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	PROGRAM KEWB PROJECT KEWB		

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SUBJECT:

VELOCITY OF SOUND IN WATER CONTAINING GAS BUBBLES

CONTENTS:

I STATEMENT OF PROBLEM

The velocity of sound in water containing bubbles of hydrogen gas is computed as a function of the volume fraction of gas for various bubble sizes, in order to illustrate that the marked reduction in sound velocity caused by the presence of gas voids is not as great if the gas is present as very small bubbles. The results are applied in some qualitative observations about inertial pressures in fast excursions in the KEWB reactor.

II INTRODUCTION

It has been observed that the behavior of the KEWB reactor during fast excursions (e.g., reactor periods from 2 to 20 milliseconds) is not very sensitive to the magnitude of transient pressures which exist during the power bursts. In particular, 2-millisecond excursions yield nearly the same peak power when the core vessel is initially filled as is observed when the core is initially only 85 percent filled, even though the maximum inertial pressure indicated at the bottom of the vessel is about 30 atmospheres in the full core and only 10 atmospheres in the underfilled core. See Table I.

Computations of transient reactivity based on the observed short-period power traces show that the majority of the reactivity which has been compensated at the instant of peak power is due to gas voids. It therefore seems reasonable to conclude that the gas present at the time of peak power is predominantly in the form of small bubbles, whose internal pressure is appreciably elevated by surface tension, and whose volume is therefore relatively insensitive to pressure changes up to 30 atmospheres in the fluid. This conclusion is consistent with the observation that the plot of peak power vs reciprocal period does not show any upward curvature in the period region 10 to 20 milliseconds, where inertial pressure first appears. (See Figure 1) The conclusion is also consistent with the observation that the volume of gas present at the instant of peak power is much smaller than the amount predicted from the observed energy release, though it is likely that some of this difference is accounted for by gas which is still dissolved.

On the other hand, the presence of inertial pressures, whose peak values are roughly proportional to the reciprocal period squared (see Figure 2*), requires an appreciable increase in compressibility of the core solution. It has been pointed out previously[1] that no inertial effect would be observed for millisecond periods unless gas voids are present. However, even for large void fractions, the presence of gas in the form of very small bubbles will not produce sufficient increase in compressibility because of surface tension; a bubble of 10^{-6} cm radius in water will have an internal pressure 150 atmospheres greater than the pressure in the surrounding fluid.

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* Peak pressures plotted are for the bottom transducer, which is submerged in all cases. In the underfilled case, the upper transducer is initially above the liquid level, and the pressure impulse it receives is not a measure of the inertial effect. In the full core, this upper transducer responds very much like the lower, the only difference being a consistently lower amplitude (roughly 10 percent), indicating that most of the gradient in inertial pressure is near the liquid surface - as predicted in Reference 1.

A simple criterion for the presence of inertial pressure[1] is that the reactor period be comparable to the time required for sound to traverse the core diameter. The purpose of this memorandum is to compute the speed of sound in water containing gas bubbles as a function of void fraction and bubble radius, and to apply the result to observations of inertial pressures in KEWB.

III THEORY

The velocity of sound is taken to be

$$V_s = \sqrt{1/k\rho} \tag{1}$$

where k is the compressibility and ρ the density of the overall system. The compressibility may be written

$$k = -\frac{1}{V}\frac{dV}{dp_0}$$

where V is the total volume and p_0 the pressure in the fluid. Writing the total volume as

$$V = V_\ell + V_g ,$$

where the subscripts refer to liquid and gas respectively, and defining the volume fraction of gas by

$$f = V_g/V,$$

we have

$$k = -\frac{1-f}{V_\ell}\frac{dV_\ell}{dp_0} - \frac{f}{V_g}\frac{dV_g}{dp_0} .$$

This may be re-written as

$$k = (1-f)k_0 - \frac{f}{V_g}\frac{dV_g}{dp_0} , \tag{2}$$

where k_0 is the compressibility of water alone (50×10^{-6} atm^{-1}).

