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in Hadron Production as a Possible Origin of the
Weak and Electromagnetic Interactions

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INSTABILITY OF COLLECTIVE STRONG-INTERACTION PHENOMENA
IN HADRON PRODUCTION AS A POSSIBLE ORIGIN OF THE
WEAK AND ELECTROMAGNETIC INTERACTIONS*

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ABSTRACT

A systematic calculus of long-range Regge cut effects in multiparticle production is constructed in the form of an infra-red-divergent stochastic field theory. Total cross-sections and two-body overlap integrals in such a theory may depend very sensitively upon internal quantum-numbers of incident particles, resulting in a strong symmetry breaking at ultra-high energies. Such symmetry violations will influence low energy processes through dispersion relations, and a bootstrap of weak interactions becomes possible. A rough analytic estimate of the scale of thresholds for such effects yields a BCS-type gap equation, which expresses the scale of weak and electromagnetic couplings in terms of purely strong-interaction parameters.

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I. INTRODUCTION

Recent attempts,⁽¹⁾ in the context of local field theories, to unify all interactions (strong, electromagnetic, and weak) have led to serious consideration of the possibility that all these interactions become indistinguishable at sufficiently small distances, or large momenta. If this were true, then methods applicable to strong interactions such as the self-consistent S-matrix approach ("bootstrap") should be equally well relevant for the other interactions, leading to the expectation that symmetries combining all interactions would be found, as in the strong-interaction dynamics alone.⁽²⁾ This phenomenon cannot be seen in a low energy ("old") bootstrap theory, since the weak and electromagnetic forces are negligible compared to the strong at low energies. However, a self-consistent S-matrix theory which relies on high-energy, high-multiplicity intermediate states⁽³⁾ should make manifest such an interplay between classes of interactions.

The multiperipheral model (MPM)⁽³⁾ provides the simplest multiparticle S-matrix theory, but does not appear at first glance to provide any natural mechanism for generating very weak couplings; all Regge-pole output couplings are of order unity intrinsically, unless accidental cancellations appear. This circumstance is connected with the finite separation of singularities in the l -plane, which is required by the usual MPM.

The inclusion of long range rapidity correlations, in addition to the usual short range (threshold and resonance) correlations, changes this picture qualitatively, at least at ultra-high energies. There are presently available three theoretical approaches to the problems which is essentially that of the Pomeron structure: Gribov Reggeon calculus,⁽⁴⁾ Dual-resonance models including loops,⁽⁵⁾ and Eikonal methods, both phenomenological⁽⁶⁾ and field-theoretic.⁽⁷⁾ We choose to discuss a phenomenological theory, partially

developed in collaboration with G. Thomas, ⁽⁶⁾ as we believe it is capable of elucidating the issues in a direct and simple fashion. The Gribov Reggeon calculus is, at present, too arbitrary when it is made sufficiently flexible to include our phenomena. The dual-loop method is not sufficiently developed to handle the infra-red divergences we investigate. The field-theoretic Eikonals must be based on a specific microscopic field theory, e.g. ϕ^3 , which may not produce the hadron correlations at the two-body level (e.g. resonances) seen in experiments at modest energies, and thus may have no phenomenological contact with present-day data. For these reasons we develop further, from ref. (6), the formalism of "multi-Eikonal" semi-phenomenological theory; then we can construct a mathematical example of a high-threshold collective phenomenon, which is an essential link, in our bootstrap viewpoint, between strong-interaction dynamics and weak interactions (including electromagnetism and, possibly, gravity). By formulating the theory in functional-integral language, ^(8, 9) it is possible to pick up many symmetry-breaking techniques developed for infra-red divergent field theories ^(10, 11, 12) and critical phenomena in statistical mechanics. ⁽¹³⁾

Our picture of weak and electromagnetic interactions involves no fundamental particles. In particular, leptons, w-mesons, and the photon must all be Regge poles generated in the t-channel, although we do not reach that level in this paper, which deals only with hadron-hadron interactions.

The basic mechanism for generation of self-consistent weak interactions is envisioned as follows:

Consider a $2 \rightarrow 2$ hadron reaction, $a + b \rightarrow a'' b''$. Let these hadrons carry internal quantum numbers, such as SU(3) indices, which we believe represent symmetries of the strong-interaction S-matrix, to first order in a weak coupling, g_w^2 .

