Impact Plate Test Problem

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TP106
Impact Plate Test Problem

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1. Problem Definition

A flyer plate composed of 2024 Aluminum is moving at a fixed velocity \( U_0 = 0.180 \text{ cm/µs} \) towards a target composed of Tantalum. The Tantalum target is at rest \( (v=0 \text{ cm/µs}) \).

The key objective of the proposed test problem (TP) is to verify Staggered Grid Hydrocodes (SGH) for metal impact with elastic collision and evaluate slide treatment as well as advection schemes. Figure 1 shows a schematic of the suggested problem configuration.

The proposed test problem will compute and compare the following quantities:
1. the shock pressure generated in the two materials at the interface due to impact,
2. the shock speed moving in each material

Objective is to perform hydro computations of the flyer plate impact until \( t = 3.00 \mu s \).

The computational domain is defined as follows:
(i) \( x\)-direction: \(-1.025, +1.025 \text{ cm} \)
(ii) \( y\)-direction: \(0.0, 0.025 \text{ cm} \)
(iii) \( z\)-direction: \(0.0, 0.025 \text{ cm} \) (for 3-D)

Aluminum flyer plate extends from \([-1.025,-0.025]\text{cm}\) and Tantalum plate extends from \([0.025,+1.025]\text{cm}\). The initial gap separating the plates extends over a distance of 0.05cm. This prevents the materials from interacting at the start of the simulation. The initial mesh resolution consists of 40 zones in each material. Eulerian/ALE simulations will have a void material in the gap separating the plates; while Lagrangian computations will use slide surfaces.

The plates collide at the simulation time of \( t = 0.05 \text{ cm/0.18 cm/µs} = 0.277778\mu s \).

The following boundary conditions are imposed:
(a) Slide surfaces at material interface in \( x \) for Lagrangian mode, symmetry planes at \( x=-1.025\text{cm} \) and \( +1.025\text{cm} \)
(b) Symmetry planes at \( y = 0 \) & \( 0.025\text{cm} \)
(c) Symmetry planes at \( z = 0 \) & \( 0.025 \text{ cm} \) (for 3-D)
The initial conditions are imposed as follows:

1. Density: $\rho_1 = 2.785$ g/cc; $\rho_2 = 16.654$ g/cc
2. Velocity Components: $u_1 = 0.18$ cm/µs; $u_2 = v_1 = v_2 = w_1 = w_2 = 0.0$
3. Energy: $e_1 = e_2 = 0$

The masses for each plate are: $M_{Al} = 1.740625$ mg; $M_{Ta} = 10.403125$ mg. These values will be used to verify that the initial masses are properly set.

1.1 EOS Model:
The material EOS is based on a Gruneisen model with the following values

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho_0$ (g/cm$^3$)</th>
<th>$c_0$ (cm/µs)</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024 Aluminum</td>
<td>2.785</td>
<td>0.5328</td>
<td>1.338</td>
</tr>
<tr>
<td>Tantalum</td>
<td>16.654</td>
<td>0.3414</td>
<td>1.201</td>
</tr>
</tbody>
</table>

Hence, the Gruneisen EOS model reduces to: $P = \rho_0 c_0^2 \mu/[1 - (s - 1) \mu]$.

1.2 Strength Model
The material strength model is based on the modified Steinberg-Cochran-Guinan (SG) model. This form includes strain, pressure and thermal softening terms. It is modified from the full Steinberg-Cochran-Guinan model (D.J. Steinberg, S.G. Cochran, and M.W. Guinan, “A Constitutive Model for Metals Applicable at High-Strain Rate”, J. Appl. Phys. 51(3), 1498 (1980)) in that the thermal softening is expressed in terms of a constant melt energy threshold.

This form does not require that temperatures be obtained from the SESAME EOS and appears to do fairly well in comparison to the full model given next below. The yield strength and shear modulus are calculated as follows:

$$Y = \min\left(Y_{Max} Y_0 \left[1 + \alpha \left(\varepsilon_p^0 + \varepsilon_p^p\right)\right]^{\gamma} \left[1 + \gamma P \left(\frac{\rho}{\rho_0}\right)^{1/3} \frac{E}{(E_m - E)}\right]^{1/3} \left(\frac{E}{E_m - E}\right)\right]$$

$$G = \min\left(G_{Max}, G_0 \left[1 + \gamma' P \left(\frac{\rho}{\rho_0}\right)^{1/3} \frac{E}{(E_m - E)}\right]^{1/3} \left(\frac{E}{E_m - E}\right)\right]$$

where $\varepsilon_p$ is the time-integrated equivalent plastic strain calculated by the code. This form requires both the initial and maximum yield strength and shear modulus. A von Mises yield criterion is used - that results in a "radial return" to the yield surface. Here, $Y_0$ and $G_0$ are the initial values for the yield strength and shear modulus; while $Y_{Max}$ and $G_{Max}$ are the maximum permitted values as reflected in the equations above. The units for $Y$ and $G$ are in Mbar.

