A COMPARISON OF THE D-T NEUTRON WALL LOAD DISTRIBUTIONS IN SEVERAL TOKAMAK FUSION REACTOR DESIGNS

BY

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A Comparison of the D-T Neutron Wall Load Distributions in Several Tokamak Fusion Reactor Designs

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Abstract

The distributions of the neutron angular and scalar flux and current around the wall of three proposed tokamak fusion reactor designs are investigated in detail. The calculational method involves a numerical solution of the integral form of the neutron transport equation using a ray tracing process. The wall loading in a circular cross section tokamak and in two non-circular tokamaks, the Princeton Reference Design and the University of Wisconsin UWMAK, are compared for three different plasma source distributions. The variation of the angular neutron flux at different wall points is investigated for each design. Neutron wall load peaking factors are also calculated and compared for each design, and are found to be sensitive to both the wall shape and plasma source. The divertors in the two non-circular designs are studied for neutron streaming losses and for the wall load in those regions.
I. Introduction

The increased interest in fusion that has occurred in recent years has led researchers to the realization that one of the major technological problems in building a fusion reactor will be the lifetime of the first wall and structural material surrounding the plasma. The lifetime will be limited primarily by the particle flux incident on the wall, of which the neutron flux will be an important component. Early fusion reactor studies(1) envisioned neutron wall loadings of $10 \text{ MW/m}^2$ or more. However, more recent and detailed designs(2,3) have been more conservative, requiring that the neutron flux on the first wall be limited to about $1 \text{ MW/m}^2$.

One reason supporting this lower value of the neutron wall load is recent experimental evidence(4) that indicates that wall erosion due to neutron sputtering may be more important than first thought. An estimate of the degradation of a stainless steel first wall due to ion and neutron bombardment has been reported(5) which predicts that the 14 MeV neutrons will account for two thirds of the total wall erosion and that the total rate of erosion at the desired reactor power level will be about 2-3 times that tolerable for a 30 year first wall lifetime. In addition, the angular dependence of the neutron flux may also be important, since for ions it has been reported(6) that the
sputtering yield is larger for angles more tangent to the material surface. Hence the erosion rate may be enhanced at wall points where there is a large grazing component of the angular flux. It seems clear that while more experimental work is needed on 14 MeV neutron bombardment effects to accurately predict sputtering yields, the effect will probably be significant in a D-T fusion reactor.

Besides these surface effects, there are bulk phenomena which are critically dependent on the neutron wall load. Helium gas production within the wall material is projected to result in swelling and loss of ductility exceeding reasonable design limits, so that replacement of the wall structure has a major impact on economic analyses. "Hot spots" of greater than average flux will aggravate this damage problem.

Of course, the neutron wall load, measured by the fusion neutron current, is an important reactor parameter in its own right, without reference to damage mechanisms. The wall load determines the local power and thereby sets the requirement for blanket cooling. Variations from the average will scale the power density, with a direct impact on blanket design.

Since the wall loading is such a critical parameter, it will be important to learn of any variations of the neutron flux, current, and angular flux at different points on the wall of a fusion reactor. To investigate these
effects, we have performed a detailed, three-dimensional calculation of the distribution of the angular and scalar flux and current around the wall of a tokamak fusion reactor. The effect of the toroidal geometry is implicitly included in the analysis by numerically solving the integral form of the neutron transport equation for streaming of neutrons from the plasma to the wall. The method uses a ray-tracing process which is essentially independent of both the plasma source distribution and the shape of the first wall, so that it is applicable to nearly any tokamak reactor design. This is in contrast to previous studies(7,8) of the wall load distributions in tokamaks, which have been limited to simplifying assumptions on both the plasma source (uniform) and the wall shape (circular or rectangular). Our method is not restricted by such assumptions.

In this paper we report on our analysis of the neutron wall load distributions in three different reactor designs: a circular cross section tokamak; the Princeton Reference Design (PRD) tokamak(2); and the Wisconsin UWMAK tokamak(3). The designs are compared for three plasma source distributions - uniform, quadratically peaked, and peaked with shift to greater major radius. The last approximates the outward shifting of the magnetic flux surfaces in a tokamak. The designs are compared for "hot spots" about the wall due to peaking of the wall loading. The angular distribution of the flux at different wall points is also presented.
The theory used and the numerical method of solution are discussed in Section II. The plasma source distributions that were analyzed are given in Section III. The results and conclusions for the wall load distributions in circular and non-circular cross section tokamaks are presented in Sections IV-VI.

II. Method of Solution

The basis of our calculation of the neutron wall flux is the integral form of the neutron transport equation, which may be written as (9)

\[ F(\vec{r}, \Omega) = \int_0^\infty S(\vec{r} - \rho \Omega, \Omega) \cdot \exp \left[ - \int_0^\rho \Sigma(\vec{r} - \rho' \Omega) d\rho' \right] d\rho \]

where \( F(\vec{r}, \Omega) \) is the neutron angular flux at the point \( \vec{r} \) and in the direction \( \Omega \); \( \Sigma(\vec{r}') \) is the macroscopic absorption cross section; and \( S(\vec{r}', \Omega) \) is the neutron source per unit volume and solid angle.

For our case, we are interested only in the streaming of neutrons from the plasma to the wall, so that there are no interactions and \( \Sigma = 0 \). Also, we shall consider only isotropic plasma sources so that the source is written as

\[ S(\vec{r}', \Omega) = \frac{1}{4\pi} s(\vec{r}') \]
where \( s(r') \) is the source per unit volume. If we then let \( r = r_w \), a wall point, the equation for the angular flux at that particular point \( r_w \) and in the direction \( \Omega \) reduces to

\[
F(r_w, \Omega) = \frac{1}{4\pi} \int_{\text{within vacuum vessel}} s(r - \rho, \Omega) d\rho
\]  

(1)

Thus this is really just geometry – adding up all the source neutrons that reach \( r_w \) with direction \( \Omega \).