Choose a set of such indices for which $ab \rightarrow a''b''$ does not proceed via the usual strong interactions (at low energies). Then write a forward dispersion integral for this (symmetry-violating) amplitude:

$$A = \frac{1}{2\pi} \int \frac{ds'}{s' - s} \operatorname{Im} A(s') \quad . \quad (1)$$

Since A does not represent a strong amplitude, $\operatorname{Im} A$ will vanish at low energies except for "normal" symmetry-breaking terms of order g_w . However, if "spontaneous" breakdown appears, at some very large s_c , $\operatorname{Im} A$ will become of order unity (from singularities near the Pomeron) above s_c . Combining these regions, we obtain (for $s \cong 0$)

$$A(0) \sim X g_w^2 + \frac{1}{\pi} \int_{s_c}^{\infty} \frac{ds'}{s'} \operatorname{Im} A(s') \quad . \quad (2)$$

Here X represents the low-energy dispersion-integral contribution; the notation is chosen to match that of Dashen and Frautschi.⁽¹⁴⁾ They pointed out that hadronic weak interaction structure was dependent, in an important way, on the nearby singularities of the S -matrix; strong interaction X -matrices could enhance, for example, the $SU(3)$ -octet contributions, and they showed that self-consistency requirements of the strong-interaction bootstrap required the weak currents to satisfy closed commutation relations. However, they had no way to determine the scale of the weak (or electromagnetic) interactions.

If we identify $A(0)$ as a weak nonleptonic process amplitude (thus identifying the normalization of g_w^2), we obtain

$$(1 - X) g_w^2 \sim \int_{s_c}^{\infty} \frac{ds'}{s'} \operatorname{Im} A(s') \quad . \quad (3)$$

A specific form for $\text{Im } A(s)$ would require knowledge of the nature of the singularities near the Pomeron. Here we simply assume $\text{Im } A$ decreases slowly like a soft cut contribution, with a cut-discontinuity vanishing at the branch point;

$$\text{Im } A(s) \sim \frac{1}{(\ln s)^{1+\tau}} \quad (\tau > 0) \quad \text{for } s \gg s_c. \quad (4)$$

Then the scale of the weak coupling is set by $Y_c^{-1/2}$.

In this paper we try only to estimate s_c ; we will assume $\tau = 1/2$, simply to give a specific content to our calculation. To accomplish this task it is necessary to develop a systematic theory of collective hadron-production phenomena at high energies. We accomplish this in Section II, which may be read independently as a strong-interaction theory.

II. MULTI-EIKONAL FORMULATION WITH FUNCTIONAL-INTEGRAL METHOD

In reference 6, a pairwise-factorizable structure for multiparticle S-matrix elements was proposed, motivated in part by a semiclassical picture of particle production including mutual pairwise correlations. We are now concerned with total cross-sections and overlap functions at $t = 0$, and we may simplify the formalism of reference 6 by dropping transverse coordinates. The basic (postulated) structure of n -particle production in (ab) collisions is then given by:

$$\sigma_n(Y) = \sigma_0(Y) \cdot \frac{1}{n!} \int_{-Y/2}^{+Y/2} dy_1 \cdots dy_n \prod_i |G(y_i|ab)|^2 \prod_j \exp[V(y_i - y_j)] \quad (5)$$

where we have denoted $|K|^2$ of ref. 6 by $\exp V$, and assumed V is translation-invariant in rapidity. Note that V , as well as σ_0 and $|G|^2$, are determined by a knowledge of the two-particle exclusive production differential cross-sections.

We wish to investigate dynamical possibilities which allow the overlap functions to be very sensitively dependent upon the quantum-numbers of a and b . To carry this out qualitatively, it is sufficient to consider an average $|G|^2$ independent of y , but very weakly dependent (in the middle of the central region) on the quantum-numbers of a and b . We would expect the influence of a and b on $|G|^2$ to decrease indefinitely, at $y = 0$, as Y increases. However, this decrease need not be rapid, and we assume that its effect can be represented for our purposes here by its average over y . Thus we write

$$|G|^2 = g \quad (6)$$

where g is weakly perturbed by quantum-numbers of a and b , but independent of Y .

