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QUARTERLY PROGRESS REPORT NO. 13

"Plasma Instabilities and Waves Excited
by an Anisotropic Energy Distribution
Produced by Electron Cyclotron Resonance"

U.S. Atomic Energy Commission
Contract No. AT(11-1) 1695

REPORT NO. COO-1695-27

June 15, 1970

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Madison, Wisconsin 53706

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Plasma Instabilities and Waves Excited by an Anisotropic Energy
Distribution Produced by Electron Cyclotron Resonance

Contract No. AT(11-1) 1695

U.S. Atomic Energy Commission

J. L. Shohet
J. C. Sprott
S. R. Seshadri
S. J. Gitomer
D. G. S. Greene
K. A. Connor
K. E. Trnka
R. T. Pedley

Principal Investigator (on leave)
Co-Investigator
Co-Investigator
Consultant
Graduate Research Assistant
Graduate Research Assistant
Student Technician
Student Technician

June 15, 1970

Department of Electrical Engineering
The University of Wisconsin
Madison, Wisconsin 53706

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1. INTRODUCTION

J. C. Sprott

The period of March 15, 1970 to June 14, 1970 has been a busy one, both theoretically and experimentally. Computer simulation of electron cyclotron heating and of rf confinement have been extended. The effect of varying the plasma density, and hence the importance of space charge electric fields, has been studied.

Calculations of the free-free bremsstrahlung from an anisotropic, relativistic plasma have been continued. The validity of the various approximations used in the derivation have been studied and corrections made for the finite mass of the nucleus. Calculations of the polarization of the bremsstrahlung have been carried out, and this promises to be a useful tool for determining plasma anisotropy.

The experimental program has been enhanced by improving the reproducibility of the plasma by reducing the ripple on the magnetic field and magnetron high voltage supply by an order of magnitude and by the construction of a 50 watt CW microwave source. Measurements of the heating efficiency have been made in order to compare with the theoretical predictions in the previous Quarterly Progress Report. Flute instabilities have been observed and studied. The CW microwave source has been used to produce plasmas and to study off resonance heating. Upper off-resonance heating is relatively inefficient, and lower off-resonance heating drives the plasma out of the machine by a process not yet understood.

The electromagnetic instability previously observed has been studied further to determine the scaling of frequency with heating frequency and mirror ratio. Comparisons are being made with other similar experiments and with theoretical predictions.

II. COMPUTER SIMULATION

J. L. Shohet

Experimental research has shown that effective heating of a plasma is possible by means of electron cyclotron resonance.¹ When the r.f. heating field is oriented perpendicularly to a uniform d.c. magnetic field, highly anisotropic plasmas are produced. Theory to explain the heating mechanism is largely based on three assumptions. They are: (1) At any given instant of time, it is equally likely for a particle to be decelerated as it is to be accelerated. (2) Since a net heating occurs; some sort of collisional process must "throw" the electrons into the proper phase to produce a net heating effect. (3) After heating, a non-Maxwellian distribution is often the result, usually having a collection of "hot" and "cold" particles.

It is the purpose of this note to show that a computer simulation of a plasma in the resonance (heating) plane will satisfy the above three assumptions. The basic method of solution follows the individual motions of a collection of "super-particles" in a two-dimensional region. Super-particles are essentially clouds or collections of real particles, and have the same charge-to-mass ratio as real particles. Both super-ions and super-electrons will be used.

The interaction region is immersed in a d.c. magnetic field. The external a.c. electric field may be made either uniform or into a plane section of the TE₁₁₁ mode of a microwave cavity. Initially, all particles are placed where desired in the interaction plane. From their initial positions, we may obtain the self-consistent electric field.² The initial velocities of the super-particles are given a Maxwellian distribution.

Each super-particle is assumed to obey the following finite difference equation, which includes a relativistic mass correction:

$$V_{j+1,x} = \frac{\beta\delta + \gamma}{1 + \beta^2} \quad \text{and} \quad V_{j+1,y} = -\frac{\beta\gamma + \delta}{1 + \beta^2}$$

where

$$\beta = \frac{q}{m_0} \frac{\Delta t B_0}{2} \quad \alpha = \left(1 - \frac{V_{j,x}^2 + V_{j,y}^2}{2} \right)^{1/2}$$

$$\gamma = v_{j,x} + \frac{q}{m_0} \Delta t \alpha E_{j,x} + \beta v_{j,y}$$

$$\delta = v_{j,y} + \frac{q}{m_0} \Delta t \alpha E_{j,y} - \beta v_{j,x}$$

m_0 is the rest mass; B_0 is the d.c. magnetic field, Δt is the time step and q is the charge. $v_{j,x}$ and $v_{j,y}$ are the known x and y components of velocities. The subscript $j+1$ refers to the values of quantities Δt units of time later. The electric field in the above expressions is the sum of the self-consistent and the external r.f. electric fields. Once the new velocities are determined, the changes in position may be obtained to produce the motion. After each calculation, the new averages of particle kinetic and potential energy are computed, and the velocity distribution is examined to observe its changes.

