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QUARTERLY PROGRESS REPORT NO. 12

"Plasma Instabilities and Waves Excited
by an Anisotropic Energy Distribution
Produced by Electron Cyclotron Resonance"

U.S. Atomic Energy Commission
Contract No. AT(11-1) 1695

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Madison, Wisconsin 53706

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Plasma Instabilities and Waves Excited by an Anisotropic Energy
Distribution Produced by Electron Cyclotron Resonance

Contract No. AT(11-1) 1695

U.S. Atomic Energy Commission

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I. INTRODUCTION

J. C. Sprott

Theoretical work during the period December 15, 1969 - March 14, 1970 has centered on two major problems: 1) calculation of rf heating rates by analytical means and by computer simulation, and 2) a continuation of the calculations of the free-free bremsstrahlung from a relativistic anisotropic plasma.

The analytical heating calculations treat the case of a cold, tenuous plasma in an arbitrary non-uniform magnetic field. The heating rate is expressed in terms of the magnetic field gradient in the vicinity of cyclotron resonance and the density distribution. The results are in good agreement with other theoretical works and the limitations do not appear to be too restrictive.

Computer simulation has been used to study electron cyclotron heating in a uniform magnetic field. Electrons and ions are free to move in the two dimensions perpendicular to $\vec{B}$ under the influence of the external dc magnetic and rf electric fields and the self-consistent space charge electric field. The results have shown heating of both electrons and ions and indication of an instability after the rf is turned off.

Calculations are being made of the polarization of the bremsstrahlung and corrections to the bremsstrahlung spectra due to the finite mass of the nucleus.

The experimental program has concentrated on developing Langmuir probe techniques for use on the magnetic mirror device. The probes work surprisingly well, and do not appear to perturb the plasma appreciably. Detailed measurements of floating potential, density, and temperature of the cold plasma component in the afterglow have been made vs position and time for a variety of heating conditions.

II. CALCULATION OF ELECTRON CYCLOTRON HEATING RATES

J. C. Sprott

A. Theoretical model.

Consider a plasma confined in a cavity of arbitrary size and shape by a static non-uniform magnetic field $\vec{B}(\vec{r})$ in the presence of an rf electric field $\vec{E}_0(\vec{r}, t)e^{i\omega t}$. Assume a plasma density sufficiently low that the waves freely penetrate the plasma and a plasma pressure sufficiently low that the magnetic field is unperturbed (i.e., $\beta = 2\mu_0 p/B^2 \ll 1$).

We define a function ψ which is constant on a flux surface and has a value equal to the magnetic flux enclosed by that surface. The magnetic field lines lie in the surfaces and may or may not be confined in the cavity. An orthogonal coordinate system will be used in which $\hat{\psi}$ is a unit vector normal to a flux surface and $\hat{l}$ is a unit vector in the direction of $\vec{B}$. This system is convenient because plasma energy flows readily in the $\hat{l}$ direction and because in many axisymmetric and linear geometries the coordinate orthogonal to $\hat{l}$ and $\hat{\psi}$ is ignorable.

The power absorbed per unit volume in a resistive dielectric medium can be expressed in terms of a real tensor conductivity $\vec{\sigma}$ as

$$\frac{dP}{dV} = (\vec{\sigma} \cdot \vec{E}) \cdot \vec{E}, \quad (1)$$

or, in our coordinates,

$$\frac{dP}{d\psi} = \int (\sigma_{\perp} E_{\perp}^2 + \sigma_{\parallel} E_{\parallel}^2) dl / B. \quad (2)$$

The perpendicular and parallel conductivities of a cold, tenuous plasma can be expressed in terms of a collision frequency ν for $\nu \ll \omega$ as

$$\sigma_{\perp} = \epsilon_0 \omega_p^2 \nu (\omega^2 + \omega_c^2) / [(\omega^2 - \omega_c^2)^2 + 4\omega^2 \nu^2]$$

and

$$\sigma_{\parallel} = \epsilon_0 \omega_p^2 \nu / \omega^2,$$

where ω_p is the plasma frequency and ω_c is the cyclotron frequency. For $v \ll \omega$, most of the contribution to the integral in equation (2) comes from the resonance at $\omega = \omega_c$, and we can expand $B(\ell)$ in a Taylor series about its resonance value, retaining only the first order term:

$$B(\ell) = B_o + \ell \left| \nabla_{\parallel o} B \right|,$$

where $\ell=0$ is taken where $B = m\omega_c/e$ and the subscript "o" refers to the value of a quantity at $\ell=0$. If the electric field and density vary slowly over the width of the resonance, the integral in equation (2) can be evaluated for $-\infty < \ell < \infty$ to get the result

$$\frac{dP}{d\psi} = \frac{\pi n_o e E_{\perp o}^2}{2 B_o \left| \nabla_{\parallel o} B \right|}. \quad (3)$$

