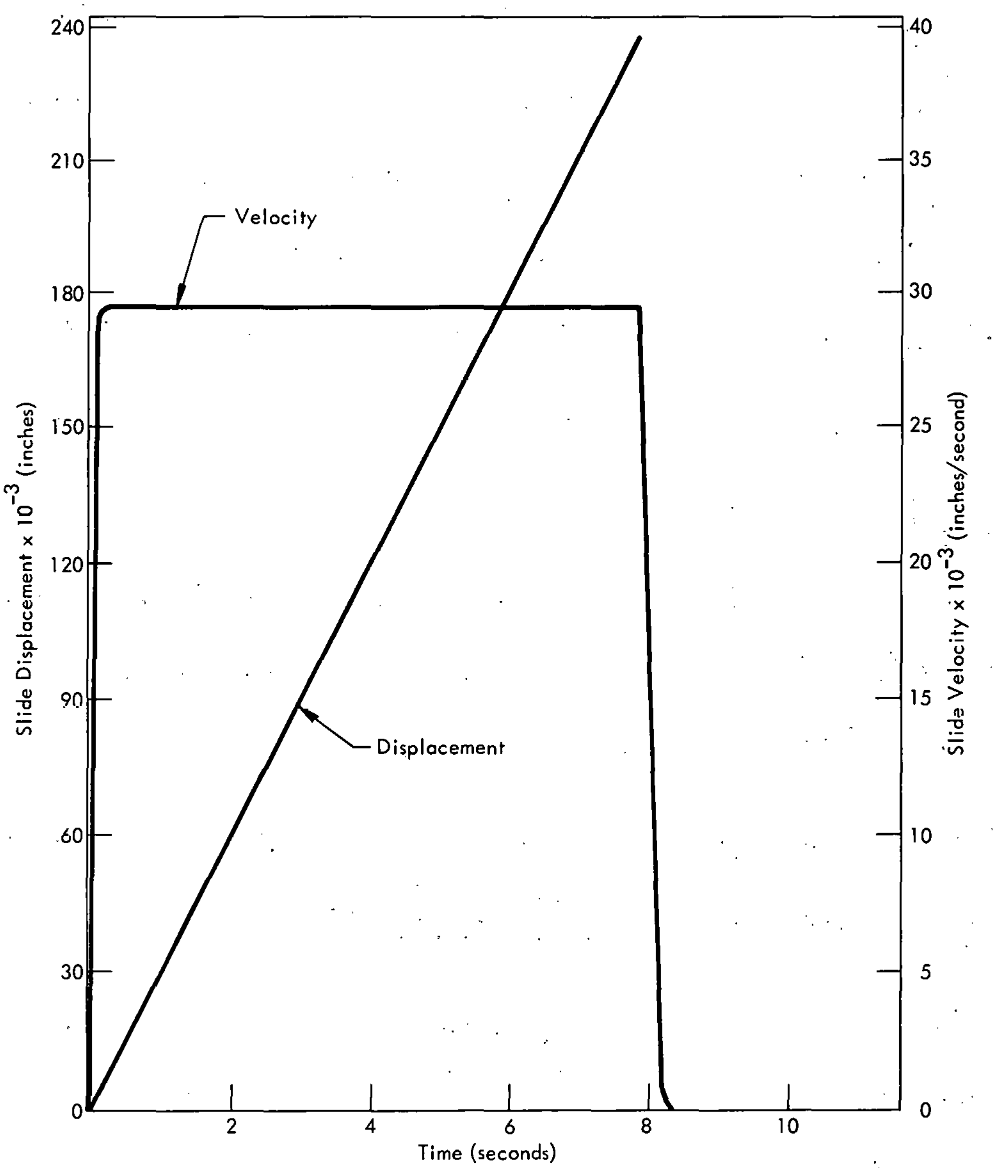


FIGURE 12

OUTPUT FOR FIRST CORRECTION CYCLE WITH THE PART 0.250 INCH OFF CENTER AND THE SYSTEM CRITICALLY DAMPED (K_1 = 0.01 AND A = 700)

$$y(t) = 0.0105 = (2.46)\,10^{-4}\,(e^{-118t} + 118t - 1), \text{ or}$$

$$t = 0.37 \text{ sec.}$$

The displacement and velocity are given by Equations 40 and 41, but

$$y(t) \le 0.0105, \text{ and}$$

$$t \le 0.37.$$

At this point in time, the outer loop is effectively closed and the output is given by Equation 43 with:

$$y(o) = 0.0105 \text{ inch,}$$

$$v(o) = 0.029 \text{ inch/sec., and}$$

$$E_Y = 3.5 \text{ volts.}$$

Therefore:

$$y(t') = \frac{8.33\ (3.5)}{2,450}\left[1 - \frac{(91e^{-27t'} - 27e^{91t'})}{64}\right] + 0.0105\left[\frac{91e^{-27t'} - 27e^{-91t'}}{64}\right] + 0.029\left[\frac{e^{-27t'} - e^{-91t'}}{64}\right]. \tag{46}$$

$$t' \le 0.37,$$

therefore:

$$t' = t - 0.37,$$

$$y(t') = 10^{-3} (11.9 - 1.55e^{-27t'} + 0.137e^{-91t'}), \text{ and} \tag{47}$$

$$v(t') = 10^{-3} (41.8e^{-27t'} - 12.5e^{-91t'}). \tag{48}$$

The slide position and velocity for the second correction cycle is shown in Figure 13. Notice the different scales than those used in Figure 12 (Page 47).

Third Correction Cycle

For the third and last correction cycle, $E_Y = (0.05)^2\ 70 = 0.175$ volt. The amplifier will be operating in the linear portion for the complete cycle and the output will be given by Equation 43 with all initial conditions zero and $t' = t$:

$$y(t) = 10^{-3} (0.595 - 0.845e^{-27t} + 0.251e^{-91t}), \text{ and} \tag{49}$$

$$v(t) = (22.8)\, 10^{-3} (e^{-27t} - e^{-91t}). \tag{50}$$

The results are plotted in Figure 14.

Performance with Critical Damping

Remembering that the rotary table speed is adjustable from 1.0 to 25 revolutions per minute and the fact that one table revolution per sample is required, then the time required for three samples varies from 180 to 7.2 seconds. Adding all the times required for the three correction cycles shown in Figures 12, 13, and 14 totals 9.0 seconds. During this time, 8.45 seconds

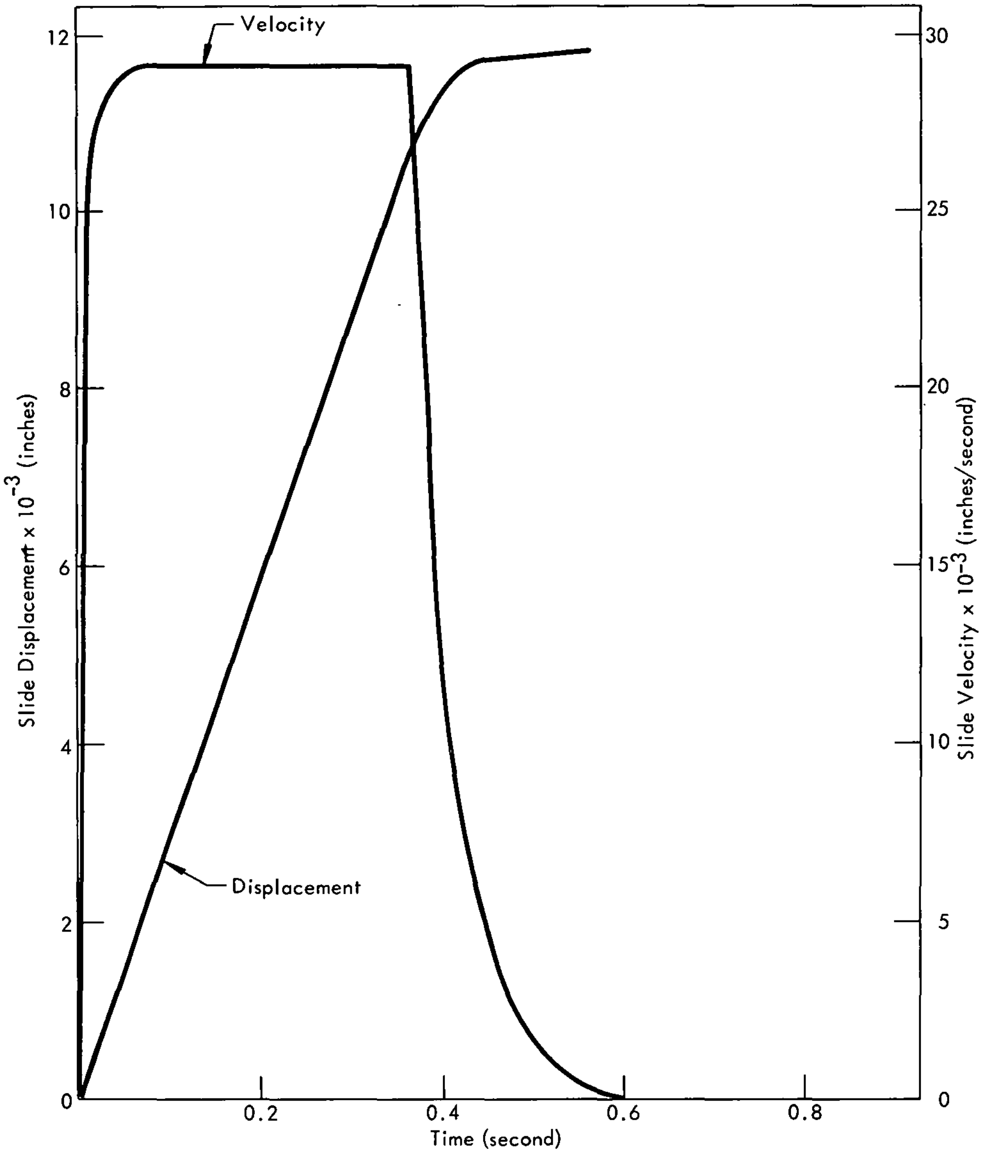


FIGURE 13

OUTPUT FOR SECOND CORRECTION CYCLE WITH THE PART 0.0125 INCH OFF CENTER AND THE SYSTEM CRITICALLY DAMPED (K_1 = 0.01 AND A = 700)