The specific problem consisted of a collection of 900 each, super-ions and super-electrons. The d.c. magnetic field was set to 3000 Gauss and the r.f. field amplitude was set to 100,000 volts/meter. By varying the number of real particles per super-particle; the plasma frequency could be changed. Results for the case when $\omega_{pe}/\omega_{ce} \sim .01$ are shown in Figure 1.

The total kinetic energy of the electrons increases with time. The ions do not increase in energy, and, when the heating is removed, the electron kinetic energy remains constant. Figure 2 shows the electron velocity distribution at 3 different times. The solid lines are Maxwellians, whose temperatures are fixed by the relation: $1/2 m \langle v^2 \rangle$, where $\langle v^2 \rangle$ is the average of v^2 . Close fits to the Maxwellians are seen. The selfconsistent field (collisions) is necessary for this result; without it, a drift, but no heating, was observed. When the plasma frequency was increased, the velocity distribution rapidly becomes non-Maxwellian, and the ions are accelerated as well. Under these conditions, collisional effects have begun to dominate³ and are over-emphasized from reality by about the ratio of the number of real particles per super-particle. In order to observe heating at higher plasma frequencies, it is necessary to increase the number of super-particles, thereby lowering the number of real particles per super-particle for a given value of plasma frequency.

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2. R.W. Hockney, Phys. Fluids 9, 1826 (1968).

TOTAL KINETIC
ENERGY - JOULES

R.F. CYCLOTRON
HEATING
REMOVED
(CASE A)

R.F. CYCLOTRON
HEATING
REMOVED
(CASE B)

3. R.W. Hockney, Proc. Conf. of Computational Physics, Culham (1969).

III. BREMSSTRAHLUNG CALCULATIONS

D. G. S. Greene

The following highlights from the relativistic quantum mechanical derivation of the Heitler¹ differential cross section for bremsstrahlung will indicate the techniques of improving the approximations and also indicate the derivation of the percent polarization calculation.

A. The transition probability per unit time is:

$$w = \frac{1}{h} |K_{FO}|^2 \rho_F \quad (1)$$

$$\text{where: } K_{FO} = \sum \left(\frac{V_{FI} H_{IO}}{E_0 - E_I} + \frac{H_{FII} V_{II0}}{E_0 - E_{II}} \right) \quad (2)$$

K_{FO} is the matrix element describing the transition from initial state 0, to final state F. It consists of two interactions, V and H. V is the interaction of the electron and the atomic field; H is the interaction of the electron and the radiation field which produces the photon K. I is the intermediate state when V follows H. A photon is present in state I. II is the intermediate state when V precedes H. There is no photon in state II. The summation is over all possible intermediate states of both positive and negative energies and over all electron spin directions,

E_0 = total energy of the incident electron (zero subscripts refer to the incident electron and no subscript refers to the scattered electron),

ρ_F is the density of final states,

$$\rho_F dE = \frac{pE d\Omega dE K^2 d\Omega_K dK}{(hc)^3 (hc)^3} \quad (3)$$

p = electron momentum times c (the speed of light).

The cross section is:

$$ds = w L^3 / v_0, \quad (4)$$

where: L^3 is the volume occupied by the colliding particles. This is the

volume over which the wave functions are normalized. L^3 is set equal to 1. The calculation of V_{ij} and H_{ij} is the heart of quantum electrodynamics. Without the details of the derivation:

$$H_{ij} = \text{coef. } (u_i^* \alpha u_j) \quad (5)$$

$$V_{ij} = \text{coef. } (u_i^* u_j) / |\vec{p}_j - \vec{p}_i|^2 \quad (6)$$

where: u_k is the wave function of the k^{th} state, $\vec{p}_k$ is the momentum (times c) of the electron in the k^{th} state (Here $\vec{p}_j - \vec{p}_i = \vec{q}$ is the momentum transferred to the nucleus), and α is the component of $\vec{\alpha}$ in the direction $\vec{q}$, which is the direction of the polarization of K . $\vec{\alpha}$ appears in the relativistic wave equation:

$$H\psi = (\vec{\alpha} \cdot (\vec{p} - e\vec{A}) + \beta U_0 + e\phi)\psi. \quad (7)$$

The summation to yield $|K_{FO}|^2$ follows the techniques leading to the Klein-Nishina formula. In particular, terms such as $(\vec{\alpha} \cdot \vec{p}_K)(\alpha)$ appear which yield $\vec{p}_K \cdot \vec{e}$ during the summation procedure. These dot products are summed over the two final polarization states to yield the Heitler differential cross-section for bremsstrahlung.