In addition, this SG form requires a list of constants that appear in the equations above as follows: $\text{strcon}=\alpha, \varepsilon_p^0, \beta, \delta, E_m, \gamma, \gamma'$; where

$\alpha =$ strain-hardening coefficient,
$\varepsilon_p^0 =$ prestrain for wrought materials,
\[ \beta = \text{strain-hardening exponent, thermal-softening coefficient,} \]
\[ E_m = \text{melt energy (Mbar cm}^3/\text{g),} \]
\[ \gamma = \text{pressure-hardening coefficient for yield function (Mbar}^{-1}) \]
\[ \gamma' = \text{pressure-hardening coefficient for shear modulus (Mbar}^{-1}) \]

The following values for S-G strength model should be used for the two materials considered:

**2024 Al:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{strcon} )</td>
<td>310.0, 0.0, 0.185, 0.001, 0.00796, 6.52, 6.52,</td>
</tr>
<tr>
<td>(y_0)</td>
<td>0.0026,</td>
</tr>
<tr>
<td>(y_{max})</td>
<td>0.0076,</td>
</tr>
<tr>
<td>(g_0)</td>
<td>0.2860,</td>
</tr>
<tr>
<td>(g_{max})</td>
<td>0.6470</td>
</tr>
</tbody>
</table>

**Tantalum**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{strcon} )</td>
<td>10.0, 0.0, 0.1, 0.001, 0.00546, 1.45, 1.45,</td>
</tr>
<tr>
<td>(y_0)</td>
<td>0.0077,</td>
</tr>
<tr>
<td>(y_{max})</td>
<td>0.0110,</td>
</tr>
<tr>
<td>(g_0)</td>
<td>0.6900,</td>
</tr>
<tr>
<td>(g_{max})</td>
<td>0.9860</td>
</tr>
</tbody>
</table>

### 2. Analytical Solution

The collision of the two plates will form a shock in the Tantalum whose Hugoniot is:

\[
P = \rho_0 \ c_0 \ u + \rho_0 \ s \ u^2
\]

\[
= 5.6857 \ u + 20.0015 \ u^2
\]

The collision will also form a shock in the Aluminum whose Hugoniot is:

\[
P = \rho_0 \ c_0 \ (U_0 - u) + \rho_0 \ s \ (U_0 - u)^2
\]

\[
= 1.4838 \ (0.180 - u) + 3.7263 \ (0.180 - u)^2
\]

\[
= 0.38782 - 2.8253 \ u + 3.7263 \ u^2
\]

The pressures and particle velocities are the same at the materials interface; the two Hugoniot equations can be equated. This leads to:

\[
16.2751 \ u^2 + 8.5110 \ u - 0.38782 = 0
\]

Solving the quadratic yields one positive root of the particle velocity:

\[
u = 0.04217 \ \text{cm} / \ \mu\text{s}
\]

The pressure can be directly computed from either Hugoniot with the following value:

\[
P_1 = 0.2753 \ \text{Mbar}
\]
The shock velocities in the Aluminum and Tantalum are then obtained:

\[
U_{\text{Aluminum}} = c_0 + s u = 0.5892 \text{ cm} / \mu\text{s}
\]

\[
U_{\text{Tantalum}} = c_0 + s u = 0.3920 \text{ cm} / \mu\text{s}
\]

The two shocks will be moving in opposite directions. It should be noted that the analytical solution does not account for any material strength.

3. Proposed Simulations and Analysis:

Perform Hydro calculations for the geometry described in Section 1 and compute the Pressure as well as the shock velocities.

Six Lagrangian tracer (marker) particles will be defined in the simulation. Tracers 1-3 are placed in the Aluminum flyer plate. Tracers 1 and 2 are near the head of the flyer and tracer 3 is located near the tail of the flyer plate. Tracers 4-6 are placed in the Tantalum target in a similar manner. The 6 Lagrangian tracers’ coordinates are set as follows (x,y,z values in cm):

1:  -0.0375,  0.0125,  0.0125
2:  -0.0625,  0.0125,  0.0125
3:  -0.9875,  0.0125,  0.0125
4:   0.0375,  0.0125,  0.0125
5:   0.0625,  0.0125,  0.0125
6:   0.9875,  0.0125,  0.0125

Perform Hydro computations and the following analysis is proposed—one task is required and the others are optional:

**REQUIRED TASK**

R.1 Perform a mesh resolution analysis with 82x2x2, 164x4x4, 328x8x8 and 656x16x16 zones in 3-D. The timestep size is based on a CFL condition of 0.25. Use your “favorite” Advection Scheme: Lagrange/Eulerian/ALE motions. For Lagrange, a slide surface treatment is required. For Eulerian/ALE, void material should fill the region between the plates. Invoke monotonic Q for artificial viscosity model.

- Use the Lagrangian tracers to determine the pressure and shock velocities. Compare to the analytical values.
- Tabulate the L1 & L2 errors of the Pressure and Shock Velocity for the Lagrange tracers in each material.

**OPTIONAL TASKS**

O.1 Perform the 3-D computations for the plates rotated by 45 degrees. For ALE/Eulerian calculations, the blocks are “shaped” in and the background mesh is based on a larger domain with mesh resolution commensurate with the ones in Step R.1.
0.2 Perform the calculations for 2-D planar configuration with horizontal impacting plates using a mesh resolution of 82x2, 164x4, 328x8 and 656x16 zones.

0.3 Repeat Step 0.1 with 2-D computations for the plates rotated by 45 degrees.

0.4 Perform a timestep resolution analysis for 82x2x2 mesh with $\Delta t = 10^{-3}$ and $10^{-4}$ µs.

**4. Typical Results with ALE3D & PAGOSA**

ALE3D computations are performed on the TP problem and the results obtained from Task R1 are summarized below in Figures 3 and 4.

Results obtained using PAGOSA are shown in Figures 5 and 6.
Figure 1. Problem set-up for flyer plate impact on a stationary target. The flyer is composed of 2024 Aluminum and the target is made of Tantalum.
Figure 2. x-t diagram for flyer plate impact on a stationary target.
Figure 3. Time variation of pressure signals for two tracers using ALE3D with several mesh resolutions.
Figure 4. Close-up view of the pressure signals using ALE3D.
Figure 5. Time variation of pressure signals for two tracers using PAGOSA for a mesh of 40 zones.
Figure 6. Close-up view of the pressure for the two tracers using PAGOSA. The analytical value is 0.2753Mbar.