The overlaps, total cross-sections, and inclusive densities are obtained⁽⁶⁾ from the generating functional:

$$F[\xi] = \sum_n \frac{g^n}{n!} \int_{-Y/2}^{+Y/2} dy_1 \cdots dy_n \exp \left[\sum_{i,j=1}^n V(y_i - y_j) \right] \prod_{i=1}^n \xi(y_i) \quad (7)$$

In particular, $\sigma_T = \sigma_0 \cdot F[1]$.

To develop a systematic calculus including long-range contributions in V , from Regge-Pomeron cuts, we assume V can be divided into two terms:

$$V = V_S + V_L \quad (8)$$

in which V_S is short range in rapidity (but arbitrarily strong), while V_L is weak but very long range, and of one sign only.

We will parametrize V_L as follows:

$$V_L(y) = \int_0^\infty \frac{dk}{2\pi} \frac{\cos(ky)}{U(k) + \lambda^{-\sigma} k^\sigma + \mu^2} \quad (9)$$

where $\sigma < 2$ (we will assume $\sigma \approx 1/2$), $U(k) = 0$ at $k = 0$ and contains only powers of k greater than 2, and is included only to provide convergence for k integrals as $k \rightarrow \infty$. Note k is essentially il along the Sommerfeld-Watson contour in the l plane; our parametrization exhibits Regge branch points as the dominant characteristic as $y \rightarrow \infty$; $V_L(y) \xrightarrow[y \rightarrow \infty]{} y^{-1-\sigma}$.

Potentials (exclusive pair correlations) of this form have been studied in statistical-mechanics models of phase transitions in one-dimensional systems.⁽¹⁵⁾

We define the short-range-ensemble generating function

$$F_S[\xi] = \sum_n \frac{g^n}{n!} \int dy_1 \cdots dy_n \exp \left[\sum_{ij} V_S(y_i - y_j) \right] \prod_i \xi(y_i) \quad (10)$$

and its leading Regge intercept, as a functional of ξ ,

$$\alpha_s[\xi] \equiv \frac{1}{Y} \ln F_S[\xi] ; \quad (11)$$

at large Y , α_s becomes independent of Y .

If we assume V_S connects only nearest neighbors in rapidity we can obtain an explicit analytic solution for constant ξ , using MPM methods with a one-dimensional kernel: $g \exp [V_S(y)]$.

Now we can apply the formalism, discussed by Wiegel,⁽⁹⁾ of Gaussian functional integration to obtain the following formula:

$$\ln F[\xi] = \int \delta \phi \exp \left\{ - \int_{-Y/2}^{+Y/2} dy \left[\mathcal{L}_0[\phi(y)] + \mathcal{L}_1[\phi(y)] \right] \right\} + (\text{terms independent of } \xi) \quad (12)$$

with

$$\mathcal{L}_0[\phi] = \frac{1}{2} \lambda^{-\sigma} (\Delta^{\sigma/2} \phi)^2 + \frac{1}{2} \mu^2 \phi^2 + \mathcal{L}_u[\phi] \quad (13)$$

where $\mathcal{L}_u[\phi]$ contains higher y -derivatives, from $U(k)$, and is quadratic in ϕ ; and

$$\mathcal{L}_1[\phi] = -\alpha_s[\xi e^\phi] \quad (14)$$

contains all ξ -dependences.

We may note that the functional-integral representation (12) can be interpreted as a probability distribution for random functions $\phi(y)$ describing the structure of individual events in rapidity as discussed by Krzewicki,⁽¹⁶⁾ with the identification

$$\frac{dN}{dy} \propto \rho_S(e^\phi(y)) \quad (15)$$

at "modest" energies where the influence of the derivative terms in $\mathcal{L}_0$ can be ignored.

Here, $\rho_S(z)$ is the density function of the short-range ensemble, defined as

$$\rho_S(z) = z \frac{d}{dz} \alpha_s(z) \quad (16)$$

Following Weigel,⁽⁹⁾ and Jalickee, Siegert, and Vezzetti,⁽¹⁶⁾ we further assume that both Y and the range of variation in V_L are much longer than any correlation length in the short-range ensemble. This enables us to sensibly approximate the functional $\alpha_s[\xi]$ with the function $\alpha_s(z)$, where z is a constant; the latter function is simply $\alpha_s[\xi = z]$.