We now define the direction \( \Omega \) locally in terms of an azimuthal angle \( \theta \), relative to the normal to the wall at \( r_w \), and a polar angle \( \phi \), with \( \mu = \cos \theta \). Hence the angular flux may be written \( F(r_w; \mu, \phi) \). By integrating over \( \phi \), we obtain the "azimuthal" flux as a function of \( \mu \), i.e.

\[
F(r_w, \mu) = \int_0^{2\pi} F(r_w; \mu, \phi) d\phi
\]  

(2)

The total scalar flux at \( r_w \) is then found as

\[
F(r_w) = \int_{-1}^{+1} F(r_w, \mu) d\mu = \int_{4\pi} F(r_w, \Omega) d\Omega
\]  

(3)

Another quantity of interest is the neutron current \( J \) across a surface perpendicular to a particular direction \( \Omega' \), defined by

\[
J(r_w, \Omega') = \int_{4\pi} F(r_w, \Omega) \cdot \Omega' \ d\Omega
\]
Choosing \( \Omega' \) to be parallel to the \( \theta = 0 \) axis (perpendicular to the wall) we have the standard definition of the current through the wall since \( \Omega \cdot \Omega' = \cos \theta = u \). Hence the wall current \( J \) is

\[
J(r_w) = \int_{-1}^{+1} \mu F(r_w, \mu) d\mu
\]

Thus there are four basic quantities of interest at each wall point \( r_w \), given by equations (1) - (4): the angular flux \( F(\mu, \phi) \); the azimuthal flux \( F(\mu) \); the scalar flux \( F \); and the current \( J \). It is relatively straightforward to analytically solve equation (1) for \( F(\mu, \phi) \) for specialized choices of the plasma source and wall shape\((7,8)\); however, for more generalized cases it would be extremely difficult. The solution is complicated by the problem of determining when the path out from \( r_w \) along \( \Omega \) crosses the torus wall somewhere further down the torus. For these reasons, it was decided to solve for \( F(\mu, \phi) \) using numerical methods based on a ray tracing scheme.

The geometry and coordinate systems used in the numerical solution for \( F(\mu, \phi) \) are shown in Fig. 1 for an arbitrary wall shape. To account for the toroidal geometry, it is convenient to define three coordinate systems— one at the wall point, one at the center of the torus, and one at the plasma center. The primary system at the torus center is composed of the cylindrical coordinates \((R, \lambda, Z)\). A
second system consists of the polar coordinates \((r, \chi)\) measured from the plasma center in any \((R, Z)\) plane, and defines a position in that plasma cross section. At the wall point \(P(R, \lambda, Z)\) with outward normal \(\hat{n}\) a spherical coordinate system \((\rho, \theta, \phi)\), or equivalently \((\rho, \mu, \phi)\), is used; \(\theta\) is the azimuthal angle measured from the normal, while \(\phi\) is the polar angle measured from the \((R, Z)\) plane \((\phi = \pi\) intersects the \(Z\)-axis).

Two principal lengths are shown in Fig. 1 - the torus major radius \(R_t\) and the plasma radius \(r_p\). From these we define the aspect ratio \(A = \frac{R_t}{r_p}\). The specialization of the geometry to a circular cross section torus is shown in Fig. 2. The same three coordinate systems are used, but now a third scale length, the wall radius \(r_w\), has been introduced. As a measure of the size of the plasma in relation to the torus cross section, we define the quantity \(Y = \frac{r_w}{r_p}\). Note that for the non-circular wall shape the wall point \(P\) is defined by its specific coordinates in the \((R, Z)\) plane, while for the circular cross section \(P\) is defined by \(R_t, \chi,\) and \(r_w\).

In the calculations reported here we have assumed that both the plasma and the vacuum wall are axisymmetric. Therefore there is no variation with the toroidal angle \(\lambda\) in any of the neutron distributions. This assumption is not at all fundamental to the numerical method.
For the numerical solution of the angular flux $F(\mu, \phi)$ at any particular wall point, we first discretize the angular space $(\mu, \phi)$ by dividing $\mu$ from 0 to +1 into $N$ segments and $\phi$ from 0 to $\pi$ into $3N$ segments. (Negative values of $\mu$ need not be considered because of the local orientation of $\theta = 0$ with the outward normal; while toroidal symmetry ensures that all distributions will be even in $\phi$, and only the half-range need be considered.) The ray tracing process then entails choosing a particular direction $(\mu_1, \phi_1)$ and following the path out from the wall point $P$ in discrete steps $\Delta \rho$, sampling the source strength whenever the ray is within the plasma, until the path comes to the torus wall (denoted by point $P'$), at which point it stops. According to Eq. 1, the sum of all the sources encountered along this ray, multiplied by the step size, is the angular flux $F(\mu_1, \phi_1)$ for the direction $(\mu_1, \phi_1)$. A different direction is then chosen and the process is repeated, so that in total there are $3N^2$ rays traced. Studies on the effect of the variation of the mesh size $N$ on the accuracy of the solution indicated values of $N = 20$ are reasonable. (10)

Note that this numerical solution for $F(\mu, \phi)$ simply involves converting back and forth from different coordinate systems as the ray is traced out, and thus is really independent of both the plasma source and the wall shape. Hence in our study we were able to investigate such non-circular wall shapes as the PRD and UWMAK. To analyze
such shapes, we used a numerical interpolation scheme based on a cubic spline which was fitted to given data points from the wall shape. Thus, for any position along the major axis $R$, we could find the wall height $Z_w$, which by comparison would indicate whether a ray being traced to $(R, Z_p)$ had crossed the torus wall. Of course, for simpler wall shapes such as circular, in which the curve has an analytical fit, such an interpolation scheme is not necessary and the solution is easier and faster.