The corresponding heating rate is

$$\frac{d\bar{W}}{dt}(\psi) = \frac{dP}{d\psi} \int \frac{nd\ell}{B} = \frac{\pi n_o e E_{\perp o}^2}{2 B_o \left| \nabla_{\parallel o} B \right| \int nd\ell/B}. \quad (4)$$

B. Comparison with other works.

The same result can be obtained by a simple phenomenological argument. The average energy gained by an electron during one transit through resonance can be written as

$$\overline{\Delta W} = \frac{1}{2} \pi \overline{\Delta v_{\perp}^2} = \frac{e^2 E_{\perp o}^2}{2m} T^2,$$

where T is the time during which the electron stays in phase with the electric field, or the time required for the electron to cross the resonance region. Kawamura and Terashima¹ used the above expression to calculate the heating rate in the TP-M machine at Nagoya University (Japan). Lichtenberg, et al.² have also proposed a similar expression, differing only by a numerical factor of order unity.

The transit time of an electron through resonance can be approximated by a method suggested by Guest³ and by Ard⁴:

$$\int_0^T (\omega_c - \omega) dt = \frac{\omega |\vec{v}_{\perp 0} B|}{B_0} \int_0^T v_{\parallel} t dt = \frac{\pi}{2},$$

to get the result,

$$T = \sqrt{\frac{\pi B_0}{v_{\parallel 0} \omega |\vec{v}_{\perp 0} B|}}. \quad (5)$$

The resulting energy gain is

$$\overline{\Delta W} = \frac{\pi e E_{\perp 0}^2}{2 v_{\parallel 0} |\vec{v}_{\perp 0} B|}.$$

Kuckes⁵ arrived at an identical result by solving explicitly the equation of motion of an electron that moves through the resonance region with constant parallel velocity in a field with a constant gradient parallel to $\vec{B}$.

The power absorbed by a group of particles is found by multiplying the average energy gained during one transit by the flux of particles through resonance:

$$dP = \overline{\Delta W} n_0 v_{\parallel 0} dA = \frac{\pi n_0 e E_{\perp 0}^2}{2 B_0 |\vec{v}_{\perp 0} B|} d\psi.$$

The result is identical to equation (3).

Several authors have considered the special case of a straight magnetic field of the form

$$B(Z) = B(0)(1 + \beta Z^2), \quad (6)$$

in which electrons execute sinusoidal oscillations:

$$Z(t) = Z_M \sin \omega_{\beta} t,$$

where

$$\omega_{\beta} = \sqrt{\beta} v_{\perp}(0).$$

For a group of electrons that mirror at Z_M , the density distribution is given by

$$n(Z) = \begin{cases} \frac{n_0 B(Z) v_{H0}}{B_0 v_H(Z)} & \text{for } Z \leq Z_M \\ 0 & \text{for } Z > Z_M \end{cases}$$

where $v_H(Z) = v_{H0} \sqrt{1 - Z^2/Z_M^2}$.

Substituting into equation (4) gives

$$\frac{d\bar{W}}{dt} = \frac{eE_{\perp 0}^2}{2B_0 \sqrt{\delta}} \frac{r}{r-1}, \quad (7)$$

where $\delta = \frac{Z_M^2}{Z_0^2} - 1$ and $r = B_0/B(0)$.