B. Approximations in this derivation and the corrections are as follows:

u_k are taken as plane waves (the Born approximation).

This is a good approximation at high energies where:

$$1 \gg \xi \text{ and } 1 \gg \xi_0,$$

$$\text{where: } \xi_1 = \frac{ze^2}{h\nu_1}. \quad (8)$$

V is the first order term of an expansion in powers of ξ_1 . A difficult way to correct for this is to calculate the higher order terms in the expansion. Consider two possible intermediate states and calculate terms like HHV, HVH, VHV, etc. This is a monumental task of many weeks. Furthermore, there is no guarantee that additional corrections such as HHVV, HVVV, etc., because of the multiplicity of such terms, are not equally important. An easier way to correct is to compare to the calculations of Sommerfeld at non-relativistic

energies (Sommerfeld uses the exact non-relativistic wave functions). A comparison shows that the angular dependence remains the same, but the total intensity has to be multiplied by the Elwert factor:

$$f(\xi, \xi_0) = \frac{\xi}{\xi_0} \frac{(1 - \exp(-\xi_0))}{(1 - \exp(-\xi))} \quad (9)$$

This correction is labeled "fix" on figures 3 and 4. $f(\xi, \xi_0)$ is always greater than 1 and increases toward lower incident electron energies and toward higher photon energies. At the characteristic temperatures considered, the correction due to $f(\xi, \xi_0)$ at the electron energies which produce the most photon intensity is as much as 10% (see figure 3). Considering the total photon intensity for all electron energies, the correction is about 6% (see figure 4). However the absolute value of photon intensity is not easily determined, whereas the measured quantity, the slope of the log of photon intensity versus photon energy, is not greatly affected by $f(\xi, \xi_0)$. The slope of the corrected curves are slightly less by about 0.8% (see figure 4).

A second approximation is that the nucleus has infinite mass. This correction was discussed in the previous quarterly progress report.²

A third approximation is that the nucleus is a point charge scattering center. Screening corrections involve a recalculation of V_{1j} given in equation (6). In general

$$V_{1j} = \int \psi_1^* V \psi_j d\tau \quad (10)$$

For equation (6) V was taken as the coulomb potential for a point charge:

$$V = Z e^2 / r_{12}. \quad (11)$$

A screened potential which considers the atomic electrons has the form:

$$V = \frac{Z e^2}{r_{12}} \exp(-r_{12}/a), \quad (12)$$

where for helium: $Z = 2$

$$a = .30 a_0 = .30 \hbar^2 / m e^2.$$

Figure 3(b)

Photon Intensity versus Electron Energy

Photon energy is 10 keV

for the Loss Cone and Z Direction Beam

Figure 4(a)

X-Ray Spectra

for Loss Cone & X(3.1) distributions

Electron Energy Distributions:

$$12 \sim \exp [-X(1.55)]$$

$$1 \sim \exp [-X(3.1)]$$

$$32 \sim \exp [-X(15.5)]$$

$$10 \sim \exp [-X(21.7)]$$

$$18 \sim \exp [-X(31.0)] = I$$

$$X(kT_e) = E_0 / (kT_e \cos^2 A + 31.0 \sin^2 A)$$

Figure 4(b)

The V_{1j} resulting from this screened potential differs from equation (6) in that the denominator q^2 is replaced by $q^2 + \alpha$, where α is $(\pi/a)^2$. $a = 0.3 a_0 = 3 \times 10^{-11}$ meters.

The equation for the x-ray intensity containing the screening correction has been integrated over the angles of the scattered electron (θ and ϕ) and programmed for numerical integration over the incident electron angles (polar angle, A , and azimuthal angle, B) and energy (E_0) to yield the x-ray spectrum.

Bremsstrahlung from electron-electron encounters (where the electrons are bound to the atom), and the nonzero size of the nucleus may be considered as corrections to the screening correction. These corrections would require a more accurate expression for the atomic wave functions, which in turn would modify V , leading to additional smaller correction terms in the denominator of equation (6).

