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Survey of the Radiation Levels
in the Containment Vessel
of the Enrico Fermi Atomic Power Plant

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Part V - Gamma Radiation Levels on the Operating Floor of the Containment Building.
a. Levels Above the Equipment Compartment

Summary
This report, Part V-a of Technical Memorandum No. 16, presents the results of a survey of calculated gamma-ray levels at many points on the surface of the operating floor of the containment building for the Enrico Fermi reactor. That portion of the floor surveyed lies directly above the equipment compartment. The calculations were made with the aid of an IBM -650 electronic computer.

The main source of radioactivity which gives rise to gamma radiation above the floor is the radioactive sodium-24 in the primary coolant system. This system was considered to be completely filled with sodium, and activated to an equilibrium activity of 0.05 curies/ce, which corresponds to infinite reactor operation at 500 megawatts power. No fission product contamination was considered for these calculations. The operating floor is 5 feet thick and of concrete and steel, as shown in Figure 1.

The results of the survey, presented in Table II, indicate that above the equipment compartment the surface dose on the operating floor will in no case exceed $0.9 \mathrm{mr} / \mathrm{hr}$ at the expected full operating power of 430 megawatts.

Included as appendices are derivations and methods of corrections': from one set of concrete and steel thicknesses to another.


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Approved By:
H. E. Hüngerford

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Survey of the Radiation Levels in the Containment Vessel of the Enrico Fermi Atomic Power Plant

Being Issued:
Part V - Gamma Radiation Level in the Operating Area of the Containment Building
a. Levels above the Equipment Compartment

Issued September 4, 1958:
Part I - Gamma Radiation Levels in the Equipment Compartment Due to Primary Coolant Activity and Associated Fission Product Contamination

To Be Used at a Future Date:
Part II - Gamma Radiation Levels in the Equipment Compartment Due to the Storage of Various Radioactive Pieces of Equipment

Part III - Gamma Radiation Levels in the Reactor Compartment
Part IV - Neutron Radiation Levels in the Reactor Compartment During Plant Operation

Part Vb - Gamma Radiation Levels in the Operating Floor Above the Reactor Compartment

Part VI - An Estimate of Neutron and Gamma Streaming into the Operating Area of the Containment Building.
I. Description and Results ..... 5
A. Design Basis ..... 5
B. Purpose of this Survey ..... 7
C. Description of Survey Program ..... 7
D. Conditions of the System ..... 10
E. Results of the Study ..... 10
II. Calculational Bases and Methods ..... 10
A. General Description ..... 10
B. Orientation of Sources and Detection Points ..... 17
C. Total Dose at Point P ..... 17
APPENDICES
Appendix A - Raw Data - Machine Calculation Results ..... 23
Appendix B - Corrections to Other Thickness ..... 25
Appendix C - List of Symbols ..... 27
Appendix D - Derivation of the Dose at Point P Directly Above ..... 32
a Vertical Line Source
Appendix E - Derivation of the Dose at Point $P$ at a Skew Angle ..... 35
Above a Vertical Line Source
Appendix F - Derivation of the Dose at Point P Above a Horizontal ..... 39
Line Source - Case I
Appendix G - Derivation of the Dose at Point P Above a Horizontal ..... 43 Line Source - Case II
Appendix H - Derivation of the Dose at Point P Above a Horizontal ..... 46 Line Source - Case III

## LIST OF TABLES

Page No.8
Table I - Radiation Sources in Equipment Compartment
Table II - Radiation Levels on the Surface of the Operating ..... 11-16 Floor Directly Above the Equipment Compartment
Table A-1 - Raw-Data, Uncorrected Results of Machine Calculations Radiation Levels on Operating Floor for a Steel Thick- ness of 6-3/4 Inches and a Concrete Thickness of 53-1/4 Inches ..... 24
LIST OF FIGURES

1. Plan View of Operating Floor, Showing Steel Thicknesses ..... 6
2. Grid of Equipment Compartment on Operating Floor ..... 10
3. Vertical Source, Configuration 1 ..... 18
4. Vertical Source, Configuration 2 ..... 19
5. Horizontal Source, Configuration 1 ..... 20
6. Horizontal Source, Configuration 2 ..... 21
7. Horizontal Source, Configuration 3 ..... 22

## I. Description and Results

A. Design Basis

The operating floor shield of the containment building for the Enrico Fermi reactor has been designed so as to allow continuous access to the building at all times during full-power operation of the reactor. The maximum weekly external radiation exposure to which plant personnel working in unregulated areas will be subjected is 0.03 rem on the basis of a 40 -hour week. Translated into a design criterion, the maximum permissible level due to radiation leaking through the operating floor of the containment building has been set at a rate of 0.75 millirem $/ \mathrm{hr}$.