Likewise, the method is essentially independent of the plasma source shape, although for this study we have specialized to a circular cross section plasma for want of a better approximation. However, the source distribution within this plasma can be fairly arbitrary, as detailed in the next section. Note finally that the method allows one to calculate the angular flux for several plasma sources using only one ray trace. This results in a significant reduction in computing time, since it is not necessary to repeat the ray process for each source.

III. Plasma Sources

We have used three different circular plasma source distributions - uniform, peaked, and shifted - in our investigation of the neutron wall load. We assume they are isotropic sources and that the plasma is axisymmetric, so
that the source strength is a function only of \((r, \chi)\), the location in the plasma cross section.

For the uniform source, we assume \(s(r, \chi)\) is simply constant within the plasma for \(r < r_p\). In the peaked plasma source distribution, we assume the source is a quadratic in \(r\), that is

\[
s(r, \chi) = 1 - \frac{r^2}{r_p^2}, \quad r \leq r_p
\]

For the shifted source, we make a first-order allowance for the effect of the toroidal geometry and plasma properties on the neutron source strength. This effect is an outward shifting of the magnetic flux surfaces, on which the plasma density and temperature, and therefore the source, are nearly constant. To first order, the source strength is a constant on circles in the \((R, Z)\) plane that are shifted to progressively greater \(R\), rather than being concentric at \(R_t\). As shown in Fig. 3, we denote the radius of the shifted flux surface by \(\psi\) and the size of the shift by \(f(\psi)\). We then approximate the plasma shift by the simple expression

\[
f(\psi) = \epsilon \frac{r_p^2 - \psi^2}{r_p}
\]
where $\varepsilon$, a constant less than one, is the maximum fractional shift.

To determine $\psi$ for a point given by $(r, \chi)$ (or by $y$ and $z$, where $y = r \cos \chi$ and $z = r \sin \chi$), we write

$$\psi^2 - z^2 + (y - f)^2$$

Substituting equation (6) for $f(\psi)$ into equation (7), we then have a fourth order equation for $\psi$ which can be solved in terms of the point $(r, \chi)$ or $(y, z)$. We then write the shifted source strength as

$$s(r, \chi) = 1 - \frac{\psi^2}{r_p^2}$$

We approximate the size of the shift parameter $\varepsilon$ by (11)

$$\varepsilon \approx \frac{1 + 4 \beta_p}{8A}$$

where $A$ is the torus aspect ratio and $\beta_p$ is the poloidal plasma beta, equal to the ratio of the plasma pressure to the poloidal magnetic field pressure. Note that the magnitude of $\varepsilon$ is determined by both a geometric factor $(A)$ and a plasma pressure term $(\beta_p)$. We see also that for $\varepsilon = 0$, $\psi$ becomes identical to $r$ so that the shifted source strength reduces to that of the simple peaked source given
by equation (5). The shifted source is thus represented by a quadratic whose peak is shifted by a distance $r_p$ from the nominal plasma center at $R_t$. Profiles of the shifted source strength for a value $\varepsilon = 0.5$ are shown in figure 3b.

In order to compare the fluxes calculated from the different sources, it is necessary to normalize each of the volumetric source distributions $s(r,\chi)$ so that the total number of neutrons $S_T$ from the plasma is a constant, where

$$S_T = 2 \pi R_t \int_0^{2\pi} \int_0^{r_p} d\chi dr r s(r,\chi)$$

Thus each of the volumetric sources is scaled by a constant, so that $S_T$ is the same for each. For convenience we used $S_T = 1.0$, so that the calculated fluxes were per one source neutron in the entire plasma volume. The normalized distributions for each source in the circular plasma cross section are shown in Fig. 4. From equation (10) we found that the maximum value of the peaked source was twice the value of the uniform source, while for a shift parameter $\varepsilon = 0.5$, the maximum value of the shifted source was approximately 95% that of the peaked source.

**IV. Circular Cross Section Tokamaks**

For the first part of our analysis, we investigated the neutron wall load distributions in a circular cross
section tokamak. Since Daenner(7) had already done this for the uniform source case, it was of interest to see if our calculation was in agreement with his analytical method. Performing our analysis for the same size torus with the uniform source, we found excellent agreement between the two techniques(10). Once this was accomplished, we began an investigation of the effect on the neutron wall load due to non-constant plasma sources.

As a circular approximation to some non-circular tokamak designs currently being proposed(2,3), we chose a torus with major radius $R_t = 10.0$ m, aspect ratio $A = 3.0$, and effective wall to plasma radius ratio $Y = 1.1$. The three alternate sources - uniform, peaked, and shifted - were used in the analyses, with a shift parameter $\varepsilon = 0.5$. This value of $\varepsilon$ and the functional form of equation (6) were chosen to approximate the plasma model of ref. 2. Considering the limited validity of the first-order shift theory, this value of $\varepsilon$ may be slightly large, but it will provide an approximate upper limit on the effect of the shifted source on the wall load. For each of the sources we calculate the current $J$, scalar flux $F$, angular flux $F(\mu,\phi)$, and azimuthal flux $F(\mu)$ at various wall points determined by their poloidal angle $\chi$, ranging from $0^\circ$ on the outside of the torus to $180^\circ$ on the inside.
The resulting angular flux $F(\mu, \phi)$ for various values of $\chi$ showed the expected property of symmetry in $\phi$ for $\chi = 0^\circ$ and $180^\circ$. For intermediate values of $\chi$, the flux tended to peak for an angle $\phi = 60^\circ$, due to the fact that these points were influenced more by sources further around the torus. These results are discussed in more detail in ref. 10. If we next integrate over all angles $\phi$ as in equation (2), we obtain the azimuthal flux $F(\mu)$ at the wall, where $\mu = \cos \theta$ and $\theta$ is the angle of incidence relative to the wall normal. In Fig. 5 some of the results of this calculation are plotted. Fig. 5a illustrates the angular flux $F(\mu, \phi)$ at the wall point $\chi=90^\circ$, for the shifted source. Here constant magnitude contours are plotted on a polar grid with $\phi$ as polar angle and $(1-\mu) = (1-\cos \theta)$ as the radius. (Elements of area on this grid distort the solid angle element $d\mu d\phi$, but the flux pattern relative to the torus is easier to grasp than with a rectangular $(\mu, \phi)$ plot.) The peaking of the angular flux at $\theta=70^\circ$ and $\phi=62^\circ$ is due to the greater thickness of source traversed by a ray projected obliquely back along that line and partly around the torus.