Grawe⁶ has treated the same problem by solving explicitly the equation of motion of a relativistic electron in a parabolic magnetic field. In the non-relativistic limit, his result is

$$\frac{dW}{dt} = \frac{\pi e E_{\perp 0}^2}{2B_0} \left(\frac{\omega}{\omega_\beta}\right)^2 J_{\alpha(1-\delta)}^2[\alpha(1+\delta)],$$

where

$$\alpha = \frac{\omega}{\omega_\beta} \frac{r-1}{4r} \gg 1.$$

In order to compare the results, consider the case $\delta=1$, for which the Bessel function can be approximated by

$$J_0^2\left(\frac{\omega}{\omega_\beta} \frac{r-1}{2r}\right) \approx \frac{2}{\pi} \frac{\omega_\beta}{\omega} \frac{r}{r-1} \cos^2\left[\frac{\omega}{\omega_\beta} \frac{r-1}{2r} - \frac{\pi}{4}\right].$$

The solution is a rapidly oscillating function with a mean value of

$$\frac{d\bar{W}}{dt} = \frac{eE_{\perp 0}^2}{2B_0} \frac{r}{r-1},$$

which is identical to equation (7) with $\delta=1$.

If a significant number of electrons mirror exactly at the resonance, it is necessary to generalize the method slightly by leaving the density inside the integral in equation (2), and by introducing a phenomenological resonance width of $v \sim 1/T$ where T is given by equation (5). The calculated heating rate for these particles is then

$$\frac{d\bar{W}}{dt} = 0.21 \frac{eE_{\perp 0}^2}{B_0} \left(\frac{\omega}{\omega_B}\right)^{1/3} \left(\frac{r}{r-1}\right)^{2/3}, \quad (8)$$

in good agreement with the result of Grawe⁶ ($\delta=0$) and of Guest³.

C. Application to a magnetic mirror.

Consider a magnetic mirror field of the form

$$B(Z) = \frac{1}{2} B(0) [(R+1) - (R-1) \cos KZ] \quad (-\pi < KZ < \pi)$$

and a loss cone electron energy distribution given by

$$f(W_{\perp}, W_{\parallel}, Z) = \begin{cases} A e^{-(W_{\perp} + W_{\parallel})/kT} & \text{for } \frac{W_{\parallel}}{W_{\perp}} < \frac{RB(0)}{B(Z)} - 1 \\ 0 & \text{for } \frac{W_{\parallel}}{W_{\perp}} > \frac{RB(0)}{B(Z)} - 1 \end{cases}$$

The density distribution is obtained by integration:

$$n(Z) = \int_0^{\infty} \int_0^{\infty} \frac{W_{\parallel}}{\frac{RB(0)}{B(Z)} - 1} A e^{-W_{\perp}/kT} e^{-W_{\parallel}/kT} dW_{\perp} dW_{\parallel} = n(0) \frac{RB(0) - B(Z)}{(R-1)B(0)} \quad (9)$$

The total number of electrons in the flux shell is

$$\int_0^{\pi/K} \frac{n(Z) dZ}{B(Z)} = \frac{\pi n(0)}{KB(0)} \frac{\sqrt{R}-1}{R-1}.$$

Substituting into equation (4) gives a heating rate of

$$\frac{d\bar{W}}{dt} = \frac{eE_{\perp 0}^2}{2B_0(\sqrt{R}-1)} \sqrt{\frac{R-r}{r-1}}, \quad (10)$$

where $r = B_0/B(0)$.

We can define a perpendicular and parallel "temperature" at $Z=0$ by

$$kT_{\perp, \parallel} = \frac{A}{n(0)} \int_0^{\infty} \int_{\frac{R-1}{R}}^{\infty} \frac{W_{\parallel}}{R-1} W_{\perp, \parallel} e^{-W_{\perp}/kT} e^{-W_{\parallel}/kT} dW_{\perp} dW_{\parallel}$$

to get the result:

$$kT_{\perp} = \left(\frac{R+1}{R}\right)kT$$

$$kT_{\parallel} = \left(\frac{R-1}{R}\right)kT.$$

We expect, therefore, a midplane anisotropy of

$$\theta = T_{\perp}/T_{\parallel} = \frac{R+1}{R-1} \quad (11)$$

The heating rate for an arbitrary anisotropy can be calculated in a straightforward way by generalizing the assumed distribution function, but the resulting equation is quite complicated and unenlightening. Qualitatively, a large anisotropy enhances the heating if resonance occurs near the midplane and reduces the heating for resonance near the mirror throats.

Electrostatic potential gradients can also be included in the model by simply adding a Boltzmann factor, $\exp(-e\Phi(Z)/kT)$ to the density distribution in equation (9).