C. To calculate the polarization, consider the terms containing $\vec{p} \cdot \vec{e}$ which were described following equation (7). $\vec{e}$ is first taken in the $\vec{z}$ direction to yield $dI(P_z)$, then in the $\vec{\theta}$ direction to yield $dI(P_\theta)$.

Percent polarization is defined as:

$$P(K) = \frac{\int dI(P_z) - \int dI(P_\theta)}{\int dI}, \quad (13)$$

where the integrations are over $d\theta$, $d\phi$, dA , dB , and dE_0 . $P(K)$ is calculated for the perpendicular X-rays. The parallel X-rays have no plane polarization (consider the cylindrical symmetry). Figure 5 shows some graphs of $P(K)$.

If the integrations of equation (13) are not carried out over E_0 , the result is $P(E_0, K)$ which is shown in figure 6. $P(E_0, K)$ is not observable, but serves to show that polarization due to low energy electrons is opposite to the polarization due to high energy electrons. However, the polarization due to the entire distribution of electron energies is determined by the lower energy electrons for the "low" energy distributions studied. This accounts for the higher characteristic temperature electron energy distributions producing less polarization than the lower temperature distributions in figure 5. The dashed asymptotes on figure 6 are at electron energies equal to the electron rest energy plus the photon energy. No real photons may be produced below this energy. The asymptotic values of $P(E_0, K) = 1$, the classical value for

Polarization vs. Photon Energy

Figure 5 (a)

$$X(K_H) = -E_0 / (KT_H \cos^2 A + 31.0 \sin^2 A)$$

$$\text{Distribution Function} = \exp(-X(K_H))$$

Polarization vs. Photon Energy

Figure 5(b) (continued)

Figure 5(c) (continued)

Figure 6(a)

Polarization versus E_0
for various photon energies

$$\text{distribution} = \exp(-E_0 / (3.1 \cos^2 A + 31.0 \sin^2 A))$$

Photon Energies

10 keV

21 keV

78 keV

136 keV

600

700

800

900

1000

1100

1200

Total Electron Energy (keV) →

Polarization →

-.10

1.0

.8

.6

.4

.2

0

-.2

-.4

-.6

-.10

Figure 6(b)

Polarization versus E_0
for various photon energies

Beam Distribution

Figure 6(c)

Polarization versus E_0
for various electron energy distributions
Photon Energy = 136 keV

the polarization of the perpendicular X-rays from non-relativistic electrons.

Note that polarization measurements will help in distinguishing between the distributions $X(kT_{\parallel})$ which gave nearly identical X-ray intensity anisotropies in figure 4(b). Other coincidences in X-ray intensity anisotropies may be resolved by polarization measurements.

References

1. W. Heitler, Quantum Theory of Radiation, 3rd edition (University Press, Oxford, 1954).
2. University of Wisconsin Report No. COO-1695-26, Quarterly Progress Report No. 12, March 15, 1970.

IV. ELECTRON CYCLOTRON HEATING EXPERIMENTS

J.C. Sprott and K.A. Connor

A. Measurement of heating efficiency

The theoretical heating model in the last Quarterly Progress Report predicted that the electron cyclotron heating efficiency (power absorbed by the plasma/ input power to the cavity) should be proportional to electron density at low densities and should approach 100% when the average density exceeds a value given by

$$\bar{n} = \frac{\epsilon_0 m \omega^2}{Q_0 G e^2}$$

where ω is the rf frequency, Q_0 is the unperturbed Q of the cavity, and G is a factor of order unity that depends on the geometry of the field, the location of the resonance zones, and the density distribution.

The cavity Q was measured by means of a 10mW swept 3cm klystron which injected microwaves through the same ports used for the high power heating. The cavity electric field was measured by a coupling loop. The resulting signal for a typical cavity mode near the heating frequency in the absence of plasma is shown in fig. 7(a). The Q calculated from $f/\delta f$ is about 1000. This value is about a factor of five lower than the Q estimated from $V/A\delta$, where V , A , and δ are the cavity volume, surface area, and skin depth respectively, but the discrepancy is reasonable when one considers microwave leaks in the cavity and contamination of the walls. From the measured Q , we conclude that total absorption of the rf power should occur for $\bar{n} > 10^9 \text{ cm}^{-3}$. This condition is marginally satisfied in the present experiment.