The gas pressure will be given by

$$p_i = p_0 + 2\sigma/r , \tag{3}$$

where σ is the surface tension of water and r is the bubble radius. We assume also the ideal gas law

$$p_i V_g = NRT , \tag{4}$$

where N is the total number of gas molecules present, implying an assembly of bubbles of equal size. Further, if there be n bubbles of individual volume v, we have

$$V_g = n v \; ; \tag{5}$$

$$v = \frac{4}{3}\pi r^3 \, . \tag{6}$$

Equation (3) yields

$$p_o = p_i - 2\sigma/r \, ,$$

$$p_o = NRT/V_g - 2\sigma\left(4\pi/3v\right)^{1/3} ,$$

$$p_o = NRT/V_g - 2\sigma\left(4\pi n/3V_g\right)^{1/3} .$$

The derivative with respect to V_g is

$$\frac{dp_o}{dV_g} = -NRT\,V_g^{-2} + \frac{2\sigma}{3}\left(\frac{4\pi n}{3}\right)^{1/3} V_g^{-4/3} .$$

Recourse to Equations (3) through (6) leads to the result

$$-V_g \frac{dp_o}{dV_g} = p_o + 4\sigma/3r \, ,$$

and substitution into Equation (2) yields*

$$k = k_o(1-f) + f/\left(p_o + 4\sigma/3r\right) . \tag{7}$$

The density of the mixture is M/V, the ratio of total mass to total volume. We have

$$M = \rho_l V_l + \rho_g \frac{p_o + 2\sigma/r}{p_o} V_g \, ,$$

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* It may be argued that the adiabatic compressibility, rather than the isothermal, should be used in computing the speed of sound. The question is deferred for later consideration.

$$M = \rho_\ell (1-f)V + \rho_g \frac{p_0 + 2\sigma/r}{p_0} fV ,$$

where ρ_ℓ and ρ_g are liquid and gas densities at pressure p_0. The overall density is therefore

$$\rho = \rho_\ell (1-f) + \rho_g \frac{p_0 + 2\sigma/r}{p_0} f . \tag{8}$$

Using Equations (7) and (8) in Equation (1), the speed of sound is

$$V_s = \left\{ \left[k_0 (1-f) + \frac{f}{p_0 + 4\sigma/3r} \right] \left[\rho_\ell (1-f) + \rho_g \frac{p_0 + 2\sigma/r}{p_0} f \right] \right\}^{-1/2} . \tag{9}$$

Computations of V_s using Equation (9) are presented in Figure 3. The numbers employed are:

$k_0 = 50 \times 10^{-6}$ atm^{-1} = 50×10^{-12} cm^2/dyne
p_0 = 1 atm = 10^6 dynes/cm^2
σ = 75 dynes/cm
ρ_ℓ = 1 gm/cm^3
ρ_g = 10^{-4} gm/cm^3

The gas density chosen is for hydrogen, but the result is completely independent of ρ_g for all interesting cases. In fact, for all values of f not extremely close to the end points f=0 and f=1 we find

$$V_s = \left[\frac{\rho_\ell}{p_0 + 4\sigma/3r} f(1-f) \right]^{-1/2} , \tag{10}$$

and the minimum speed of sound is immediately identified as being at the maximum of the function f(1-f), namely f=½.

The interesting special case of $r=\infty$ (bubbles large enough to have internal pressure p_0, say 10^{-3} cm or larger) has therefore a minimum sound speed of

$$V_s = 2\sqrt{p_0/\rho_\ell}$$

which is 20 meters/sec for water at one atmosphere, independent of the kind of gas. This result is confirmed in experiments by Karplus[2] at Armour

Research Foundation, who states:

"Minimum sound velocity in a water air mixture occurs at 50 per-cent concentration where the velocity is approximately 22 m/sec."

Note also the straight-line portion of each curve, a range from f=0.2 down to some smaller value depending on r. This is the range for which Equation (10) reduces to

$$V_s \simeq \sqrt{\frac{p_o + 4\sigma/3r}{\rho_\ell f}} \quad . \tag{11}$$

This approximation fails for very small f when the term in k_o in Equation (9) becomes important.