D. Limitations and discussion.

The use of a steady state, cold plasma conductivity to calculate the heating rate is justified if

$$\omega \gg \nu = \frac{1}{T} = \sqrt{\frac{\nu_{HO} \omega |\nabla_{HO} B|}{\pi B_0}},$$

or

$$\nu_{HO} \ll \omega B_0 / |\nabla_{HO} B|. \quad (12)$$

This condition is generally well satisfied for non-relativistic plasmas in typical magnetic field geometries.

Calculation of the heating rate requires a knowledge of the electric field $E_{\perp 0}$. The vacuum electric field is perturbed locally at $\omega = \omega_c$ because of the resonance in the dielectric constant, which in the absence of collisions can be written as

$$\epsilon = \epsilon_0 \left[1 - \frac{\omega_p^2}{\omega(\omega - \omega_c)} \right]$$

This local perturbation is negligible provided

$$\int_{-\infty}^{\infty} \ln \left| \frac{\omega}{c} \sqrt{\frac{\epsilon(z)}{\epsilon_0}} \right| dz = \frac{\omega}{c} \int_0^{\omega_p^2 B_0 / \omega^2 |\nabla_{HO} B|} \sqrt{\frac{\omega_p^2 B_0}{\omega^2 |\nabla_{HO} B| z}} - 1 \, dz = \frac{\pi \omega_p^2 B_0}{2 \omega c |\nabla_{HO} B|} \ll 1,$$

$$\text{or } \omega_p^2 \ll \omega c |\nabla_{\parallel 0} B| / B_0. \quad (13)$$

The low density plasmas produced by electron cyclotron heating generally satisfy this requirement.

In addition to this local perturbation, the mean square electric field in the cavity is reduced because the cavity Q is lowered in the presence of plasma. The input power P_0 is given by

$$P_0 = \frac{\omega \epsilon_0 \overline{E^2} V}{Q_0} + \int \frac{dP}{d\psi} d\psi,$$

where the first term on the right is the power absorbed by the cavity and the second term is the power absorbed by the plasma. The second term can be written, according to equation (3), as

$$\int \frac{dP}{d\psi} d\psi = \frac{\overline{n} e \overline{E^2} V G}{B_0},$$

where

$$G = \frac{\pi}{2V} \int \frac{n_0}{n} \frac{E_{\perp 0}^2}{E^2} \frac{d\psi}{|\nabla_{\parallel 0} B|}$$

is a factor of order unity that depends on the geometry of the field and density distribution. For $n = \text{const}$ and $E_{\perp 0}^2 = \frac{2}{3} \overline{E^2}$, we have

$$G = \frac{\pi}{3} \frac{B_0}{V} \frac{dV}{dB} \Big|_{B_0}.$$

The mean square electric field in the presence of plasma is given by

$$\overline{E^2} = \frac{P_0}{\frac{\omega \epsilon_0 V}{Q_0} + \frac{\overline{n} e V G}{B_0}},$$

and the resulting heating efficiency is

$$\eta = \frac{1}{P_0} \int \frac{dP}{d\psi} d\psi = \frac{1}{1 + n_c / \overline{n} Q_0 G}, \quad (14)$$

where $n_c = \epsilon_0 m \omega^2 / e^2$. Note that at low densities, the efficiency is proportional to density, but for $\bar{n} > n_c / Q_0 G$, the plasma loading dominates the cavity Q , and total absorption of the rf is expected. At much higher densities ($\bar{n} > n_c$), the rf fields are excluded from the plasma, and the efficiency presumably decreases.

References

1. T. Kawamura and Y. Terashima, in Institute of Plasma Physics, Nagoya University Annual Review, page 103 (1968).
2. A. J. Lichtenberg, M. J. Schwartz, and D. T. Tuma, Plasma Physics 11, 101 (1969).
3. G. E. Guest, in Oak Ridge National Laboratory report ORNL-4150 (1967), page 29.
4. W. B. Ard, in Oak Ridge National Laboratory report ORNL-4401 (1968), page 41.
5. A. F. Kuckes, Plasma Physics 10, 367 (1967).
6. H. Grave, Plasma Physics 11, 151 (1969).

III. COMPUTER SIMULATION

J. L. Shohet

The work reported in this section is the result of an effort to simulate the electron cyclotron heating of a plasma on a computer.¹ The model used is collisionless but the self-consistent electric field of the "super-particles" making up the plasma is calculated and used, along with an external rf electric field and a dc magnetic field. The particles are given a Maxwellian velocity distribution and their mass is relativistically changed through each time step.