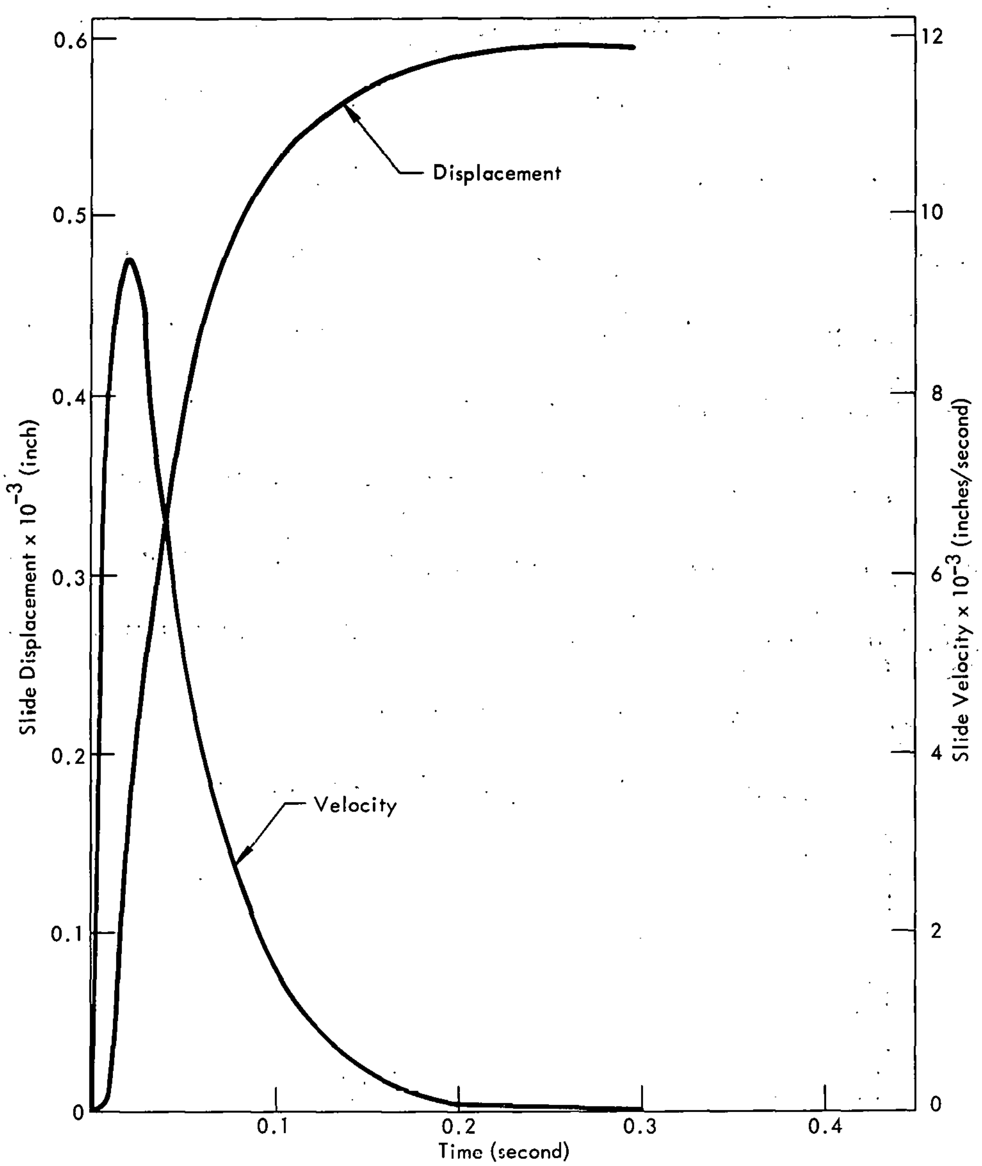


FIGURE 14

OUTPUT FOR THIRD CORRECTION CYCLE WITH THE PART 0.000625 INCH OFF CENTER AND THE SYSTEM CRITICALLY DAMPED (K_1 = 0.01 AND A = 700) are consumed with the motor running full speed under saturated conditions. The total acceleration and decelerating time amounts to less than 0.5 second, which is about 3 per cent of the total sampling and correction time. There is obviously no point in compensating the system in order to make it faster since a 100 per cent increase in system response time would only reduce the total time by 1.5 per cent. However, it is worthwhile using a frequency-shaping network along with velocity feedback in order to increase the inner loop gain which will reduce the steady-state error. If additional outer-loop compensation is needed for proper damping, it will be considered later. The transient behavior of both loops must be controlled since they operate separately part of the time and as a multiple loop for the remaining time.

CHAPTER VI

SYSTEM COMPENSATION

The inner loop will be compensated first to produce a relatively large gain for good overall steady-state accuracy. However, the transient response to a step input must be satisfactory since for large initial part off-center values the outer loop is effectively open circuited and the inner loop is operating alone. Once the inner loop is compensated, the outer loop will be analyzed and compensated independently.

Compensated velocity feedback will be used for controlling the performance of the inner loop.

Position Error

The system transfer function given by Equation 36 with K_1 replaced by the more general compensator transfer function is:

$$\frac{Y(S)}{R(S)} = \frac{0.0119A}{S\,(S + 53 + 9.3A\ G_C(S)) + 3.5A} . \tag{51}$$

The steady state output is:

$$y(t)_{SS} = \lim_{S \to 0} SY(S) = \lim_{S \to 0} \frac{0.0119A\ E_Y}{S(S + 53 + 9.3A\ G_C(S)) + 3.5A}, \text{ or}$$

$$y(t)_{SS} = \frac{0.0119A\ E_Y}{3.5A}.$$

But:

$$e(t)_{SS} = E_Y - 295\,y(t)_{SS}.$$

Therefore,

$$e(t)_{SS} = E_Y \left[1 - \frac{295\,(0.0119)\,A}{3.5A}\right] = 0.$$

This is the ideal case of zero position error. However, due to a small amount of static friction in the slides and nonlinearities in the motor near zero speed, the motor stops when its control winding goes below 20 volts.

Before proceeding further, the motor electrical time constant, which has been neglected so far, will be added to the motor transfer function since it will affect the high-frequency stability:

$$G_m(S) = \frac{150\,(417)}{S\,(S + 53)(S + 417)} = \frac{62{,}500}{S\,(S + 53)(S + 417)}. \tag{52}$$

Observing Figure 10 (Page 32) and using Equations 52 and 25 (Page 34), the motor voltage-to-input ratio is:

$$\frac{E_m(S)}{E_\epsilon(S)} = \frac{62{,}500\,A}{S\,(S + 53)(S + 417)}.$$

The steady state value of this ratio is:

$$\frac{e_m(t)_{SS}}{e_\epsilon(t)_{SS}} = \lim_{S \longrightarrow 0} \frac{62{,}500A}{(S + 53)(S + 417)} = 2.83\,A.$$

But,

$$e_\epsilon(t)_{SS} = e_e(t)_{SS} - e_C(t)_{SS}.$$

At the point in question, the motor is stopped, therefore:

$$e_e(t)_{SS} = \frac{e_m(t)_{SS}}{2.83A} = \frac{20}{2.83A} = \frac{7.07}{A}. \tag{53}$$

From this expression it is easily seen that the larger the gain A the closer $e_e(t)_{SS}$ approaches the ideal value of zero. This value for $e_e(t)$ would give exact centering with no error. Since the maximum error allowed is 0.0001 inch, which is equivalent to $e_e(t) = 0.028$ volt, the minimum value for the gain is:

$$0.028 = \frac{7.07}{A}, \quad A = 252.$$

In order to achieve the best accuracy, it is desirable to maximize the gain, insure stability, and keep the overshoot within reasonable limits. However, if the gain is increased without limit, the amplifier will become saturated with noise. If the input error signal is limited to 3.5 millivolts, the maximum usable gain is 2,000. This value will produce an ultimate centering error of 12.5×10^{-6} inch, well beyond the mechanical accuracy of the inspection machine.

Choosing the Inner Loop Compensator

The compensator must be chosen such that the system is stable with all values of gain from 1.0 to some maximum value. Since the amplifier operates under saturated conditions part of the time, with the associated gain reduction, a conditionally stable system will tend to oscillate near saturation. The largest time constant in the amplifier, which has been ignored up to now, is:

$$T_A = 0.00059.$$

Thus, the amplifier transfer function is:

$$G_A(S) = \frac{A}{1 + 0.00059S}.$$

The forward minor loop transfer function is:

$$G_A(S)\, G_m(S) = \frac{2.83A}{S\,(1 + 0.0189S)(1 + 0.0024S)(1 + 0.00059S)}. \qquad (54)$$

The minor loop feedback transfer function is:

$$G_C(S)\, G_T(S) = 0.062K_1\, S\, G_C(S).$$

Let:

$$G_C(S) = 1.0 \text{ and } K_1 = 0.01,$$

then:

$$\frac{1}{G_C(S)\, G_T(S)} = \frac{1{,}610}{S}. \qquad (55)$$

A Bode plot of these two relationships is shown in Figure 15 with A = 2,000.