The shield floor consists of a total thickness of 5 Ceet in most areas, consisting of from $3-1 / 2$ inches to 10 inches of steel plate on the underside of the shield, with the remaining thickness being concrete having a dry density of $150 \mathrm{lb} / \mathrm{ft}^{3}$ (1).

Serpentine concrete
has been chosen for the high-temperature areas which will exist around the access plugs in the floor for the primary coolant heat exchangers and primary sodium pumps. This concrete, developed for PRDC by the Toledo Tésting Laboratory, has a density of $130 \mathrm{lb} / \mathrm{ft}^{3}$ and a compressive strength of around 2500 psi. The serpentine rock aggregate, actually a type of asbestos ore, has the ability to hold its hydrated water to temperatures as high as 900 F , and therefore to keep its neutron shielding effectiveness to these temperatures. It should be noted that in the serpentine concrete areas; the steel thicknesses required are greater than in other areas because of the low density of serpentine concrete.

The present report is concerned with the radiation levels at the surface of that portion of the operating floor above the equipment compartment. (3), (4)
(1) At the time that the design of this operating floor shield was set, the most reliable information which was available indicated that $150 \mathrm{lb} / \mathrm{ft}^{3}$ concrete could be easily obtained by use of a particularly heavy grade of local trap rock. It has not been established that this density is possible to achieve consistently in large quantities with this aggregate. Some magnetite ore will be added to achieve this density.

A description of this concrete is given in an article entitled "New Shielding Material for High Temperature Application" by H. E. Hungerford, R. F. Mantey, and R. Van Maele, Nuclear Science and Engineering, November 1959.

Section $V \not b$; Radiation Levels on Operating Floor Above the Reactor Compartment, will be issued at a future date.
(4) The secondary shield wall (Figure 1) divides the lower building area into an inner reactor compartment and an outer equipment compartment.

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The maximum level of the neutron flux which is expected to be incident on the underside of the operating floor in the equipment compartment is less than $1 \times 10^{4} \mathrm{n} / \mathrm{cm}^{2}$-sec (5). Part I of the survey (6) indicated that the highest $\gamma$-ray flux on the underside of the operating floor due to radioactive sodium-24 is of the order of $5 \times 10^{4} \mathrm{r} / \mathrm{hr}$. Fission product impurities present to the extent taken in Case IV of Part I (7) could raise this level by as mach as a factor of 10 at any given point.

The steel and concrete thicknesses were determined by means of detailed hand calculations which considered separately and collectively all the important radiation sources which contribute radiation to a given area of the floor above the shield. With the aid of the results of these calculations, the steel requirements are satisfied in various areas by the use of from $3-1 / 2 \mathrm{ft}$ to 4 ft of concrete. The remainder of the concrete thickness and the steel thickness requirements then are necessary because of the nature and fintensity of the gamma radiation below the floor and the maximum design dose rate permitted above the operating floor.

## B. Purpose of this Survey

This survey was undertaken as a check on the previous hand calculations and to detect any hidden'weaknesses which may exist in the floor as designed. Since the floor is a relatively large area, there could conceivably exist points not covered by the hand calculations.
C. Description of Survey Program

The plan of the survey follows closely that outlined in Part I. The gamma radiation penetrating the operating floor comes primarily from the radioactive sodium-24 in the primary coolant loops.

The sources listed in Table I are the same as those used for Part I. A grid consisting of radial lines and concentric circles was overlaid on a plan view of the equipment compartment as shown in Figure 2 . Since it was previously established that the total thickness of the shield was determined only by the gamma radiation levels existing below the floor, no survey on neutron levels above the operating floor has been made. (8)
(5)

This value is a mid-compartment figure. Levels at the ceiling of the lower compartment will be slightly less than this.
(6)

Part I was issued Sept ember 4, 1958. See Chart 14 of Part I for expected fullpower radiation levels at the underside of the operating floor.
(7)
i.e., a contamination level of 0.025 curies/cc (Chari II of Part I).
(8)

The only sources of neutrons above the operating floor are possible streaming paths. Possible neutron levels from these paths will be presented in Part VI. The statement of the above paragraph refers only to the shield above the equipment compartment.