Fig. 5b is a perspective view of the azimuthal flux $F(\mu)$ for the shifted source as a function of $\chi$ and of $\mu = \cos \theta$ (for $15^\circ \leq \theta \leq 88^\circ$). Several interesting observations on the behavior of $F(\mu)$ with $\chi$ can be made. First, it can be that there is no flux tangent to the wall ($\theta = 90^\circ$) until $\chi$ is greater than about $90^\circ$, at the top of the torus. For small
values of $\chi$, there is a sharp decrease in $F(\mu)$ as $\theta$ increases. Hence most of the flux at these points is directed fairly perpendicular to the wall. Note also that the $\theta$ of maximum $F(\mu)$ increases as $\chi$ increases; that is, $F(\mu)$ at $\chi = 0^\circ$ has its maximum at $\theta \approx 35^\circ$, while at $\chi = 45^\circ$ the maximum is at $\theta \approx 47^\circ$. However, for points toward the inside of the torus ($\chi > 90^\circ$) this shifting of the maximum $F(\mu)$ has ceased. At those wall points the maximum is more normal to the wall ($\theta \approx 0 -15^\circ$), although the magnitude has decreased and instead the flux is more evenly distributed over all $\theta$.

Thus we can see that the angular flux as a function of $\theta$ is very highly peaked for wall points on the outside of the torus, while for inner wall points the flux is much smoother and extends over all values of $\theta$. This variation of the angular flux about the wall of a tokamak fusion reactor could be an important consideration in the first wall lifetime, as previously discussed. If experiments should show that greater material erosion occurs for a grazing (tangential) flux, then our results would indicate that wall erosion will be enhanced on the inner parts of the wall.

It is also interesting to see what effect the angular flux distribution will have on the neutron fluxes in a blanket surrounding the reactor. To investigate this
effect, we used the results of $F(\mu)$ at different wall points $\chi$ to generate an angular source condition for input to the discrete ordinates transport code ANISN(12). In particular, six different shell source angular distributions of 14 MeV neutrons were used: four were generated from $F(\mu)$ at $\chi = 0^\circ$, $60^\circ$, $120^\circ$, and $180^\circ$ for the shifted source; one was isotropic; and the last was a "perpendicular" beam source. Slab geometry with $S_8$ angular quadrature was used. The composition of the blanket was that of the standard or "benchmark" blanket(13) of lithium, carbon, and niobium. For comparison, the total tritium breeding ratio $T$ and its components $T_6$ and $T_7$ due to the $^{6}\text{Li}$ and $^{7}\text{Li}$ tritium breeding reactions were calculated. The results of each source on the breeding parameters are given in Table I.

It is seen that the total $T$ is found to be nearly the same for all sources, with agreement within 1%. However, there is a larger difference in the partial contributions $T_6$ and $T_7$ for the various sources. The breeding from the $^{6}\text{Li}$ reaction varies by 2%, while for the $^{7}\text{Li}$ reaction there is a 5% variation. $T_6$ is largest for the sources at the wall points $\chi = 120^\circ$ and $180^\circ$, while the $T_7$ is highest for $\chi = 0^\circ$. The reason for this difference is the variation of $F(\mu)$ with $\chi$, as shown in Fig. 5, and also because of the nature of the different Li reactions. The $^{6}\text{Li}$ reaction has a large cross section at thermal neutron energies, while the $^{7}\text{Li}$ is a threshold reaction requiring neutron energies
larger than 2 MeV. Hence for a source of 14 MeV neutrons more perpendicular to the wall, as in the $\chi = 0^\circ$ case and the beam case, the $T$ is large because more of the high energy neutrons are able to penetrate through the niobium first wall into the lithium. Conversely, for a more uniform neutron source (such as at $\chi = 120^\circ$ or $180^\circ$ or the isotopic source) there is more thermalization of the high energy neutrons as they enter the blanket at oblique angles and are scattered, and thus there is more tritium production from the Li$^6$. This difference in the neutron energy spectrum could be an important factor in optimization of the blanket for tritium breeding. It could also have an effect on the nuclear heating rates in different regions of the blanket, depending on the sensitivity of the various cross sections to specific energy ranges in the neutron spectrum.

Proceeding, we integrate the azimuthal flux $F(\mu')$ over all $\mu$ to compute the scalar flux $F$ and the current $J$ at each wall point as in equations (3) and (4). The results, shown in Fig. 6, reveal the different behavior of $F$ and $J$ for each source. Both the uniform and peaked source were found to have a maximum flux slightly inside the torus, at $\chi = 105^\circ$, with the uniform source producing the larger flux. The maximum flux for the uniform source is 7.7% greater than the minimum (at $\chi = 0^\circ$), while for the peaked source the maximum is 10.4% larger. However, the shifted source flux is much different. For that case, the maximum occurs at $\chi = 0^\circ$ and
is 42.7% larger than the minimum at $\chi = 180^\circ$. Since the source intensity for the shifted source is strongest at the outer edge of the plasma, the outer wall points feel a stronger flux than those on the inside. This effect is also evident for the shifted source current, where there is a large difference of 61.3% between maximum and minimum. The current for the uniform and peaked source are nearly the same, with maxima occurring at $\chi = 25^\circ$ which are about 17% larger than the minima at $\chi = 180^\circ$. The reason for this behavior of the current vs. $\chi$ can be understood from Fig. 5, in which we found that $F(\mu)$ was more normal to the wall point for smaller values of the wall angle $\chi$.