In order for a system of the form G/1 + GH to be stable:

$1 + GH \geq 0$, and

$GH \geq -1.$

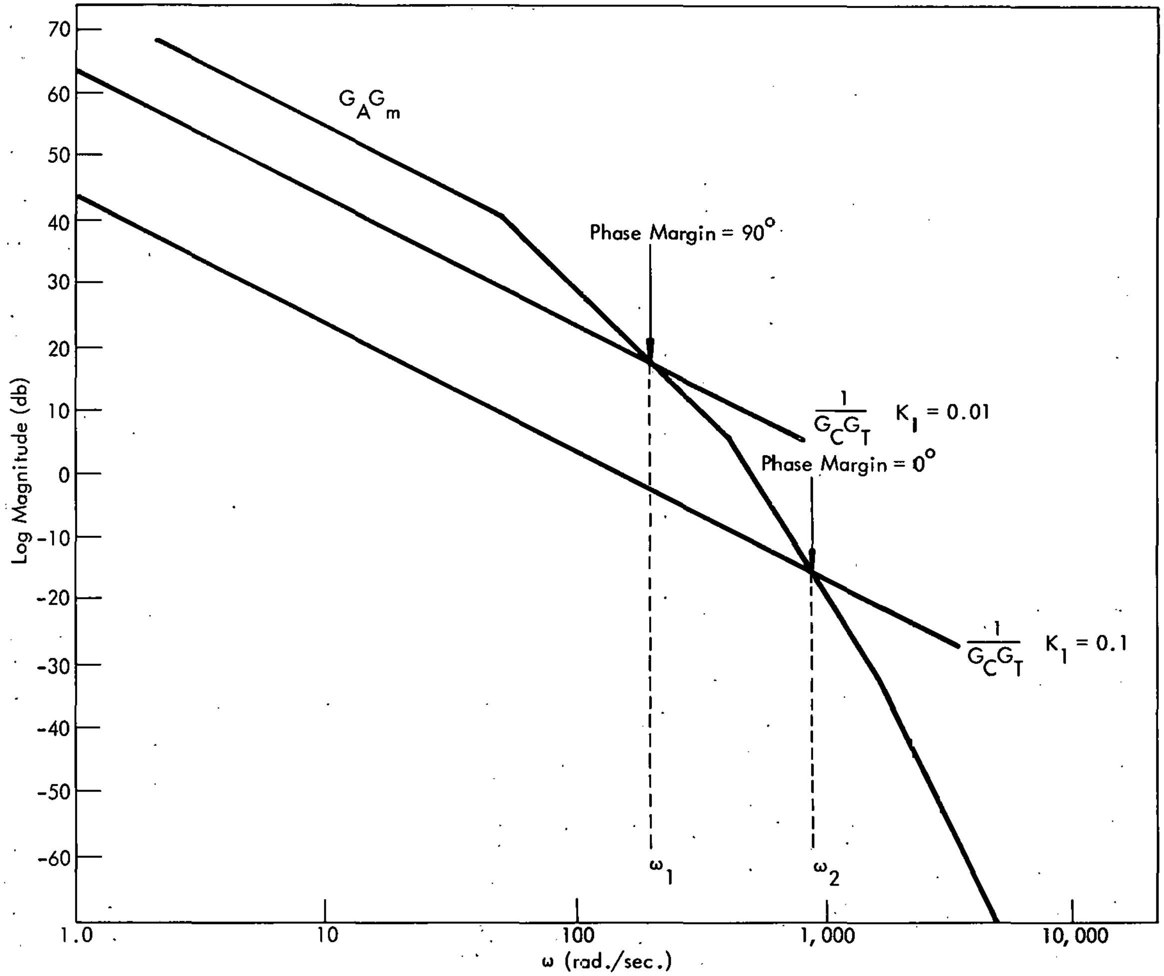


FIGURE 15

LOG MAGNITUDE OF THE INNER LOOP, FORWARD, AND INVERSE FEEDBACK TRANSFER FUNCTIONS (UNCOMPENSATED A = 2,000, G_C = 1.0, K_1 = 0.01, AND K_1 = 0.1)

This relationship will be true if the phase angle of GH is greater than $-180°$ when the magnitude is equal to 1. To prevent plotting the phase relationship for GH, an approximate formula for the phase is given by:[5]

$$\text{SLOPE}\left(\frac{\text{db}}{\text{oct.}}\right) = \frac{(\text{PHASE ANGLE})(12)}{180°}\left(\frac{\text{db}}{\text{oct.}}\right).$$

Since a slope of 6 db per octave produces a limiting phase shift of 90°, the 180° phase shift point is associated with a 12 db per octave slope of the log-magnitude curve. By plotting the inverse of the feedback transfer function the $|GH| = 1$ occurs when the log-magnitude plots of G and 1/H cross. Thus,

$$|G| \; \frac{1}{|H|} = 1, \text{ or,}$$

$$|G| = |H| .$$

At this point,

$$\angle G + \angle H \geq -180°,$$

the slope

$$|GH| = \text{SLOPE } |G| + \text{SLOPE } |H| , \text{ or,}$$

$$\text{SLOPE } |GH| = \text{SLOPE } |G| - \text{SLOPE } 1/|H| \leq 12 \text{ db/oct.}$$

This method allows a more direct evaluation of H, the feedback transfer function. If the difference in the slopes is 12 db/oct., the system has no phase margin to allow for parameter variations. Adding an additional term for phase margin:

$$\text{SLOPE } |GH| = \text{SLOPE } |G| - \text{SLOPE } 1/|H| \leq (12 - \varphi_m) \text{ db/oct.},$$

where φ_m is the desired phase margin expressed in units of slope. For instance, a margin of 45° is equivalent to 3.0 db/oct. difference in slope.

By observing Figure 15 (Page 57), it is apparent that stable operation can be obtained by limiting $K_1 = 0.01$. However, the point where the open loop gain becomes unity occurs at $\omega_1 = 200$. If an attempt is made to raise this frequency to $\omega_2 = 840$ by increasing K_1 to 0.1, the phase margin goes to zero and the system is unsatisfactory. This fact is more apparent in the root locus plot shown in Figure 16. This plot was made with the gain fixed at 2,000 and K_1 adjustable from zero up to the point where the real part of the roots cross over into the right half plane. Notice that this situation occurs at a low value of $K_1 = 0.14$. With $K_1 = 0.01, \zeta = 0.67$, which is still somewhat low.

If the plot of:

$$\frac{1}{|G_C(j\omega)\, G_T(j\omega)|}$$

can be made to slope downward at 12 db/oct. in the region where the forward gain is large and then slope at 6 db/oct. near the open loop unity gain point, an improvement in performance can be obtained. In order for the system to be reasonably fast, the point where the two curves cross should be at a high frequency.

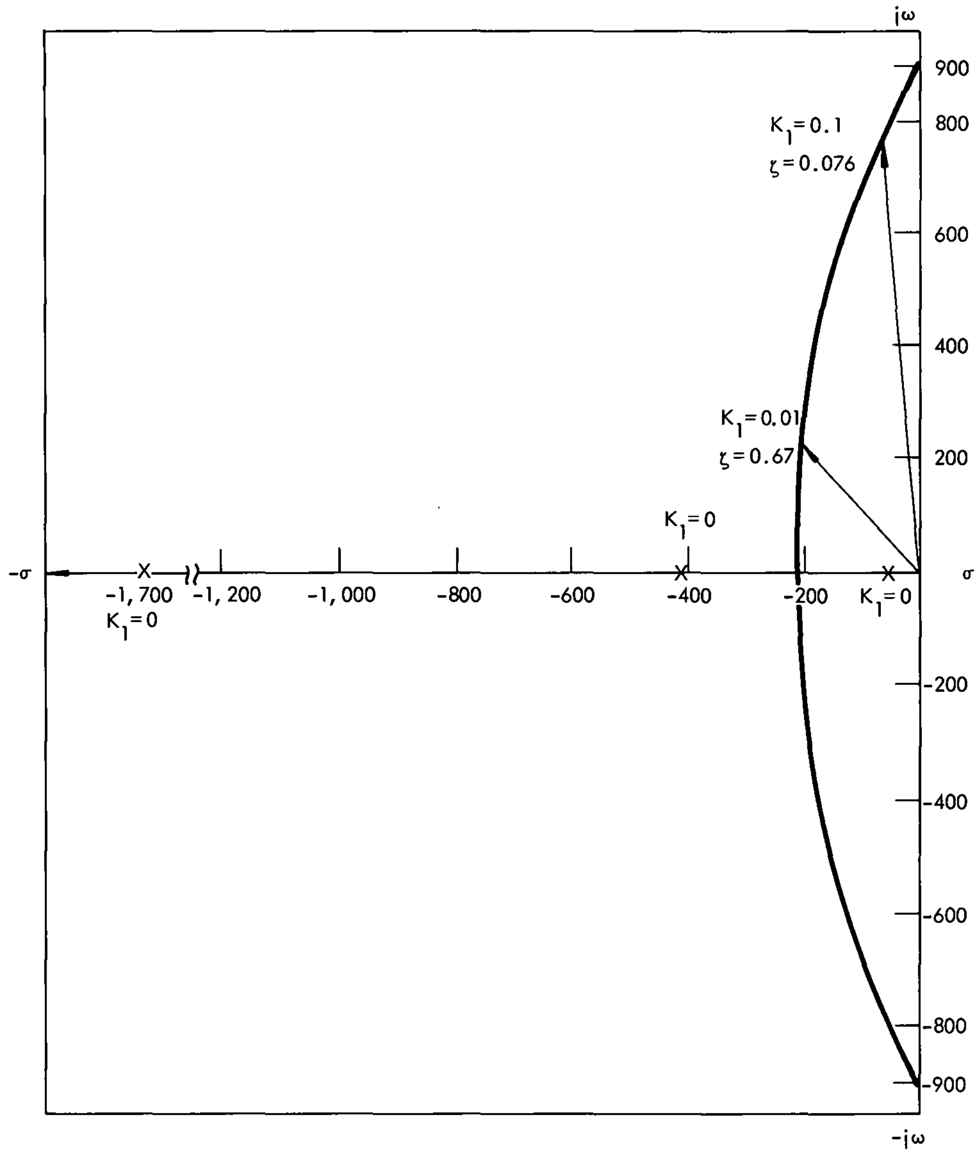


FIGURE 16

INNER CLOSED-LOOP ROOT LOCUS
($A = 2{,}000,\ 0 \le K_1 \le 0.14$)

The feedback transfer function needs to be of the form:

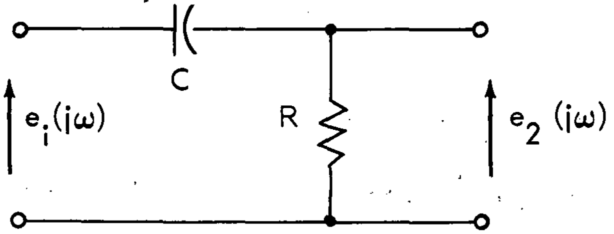
$$G_T(j\omega)\, G_C(j\omega) = \frac{K T_1 (j\omega)^2}{1 + T_1\, j\omega}.$$

Since:

$$G_T(j\omega) = K_1\, j\omega, \text{ then}$$

$$G_C(j\omega) = \frac{T_1\, j\omega}{1 + T_1\, j\omega}.$$

This equation can be achieved by a single lead network:



$$G_C(j\omega) = \frac{E_2(j\omega)}{E_1(j\omega)} = \frac{RC\, j\omega}{1 + RC\, j\omega}, \text{ and}$$

$$G_T(j\omega)\, G_C(j\omega) = \frac{0.062\, K_1\, RC(j\omega)^2}{1 + RC\, j\omega} = \frac{0.062\, K_1\, T_C (j\omega)^2}{1 + T_C\, j\omega}.$$

The selection of K_1 and T_C is largely by trial and error. The open-loop transfer function is:

$$G_o(S) = G_A(S)\, G_m(S)\, G_T(S)\, G_C(S), \text{ or}$$

$$G_o(S) = \frac{350\, K_1\, T_C\, S}{(1 + 0.019S)(1 + 0.0024S)(1 + 0.00059S)(1 + T_C S)}.$$

Notice that the former pole at the origin has been replaced by an open loop zero. This replacement will have the desired effect of moving the root locus to the left, but at the expense of adding a real root near the origin which will produce a slowly decaying exponential in the time solution. If the magnitude of this root can be kept reasonably large, the system will be satisfactory.