## TABLE I

## Individual Sources

In Equipment Compartment

|  | Sources | Horizontal |  |  | Vertical |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Loop I | Loop II | Loop III | Loop I | Loop II | Loop III |
| 1. | 6-inch pipe | 1 | 1. | 3 | 1 | 1 | 1 |
| 2. | 14-inch pipe | 2 | 2 | 2 | 1 | 1 | 1 |
| 3. | 16-inch pipe | 1 | 0 | 0 | 0 | 0 | 0 |
| 4. | 30-inch pipe | 3 | 3 | 3 | 1 | 1 | 1 |
| 5. | Sodium Pump | 0 | 0 | 0 | 1 | 1 | 1 |
| 6. | IHX | 0 | 0 | 0 | 1 | 1 | 1 |
| 7. | Sodium overflow tank |  |  | rtical sour | e only |  |  |


| Total Horizontal Sources | - | 21 |
| :--- | :--- | :--- |
| Total Vertical Sources | - | 16 |
| Total Sources | - | 37 |

## Grid of Operating Floor Above Equipment Compartment



Figure 2
$-9-62300$

Calculations of the dose rate to be expected at the surface of the operating floor were made at each of the more than 200 grid points designated by the intersection of the radial lines and the concentric circles. A program was developed for running these calculations on the IBM-650 electronic calculator, using the formulas developed in section II and in the appendices to this report.

The machine calculations were run using a single steel thickness of 6-3/4 inches -- the largestsingle thickness of steel used (See Figure l) in the operating floor shield. The corresponding concrete thickness is $531 / 4$ inches. After an inspection of the results, the dose rate values for all points over areas of different concrete and steel thicknesses were corrected to their proper thicknesses, by hand calculation, using the ratio formulas developed in Appendix B.

## D. Condition of the System

The sodium primary coolant system at operating power of 500 megawatts (9) will contain radioactive sodium calculated to have a specific activity of 0.05 curies/cc. The dominant gamma-ray which contributes almost all the dose has an energy of 2.76 mev . No fission product contamination was assumed in these calculations, because the proportion of fission products which may exist. in the quantities assumed in Part I of this report, and which have 8 -ray energies above that of the sodium gamma-ray is negligibly small.
E. Results of the Study

The results of this set of calculations are presented in Table II, which show the expected gamma radiation levels at the surface of the operating floor at a reactor operating power level of 500 megawatts.

This survey shows that, for the most part, the radiation levels are within the design limits. It is, of course, impossible to design so that every point will be exactly at the design limit. Effort is made to keep the point of highest radiation level at the design limit in each area of the floor. This means that other nearby points will fall somewhat below the limit.

The machine calculations indicated a few points that were slightly over the design limit of $0.75 \mathrm{mrem} / \mathrm{hr}$ at the 500 megawatt shield design power of the reactor. When correction is made to the correct reactor operating powers of 300 mw and 430 mw , respectively, most of the points fall below the design limit.

It may be concluded, as a result of this study, the operating floor shield will perform as designed and that the radiation levels will at no point be more than $15 \%$ greater than the design limits.
II. Calculational Bases and Methods
A. General Description

All the calculations weremade assuming line sources. The same line sources were used as those for Part I of the survey. Each piece of equipment, and horizontal and vertical run of coolant piping, was treated as a separate source (see Table I). Pipe elbows were divided appropriately between the horizontal and the vertical runs.
(9) First full-power operation will be 300 megawatts. Later this will be raised to 430 mw . Use of the 500 -megawatt figure for design purposes allows a safety factor of about $15 \%$ 。