For comparison, in Fig. 6 we have also shown the nominal value of the current or wall load, found by dividing the total number of source neutrons by the wall area. It is evident that the neutron current is about equal to the nominal value at the wall angle $\chi = 80^\circ$, but is larger at wall points toward the outside of the torus and smaller for points toward the inside. Since this nominal current is the standard value usually quoted as the wall load for a power reactor design, it is interesting to investigate this variation in the wall load in more detail.

To do this, we define a wall load peaking factor $\Delta$ as

$$\Delta(\chi) \equiv \frac{J(\chi)}{J_{\text{nom}}} - 1 \quad (11)$$
where $J_{\text{nom}}$ is the nominal current (equal to $0.6902 \times 10^{-3}$ neutrons/m$^2$ in Fig. 6). and $J(X)$ is the calculated neutron current at the wall angle $\chi$. The peaking factor $\Delta$ is plotted in Fig. 7 for both the peaked source and the shifted source. Note that for the peaked source the wall load can be 6% larger than nominal towards outside points and about 10% smaller at inside points. The peaking effect was found to be much stronger for the shifted source, with asymmetries of nearly 25% in the neutron wall loading relative to the nominal. As noted before, the value of $\varepsilon = 0.5$ used here may be somewhat large for a realistic tokamak, and hence should provide an upper bound on the peaking effects.

Note that for some intermediate value of $\varepsilon$ it might be possible to obtain a nearly constant neutron flux about the wall. This may be seen by comparing the difference in the fluxes shown in Fig. 6 when going from a peaked source (i.e. $\varepsilon = 0$) to the shifted source with $\varepsilon = 0.5$. Since the shift was found to have such a pronounced effect, a study of the variation of the flux with changes in the shift parameter $\varepsilon$ was suggested.

The variation of the flux and current vs. the shift parameter $\varepsilon$ is shown in Fig. 8 for three different wall positions - outside, top, and inside. It can be seen that the variation with $\varepsilon$ is smooth, that is the transition is
not abrupt when going from $\varepsilon = 0$ to 0.5. Note that values of $\varepsilon$ from 0.1 - 0.2 would seem to be the most interesting as far as obtaining a nearly constant flux about the torus wall. The current does not show the same effect, that is it always decreases with increasing wall angle. This is due to the greater perpendicularity of the flux for points on the outside of the torus, as shown in Fig. 5.

These calculations for the neutron load on the wall of a circular cross section tokamak have revealed interesting variations in the angular flux incident on the wall. The asymmetry in the scalar flux and current, particularly as a result of the shifted source, should be an important consideration in the first wall lifetime of the fusion reactor.

V. Non-Circular Cross Section Tokamaks

In order to extend our analysis to non-circular wall shapes, we needed a more sophisticated treatment of the ray wall-crossing test. This was done by using a numerical interpolation scheme for the wall based on a spline fit of a number of given data points which defined the wall shape. The two designs that were studied were the Princeton Reference Design (PRD)(2) and the Wisconsin UWMAK.(3)
Princeton Reference Design

The PRD, shown in Fig. 9, is a D-T fueled reactor designed to produce 5305 MW of thermal power. The blanket surrounding the plasma uses the molten salt flibe to breed tritium and is helium gas cooled. The reactor is designed to operate with a single-null poloidal field divertor, which results in the non-circularity of the wall. A major point of interest is the angular distribution and magnitude of the neutron wall load at points near the divertor channel, as well as any major peaking in the flux at other points on the wall.

For the neutron source in the PRD, we used a circular plasma of radius 3.25 m centered at the major radius of 11.0 m. This results in an average plasma aspect ratio of 3.38. Since it was not possible to measure the wall position simply by a poloidal angle $\chi$, we instead denote the wall point by its coordinate $R$ along the torus major radius. All three plasma source distributions given in section III were used, but with major emphasis on the shifted source with $\omega=0.5$.

Contour plots of the angular flux $F(\mu, \phi)$ from the shifted source are shown in Fig. 10 for four wall points: a) $R=14.6$ m, at the mid-plane on the outside of the torus; b) $R=11.0$ m, at the top; c) $R=5.0$ m, at the mid-plane on the
inside; and d) R=4.5 m, on the inner wall of the divertor. Figures 10b and 10c resemble their counterparts from the circular cross section case, illustrating, respectively, the importance of streaming down the torus arms and the restricted vertical extent of the source.

Streaming around the torus has the greatest impact at the midplane on the outside, as shown by the concentration of contours in Fig. 10a. The sharp drop in the flux at μ=.94 (θ=20°) and φ=±90° is due to the interception of rays with θ<20° by the central core of the torus. The sharp change in flux as a function of φ is due to a similar interception by the wall around R=6 m, before it diverges into the divertor. The angular flux at R=4.5 m, while even more peculiar, is readily explained. A ray normal to the wall from this point does not intercept the plasma at all, so that the flux can be non-zero only for a restricted range of oblique angles.

After integrating the angular flux over all φ, the azimuthal flux distributions F(μ) at six different points on the wall of the PRD are shown in Fig. 11. These distributions are for the shifted plasma source, but, as in the circular cross section torus, the distributions for the other sources were similar although the magnitudes were slightly different. The two points R = 4.5 m and 5.25 m are particularly interesting since they are along the divertor channel, with the former being on the lower portion and the
latter on the upper portion. As expected, there is no component of the flux normal to the wall, since neither are in the direct line of sight of the plasma. However, both points do have neutrons incident at angles more tangential to the surface due to the toroidal curvature. Although the magnitude of the flux at these points is less than that at other wall points directly facing the plasma, there is still a significant grazing component which could lead to erosion problems in the divertor.