The RMS electric field in the cavity in the absence of plasma is given by

$$E_{\text{RMS}} = \frac{P_0 Q_0}{E_0 \omega V} = 1.8 \text{ kV/cm for } P_0 = 100 \text{ kW.}$$

We expect negligible reflection of the microwave power for

$$\lambda^3 \ll \frac{8\pi V}{Q}$$

or $f \gg 5.2 \text{ GHz}$. For the normal heating frequency (8.35 GHz), this condition

Typical
Cavity Mode

(a)

Incident Power

Diamagnetic Signal

(b)

1 μ sec/Div

FIGURE 7.

is not well satisfied, and the reflected power is $\sim 33\%$ of the incident power in the absence of plasma. The reflected power drops to $\sim 22\%$ when a plasma with $\bar{n} \sim 10^{10} \text{ cm}^{-3}$ is present.

The measured peak input power to the cavity as determined by measurements of the incident and reflected power is $\sim 40 \text{ kW}$ and has the waveform shown in fig. 7(b). The heating rate as measured by a diamagnetic loop near the midplane is given by

$$\frac{d}{dt} (nV_k T_e) = \frac{BV\epsilon}{\mu_0 NA}$$

where B is the magnetic field strength, ϵ is the loop voltage, N is the number of turns in the loop, and A is the loop area. The diamagnetic signal shown in fig. 7(b) gives a heating rate of $\sim 12 \text{ kW}$ implying an efficiency of $\sim 30\%$, although this is probably an underestimate because of frequency response limitations in the loop and because of power losses by ionization and scattering into the loss cone during the heating pulse. The frequency response of the diamagnetic loop was measured by inductively coupling a sinusoidal voltage to the loop. The upper frequency limit of the loop was $\sim 400 \text{ kHz}$ in agreement with the value calculated from $f_c = R/2\pi L$. A correction was made to the signal in fig. 7(b) to compensate for the bandwidth of the loop. The diamagnetic signal and the ion saturation current to a probe on axis are proportional to the input rf power over the range tested (about a factor of two).

B. Observation of flute instabilities

An anisotropic, low β plasma is expected to be flute unstable if

$$\int \frac{p_\perp + p_\parallel}{B^2 R r} dl < 0,$$

where $p_\perp$ and $p_\parallel$ are the plasma pressure perpendicular and parallel to $\vec{B}$, R is the radius of curvature of the line of force, r is the distance to the axis of symmetry, and l is an element of length parallel to $\vec{B}$. The above equation is satisfied in the present experiment, and so we expect to see flute instabilities.

Figure 8 shows a radial profile of density $\langle n \rangle$ and density fluctuation $\langle \delta n \rangle$ 3 msec after the heating pulse as determined from the ion saturation current to a Langmuir probe at an axial position where $B(2)/B(0) = 1.013$ for a mirror ratio of 1.02. The annular structure appears to be a general feature of the

FIGURE 8

plasma and is independent of the position of the heating zones. The fluctuations occur predominantly in the region where $\frac{dn}{dr} < 0$, and satisfy the condition

$$\frac{\langle \delta n \rangle}{\langle n \rangle} = \frac{e \langle \delta V_f \rangle}{k T_e} = 100\%,$$

where V_f is the floating potential. We note that the fluctuations occur only on the boundary of the plasma, and that the density gradient near the mirror axis is in the favorable direction for containment. The fluctuations are reasonably sinusoidal with a frequency of ~ 100 kHz just after the heating pulse ~ 10 kHz after 3 msec.

In order to show that these fluctuations are indeed flute modes, the perpendicular and parallel wavelengths were compared using a floating double Langmuir probe with a tip separation of 0.5 cm. The measured electric field in the θ -direction ($\vec{A}$ to $\vec{B}$) and in the Z -direction ($\vec{I}$ to $\vec{B}$) are shown in fig. 9. The parallel electric field is much smaller than the perpendicular field, as would be expected for a flute. An exact measurement of the wavelength was not attempted, although we can conclude from the observations that $\lambda_{\parallel} \gg \lambda_{\perp} > 0.5$ cm.

Flute instabilities can be suppressed by the addition of a linear multiple field (Ioffe bars) to produce a minimum- B geometry. Calculations were performed to determine the required strength for a quadrupole field. With $R = 1.02$, an absolute minimum B can be obtained in the present apparatus with $\sim 18,000$ amperes per rod. A quadrupole field with 290 amperes per rod was tried with no observable effect on the fluctuations.