The approximate value of T_C can best be determined by plotting the log magnitude of the forward and inverse transfer functions. These curves are given in Figure 17 for $K_1 = 1$ and two different values of T_C. To determine what the transient solution will be, the root loci for these two cases are needed.

Transient Response of the Inner Loop

The inner closed-loop transfer function with a gain of 2,000 and compensated feedback is:

$$\frac{\theta_m(S)}{E_e(S)} = \frac{a_1 S + a_0}{b_5 S^5 + b_4 S^4 + b_3 S^3 + b_2 S^2 + b_1 S + b_0}, \tag{56}$$

where:

$$a_0 = \frac{2 \times 10^{11}}{T_C},$$

$$a_1 = 2 \times 10^{11},$$

$$b_0 = 0.0,$$

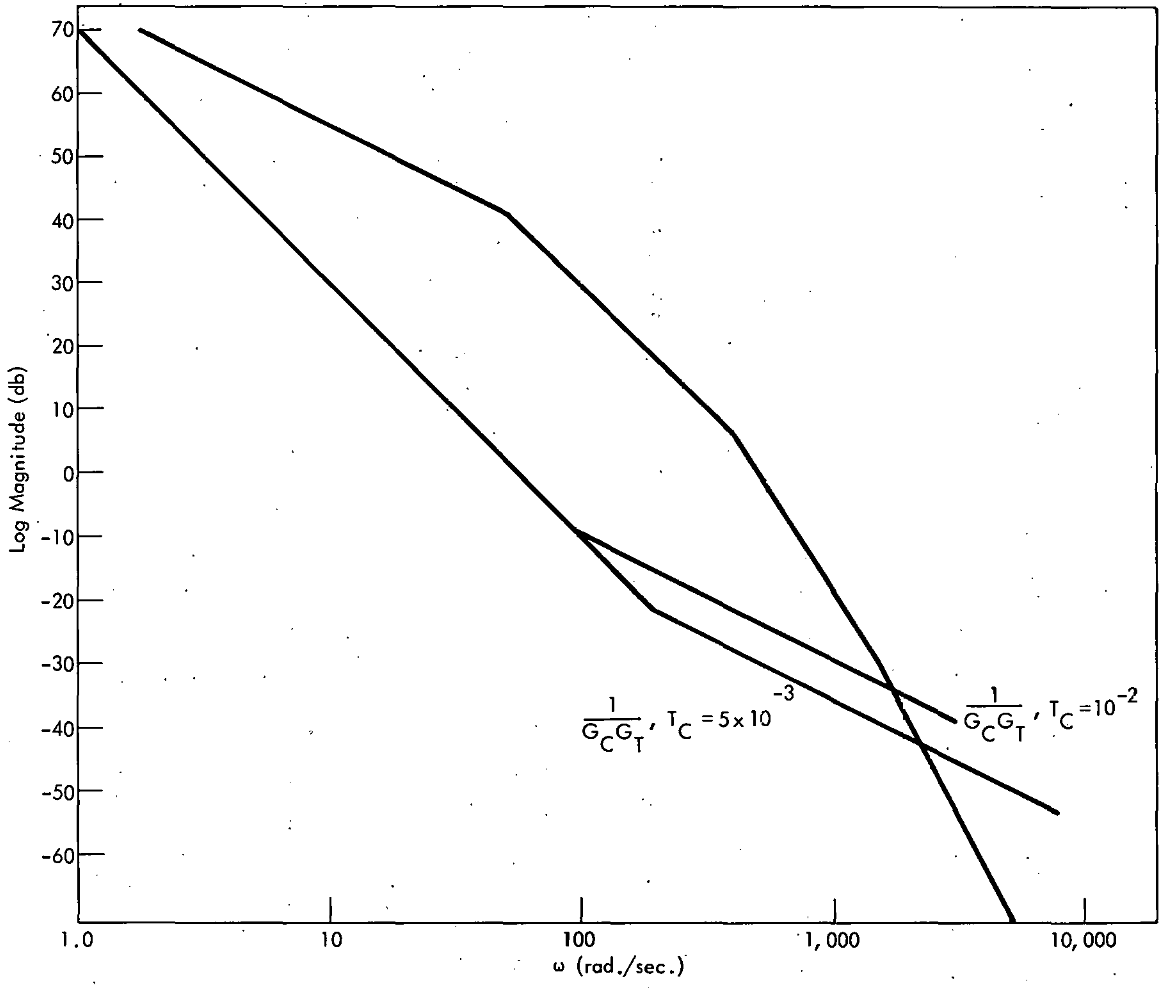


FIGURE 17

LOG MAGNITUDE OF THE INNER LOOP, FORWARD, AND INVERSE FEEDBACK TRANSFER FUNCTIONS ($A = 2{,}000$, $G_C G_T = 0.062\, T_C j\omega^2/1 + T_C j\omega$, $K_1 = 1.0$)

$$b_1 = \frac{3.75 \times 10^7}{T_C},$$

$$b_2 = 3.75 \times 10^7 + \frac{8.22 \times 10^5}{T_C} + 1.31 \times 10^{10} K_1 T_C,$$

$$b_3 = 8.22 \times 10^5 + \frac{2.17 \times 10^3}{T_C},$$

$$b_4 = 2.17 \times 10^3 + \frac{1}{T_C}, \text{ and}$$

$$b_5 = 1.$$

The root locus for the case of $T_C = 5 \times 10^{-3}$ is shown in Figure 18. With $K_1 = 1$, which is the largest value it can have without using an amplifier in the feedback path, the system is damped more than is desired. However, a value of $\zeta = 0.84$ with a real root of -33 and an undamped natural frequency of 280 rad./sec. would be satisfactory. A slightly better transient solution is obtained with $T_C = 10^{-2}$, which is shown in Figure 19. Here $\zeta = 0.707$ with $K_1 = 0.92$ and the undamped natural frequency, $\omega_n = 350$ rad./sec., is higher than the previous case. The real root has dropped to -17.

The inner closed-loop transfer function in factored form can be found from Equation 56 and Figure 19 as:

$$G_1(S) = \frac{2 \times 10^{11} (S + 100)}{S(S + 17)(S + 1{,}750)(S + 250 + j250)(S + 250 - j250)}. \qquad (57)$$

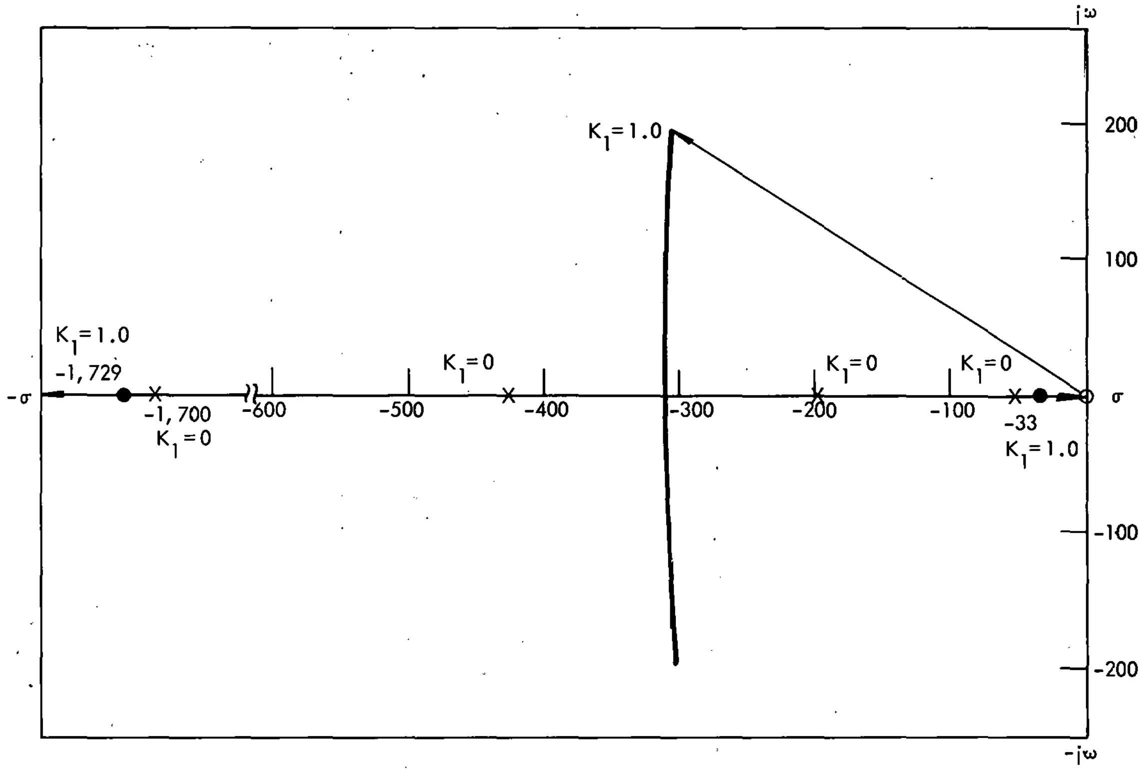


FIGURE 18

INNER CLOSED-LOOP ROOT LOCUS ($A = 2,000$, $0 \leq K_1 \leq 1.0$, $T_C = 5 \times 10^{-3}$, $\zeta = 0.84$)