The standard approach to an evaluation of a functional integral utilizes a saddle-point integration around stationary values of the integrand with respect to variations in the functional variable, ϕ . We denote these stationary values by $\hat{\phi}$; they satisfy $\frac{\delta \mathcal{L}}{\delta \phi} = 0$, where $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$, which yields the nonlinear pseudo-differential equation:

$$\lambda^{-\sigma} \nabla^\sigma \phi + \mu^2 \phi = -\rho_S(\xi e^\phi) \quad (17)$$

+ (terms linear in ϕ with higher derivatives, from $\mathcal{L}_u$)

where ρ_S is defined by (16). If many solutions exist, we consider the one which minimizes $\int \mathcal{L} dy$, as it should give the dominant contribution to F as $Y \rightarrow \infty$. Boundary conditions for (17) will be set by the behavior assumed for densities at $y = \pm Y/2$, the phase-space edges. They may depend upon the quantum-numbers of a and b , and thus provide another mechanism for perturbation of the phase transitions. We will not here examine the possibilities of a sensitive dependence of F on such perturbations; this remains an open, and difficult, question. Instead, we assume periodic boundary conditions, for simplicity.

It is often expected in such problems that the dominant $\hat{\phi}$ will be a constant. We will assume that is true here, again for simplicity. This fact will be verifiable with any given V_S and V_L .

Near criticality, sums over integrations around other nondominant $\hat{\phi}$'s may become more important than the dominant $\hat{\phi}$. This point is discussed by Wiegel,⁽⁹⁾ in the context of the Van der Waals fluid. We neglect this possibility here, only to simplify the discussion.

Expanding $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$ to fourth order in $\psi \equiv \phi - \hat{\phi}$, we obtain:

$$\mathcal{L}(\psi) = -a_s(\xi e^\phi) + \frac{1}{2}\mu^2 \phi^2 + \frac{1}{2}\lambda^{-\sigma} (\nabla^{\sigma/2} \psi)^2 + \frac{1}{2}(\mu^2 - C_2)\psi^2 - \frac{C_3}{3!}\psi^3 - \frac{C_4}{4!}\psi^4 + \mathcal{L}_u(\psi) \quad (18)$$

where the $C_k(\xi)$ are defined by:

$$C_k(\xi) = \frac{\partial^k a_s(z)}{\partial (\ln z)^k} \bigg|_{z = \xi e^{\hat{\phi}}} \quad (19)$$

The first two terms in $\mathcal{L}$ give the leading Regge intercept in the "mean-field" approximation. If V_S is taken as a hard-core repulsive correlation, we would obtain from these terms exactly the Vander Waals approximation for σ_n studied by Thomas and the author.⁽¹⁸⁾ We see that V_L plays an important

quantitative role even at "moderate" energies (a few hundred GeV), in determining the location of the leading effective (Pomeron) intercept, through the value of μ^2 .

If we were to take $\sigma = 2$, and drop $\mathcal{L}_u$ and C_3 , we would obtain essentially the model of Scalapino and Sugar,⁽¹⁹⁾ which does not contain the possibility of phase-transition behavior (at any nonzero λ).

Note C_2 is always positive, since $C_2 = \overline{(n - \bar{q})^2}$ in the short-range ensemble.

The "Lagrangian" $\mathcal{L}$ will be infra-red divergent when $C_2 = \mu^2$, yielding "critical" long-range correlations, as expected in Reggeon calculus⁽⁴⁾ and also on phenomenological grounds.⁽¹⁸⁾ Ultra-violet divergences are avoided with $\mathcal{L}_u$. The choice $\sigma < 1$ prevents the derivative terms from destroying the phase transitions.⁽¹¹⁾

Sensitivity of $F[\xi]$ to small perturbations in ξ near $\xi = 1$ will arise if the "vacuum" state ϕ becomes unstable for large Y .

To obtain criticality, we require positive central-region cut contributions V_L ($\mu^2 > 0$, $\lambda > 0$). Positive contributions are phenomenologically indicated by inclusive data at a few hundred GeV, although the separation into V_L and V_S is unclear. Theoretically, the inclusive cut analysis of Abramovski, Kancheli and Gribov⁽²⁰⁾ predicts positive long-range correlations, even though the cut contributions are negative in total cross-sections.