The plasma lifetime calculated for this instability, assuming a 100% fluctuation in density, and perfect correlation between δn and δE is given by

$$T = \frac{\lambda}{\langle v \rangle} = \frac{n \lambda B}{\langle \delta n \delta E \rangle} = \frac{\lambda B}{\delta E} \sim 100 \mu\text{sec}.$$

The observed lifetime is ~ 1 msec, and the density distribution remains relatively constant throughout the life of the plasma, suggesting that the radial diffusion caused by these fluctuations is small.

C. CW microwave heating

A new class of experiments has been made possible by the construction of a 50 watt, CW microwave source using a QK61 magnetron that is tunable from 2975 to 3170 MHz. This source and a similar, 100 watt, 2600 MHz CW source borrowed from another group have been used to produce plasmas by electron cyclotron heating

FIGURE 10

as well as for off resonance heating studies (see next section). The power is fed into the cavity coaxially, and the cavity is excited by a coupling loop whose position and orientation can be adjusted for optimum coupling. At these low frequencies, it is difficult to get a good impedance match to the cavity, and the reflected power is generally large.

Langmuir probe measurements show that the density increases monotonically with helium gas pressure and microwave power, and that densities of $\sim 10^{14} \text{ cm}^{-3}$ (where $\omega_p = \omega$) can be produced with 100 watts. The electron temperature is 10-20 eV and depends only slightly on pressure and on microwave power. The density is peaked near the axis, and fluctuations at $\sim 10 \text{ kHz}$ with an amplitude of 10-100% occur throughout the plasma. The percentage fluctuation does not depend strongly on gas pressure or on microwave power, in contrast with recent experiments at Los Alamos.¹

The microwave source can be pulsed off abruptly for $\sim 30 \text{ msec}$ and then turned on again. When the heating is turned off, the ion saturation current decays in $\sim 10 \mu\text{sec}$ in contrast to the $\sim 1 \text{ msec}$ observed for the 100 kW pulsed heating. The decay time is independent of gas pressure and suggests a strong loss mechanism. When the source is turned back on, the ion saturation current builds up much more slowly. At $1 \times 10^{-6} \text{ torr}$, the buildup time is $\sim 1 \text{ msec}$, and varies inversely with gas pressure in agreement with the calculated ionization time. The power absorbed by the plasma can be estimated from

$$P \sim \frac{nkT_e V}{\tau}$$

where τ is the decay time after the rf is turned off. Using $n = 10^{14} \text{ cm}^{-3}$, $kT_e = 15 \text{ eV}$, and $\tau = 10 \mu\text{sec}$, the absorbed power is $\sim 100 \text{ watts}$ suggesting a high heating efficiency as expected theoretically (see section A).

The ion saturation current to a Langmuir probe on axis was measured during CW heating as a function of the background helium gas pressure for three different values of incident microwave power. The result is shown in fig. 10. Since T_e does not depend strongly on pressure or power, the density is seen to follow approximately the relation

$$n \propto P\sqrt{p}.$$

On this curve an ion saturation current of 1 mA corresponds to a density of

FIGURE 9

$\sim 1 \times 10^9 \text{ cm}^{-3}$. When a stub tuner was added to the magnetron output, the ion saturation current could be increased to almost 100 mA for an input power of 100 watts and a pressure of 1×10^{-3} torr. These densities caused considerable heating of the Langmuir probes.

D. Off resonance heating

Recent experiments at Oak Ridge² using microwave heating both above and below the electron cyclotron frequency have produced some unexpected and spectacular effects. In these experiments, upper off resonance heating is found to be very efficient, while lower off resonance heating results in expelling the plasma from the mirror field. Similar experiments have been performed here using ≤ 100 watts of 10 cm CW microwaves and 100 kW of 3 cm pulsed microwaves with a pulse width of 1.5 μsec and a repetition rate of 360 sec^{-1} . The mirror field has been operated at mirror ratios of 1.02 and 1.19. The field strength can be adjusted to give electron cyclotron resonance at either of the two heating frequencies.

When the plasma is produced using resonant 10cm CW microwaves and the upper off resonant heating pulses are applied, no change is observed in either the visible appearance of the plasma or in the ion saturation current to a Langmuir probe. The diamagnetic signal, which gives a measure of the heating rate, was below the noise level, which is $\sim 10\%$ of the signal observed for resonant heating with the same source. If we assume nearly 100% resonant heating efficiency as argued in section A, we conclude that the efficiency of upper off resonance heating is $< 10\%$. This result is consistent with a theory by Grawe³ that predicts stochastic off resonance heating for relativistic electrons. The plasmas at Oak Ridge are highly relativistic, in contrast to the rather cold plasmas produced here. We hope to improve the sensitivity of the diamagnetic loop to the point where some heating can be observed, and then check the scaling of the heating efficiency with the electron temperature of the background plasma.