IV. APPLICATION TO KEWB

The curves presented in Figure 3 are for fluid pressures of one atmosphere, and hence can be applied strictly only to predicting the onset of inertial pressures if the initial reactor system pressure is one atmosphere. The effect of changing p_o to 15 cm Hg will be to lower most of the curve for $r = \infty$ by the factor $\sqrt{5}$ (square root of the pressure ratio), and the curves for smaller r by a lesser amount; see Equation (11). For $r < 10^{-4}$, the value of p_o has no effect, as evidenced by the reduction of Equation (11) to

$$V_s \simeq \sqrt{\frac{4\sigma}{3\rho_\ell r f}}$$

for the straight-line portion. The qualitative conclusions to be drawn here will not be affected by this factor.

Let us choose as the criterion for inertial pressure the speed of sound which makes the reactor period equal to the time for sound to cross the core diameter (30 cm) and return. (Using twice the diameter as the characteristic distance is less restrictive and possibly more realistic for predicting the onset of inertial effects.) This becomes

$$V_s < 60/\tau \; ,$$

where V_s is in cm/sec and τ is the reactor period in seconds. The largest realistic value of f may be estimated roughly from computations of reactivity due to voids at peak power[3]; it is probably less than f=0.1.

For a period of two milliseconds, this condition (V_s=30,000 cm/sec and f=0.1) is met by bubbles only slightly larger than 10^{-6} cm in radius (internal pressure 150 atm.) Actually, the onset of inertial pressure is probably at some smaller void fraction (say f=0.01) which would require bubbles of radius 10^{-5} cm if V_s = 30,000 cm/sec. Such bubbles would be somewhat sensitive to a change from 10 to 30 atmospheres in fluid pressure, since $2\sigma/r$ would be only 15 atmospheres in this case. Even here, however, the change from 10 to 30 atmospheres in fluid pressure represents a change from 25 to 45 atmospheres in internal pressure, which is less than a factor of two. Such slightly "soft" bubbles might still be consistent with the peak powers in Table II, especially when it is realized that the full core case has immediate expulsion of fuel as an additional shutdown mechanism not present in the underfilled core; if the voids were completely insensitive to fluid pressure, one would presumably see a lower peak power for the full core experiment.

For larger periods, the curves of Figure 3 indicate larger bubbles. At 10 milliseconds, the critical sound speed is 6000 cm/sec, which can exist only for $f > 0.03$* and which predicts bubbles of radius 5×10^{-5} cm at f=0.1. This latter radius corresponds to $2\sigma/r$ = 3 atmospheres, but since the inertial pressures are less than one atmosphere for the underfilled case and about 2 atmospheres for the full core, it appears that the sensivity to pressure should not be great here either.

V CONCLUSIONS

The qualitative conclusions may be summarized as follows:

1. The existance of inertial pressures is not inconsistent with the relatively small effect of these pressures on the reactor

*For initial pressure of 15 cm Hg, the curve for ∞ would be lowered by the factor $\sqrt{5}$; in this case the sound speed of 6000 cm/sec could exist for $f > 0.01$.

C8

behavior.

2. Bubble size during the crucial time up to peak power must be restricted to a fairly limited range: large enough to permit inertial pressures, yet small enough to allow appreciable pressure variation without a great effect on peak power.
3. Slower transients (10-20 millisecond periods) require much larger average bubble size than fast transients (down to 2 milliseconds) in order to account for inertial pressure.