## TABLE II

RADIATION LEVELS ON THE SURFACE OF THE OPERATING FLOOR ABOVE THE EQUIPMENT COMPARTMENT

| Floor Thickness (inches) |  |  |  |  | Dose Rate, $\mathrm{mr} / \mathrm{hr}$ at 500 Mw |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Steel | Concrete | Serpentine |  |
| $0^{\circ}$ | A | 5.75 | 54.25 |  | 4.86 (-1)* |
|  | B | 5.75 | 54.25 |  | 4.38 (-1) |
|  | C | 5.75 | 54.25 |  | 2.60 (-1) |
|  | D | 3.50 | 80.50 |  | 2.66 (-3) |
|  | E | 3.50 | 80.50 |  | 2.66 (-3) |
|  | G | 3.50 | 80.50 |  | 5.10 (-3) |
|  | H | 3.50 | 80.50 |  | 6.09 (-3) |
| $10^{\circ}$ | A | 5.75 | 54.25 |  | 2.96 (-1) |
|  | B | 5.75 | 54.25 |  | 3.92 (-1) |
|  | C | 3.50 | 80.50 |  | 8.17 (-3) |
|  | D | 3.50 | 80.50 |  | 8.64 (-3) |
|  | E | 3.50 | 80.50 |  | 8.64 (-3) |
|  | G | 3.50 | 80.50 |  | 8.64 (-3) |
|  | H | 3.50 | 80.50 |  | 8.64 (-3) |
| $20^{\circ}$ | A | 3.50 | 80.50 |  | $2.20(-3)$ |
|  | B | 3.50 | 80.50 |  | 3.31 (-3) |
|  | C | 3.50 | 80.50 |  | 4.49 (-3) |
|  | D | 3.50 | 80.50 |  | 5.67 (-3) |
|  | E | 3.50 | 80.50 |  | 6.20 (-3) |
|  | G | 3.50 | 80.50 |  | 6.42 (-3) |
|  | H | 3.50 | 80.50 |  | 6.37 (-3) |
| $30^{\circ}$ | A | 3.50 | 80.50 |  | 1.01 (-3) |
|  | B | 3.50 | 80.50 |  | 1.65 (-3) |
|  | C | 3.50 | 80.50 |  | 2.43 (-3) |
|  | D | 3.50 | 56.50 |  | 3.13 (-3) |
|  | G | 3.50 | 80.50 |  | 2.97 (-3) |
|  | H | 3.50 | 80.50 |  | 3.15 (-3) |
| $40^{\circ}$ | A | 5.75 | 54.25 |  | 7.30 (-2) |
|  | B | 5.25 | 54.75 |  | 1.53 (-1) |
|  | C | 5.25 | 54.75 |  | 2.34 (-1) |
|  | D | 5.25 | 54.75 |  | 2.92 (-1) |
|  | G | 5.25 | 54.75 |  | 3.03 (-1) |
|  | H | 5.25 | 54.75 |  | 1.25 (-1) |
| $50^{\circ}$ | A | 5.25 | 54.75 |  | 2.31 (-1) |
|  | B | 5.25 | 54.75 |  | 3.17 (-1) |
|  | C | 6.75 | 53.25 |  | 1.81 (-1) |
|  | D | 6.75 | 53.25 |  | 1.80 (-1) |
|  | G | 6.75 | 53.25 |  | 2.06 (-1) |
|  | H | 6.75 | 53.25 |  | 2.22 (-1) |

* The figures in parenthesis indicate the power of ten by which the numbers to the left are to be multiplied.