When the magnetic field is adjusted to give resonance for the 3cm source and lower off resonance heating is applied, the plasma is completely extinguished as evidenced by the absence of visible light, diamagnetism, and Langmuir probe current. Figure 11 shows how the diamagnetic signal during the 1.5 μsec heating pulse and the ion saturation current to a probe on axis in the afterglow vary with lower off resonance heating power for a typical case. In some cases, the diamagnetic signal is observed to increase first and then to decrease with increasing

FIGURE 11

off resonance power. This could be due to the fact that the loop is located away from the midplane and would therefore give a larger signal if the axial diffusion of the plasma increased.

The off resonance heating power required to extinguish the plasma has been measured as a function of various parameters. The required power increases with increasing background gas pressure and depends on microwave frequency and position of the coupling loop. Less power is required when the coupling loop is near the cavity wall than when near the axis. The frequency dependence is very pronounced for frequency changes of only $\sim 10\text{MHz}$, but this is probably due to changes in the cavity mode pattern and in the coupling efficiency. Changing the mirror ratio from 1.02 to 1.19 does not appear to affect the power required. The reflected power for the resonant source was higher (29%) with the plasma extinguished by off resonance heating than without (22.5%), but not as high as the reflected power when the pressure is reduced to the point where no plasma is produced (33%), implying that some resonance power is being absorbed in the presence of strong off resonance heating.

Langmuir probe measurements show that the flute oscillations (see section B) are not noticeably affected by off resonance heating, but the ion saturation current on axis (which normally decays smoothly) develops large amplitude (20-100%), low frequency (1-10kHz) fluctuations as shown in fig. 12. A higher frequency (20-50kHz) mode is also observed for certain off resonance heating frequencies. This behavior is suggestive of some instability excited perhaps by changes in the electron distribution function.

The time dependence was tested by pulsing off the lower off resonance source for $\sim 30\text{msec}$. When this is done, the plasma returns to normal after two pulses of the resonance power, but is rapidly extinguished again when the source comes back on.

The integrated X-ray flux from the plasma was measured with a geiger counter. The axial flux is $\sim 0.1\text{ mR/hr}$ in the absence of off resonance heating and increases monotonically with power to $\sim 6\text{mR/hr}$ just before the plasma is extinguished. The radial X-ray flux increases by only a factor of two at the same time. This behavior is suggestive of increased axial diffusion of the plasma in the presence of the off resonance heating. Further tests will be made to confirm this hypothesis.

FIGURE 12

References

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V. INSTABILITY STUDIES

S.J. Gitomer*

An instability which has previously been discussed in these progress reports has been studied further. The instability is accompanied by strong microwave emission, axial particle loss, fluctuations in diamagnetism and intense x-ray bursts.

In order to enable the unambiguous identification of this instability, a number of questions need to be answered: (1) How does the heating zone location effect its existence? (2) How does the emitted rf frequency scale with the applied heating frequency? (3) What are the plasma parameters at the time the instability occurs? After it occurs? (4) What is the plasma spatial density distribution? The measurements reported here attempt to answer these questions and will enable definitive comparison to be made with relevant instability theories.

In general features, the strong microwave emission occurs in bursts (0.1-1.0 μ sec long) within 5 μ sec after the applied heating ends. The frequency of the radiation falls in a band below the microwave heating frequency. The instability is found to occur for only midplane and off axis heating. The dependence on background pressure appears to favor the lower pressures ($6-10 \times 10^{-4}$ Torr) for which discharge is possible.

The instability was observed to occur for both 1.02 and 1.19 mirror ratios. The heating zone for instability appeared to be near the midplane for the 1.02 case; however, a $\pm 1\%$ magnet current ripple made this conclusion uncertain. At $R = 1.19$, however, the $\pm 1\%$ ripple doesn't affect the heating zone location appreciably. Figure 13 shows the range of heating zones over which instability was produced for a heating frequency of about 8.525GHz. (Mod $\vec{B}$ contours of about 0.962-0.998)