The asymptotic ($\Delta y \rightarrow \infty$) behavior of two-particle inclusive correlations is given by

$$\rho_2 - \rho_1 \rho_1 \cong [C_2(\xi)]^2 \langle \psi(y_1) \psi(y_2) \rangle \quad (20)$$

$$|y_1 - y_2| \rightarrow \infty$$

$$\text{with } \langle \psi \rangle = 0,$$

where the field-expectation-values are determined from the "Lagrangian" $\mathcal{L}$. These correlations identify the character and strength of the singularities in the neighborhood of the Pomeron.

The problem of determination of these singularities has now been formulated in a Landau-Ginzburg-Wilson⁽¹³⁾ form, to which standard techniques may be applied, including renormalization-group calculations. At "low" energies, the "loop" corrections are weak, and we find "classical" results for long-range correlations exactly as in the "spherical model" long-range calculations.⁽¹⁵⁾

Singularities near the Pomeron can be obtained in closed form if we use the model of Kac, Uhlenbeck and Hemmer,⁽²¹⁾ which corresponds to a hard-core V_S , $\lambda \ll 1$, and $r = 2$, representing a pole with intercept near unity in V_L instead of a cut. An infinite number of secondary poles, with small residues, accumulate in a neighborhood of $l = 1$ when $\lambda \rightarrow 0$.

Up to this point, the theory is purely one of strong interactions at high energies. We now wish to investigate the possibility that a second-order phase transition becomes first-order when loop corrections are important, at ultra-high energies.

III. THRESHOLD ESTIMATES

If the vacuum-state $\hat{\phi}$ becomes unstable for $Y \rightarrow \infty$, when $\xi = 1$, then an infinitesimal change in g can cause a shift to a second vacuum-state which differs by a finite amount from the original $\hat{\phi}$. This will result in a finite change in the strength of singularities (but not necessarily their location in the l -plane), as is evident from the appearance of $C_2(\xi)$ in Eq. (18). Thus a weak-interaction bootstrap becomes possible, as discussed in Section I.

For qualitative estimates of the scale of Y required for such an instability to appear, we will use a Hartree (self-consistent field)⁽²²⁾ approximation. In systems with a continuous $O(N)$ symmetry,⁽²³⁾ this approximation becomes exact as $N \rightarrow \infty$. We replace, in $\mathcal{L}$, powers of ψ higher than quadratic with expectation-values of such products, then determine such expectation-values self-consistently.

Writing

$$\Delta = \langle \psi^2 \rangle = \langle \psi(y_1) \psi(y_2) \rangle \Big|_{y_1 = y_2} \quad (21)$$

we obtain a Hartree Lagrangian

$$\mathcal{L}_H[\psi] = \frac{1}{2} \lambda^{-\sigma} (\nabla^{\sigma/2} \psi)^2 + \frac{1}{2} (\mu^2 - C_2) \psi^2 - C_3 \Delta \psi - \frac{1}{2} C_4 \Delta \psi^2 + \mathcal{L}_U[\psi] \quad (22)$$

The propagator, calculated from $\mathcal{L}_H$, is

$$\langle \psi(y_1) \psi(y_2) \rangle = \int_{Y^{-1}}^{\infty} \frac{dk}{2\pi} \frac{\cos[k(y_1 - y_2)]}{\lambda^{-\sigma} k^{\sigma} + \mu^2 - C_2 - C_4 \Delta + U(k)} \quad (23)$$

We have included the effect of finite Y here as a cutoff on the lower end of the k integral. A more precise formulation would replace the Fourier integral representations by finite sums with levels in k -space separated by $\Delta k \sim Y^{-1}$, and we would obtain expressions closely resembling those for a field theory at finite temperature^(11, 23) with Y replacing β . We assume such refinements are not necessary for our purposes; the essential qualitative features of finite β are represented with the infra-red cutoff used here.