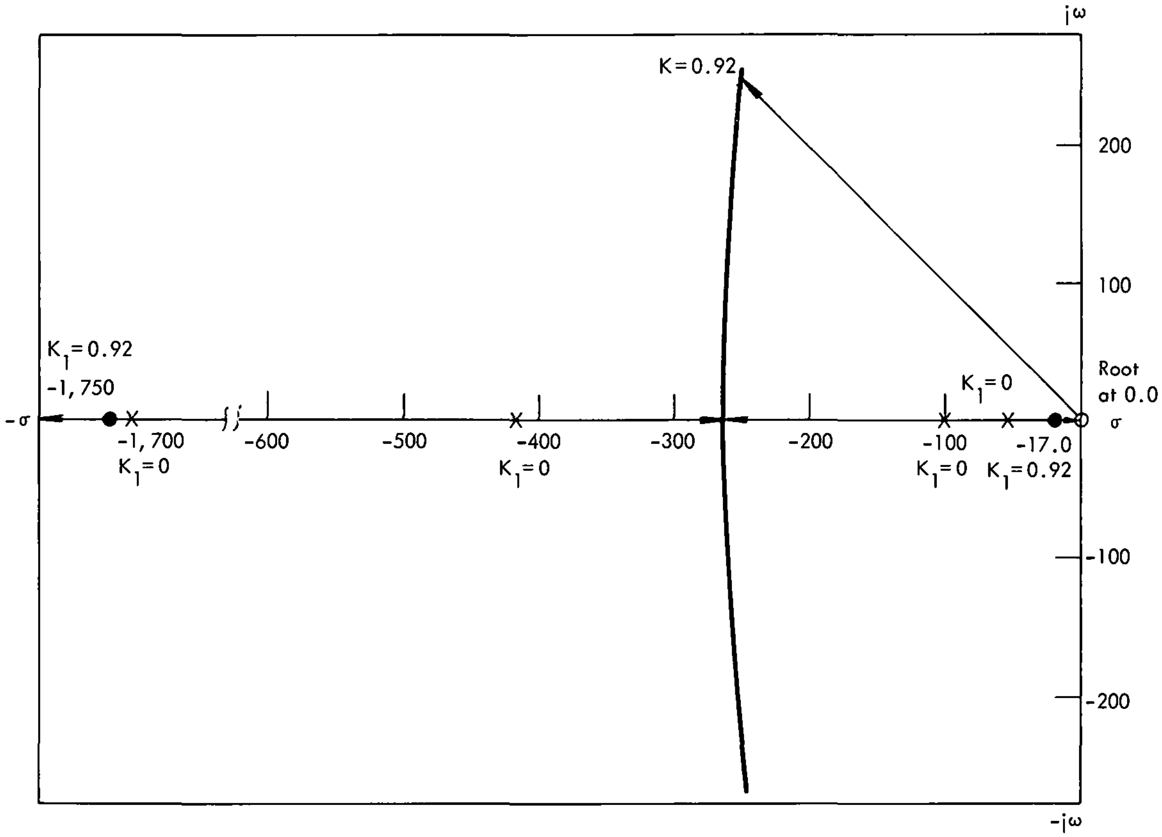


FIGURE 19

INNER CLOSED-LOOP ROOT LOCUS ($A = 2{,}000$, $0 \leq K_1 \leq 1.0$, $T_C = 10^{-2}$, $\zeta = 0.707$)

The next problem is the determination of the outer closed-loop performance.

Transient Response of the Outer Closed Loop

Since the inner closed-loop transfer function is $G_1(S)$ and letting the outer closed-loop transfer function be $G_3(S)$, then

$$G_3(S) = \frac{G_1(S)}{1 + G_1(S)\, G_I(S)},$$

where:

$$G_I(S) = \frac{0.062S}{2.63S} = 0.0236,$$

the transfer function of the integrator. The closed-loop poles and zeros of $G_1(S)$ become the open-loop poles and zeros of $G_3(S)$.

Let $G_2(S)$ be the open-loop transfer function for the outer loop. Then:

$$G_2(S) = G_I(S)\, G_1(S) = \frac{2.36\,(2)\,10^9\,(S + 100)}{S\,(S+17)(S+1{,}750)(S+250+j250)(S+250-j250)}. \quad (58)$$

A Bode plot of this equation is shown as the solid curve in Figure 20. The slope of this curve at the unity gain point is -12 db/oct. which will produce an under damped system and possibly instability. The easiest way to produce less slope in this region would be to add a lead network producing a zero somewhere between 17 and 50 and a pole at 100. This network would produce a slope of 6 db/oct. through the unity gain region. However, it is impossible to exactly cancel the zero at 100 with a pole, due to parameter variations.

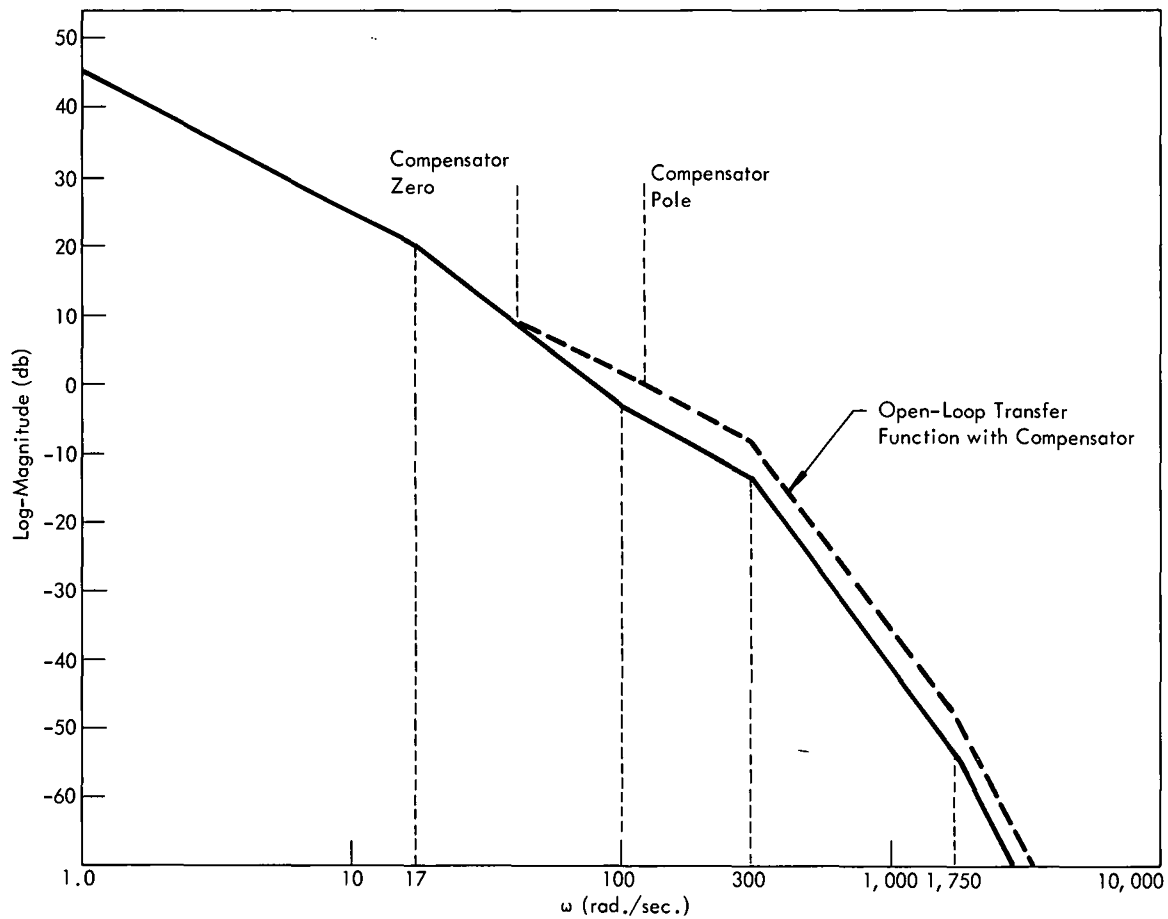


FIGURE 20

OUTER OPEN-LOOP TRANSFER FUNCTION WITHOUT COMPENSATION SOLID CURVE (THE SAME DATA, BUT WITH LEAD COMPENSATION ADDED IS SHOWN IN THE DOTTED CURVE)

Therefore, it is best to set the compensator pole about 20 per cent away from the system zero so the root produced at this point will be present in the following root locus analysis.

The root locus for the uncompensated outer closed loop is shown in Figure 21. An amplifier with gain of A_1 has been added. Notice that an operating point producing a $\zeta = 0.707$ will be very slow due to the complex roots near the origin.

Adding a lead network and an amplifier with a gain of A_1 to Equation 58 gives:

$$G_2(S) = \frac{(4.72)\,10^9\,A_1\,(S + 100)(S + Z)}{S\,(S + 17)(S + P)(S + 1{,}750)(S + 250 + j250)(S + 250 - j250)}.$$

Solving for the closed-loop transfer function:

$$G_3(S) = \frac{\dfrac{G_2(S)}{G_1(S)}}{1 + G_2(S)}, \text{ or}$$

$$G_3(S) = \frac{(2)\,10^{11}\,A_1\,(S + 100)(S + Z)}{b_6S^6 + b_5S^5 + b_4S^4 + b_3S^3 + b_2S^2 + b_1S + b_0},$$

where:

$$b_0 = 4.72 \times 10^{11}\,A_1 Z,$$

$$b_1 = 4.72 \times 10^{9}\,A_1\,(100 + Z),$$

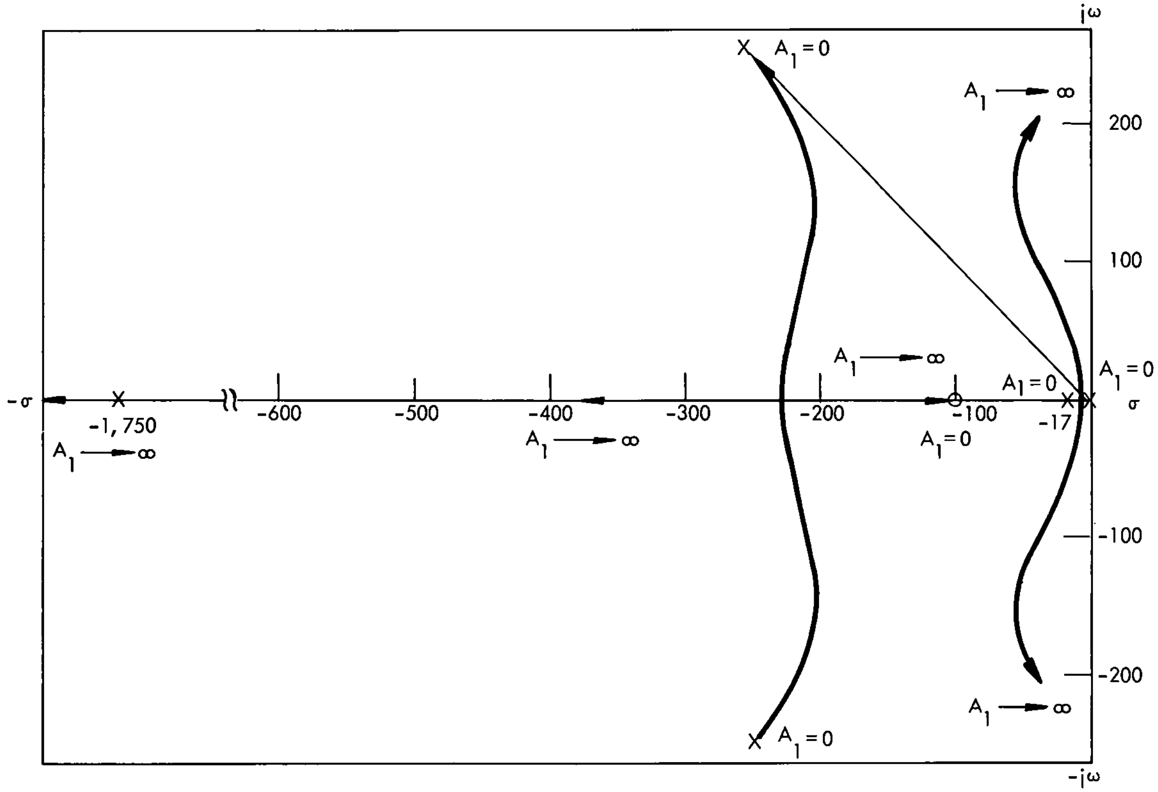


FIGURE 21

OUTER CLOSED LOOP WITHOUT COMPENSATION BUT WITH AN AMPLIFIER OF GAIN A_1 ADDED

$$b_2 = 3.73 \times 10^9 \, (1 + 0.0633\,P + 1.26\,A_1),$$

$$b_3 = 2.36 \times 10^8 \, (1 + 0.0044\,P),$$

$$b_4 = 1.04 \times 10^6 \, (1 + 0.00218\,P),$$

$$b_5 = 2.27 \times 10^3, \text{ and}$$

$$b_6 = 1.$$

Several values of Z and P were tried with A_1 ranging from 1.0 to 10. The best combination is shown in Figure 22, which is the root locus for the outer closed loop with lead compensation. The compensator zero is 40 and the pole is 120.

Selecting $\zeta = 0.707$ for the complex roots nearest the origin, A_1 is found to be 4.8. The final outer closed-loop transfer function can now be written in factored form by reading the roots from Figure 21 at $A_1 = 4.8$:

$$G_3(S) = \frac{(9.6)\,10^{11}\,(S+40)(S+100)}{(S+320)(S+1,570)(S+38+j38)(S+38-j38)(S+153+j193)(S+153-j193)} \,. \quad (59)$$

The root loci shown in Figures 18 (Page 65), 19 (Page 66), and 21 were found by computer solution using a modification to a root solving program available from International Business Machines Company, New York, as "Share" Number 2230.

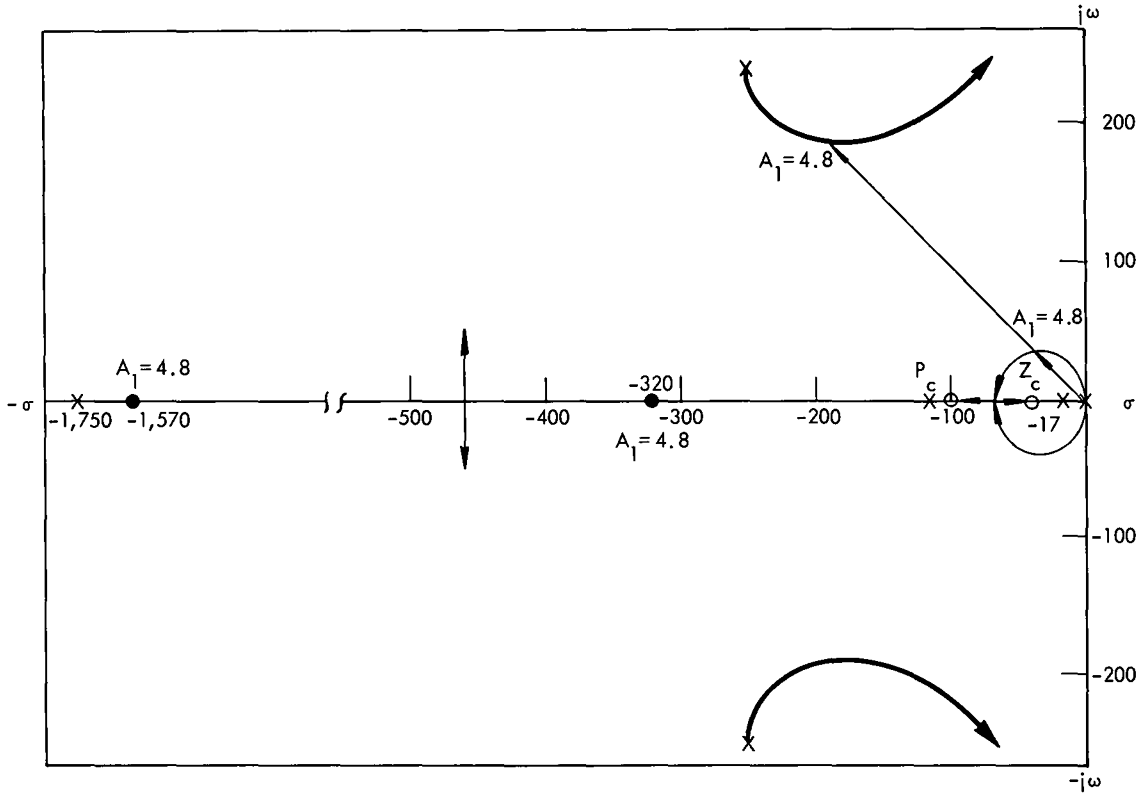


FIGURE 22

OUTER CLOSED-LOOP ROOT LOCUS WITH LEAD COMPENSATION

The program has been changed to allow any one of the coefficients to be incremented by any specified value.[a] This alteration allows root loci to be calculated by sweeping any of the system parameters. A plotting program has also been added to plot the root locus automatically on a computer-controlled plotter.

(a) W. L. Griffith, Process Analysis Group, Y-12 Plant, Union Carbide Corporation-Nuclear Division, Oak Ridge, Tennessee, made the program modifications.

CHAPTER VII

TIME RESPONSE

Inner Loop

The transfer functions of the inner closed loop in terms of the motor shaft angle and shaft speed are given by Equation 57 (Page 64) and its derivative as:

$$\frac{\theta_m(S)}{E_e(S)} = \frac{(2)\,10^{11}\,(S + 100)}{S\,(S + 17)(S + 1{,}750)(S + 250 + j250)(S + 250 - j250)}, \text{ and}$$

$$\frac{\omega_m(S)}{E_e(S)} = \frac{(2)\,10^{11}\,(S + 100)}{(S + 17)(S + 1{,}750)(S + 250 + j250)(S + 250 - j250)}.$$

The time response to a step input of magnitude E_Y is:

$$\theta_m(t) = E_Y\left[-287 + 5{,}380t + 0.027e^{-1{,}750t} + 3.59e^{-250t}\cos(250t - 21.4) + 287e^{-17t}\right], \text{ and}$$

$$\omega_m(t) = E_Y\left\{5{,}380 - 4{,}870e^{-17t} - 47.2e^{-1{,}750t} + 900e^{-250t}\left[\sin(250t - 21.4) - \cos(250t - 21.4)\right]\right\}.$$

The maximum steady-state motor speed is 365 rad./ second; therefore, the value of $e_e(t)$ which produces amplifier saturation is:

$$e_e(t) = \frac{365}{5{,}380} = 0.068 \text{ volt.} \tag{60}$$

Converting from shaft angle to slide position by the relation:

$\theta_m(t) = 4{,}000\pi y(t),$

and dropping the third term which is insignificant, the output in terms of time is:

$$y(t) = E_Y \left[-0.0228 + 0.428t + 0.0228e^{-17t} + 0.000286e^{-250t} \cos (250t - 21.4) \right], \text{ and} \tag{61}$$

$$v(t) = E_Y \left\{ 0.428 - 0.387e^{-17t} + 0.0715e^{-250t} \left[\sin (250t - 21.4) - \cos (250t - 21.4) \right] \right\}. \tag{62}$$

Assuming a maximum part off-center value of 0.250 inch initially, and remembering that the inner loop input is given by:

$$e_e(t) = E_Y - 295y(t), \tag{63}$$

where:

$E_Y = 70$ volts for an initial off-center value of 0.250 inch, the inner loop comes out of saturation, and the outer loop is effectively closed at a slide position of:

$0.068 = 70 - 295\, y(t)$, or

$y(t) = 0.237$ inch.

From Equation 61, this slide position occurs at a time given by

$0.237 = 0.068\ (0.428\,t)$, or

$t = 8.14$ sec.

At this point in time, the outer loop becomes operative with initial conditions of:

$y(o) = 0.237$ inch,

$v(o) = 0.029$ inch/sec., and

$t' = t - 8.14$ sec.

Outer Loop

The outer-loop transfer function with initial conditions of zero is given by Equation 59 (Page 71). Adding the initial conditions to this equation, the output in terms of slide position is:

$$Y(S) = \left[\frac{(7.63)\,10^7\,E_Y(S+40)(S+100)}{DS} + \frac{y(o)}{S} + \frac{v(o)}{S^2} - \frac{(8.8 \times 10^{13}\,y(o) + 3.09 \times 10^{12}\,v(o))}{DS} - \frac{8.8 \times 10^{13}\,y(o)}{DS^2} \right] e^{-8.14S}, \quad (64)$$

where:

$$D = (S+320)(S+1{,}750)(S+38+j38)(S+38-j38)(S+153+j193)(S+153-j193).$$

Taking the inverse LaPlace transform using the above initial conditions and remembering that $E_Y = 70$, the slide position as a function of time can be found. The inverse transform and numerical values for both the position and velocity were found by use of a computer. The results are plotted in Figure 23. Notice that the slide reverses direction momentarily, but the system comes to rest in about 0.2 second.