A basic description of the method used has been presented previously.

The basic problem solved is the following: the plasma is placed in a 2-dimensional rectangular plane in which the magnetic field is perpendicular and uniform to the plane. 900 super electrons and 900 super-ions are placed uniformly in the plane in one of two ways. These are: a) electrons alternated positions with ions, b) electrons and ions have the "same" coordinates initially. The following items further specify the problem.

Number of real particles per "super particle"	10^{10}
rf heating frequency	8.39×10^9 Hz
Electron cyclotron frequency	8.39×10^9 Hz
Electron plasma frequency	1.7×10^8 Hz
Electron density	$3.62 \times 10^{14} / \text{m}^3 = 3.62 \times 10^8 \text{ cm}^3$
Electron temperature (initial)	10 electron volts
Electron Debye length (initially)	1.23×10^{-3} meters = 1.23 mm
Dimension of plane	.16 meters = 160 mm
dc magnetic field	3000 gauss
rf heating field	100,000 volts/meter (TE ₁₁₁ mode)

The time step is always 1/20th of the shortest period, be it cyclotron, plasma, or rf excitation (the latter if off resonance heating is desired).

For this problem, the shortest period is the cyclotron period. At each time step the kinetic energy of the electrons and ions is computed, as well as the electrostatic energy stored in the region of interest. A photographic record of the positions of the electrons and ions is also made at each point.

Several interesting results appeared. They are: 1) there is heating of the ions as well as the electrons. This is basically due to the space charge field of the electrons; 2) the kinetic energy increases rapidly without the need for collisions; 3) the electron cyclotron instability has been observed after the heating field has been turned off. A discussion of this will follow at the end of this section.

Figure 1 shows the kinetic energy of the electrons and ions as a function of time. Note that the heating rate may be easily evaluated from these data. However, to ascertain whether true heating has occurred, it is necessary to investigate the velocity distribution after heating. If it looks Maxwellian, then heating has occurred. Figure 2 shows the electron velocity distribution after 5 1/2 heating cycles (110 time steps). A 10 electron volt and a 160 electron volt Maxwellian are also plotted on the same curve. The latter function was used because 160 electron volts is the computed electron temperature at the end of the pulses.

Figure 3 shows the electric field in the x direction at a point in the middle of the left quadrant of the interaction region. In this case the mass of ions was made only 5 times greater than that of the electrons. This was done to show the effects of the ion motion more quickly. This graph shows oscillation of the electric field at both the electron and ion cyclotron frequencies. When the heating is removed, after 110 time steps, the oscillations continue but grow in amplitude. The growth rate may be calculated. It is, in effect, an instability, similar in form to that produced

Heating field for all times, 10^5 volts/meter, TE_{111} mode
 Mass of electron 9.1×10^{-31} kg
 Mass of ion $1836 m_e$

Figure 1

in the actual experiment. Figure 4 shows the kinetic energies of the electrons and ions for this problem.

To analyze this instability we note the "experimental" conditions. There is no motion in the direction perpendicular to the resonance plane, i.e., infinite anisotropy. There are no collisions. Under these conditions the electron cyclotron instability will appear, if enough particles are moving in phase (the result of the heating process) at the electron cyclotron frequency. The growth rate of the instability has been found to be

$$\frac{\omega_p}{\sqrt{2}} \frac{\langle v_T \rangle}{c}$$

and agrees excellently with the growth rate measured from the computer.

The experimental measurements also show a similar result. The anisotropy in the real experiment has now been shown to be quite high. The collision rate is low, and under these conditions, the frequency of the instability is predicted to be, and is, less than the electron cyclotron frequency and its growth rate is also slightly less than that shown here.

Note also, that the growth rate is independent of the magnetic field. The computer simulation was tested for this relation by doubling the magnetic field. No change in the growth rate was observed. When the number of particles per super particle were lowered by a factor of 100 (to lower the plasma frequency by a factor of 10) the growth rate was observed to drop to the appropriate level.

The value of $\langle v_T \rangle$ used in calculating the growth rate is the average at the time at which the rate is measured to correct for the effects of the heating.

Figure 5 shows the initial positions of the electrons and ions. The electrons are represented by the symbol $\blacksquare$ and the ions by the symbol $\star$.