The microwave source was mechanically tuned through its range (8.5-9.2GHz) and the bands of frequencies associated with the instability were measured using a microwave spectrum analyzer. A typical such spectrum is shown in figure 14. The applied heating frequency is clearly indicated. The plot in figure 15 shows the variation of instability frequency divided by applied heating frequency as a function of applied heating frequency. For both mirror ratios, the frequency associated with the instability scales roughly linearly with the applied heating frequency. The possibility of the excitation of a particular cavity mode (as noted

* Permanent address: University of Pennsylvania, Philadelphia, Pa. 19104

FIGURE 13. CONTOURS OF CONSTANT MAGNETIC FIELD AMPLITUDE ($R = 1.190$)
SHOWING REGION OF INSTABILITY PRODUCTION (CROSS-HATCHED)

FIGURE 14. TYPICAL SPECTRUM SHOWING PLASMA MICROWAVE EMISSION FOR A BAND OF FREQUENCIES BELOW THE HEATING FREQUENCY. (HEATING PULSE FREQUENCY = 8.52GHz IS ALSO SHOWN)

FIGURE 15. NORMALIZED INSTABILITY FREQUENCY VERSUS APPLIED HEATING FREQUENCY ($R = 1.02$ and 1.19)

in reference 1) is ruled out by this behavior.

A number of diagnostic measurements were made to characterize the plasma for parameters yielding instability. Figure 16 shows electron density variation with time as determined using probe and microwave methods. The microwave method yields an average of the total (cold and hot) electron density while the Langmuir probe method yields a local value of the cold electron density. We will make the reasonable assumptions² that the microwave power pulse is of too short a duration to appreciably change the electron density and that the electron density is only hot at the end of the heating pulse. This implies a ratio $n_C/n_H \ll 1$. At the end of the heating pulse, n_C/n_H rises rapidly to a maximum value of ~ 10 at $t \approx 50 \mu\text{seconds}$, and then falls slowly to a final value ~ 0.3 just before the next heating pulse.

The radial and axial dependence of the plasma cold electron density (determined using Langmuir probes) is shown in figures 17 and 18 for two times in the afterglow (see also figure 8.) Since the cold electron component is produced by ionization of neutral helium atoms by the hot electrons, these plots display the hot electron spatial distribution also.

Previously² temperature and anisotropy measurements were made using plasma diamagnetism and $\vec{E} \times \vec{B}$ energy analyzer information. It was found, however that for the plasma studied here (1) the diamagnetism could not always be interpreted as stored energy in the plasma due to the complication of plasma motion and (2) no appreciable low energy electron signals were detectable using the $\vec{E} \times \vec{B}$ analyzer. Temperature measurements were made instead using Langmuir probes. Figure 19 shows the decay of cold electron temperature with time. This decay is not inconsistent with those shown in figure 16 for the total and cold electron densities. An estimate of hot electron temperature of 10 keV is made based on previous x-ray measurements. X-ray measurements of the hot electron energy distribution were attempted with ambiguous results. These measurements are being pursued further.

Estimates of anisotropy may be made from (1) the axial probe density data and (2) the formula $\langle W_{||} \rangle / \langle W_{\perp} \rangle = (R-1)/(R+1)$ where R = axial mirror ratio. These yield anisotropy values of the order of 100 ($R = 1.02$) and 10 ($R = 1.19$). These values probably represent lower limits on the actual anisotropy.

The only remaining piece of information needed is instability wave number. This will be determined using a fixed loop probe and a movable axial loop probe

FIGURE 16. ELECTRON DENSITY VERSUS TIME - USING FUNDAMENTAL MODE SHIFT AND LANGMUIR PROBE METHODS.

FIGURE 17. ELECTRON DENSITY VERSUS RADIUS ($z = 15.0\text{cm}$) FOR TWO TIMES IN THE AFTERGLOW. ($R = 1.19$, $f = 8.52 \text{ GHz}$)

FIGURE 18. ELECTRON DENSITY VERSUS AXIAL COORDINATE
 ($r = 0.0$ cm) FOR TWO TIMES IN THE AFTERGLOW.
 ($R = 1.19$, $f = 8.52\text{GHz}$)

FIGURE 19. ELECTRON TEMPERATURE
VERSUS TIME LANGMUIR PROBE DATA
($r = 4.0\text{cm}$, $z = 15.0\text{cm}$)

following the method used by Ikegami, et al.³

Our previous theoretical identification of the instability (an electromagnetic mode) failed to consider the spatial variation of plasma density and the presence of a background of cold plasma. Following the work of Ikegami, et al.³, Jacquinet, et al.⁴, Lee and Crawford⁵, and others, an assessment of this instability will be made using this new information to obtain a more clearcut identification.

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