For the second correction cycle, the part is off center
$0.250 - 0.240 = 0.010$ inch.

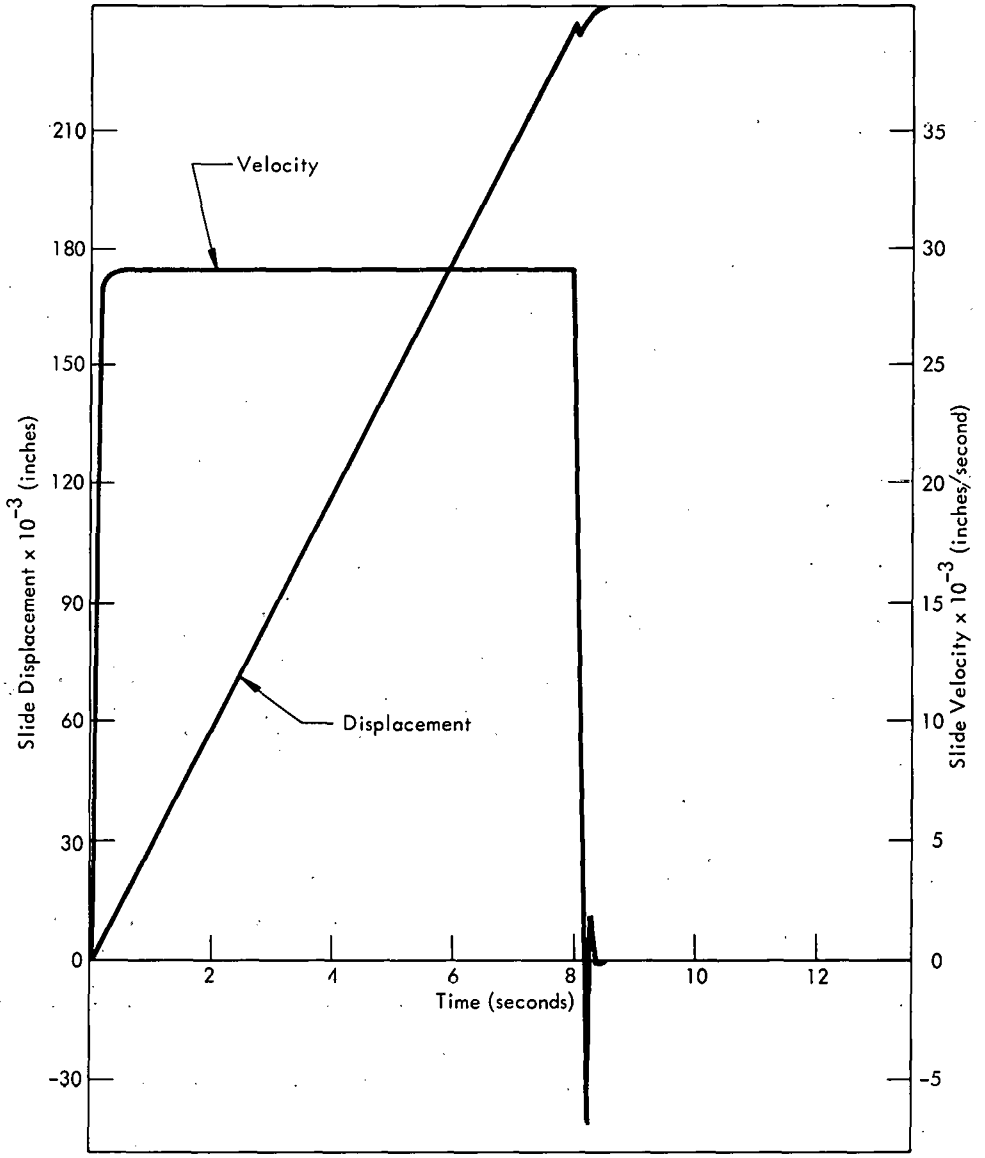


FIGURE 23

OUTPUT FOR FIRST CORRECTION CYCLE WITH THE PART 0.250 INCH OFF CENTER (COMPENSATED SYSTEM)

This value of slide error produces a value of $E_Y = 280\ (0.010) = 2.8$ volts in the integrating condenser. Again, the outer loop is ineffective due to saturation until the slide position is:

$$0.068 = 2.8 - 295\ y(t), \text{ or}$$

$$y(t) \doteq 0.00926 \text{ inch}$$

from center. The time when the slide reaches this position can be found from Equation 61 which is plotted in the first part of Figure 24. This time is:

$$t = 0.370 \text{ sec.}$$

The output is given by Equation 61 with $E_Y = 0.068$ until $t = 0.370$. After this value of time, the outer loop is closed and the response is given by Equation 64. The numerical results are given in Figure 24. Notice the scale changes. The final displacement is 0.00975 inch which leaves:

$$0.010 - 0.00975 = 0.00025 \text{ inch}$$

for the final correction.

For the third and final correction cycle,

$$E_Y = 280\ (0.000250) = 0.070,$$

which is just at the amplifier saturation point and the outer loop is closed for the complete cycle. The final slide displacement is 0.000240 inch; therefore, the final error is

$$0.000250 - 0.000240 = 0.000010 \text{ inch},$$

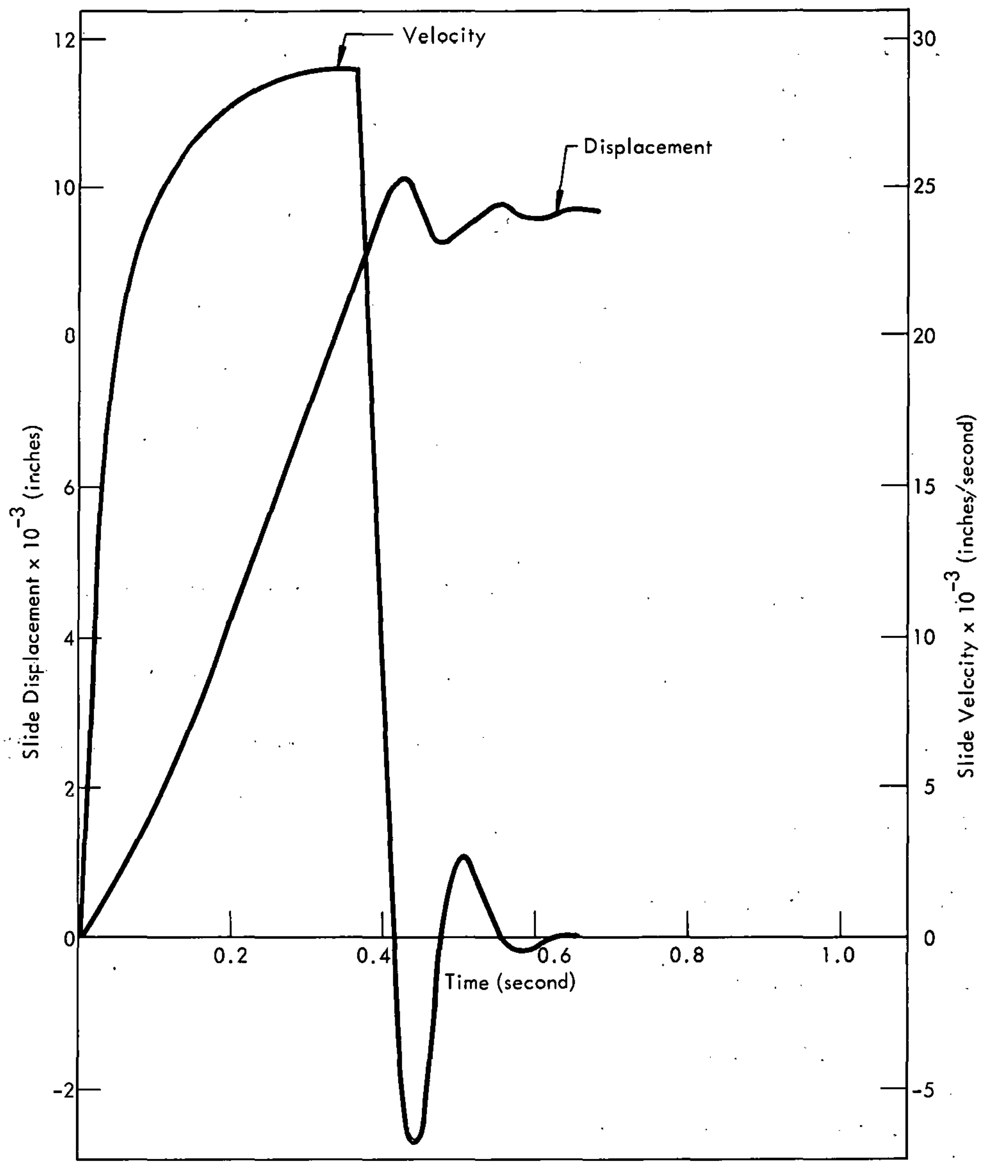


FIGURE 24

OUTPUT FOR SECOND CORRECTION CYCLE WITH THE PART 0.00926 INCH OFF CENTER (COMPENSATED SYSTEM) or 10 microinches. The results are shown in Figure 25; the final system block diagram is shown in Figure 26.