Figure 5

Figure 6 shows the positions after 5 1/2 heating cycles, and Figure 7 shows the result of 5 1/2 cycles after the heating pulse is turned off. Both of these are for the case where the ion mass is 5 times the electron mass. When the ions are allowed to have their correct mass, there is essentially no change in position detected.

It should be noted that there is very little difference in the results between the two possible starting positions, i.e., electrons on top of ions, or electrons alternating with ions. The latter condition results in somewhat greater heating rates since the self-consistent field is not zero at time=0 in this model.

The program is also being used to check for rf confinement.² In this case, the dc magnetic field is set to zero, and the particles are moved with the self-consistent field and the rf electric and magnetic fields. The electric field amplitude is set to 5000 volts per meter. Figure 8 shows the positions of the ions and electrons after 44 heating cycles. The ions have their full mass in this problem. Some slight evidence of confinement is noticed in the center region. However, it is quite weak and much more computer time will be needed to show the result.

Figure 6

Figure 7

Figure 8

References

1. The computer used is a CDC 6600 located at the Centre d'Etudes Nucleaires, in Saclay, France.
2. This work was done in conjunction with A. J. Hatch at the Argonne National Laboratory

IV. THEORETICAL CALCULATIONS OF THE FREE-FREE BREMSSTRAHLUNG FROM AN ANISOTROPIC PLASMA

D. G. S. Greene

Two problems related to free-free bremsstrahlung from a plasma containing relativistic electrons have been considered: 1) polarization of the bremsstrahlung, and 2) corrections to the bremsstrahlung spectra due to the finite mass of the nucleus. Both problems were programmed for simultaneous computer calculation.

To express the polarization, the cross-section for bremsstrahlung is derived following the technique of W. Heitler¹ without averaging over the final states of polarization. These two final states of polarization yield two cross-sections: σ_1 and σ_2 . Bremsstrahlung emission perpendicular to the magnetic field results from $\sigma_1 = \sigma_z$ and $\sigma_2 = \sigma_\theta$ and has the intensities:

$$I_z = \int I_0 dN V_0 d\sigma_z \quad \text{and} \quad I_\theta = \int I_0 dN V_0 d\sigma_\theta \quad (1)$$

where: I_z = intensity of radiation polarized in the z direction,
 I_θ = intensity of radiation polarized in the θ direction,
 I_0 = constant of proportionality, and
 dN = density of incident electrons having velocity V_0 .

The polarization is:

$$P(h\nu) = (I_z - I_\theta) / (I_z + I_\theta) \quad (2)$$

Preliminary approximate calculations indicate that P may be a measurable fraction.

The finite mass correction results from four considerations: 1) the density of final energy states for photon and electron is altered by the final energy states for the nucleus; 2) the probability of populating these final energy states of the nucleus is a product of the nuclear wave functions;

3) on deriving the Heitler cross-section the incident electron energy no longer equals the recoil electron energy plus the photon energy, so certain cancellations are no longer possible, resulting in additional terms; and
4) the energy of the recoil electron is less by the amount of energy taken by the recoil nucleus. For the range of energies examined (electrons up to 1 MeV and photons up to 80 Kev) these effects for a helium nucleus produce corrections to the cross-section of the order of .01%. Moreover, the corrections are largest where the cross section is smallest.

Reference

1. W. Heitler, Quantum Theory of Radiation, 3rd edition (University Press, Oxford, 1954).

V. LANGMUIR PROBE MEASUREMENTS IN THE MAGNETIC MIRROR MACHINE

K. A. Connor

The machine on which experiments are carried out is the same magnetic mirror device described in previous progress reports except for the thyatron trigger circuitry. The astable multivibrator has been replaced by the SCR circuit shown.

A modification of this circuit to include the functions of the delayed pulse generator and the pulse amplifiers is being constructed. The machine may be operated in either a single or double power pulse code. The experiments reported below were carried out with a 100 kW single power pulse 300 times per second.