We may replace the effect of $U(k)$ at this point with a simple ultra-violet cutoff k_0 which satisfies $Y^{-1} \ll k_0 \ll$ (short-range correlation lengths). The "gap" (self-consistency) equation for Δ then becomes

$$\Delta = \int_{Y^{-1}}^{k_0} \frac{dk}{2\pi} \frac{1}{\lambda^{-\sigma} k^{\sigma} + \mu^2 - C_2 - C_4 \Delta} \quad (24)$$

We are interested specifically in the possibility of developing a nonzero Δ with $C_2 = \mu^2$ and $\lambda \ll 1$, "critical" for small Y and "weak coupling" (of cuts).

We note it is necessary that $C_4 < 0$ to obtain a solution with $\Delta \neq 0$.

Keeping only the leading term for $\lambda \ll 1$, only the infra-red divergent part matters. The specific results depends on σ ; we choose $\sigma = 1/2$, motivated by studies of Regge-Regge cut discontinuities at t-channel elastic thresholds.⁽²⁴⁾ Then we obtain

$$\Delta = - \frac{|C_4|}{\pi} \lambda \Delta \ln (Y^{-1/2} + |C_4| \Delta \lambda^{1/2}) \quad (25)$$

If $\Delta \neq 0$, we have

$$\Delta = \frac{1}{\lambda^{1/2} |C_4|} \left[\exp \left(\frac{-\pi}{\lambda |C_4|} \right) - Y^{-1/2} \right] \quad (26)$$

for $Y > Y_c$, where

$$Y_c = \exp \left(\frac{2\pi}{\lambda |C_4|} \right) \quad (27)$$

Below $Y = Y_c$ we must take $\Delta = 0$, in the weak-coupling approximation.

This completes our estimate of a threshold scale relevant to a bootstrap of weak or electromagnetic couplings.

The significance of this result must qualified as follows. Note with $C_4 < 0$, the 4th order $\mathcal{L}(\psi)$ has a vacuum which is not stable against large fluctuations ψ , at least for $Y \rightarrow \infty$. However, if Y remains small, and if we examine the Hartree approximation, $\mathcal{L}_H$, then the vacuum $\hat{\phi}$, as determined from the saddle-point equations, remains stable ($C_4 \Delta = 0$) until Y exceeds Y_c .

For $Y > Y_c$, more complex methods must be used, presumably requiring higher powers of ψ to stabilize the shifted vacuum.

(The investigation of these phenomena can be carried out by adapting the field-theory "effective-potential" methods^(11, 12, 22) to our $\mathcal{L}_0$, and our generally nonpolynomial $\mathcal{L}_I$. We must also study multiple solutions of the saddle-point equations, and possible nonconstant solutions of them,^(11, 22) for specific short-range-correlation models.)

Given Y_c we may roughly estimate the magnitudes expected for low-energy weak couplings. Assuming the $t = 0$ dispersion relation in s for a weak amplitude converges as slowly as possible, like a soft cut (similar to V_L), and the (symmetry-breaking) contribution is of order unity above $S_c = e^{Y_c}$,

$$A_w(s \approx 0) \approx \int_{s_c}^{\infty} \frac{ds'}{s'} \cdot \frac{1}{(\ln s')^{1+\sigma}} \sim Y_c^{-\sigma} \quad (28)$$

where we ignore the Dashen-Frautschi⁽¹⁴⁾ $(1-X)$ factor for simplicity.

Thus if $A_w \approx g_w^2$, we obtain (putting $\sigma = 1/2$) $g_w^2 \approx Y_c^{-1/2}$, or:

$$g_w^2 \approx \exp \left(- \frac{\pi}{\lambda |C_4|} \right) \quad (29)$$

which expresses a weak (nonleptonic) coupling scale in terms of purely strong-interaction parameters; $\lambda^{1/2}$ measures the relative strength of cuts; and C_4 [a "small parameter" for hadron reactions in Chew's⁽²⁵⁾ nomenclature] is related to the (secondary-pole) l -plane structure through $\alpha_s(z)$. (Typical estimates in a MPM with repulsive correlations give $|C_4|$ much less than unity). A more precise estimate would require a more detailed solution of the ϕ field-theory, which would provide a theory of the Pomeron structure as well. Generalizations to include continuous

internal symmetries⁽¹²⁾ will lead to analogs of Goldstone modes associated with $\mathcal{L}(\phi)$, which we believe are associated with leptons, which will arise in this theory as t-channel Regge poles.

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