The angular flux at other wall points was found to exhibit similar properties to those in the circular design, that is, an increasing tangential flux for points toward the inner part of the torus. For the extreme inner and outer wall points \( R = 5.0 \text{ m} \) and \( 14.6 \text{ m} \), the flux is highly peaked in directions more perpendicular to the wall. This peaking is reduced at intermediate wall points such as \( R = 11.0 \text{ m} \), where the flux varies fairly smoothly with the angle of incidence. At more inward points such as \( R = 7.5 \text{ m} \), the flux is peaked at angles of about \( 40^\circ \) to the normal and extends over all angles of incidence. The detailed shape of these curves, which in some cases seems rather perplexing (e.g. the peak in the 14.6 m curve) is usually clarified by reference to the corresponding angular flux plot in Fig. 10. From these curves it can be seen that there will be a large variation in the neutron angular flux distribution about the wall of the PRD.
If we now integrate the azimuthal flux over all angles $\theta$, we obtain the scalar flux and the current as a function of the wall position $R$. These are shown in Fig. 12 for the PRD. Note that the nominal current, calculated using a circular wall of radius 3.6 m, is shown for comparison. Based on a 14 MeV neutron production rate(2) of $1.22 \times 10^{21}$ per sec, this nominal neutron current is equivalent to a wall loading of 1.76 MW/m$^2$. It was found that for the peaked (quadratic) source, the maximum scalar flux occurred near the top of the torus, similar to the circular torus case. However, for the shifted source the maximum was at the outside wall point, with a sharp decrease for points towards the inner part of the torus. The same effect was found to occur for the shifted source neutron current, but for the quadratic source the current was not nearly so sharply peaked.

From these values of the current we can calculate the wall load peaking factor $\Delta(R)$ as defined in equation (11). We find that the wall load at $R = 14.6$ m (the outermost wall point) is 2.51 MW/m$^2$ for the shifted source, which gives a peaking factor of 43% over the nominal. For the quadratic source, the maximum wall load is found to be 1.93 MW/m$^2$ at $R = 14.0$ m, corresponding to a $\Delta = 10\%$. For a more inward point such as $R = 7.5$ m, the wall load for the peaked source is 1.38 MW/m$^2$ which gives $\Delta = -21\%$; while for the shifted source it is about 1.0 MW/m$^2$, resulting in $\Delta = -43\%$. 
Hence we can see that the wall load variation in the PRD is fairly great, with differences of up to 40% above or below the nominally quoted value, depending on the wall point and the plasma source being considered. Since the value of the shift parameter $\varepsilon = 0.5$ is perhaps large for this plasma, we can estimate the wall load peaking to be somewhere between 10% and 43%, a significant amount. It is interesting to note that these variations are somewhat larger than those in the circular cross section case of Fig. 7, where maximum peaking factors of about 20% were calculated. This is a direct result of the high degree of non-circularity in the PRD wall shape.

The scalar flux and current along the divertor channel are shown in the left portion of Fig. 12. We found that at these wall points the neutron loading was greatly reduced, as would be expected due to shadowing of the plasma. This in turn implies "peaking" factors of -50% and less, which indicates that the wall load would not be a critical problem in the divertor. However, as shown earlier most of the flux at these points is incident at tangential angles. This might result in enhanced erosion even though the magnitude of the wall load is fairly low. A further consideration related to the wall load in this region is the loss of neutrons streaming up the divertor. For the PRD it was estimated(2) that about 2% of the neutrons would be lost in this manner. Using these wall load calculations, we were
able to refine this prediction by integrating the current at the divertor wall points over the appropriate wall area along the divertor channel. We find that for the quadratic source the neutron loss up the divertor was about 3.3% of the total neutron source while for the shifted source the loss was approximately 3%. Thus there is not a very great loss, but since the PRD was designed with a breeding ratio close to unity it is a critical parameter.

Wisconsin UWMAK Design

The UWMAK-I(3) design shown in Fig. 13 is a D-T fueled reactor which produces 5000 MWt. The basic structural material is stainless steel and the coolant, moderator, and breeding medium is liquid lithium. The reactor is designed with a double null poloidal field divertor producing the non-circular wall shape. The blanket design has since been improved(14) by going to a solid breeding material and helium gas cooling, but the basic wall shape is similar. The neutron source used for the UWMAK study was again a circular plasma with a radius of 5.0 m centered at the torus major radius of 13.0 m, giving an aspect ratio of 2.6. For the shifted source, a shift parameter $\varepsilon = 0.5$ was used. To denote the particular wall point in question, its coordinate $R$ along the major axis was specified, as in the PRD study.
In Fig. 14 we show the angular flux $F(\mu, \phi)$, due to the shifted source, at four wall points: a) $R=18.86$ m, at the midplane on the outside; b) $R=14.8$ m, near the top of the torus; c) $R=7.8$ m, at the midplane on the inside; and d) $R=9.4$ m, on the collector plate in the top of the divertor channel. The general features of these contour plots in Fig. 14a–c are comparable to those at corresponding points in the PRD. One notable difference is the greater extent of the flux in $\theta$ for $\phi=0^\circ$ or $180^\circ$ in Fig. 14c. This is caused by the closer approach of the plasma to the inner midplane wall in the UWMAK. Another difference is the reduction in the flux concentration and gradient for $\phi=\pm90^\circ$ in Fig. 14a. This is due to the comparatively greater radius of the central core, which thereby has a greater shadowing effect on the outer wall. The angular flux in Fig. 14d for the point on the upper divertor wall is particularly interesting, since this region appears as though it should be completely shielded from any neutrons. However, we found that the flux was non-zero there, due to some neutrons being able to reach the point in a restricted range of angles ($\theta=70^\circ, \phi=\pm60^\circ$) by streaming down the torus and up past and around the vacuum port shield.