Electron temperature, density, and floating potential were measured with Langmuir probes. The first question to be answered when one wishes to make such measurements is how the probe affects the plasma to be measured. If the plasma is perturbed too much, the information from the probe is useless. One should expect worthwhile results from Langmuir probes in our apparatus for the following reasons. First, there is no noticeable change in the visible light produced when the probe is in place for measurements. Second, the output of a shielded diamagnetic loop shows a change of less than 10% when the axial and radial probes are extended fully into the machine. Third, as is

noted below, the radial density profile is approximately symmetric while the probes are obviously asymmetrically introduced into the plasma. Finally, for the probe to operate such that its data is reasonable, the amount of charge drawn from the plasma must not be an appreciable percentage of the total charge. The total charge is given by

$$Q = neV = 8 \times 10^{-6} \text{ coulombs.}$$

V = plasma volume.

The charge drawn out by the probe is given by

$$\Delta Q = \int_0^{t_p} I_{oi}(0) e^{-t/\tau} dt = 6 \times 10^{-8} \text{ coulombs}$$

τ = decay constant

t_p = time between power pulses.

Thus, the charge drop due to the presence of the probe is less than 1% of the total.

The probes used consist of a 1/8 inch diameter brass welding rod, approximately a centimeter in length, soldered to the center conductor of an RG 58A/U cable. The cable is placed inside a section of 1/4 inch glass tubing and epoxied at both ends of the glass for vacuum seal.

The circuit shown below was used for the measurements made. R_L was 10 K Ω for the density and temperature measurements and 1 M Ω for the floating potential measurement. C is due to the capacitance of the cable and the input capacitance of the Tektronix 1A6 preamplifier, and is approximately equal to 300 pF.

Figure 9 shows the theoretical magnetic field structure in which the measurements were made. The frequency of the microwave power is 8.54 GHz corresponding to cyclotron resonance at $B=0.3050 \text{ w/m}^2$. The figure shows where the resonance zones are for two different values of magnet current. Except when indicated, measurements were made with $I_{\text{MAG}} = 287 \text{ amps}$.

Figure 10 shows the V-I characteristic of the radial probe at $R=3.5 \text{ cm}$ 100 μsec after the 2 μsec power pulse. Notice that the high electron current region does not show saturation in the expected manner. This, however, is a common effect in strong magnetic fields. The probe draws more electrons off a field line than are there so electrons have to diffuse across the field lines. The rate at which they diffuse limits the current drawn.

Figure 11 shows the V-I characteristic of the axial probe at $R=0$, $Z=15.1 \text{ cm}$, 50 μsec after the power pulse. (Z is measured from the midplane.)

The hot component of the electron temperature has been measured by other techniques--x-rays, diamagnetic loop and $\vec{E} \times \vec{B}$ analyzer. The electron temperature of the cold plasma component can be obtained from the V-I plots using the admittance probe method.¹ The method works as follows. R_s , the source resistance of the plasma, is the inverse slope of the V-I characteristic at $V=V_f$, the floating potential. The Taylor expansion for R_s gives the result that

$$R_s = \frac{kT_e}{e} \frac{1}{I_{oi}}$$

where: I_{oi} is the ion saturation current

T_e is the electron temperature.

By measuring the slope at $V=V_f$, kT_e can be calculated. Figure 12 shows kT_e vs. time at $R=3.5 \text{ cm}$, $Z=15.1 \text{ cm}$. This method was used because it avoids drawing large currents from the plasma.

FIGURE 9 CONTOURS OF CONSTANT MAGNETIC FIELD AMPLITUDE ($R=1.021$)

Location of resonance zones for $I_{MAG} = 287$ amps and $I_{MAG} = 289$ amps

$$|\vec{B}| = \# \times I_{MAG}(\text{amps}) \times 10.42 \text{ w/m}^2$$

-40 -30 -20 -10 10 20 30 40

V-I characteristic for radial Langmuir probe at
 $t = 100 \mu\text{sec}$, $R = 1.02:1$, pressure = 2.5μ ,
rep. rate = 300 sec^{-1} , $I_{\text{MAG}} = 287 \text{ amps}$

mA

ion current

Figure 10

-40 -30 -20 -10 volts 10 20 30 40

2

.1

-.1

-.2

-.3

29

V-I characteristic for axial Langmuir probe at
 $t = 50 \mu\text{sec}$, $R = 1.02:1$, Pressure = 2.5μ ,
 rep. rate = 300 sec^{-1} , $I_{\text{MAG}} = 287 \text{ amps}$

Figure 11

Electron temperature of cold component vs. time
 $I_{MAG} = 287$ amps

Figure 12

The electron temperature can be calculated in another way.² If thermal equilibrium is assumed, the current for $V_p > V > V_F$ is given by

$$j(V) = \frac{I(V)}{A} = \frac{I(V_p)}{A} \exp\left[-\frac{e(V-V_p)}{kT_e}\right].$$

When solved for the electron temperature we have

$$T_e = \left[\frac{k}{e} \frac{d(\ln I(V))}{dV} \right]^{-1}.$$

This method, being a bit more clumsy than the first, was used only as a check. For the case of $R=3.5$ cm and $Z=15.1$ cm it gave 6.5 ev vs. 6.9 ev for 50 μ sec and 5.9 ev vs. 4.8 ev for 100 μ sec, showing reasonable agreement.