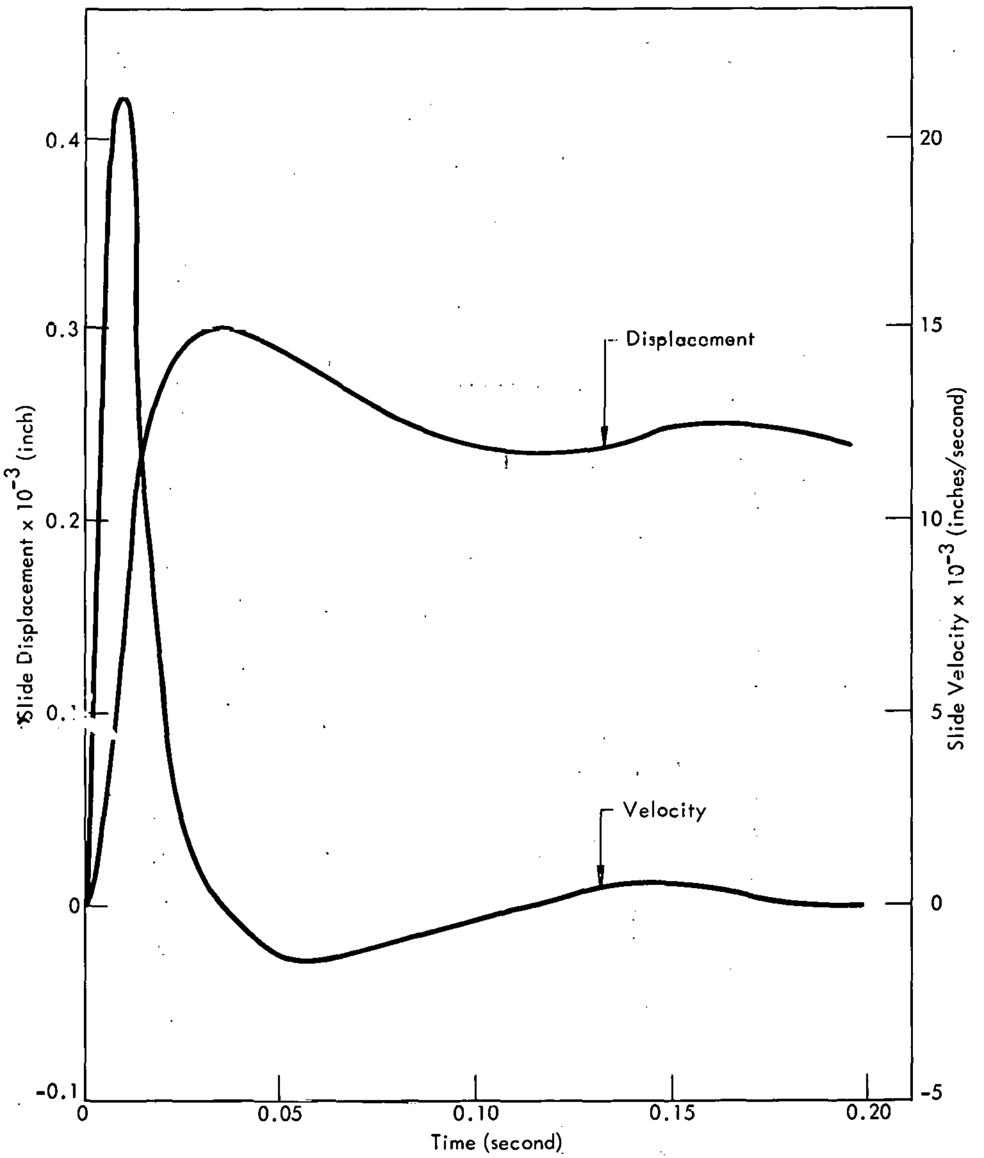


FIGURE 25

OUTPUT FOR THIRD CORRECTION CYCLE WITH THE PART 0.00025 INCH OFF CENTER (COMPENSATED SYSTEM)

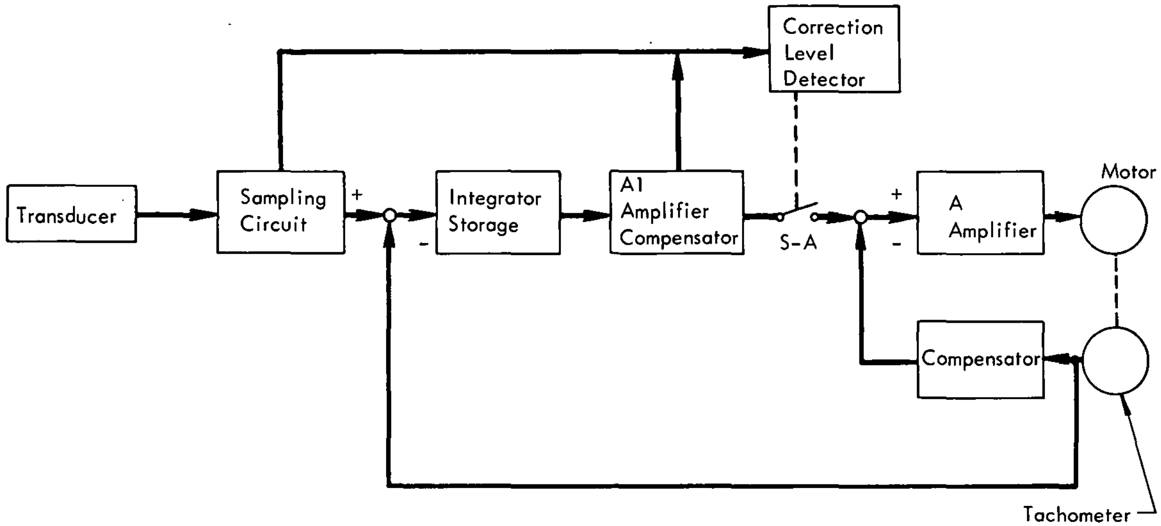


FIGURE 26

BLOCK DIAGRAM OF THE FINAL SYSTEM FOR ONE AXIS

CONCLUSIONS

The Control System is capable of centering, to within 50 microinches, any part which is a surface of revolution about its axis of rotation. However, the mechanical system has extraneous deflections and bearing free play which limits the overall accuracy to 500 microinches. To prevent the machine from continuing successive correction cycles after the part is centered to within 500 microinches, the level detector is adjusted such that any sample producing a value of E_Y less than 0.14 volt, which corresponds to a 500-microinch error, will stop the centering cycle and signal the operator that the part is centered and ready for inspection.

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2. Chestnut, Harold and Mayer, Robert W., Servomechanisms and Regulating System Design, II. New York: John Wiley and Sons, Inc., 1955, pp. 101-117.

3. D'Azzo, John J. and Houpis, Constantine H., Feedback Control System Analysis and Synthesis. New York: McGraw-Hill, Inc., 1966.

4. Garner, Murry F. and Barnes, John L., Transients in Linear Systems. New York: John Wiley and Sons, Inc., 1942, p. 342.

5. Granata, P. J. and Aasnaes, H. B., "A Modified Bode Criterion for Feedback-System Stability," EEE Circuit Design Engineering, 14:97-99, October, 1966.

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APPENDIXES

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APPENDIX A

POLYNOMIAL ROOT EXTRACTION

This program was obtained from International Business Machines Corporation, New York; Share Number 33,320. It was written by M. L. Smith of the Pan American Petroleum Corporation Research Center, Tulsa, Oklahoma. The program has been modified to allow multiple cases to be run with one parameter to be iterated between selected bounds. A plotting program has been added to allow a complete root locus to be plotted as a function of one parameter. The languages are Fortran IV and MAP; written for the IBM 7044 computer.

Program Description

This program is designed to find all the roots, both real and complex, of a polynomial of the form:

$$C_1 + C_2X + C_3X^2 + \text{---} \; C_nX^{n-1} + X^n = 0.$$

C_i are restricted to real coefficients;

$2 < n < 100.$

As a secondary feature, the real coefficients can be determined if the roots are furnished.

When using the program for root extraction, the first root is found by the Muller method, an iterative process which is continued until successively estimated roots agree within a tolerance specified by the user. The root is then refined by the Newton method, also an iterative technique. The Newton method is terminated by a user-specified tolerance. After finding the first real root, or complex pair, the polynomial is divided by the resulting linear or quadratic expression to yield a reduced polynomial. The above process is repeated on the reduced polynomial. The resulting root is then refined by using the Newton method on the original polynomial.

This process is repeated until the polynomial has been reduced to a quadratic or linear form. The roots are then found explicitly except that the Newton method is still used to refine the roots based on the original polynomial. After finding all roots, their validity is checked by computing a new set of coefficients and comparing these with the originals. If they differ by a specified tolerance, an error message is printed out, but the program continues.

The main limitation of this program is a loss of precision inherent in repeated multiply-add operations. To reduce this error, double precision is used for the Newton method, the polynomial division process, and for all calculations to determine the value of the polynomial. To avoid overflows, special techniques have been used to avoid calculating the sum of squares of large terms which are frequently encountered in complex arithmetic. To avoid underflows, a special MAP routine has been included which gives a zero on an underflow and does not limit the number of such underflows.

This program, with modifications, has been used successfully to compute and plot root loci for polynomials up to tenth order with no difficulties.

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APPENDIX B

INVERSE LAPLACE TRANSFORM COMPUTER PROGRAM

This program was obtained from the International Business Machines Corporation, New York; Share Number 1,125. It was coded for the IBM 7090 computer by E. Castaneula using Fortran II and FAP languages. The only modifications made were the addition of a plotter program and arranging the program to run multiple problems.

Method

Heaviside's method was used to solve for y(t) in the equation:

$$\frac{P(S)}{q(S)} = \int_0^\infty e^{-st}\, y(t)\, dt .$$

The equation which is solved is:

$$y(t) = \sum_{i=1}^{L} \frac{P(a_i)}{q'(a_i)} e^{a_i t} +$$

$$\sum_{i=1}^{m} e^{b_i t} \sum_{h=1}^{2} \frac{t^{h-1}}{(2-h)!(h-1)!} \left[\frac{(S - b_i)^2\, P(S)}{q(S)}\right]^{(2-h)}_{S=b_i} +$$

$$2 \sum_{i=1}^{\lambda} \sqrt{\varphi_{1i}^{\,2} + \varphi_{2i}^{\,2}}\; e^{\alpha_i t} \cos(\beta_i t + \theta_i),$$

where:

$$\varphi_{1i} = \mathrm{Re}\left[\frac{P(\alpha_i + \beta_i i)}{q'(\alpha_i + \beta_i i)}\right],$$

$$\varphi_{2i} = \mathrm{Im}\left[\frac{P(\alpha_i + \beta_i i)}{q'(\alpha_i + \beta_i i)}\right], \text{ and}$$

$$\theta_i = \tan^{-1} \frac{\varphi_{2i}}{\varphi_{1i}} .$$

The term (2-h) means the $(2-h)^{th}$ derivative of the bracketed function. The term $q'(a_i)$ is the derivative of $q(a_i)$.

Restrictions

The program restricts repeated real roots to multiplicity of two and will not solve problems with repeated complex roots.

Output

In addition to finding the numerical values of y(t) for specified values of time, the program also calculates values of the first and second derivatives of y(t) for the same specified time increments. The output is tabular values of time, displacement, and acceleration.