In Fig. 15 we have shown the azimuthal flux $F(\mu)$ at seven different points on the wall of UWMAK, where $\mu=\cos\theta$ and $\theta$ is the angle of incidence relative to the wall normal. These are all for the shifted source plasma, but the results
for the other sources are similar in their dependence on $\theta$. It is clear that there is a wide variation in the angular distribution, according to the wall point. At the innermost point, $R = 7.8$ m, the flux is peaked perpendicular to the wall and then decreases with larger $\theta$, although there is still a significant component at $\theta = 90^\circ$. Further up the inner wall, at $R = 8.6$ m, the flux is fairly constant with $\theta$ and again extends over the entire range of angles. At a point further out along the wall, such as at $R = 14.8$ m, the flux is found to be largest for angles more normal to the wall, with a sharp decrease for angles greater than about $60^\circ$, so that there is no tangential component of the flux. This effect is more evident at the outermost wall point $R = 18.86$ m, where now the flux peaks at $\theta = 30^\circ$ and then decreases, with no flux incident for angles larger than $75^\circ$. These effects are similar to those shown in Fig. 11 for the PRD, and hence both reactor designs have about the same behavior in the variation of the neutron angular flux with wall position.

The two points on the structure in the divertor channel are interesting for possible enhanced erosion problems, as discussed for the PRD. At the inner point $R = 9.4$ m, there is no component normal to the wall since it is not in direct line with the plasma. Instead, the flux ranges from $\theta = 20^\circ$ to $90^\circ$, with a fairly large peak at about $65^\circ$. The other divertor point, at $R = 10.2$ m, has an
azimuthal flux over the entire range of $\theta$ with its maximum at the perpendicular. Thus for these two divertor structure points there is a great deal of difference in the azimuthal flux, with peaks of approximately the same magnitude occurring at $65^\circ$ for the inner point and $0^\circ$ for the outer. The azimuthal flux at $R = 9.4$ m on the collector plate wall in the top of the divertor channel reveals a small but clearly non-zero component at a point that might appear at first glance to be shielded from any neutrons.

We next integrated the angular flux to find the current and scalar flux for the UWMMAK design. These are shown as a function of $R$, the wall position, for the main wall in Fig. 16 and for the divertor and vacuum port shield wall in Fig. 17. Only the peaked and shifted source profiles are shown, since the uniform source was very similar to the peaked source. These curves show that the point of maximum flux greatly depends on which plasma source is assumed. For the peaked source, the maximum flux is at the very inside of the torus, while for the shifted source with $\xi = 0.5$ the maximum is on the outside. The difference in the flux at outer wall points is similar to that found for the PRD; that is, the shifted source produces a steady increase in the flux with increasing $R$ while the quadratic source has a peak at about 14 m and then decreases.

The neutron current as a function of $R$ is also plotted in Fig. 16 and 17. To calculate the peaking
factors, we have shown the nominal current found using the "nominal" wall area(3) of UWMAK. This nominal current corresponds to a neutron wall loading of 1.25 MW/m², assuming a plasma reaction rate of $1.57 \times 10^{21}$ neutrons/sec. It can be seen that the general behavior of the neutron current is nearly the same as in the PRD; that is, the shifted source produces a steady increase in the wall load as the wall position moves outward while the quadratic source has a smaller maximum at about 17 m and is greater than the shifted source at more inward points. The maximum wall loading for the shifted source is about 1.40 MW/m², which gives a peaking factor $\Delta$ calculated from equation (11) of about 12%. The largest wall load for the peaked source is 1.31 MW/m² which implies a $\Delta$ of 5%. The wall load in UWMAK is less than the nominal value for wall points smaller than about 14 m. For example, at the innermost point $R = 7.8$ m the peaked source gives a wall load of 1.2 MW/m² and a peaking factor $\Delta$ of -4%, while the shifted source has 1.0 MW/m² and -20% peaking. Clearly at most other wall points the load is even smaller and hence the UWMAK design will not have significant peaking in the neutron wall load.

The neutron flux and current in the divertor channel, shown in Fig. 17, are of particular interest since this is an area where a low wall loading is desirable. Along the lower face of the vacuum port shield, the maximum current is about 35% to 40% below nominal, which implies a wall load of
up to 0.8 MW/m², a significant amount. Along the divertor collector plate we find maximum wall loadings at either end of the wall of about 30% of nominal, or about 0.4 MW/m². In the middle of this area, which would appear to be shielded, we find instead a small but non-zero component of the current which ranges up to 1% of nominal, or about 0.01 MW/m². Hence the UWMAK divertor channel will experience a fairly large flux of neutrons.

To illustrate how neutrons are able to reach points along the middle of the divertor collector plate, in Fig. 18 we show the projections of three neutron streaming paths in two different planes of the UWMAK. Two of the rays, beginning at point A on the outside wall in the midplane, show how a ray can either be stopped by the torus central core or can continue past it and hit the outer wall somewhere further down the torus. These paths are straight lines in the Z=0 plane, shown in the lower portion of Fig. 18, but when the ray points are projected onto the UWMAK cross section, shown in the upper portion of Fig. 18, they become curved lines. If the ray passes by the central core, as in case 2, the curve passes through a minimum value of R and then increases until it hits the outer wall. For the point D in the middle of the upper wall in the divertor channel, Fig. 18 shows how, for a restricted range in the angular mesh, a ray is able to pass around and under the vacuum port shield and penetrate the plasma source. This unanticipated toroidal effect illustrates that special care
may be required in designing protective shields for areas where it is desirable to prevent neutron streaming.