Once we know kT_e , the density may be derived. The ion saturation current may be expressed as follows

$$I_{oi} = \frac{1}{4} neV^*A \quad (T_i \leq T_e)$$

$$\text{where: } V^* = \left[\frac{8kT_e}{\pi m_i} \right]^{\frac{1}{2}} \text{ for a Maxwellian}$$

A = effective probe area.

Figures 13 and 14 show ion saturation current as functions of r , z , and t . The corresponding densities range from 10^9 to 10^{11} cm^{-3} . Notice the approximately symmetric form of the radial profile. The minimum also occurred on the axis of the machine for the case where the resonance zone was determined by $I_{MAG} = 289$ (see Figure 9). The radial profile at different times for this case is shown in Figure 15. This indicates an annular structure to the plasma which has also shown up in the visible plasma light. Similar results have been obtained by Dandl, et al.³ The annular structure has shown up for heating zones both on and off the axis.

The floating potential can be measured directly making R_L large, limited by the input impedance of the Tektronix 1A6 plug-in unit, and C

Ion saturation current - axial profile at constant time
 $R = 1.02:1$, pressure = 2.5μ , rep. rate = 300 sec^{-1}
 $I_{\text{MAG}} = 287 \text{ amps}$

○ 50 μsec
 □ 100 μsec
 △ 150 μsec

Figure 14

small, limited by cable length and the input capacitance of the 1A6. A check was made to be sure that the decay of the floating potential was not due to the R-C time constant of the measuring circuit. The circuit time constant was changed using both longer cables to increase C and a 10:1 attenuator to increase R. Both doubling the cable length and increasing the input impedance by 10 resulted in less than a 10% change in the value of the floating potential at all times. The result of the measurements is shown in Figure 16. For the worst case--occurring seldomly--the upper frequency limit of the probe is given by

$$\frac{1}{2\pi fCR_s} = 2 \times 10^3 \text{ Hz.}$$

This is large enough to allow the measurements taken to be useful.

These measurements have shown that it is possible to extend our measuring tools to include Langmuir probes. This allows us to check the results obtained against previous measurements, make spatially localized measurements of plasma parameters, and extend our knowledge of the plasma parameters to longer times. The density measured with probe techniques shows agreement to within an order of magnitude with the fundamental mode microwave cavity technique. The temperature is more difficult to compare since diamagnetic loop and $\vec{E} \times \vec{B}$ analyzer measurements work for times to about 25 μsec after the power pulse while the probe data is for 50 μsec after the power pulse and later. Figure 17 shows kT_e vs. time with an extension to short times for the same data contained in Figure 12, using data from a diamagnetic loop and microwave density measurements under similar conditions. More measurements will be made to check the various techniques more thoroughly.

Fluctuations at -40 kHz in both ion saturation current and floating potential are observed near the edges of the plasma with resulting amplitude

Floating potential - axial profile at constant time
 $R = 1.02:1$, pressure = 2.5μ , rep. rate = 300 sec^{-1}
 $I_{\text{MAG}} = 287 \text{ amps}$

Figure 16

Figure 17

variations

$$\langle \Delta n \rangle / n \approx \frac{e \langle \Delta V \rangle_f}{k T_e} \approx 30-50\%.$$

This effect will be studied when we take enough data to more completely parameterize the plasma produced in our machine.

References

1. J. C. Sprott, Rev. of Sci. Instr. 39, 1569 (1968).
2. F. F. Chen in Plasma Diagnostic Techniques, R. H. Huddleston and S. L. Leonard, Eds. (Academic Press Inc., New York, 1965) Chap. 4.
3. Oak Ridge National Laboratory Thermonuclear Division Semiannual Progress Report for the period ending December 31, 1968.