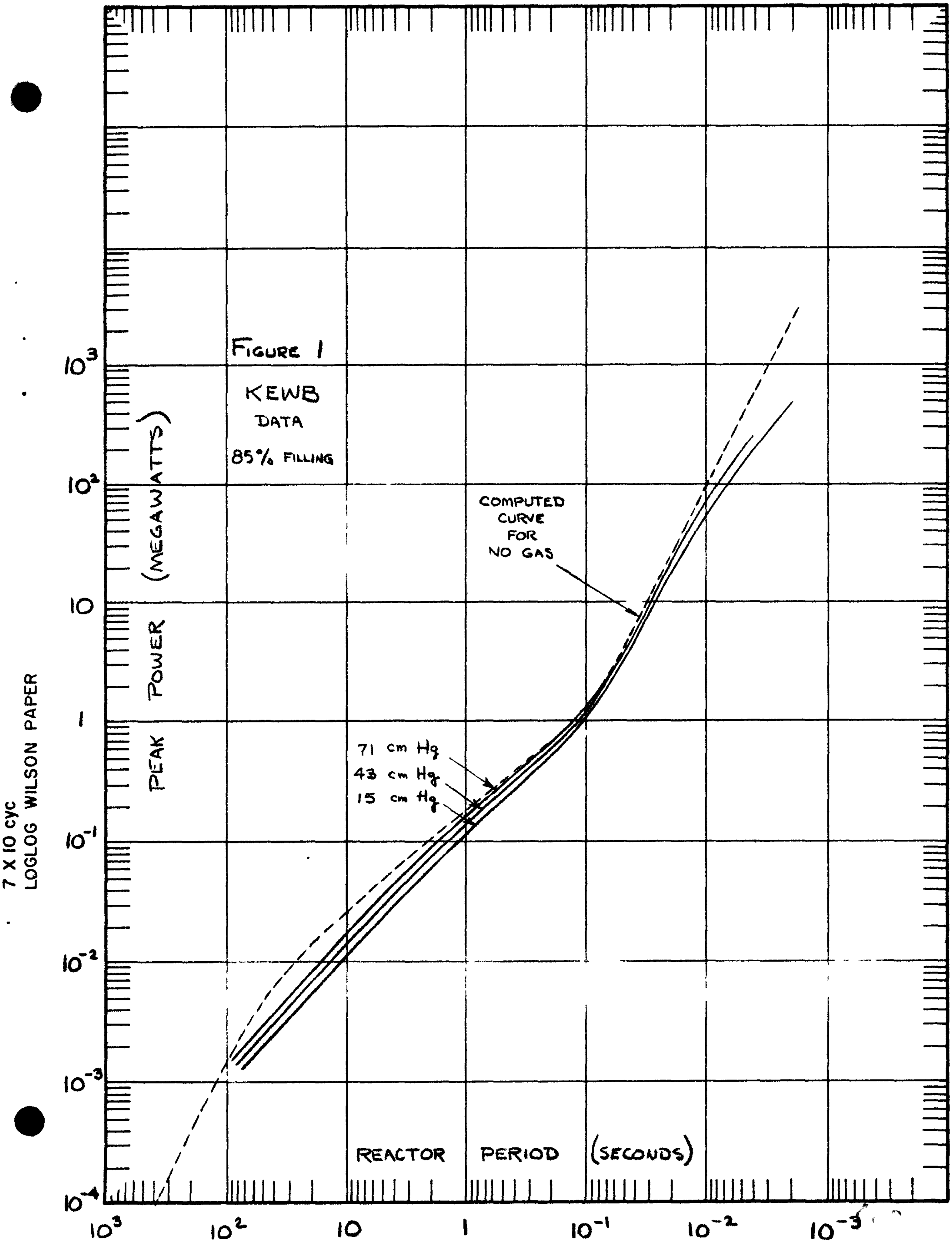
More detailed conclusions must await a much more comprehensive theory of inertial effects*. However, any model of void formation and growth during fast excursions would appear to be constrained to meet the above requirements.

* The discussion of inertial effects in Reference 1 is incomplete for two reasons: (1) the formation of bubbles and their growth to large size is assumed instantaneous; and (2) the equations are solved only for exponentially rising power, for which the differential equations are separable in space and time.

TABLE I

PEAK POWER AND PEAK INERTIAL PRESSURE AT 2-MILLISECOND PERIOD

Initial Core Filling	Peak Power (Megawatts)	Peak Inertial Pressure (atmospheres)
85%	510	10
100%	670	30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LOGLOG WILSON PAPER
FIGURE 1
KEWB
DATA
85% FILLING
PEAK POWER (MEGAWATTS)
COMPUTED
CURVE
FOR
NO GAS
71 cm Hg
43 cm Hg
15 cm Hg
REACTOR PERIOD (SECONDS)
10³
10²
10
1
10⁻¹
10⁻²
10⁻³
10⁻⁴