The most notable difference between the PRD and UWMAK results is the difference in the wall load peaking; for the shifted source, UWMAK has 12% peaking, while PRD has 43% peaking. This can be explained by the difference in the plasma chamber cross-section. In the UWMAK, the double-null divertor elongates the wall at the top and bottom of the torus; while in the PRD the single-null divertor narrows the wall towards the center of the torus. Thus in the UWMAK the central core provides a greater shadowing of the outer wall, reducing the load there. Put another way, in the PRD the relatively smaller core allows neutron streaming onto the outer wall from farther around the torus. A second effect, perhaps as important as the peaking, is that the UWMAK divertor is exposed to much higher wall loads than the PRD divertor. Note that the maximum current in the divertor region is about 65% of the nominal, a reasonably large neutron load. To estimate the neutron losses to the divertor, we can integrate the current incident on the structure over its area, as done for the PRD. We find that for the UWMAK there is about a 7% loss of neutrons due to streaming into the divertor channel. This is about twice the loss of the PRD, and is somewhat expected since in UWMAK the divertor is larger and faces the plasma directly. These losses would be important if the tritium breeding ratio were close to unity, but in the UWMAK design the tritium breeding
ratio is sufficiently greater than one so that the effect would not be critical.

VI. Conclusions

We have investigated the neutron wall load distributions in tokamaks using a numerical ray tracing scheme to solve the integral neutron transport equation. The method of solution is essentially independent of both the plasma source and the wall shape, so that arbitrary tokamak cross section designs may be studied.

In this paper we have reported on our analysis of three different wall shapes - a circular cross section, the Princeton Reference Design, and the University of Wisconsin UWMAK. For each design we studied the angular dependence of the flux and the peaking of the scalar flux and current about the torus wall for three different plasma source distributions. The angular flux distributions were similar for each design with the flux being more normal at outer points on the wall and then developing a tangential component toward the inside. However, points on the divertors in the non-circular designs had a different angular flux, depending on whether they directly faced the plasma. The peaking factors calculated from the neutron current showed a strong dependence both on the plasma source and on the wall shape. They were largest for the shifted
source and occurred on the outer wall point, with a range of 12% in UWMAK to 25% in the circular design and up to 40% in the PRD. Losses due to neutrons streaming up the divertor were estimated to be about 3% for the PRD and 7% for the UWMAK. These differences in the two non-circular designs are a result of the different poloidal magnetic field divertor configurations and their effect on the wall shape.

The variations in the neutron wall load and its angular distribution could be an important consideration in fusion reactor operation. As we showed for the circular design, the breeding ratio may differ at various regions in the blanket. Power densities will be directly scaled by the wall load. The lifetime of the first wall surrounding the plasma will be dependent on the wall flux. Our calculations indicate that the maximum load will occur on the outer points of the wall, and thus result in excessive radiation damage at those points. If future neutron sputtering experiments show that a tangential flux results in increased damage, then the inner wall points may undergo enhanced erosion due to their larger grazing flux. Thus these variations in the neutron flux about the torus wall will be significant for a number of reasons and will be an important consideration in the design and operation of tokamak fusion reactors.
Acknowledgments

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References


Table I. Breeding ratios for different angular source distributions in the benchmark blanket design.

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Fig. 1. Geometry used in the three-dimensional ray-tracing process for the wall load at point P in a torus of arbitrary cross-section.

Fig. 2. Geometry used in the ray-tracing process for the specialized case of a circular cross-section plasma and torus wall.
Fig. 3. Shifted plasma source distribution. (a) Geometry used in source calculation. (b) Contour plot of the source strength in the circular plasma cross-section for a shift parameter $\varepsilon = 0.5$.

Fig. 4. Normalized plasma source strength as a function of position in the circular plasma cross-section for the three source distributions.
Fig. 5. Circular cross-section torus neutron flux for the shifted source. (a) Contour plot of the angular flux $F(\mu, \phi)$ at $\chi = 90^\circ$. (b) Azimuthal flux $F(\mu)$ (arbitrary scale) vs. $\chi$ and $\mu = \cos \theta$; $\theta =$ angle relative to wall normal.

Fig. 6. Neutron scalar flux and current vs. wall position in the circular cross-section torus. The percent values are the difference between the minimum and maximum flux or current for each source.
Fig. 7. Neutron wall load peaking factors vs. wall position in the circular cross-section torus.

Fig. 8. Neutron scalar flux and current vs. the shifted source parameter for the outside, top, and inside wall points of the circular cross-section torus.
Fig. 9. Cross-section of the Princeton Reference Design (PRD) tokamak fusion reactor (from Ref. 2).

Fig. 10. Contour plots of the angular flux $F(\mu, \theta)$ at four wall points of the PRD. (a) $R = 14.6$ m (outside). (b) $R = 11.0$ m. (c) $R = 5.0$ m (inside). (d) $R = 4.5$ m (divertor). The circle radii are spaced every 15° in $\theta$ and correspond to $1 - \mu = 1 - \cos \theta$. 
Fig. 11. Azimuthal flux $F(\mu)$ at six wall points of the PRD.

Fig. 12. Neutron scalar flux and current (wall load) vs. wall position in the PRD.
Fig. 13. Cross-section of the University of Wisconsin UWMAK-1 tokamak fusion reactor design (reproduced from Ref. 3).

Fig. 14. Contour plots of the angular flux $F(\mu, \theta)$ at four wall points of the UWMAK. (a) $R = 18.86$ m (outside). (b) $R = 14.8$ m. (c) $R = 9.4$ m (inside). (d) $R = 9.4$ m (divertor). The circle radii are spaced every 15° in $\theta$ and correspond to $1 - \mu = 1 - \cos \theta$. 
Fig. 15. Azimuthal flux $F(\mu)$ at seven wall points of the UWMAK.

Fig. 16. Neutron scalar flux and current (wall load) vs. position along the main wall (facing the plasma) of the UWMAK.
Fig. 17. Neutron scalar flux and current (wall load) vs. position along the lower face of the vacuum port shield and along the divertor collector plate wall in the UMWAK.

Fig. 10. Projections of three neutron streaming paths in two different planes of the UMWAK.