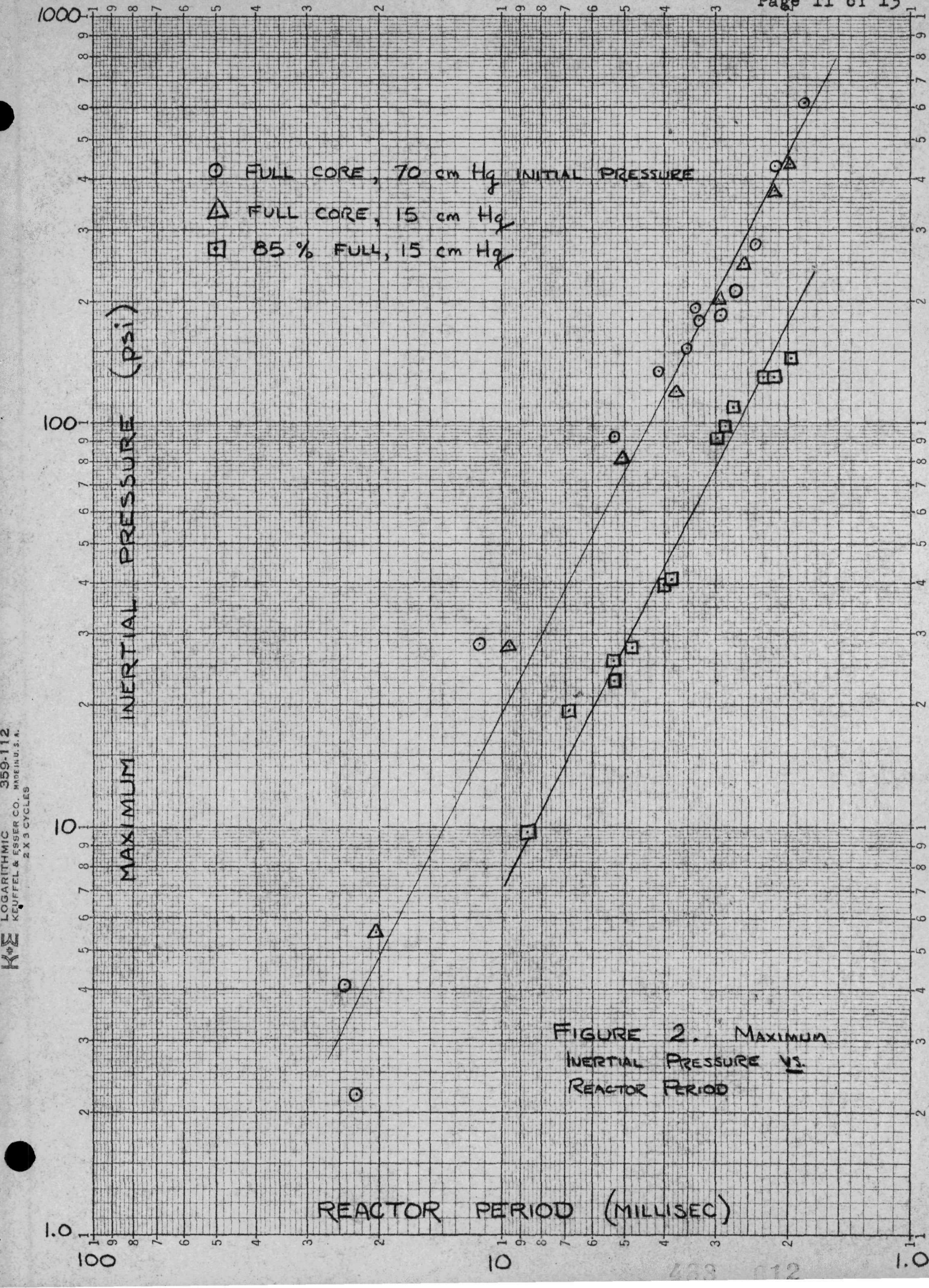

FIGURE 2. MAXIMUM INERTIAL PRESSURE VS. REACTOR PERIOD

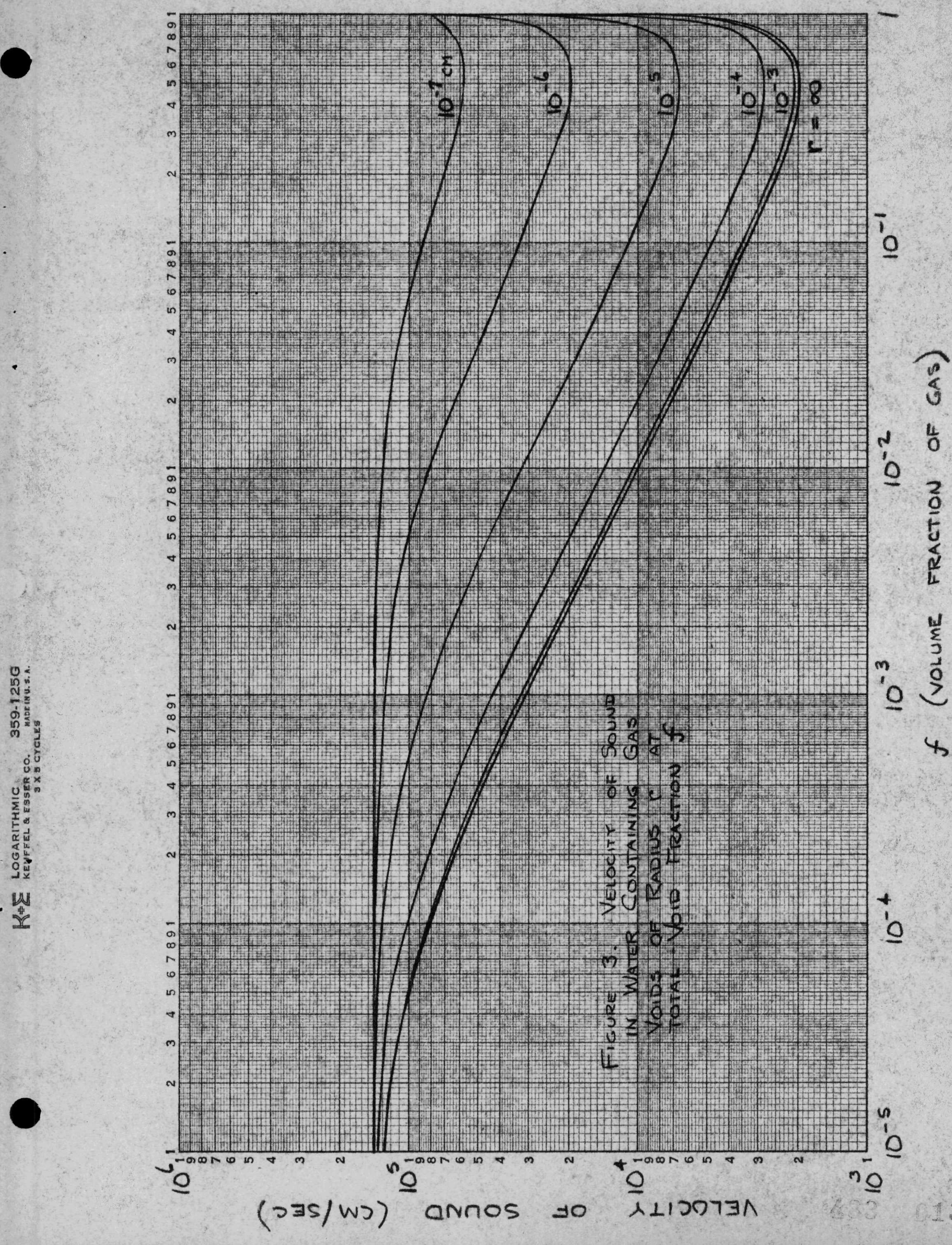

Figure 3. Velocity of Sound in Water Containing Gas Voids of Radius r at Total Void Fraction f

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2. H. B. Karplus, "The Velocity of Sound in a Liquid Containing Gas Bubbles", Paper 3-10, Second Winter Meeting, American Nuclear Society, New York, 10/57.

3. D. L. Hetrick, and D. P. Gamble, "Transient Reactivity During Power Excursions in a Water Boiler Reactor", manuscript in preparation.