[^0]| Floor Thickness (inches) |  |  |  |  | $\begin{aligned} & \text { Dose Rate, } \mathrm{mr} / \mathrm{hr} \\ & \text { at } 500 \mathrm{Mw} \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ |  | Steel | Concrete | Serpentine |  |
| $60^{\circ}$ | A | 5.25 | 54.25 |  | $4.08(-1) *$ |
|  | B | 5.25 | $54.25$ |  | 3.10 (-1) |
|  | C | 10.00 |  | 40.00 | 2.97 (-2) |
|  | D | 10.00 |  | 40.00 | 4.31 (-2) |
|  | E | 6.75 | 53.25 |  | 2.09 (-1) |
|  | G | 6.75 | 53.25 |  | 2.64 (-1) |
|  | H | 6.75 | 53.25 |  | 2.90 (-1) |
| $70^{\circ}$ | A | 5.25 | 54.75 |  | 5.97 (-1) |
|  | B | 5.25 | 54.75 |  | 4.36 (-1) |
|  | C | 11.00 |  | 58.00 | 4.75 (-1) |
|  | D | 11.00 |  | 58.00 | 4.75 (-1) |
|  | E | 10.00 |  | 40.00 | 3.04 (-1) |
|  | G | 6.75 | 53.25 |  | 4.44 (-1) |
|  | H | 6.75 | 53.25 |  | 4.66 (-1) |
| $80^{\circ}$ | A | 5.75 | 54.75 |  | 8.01 (-1) |
|  | B | 5.25 | 54.75 |  | 5.65 (-1) |
|  | C | 10.00 |  | 40.00 | 2.18 (-1) |
|  | D | 10.00 |  | 40.00 | 1.02 (-1) |
|  | E | 10.00 |  | 40.00 | 4.12 (-1) |
|  | F | 10.00 |  | 40.00 | 5.16 (-1) |
|  | G | 6.75 | 53.25 |  | 6.42 (-1) |
|  | H | 6.75 | 53.25 | . | 6.18 (-1) |
| $90^{\circ}$ | A | 5.25 | 54.75 |  |  |
|  | B | 10.00 |  | 40.00 | 1.95 (-1) |
|  | C | 19.50 |  | 51.50 | 3.96 (-3) |
|  | D | 19.50 |  | 51.50 | 3.96 (-3) |
|  | E | 10.00 |  | 40.00 | 7.43 (-1) |
|  | F | 10.00 |  | 40.00 | 7.06 (-1) |
|  | G | 6.75 | 53.25 |  | 5.72 (-1) |
|  | H | 6.75 | 53.25 |  | 5.42 (-1) |
| $100^{\circ}$ | A | 5.25 | 54.75 |  | 4.63 (-1) |
|  | B | 5.25 | 54.75 |  | 2.24 (-1) |
|  | C | 10.00 |  | 40.00 | 1.33 (-1) |
|  | D | 10.00 |  | 40.00 | 2.45 (-1) |
|  | E | 10.00 |  | 40.00 | 6.92 (-1) |
|  | F | 7.25 | 52.75 |  | 6.79 (-1) |
|  | G | 7.25 | 52.75 |  | 3.03 (-1) |
|  | H | 7.25 | 52.75 |  | 3.48 (-1) |
| $110^{\circ}$ | A | 5.25 | 54.75 |  | 6.37 (-1) |
|  | B | 5.25 | 54.75 |  | 8.11 (-1) |
|  | C | 6.75 | 53.25 |  | 4.01 (-1) |
|  | D | 6.75 | 53.25 |  | 4.44 (-1) |
|  | E | 7.25 | 52.75 |  | 5.05 (-1) |
|  | F | 7.25 | 52.75 |  | 6.37 (-1) |
|  | $\stackrel{\text { G }}{ }$ | 7.25 | 52.75 |  | 5.53 (-1) |
|  | H | 7.25 | 52.75 |  | 5.42 (-1) |

TABLE II (Continued)
Floor Thickness (inches)

| $130^{\circ}$ | A |
| :---: | :---: |
|  | B |
|  | C |
|  | D |
|  | E |
|  | F |
|  | G |
|  | H |


| Steel |
| ---: |
| 5.25 |
| 5.25 |
| 10.00 |
| 10.00 |
| 10.00 |
| 6.75 |
| 6.75 |
| 6.75 |


| Concrete | Serpentine |
| :---: | :---: |
| 54.75 |  |
| 54.75 |  |
|  |  |
|  | 40.00 |
|  | 40.00 |

A $\quad 5.25$
B $\quad 10.00$
19.50
19.50
10.00
10.00
6.75
6.75
53.25
53.25
53.25
54.75
40.00
40.00
40.00
40.00
40.00
53.25
53.25
54.75
40.00
51.50
51.50
40.00
40.00

150
A
B
6.75
6.75
10.00
10.00
10.00
6.75
6.75
6.75
53.25
53.25
40.00
40.00
40.00
$160^{\circ}$
A
B
C
D
E
G
H
B
E
G
H
6.75
6.75
10.00
11.00
10.00
6.75
6.75
40.00
58.00
53.25
53.25

A
B
C
D
G
H
6.25
6.25
6.25
6.25
6.25
6.25
53.75
53.75
53.75
53.75
53.75
53.75

Floor Thickness (inches)


- 14 -

TABLE II (Continued)
Floor Thickness (inches)


TABLE II (Continued)
 B C D E G

| $310^{\circ}$ | A |
| :---: | :---: |
|  | B C |
|  | D |
|  | G |
|  | H |
| $320^{\circ}$ | A |
|  | B |
|  | C |
|  | D |
|  | G |
|  | H |


| $330^{\circ}$ | $A$ |
| :--- | :--- |
|  | B |
|  | C |
|  | D |
|  | G |
|  | $H$ |


| $340^{\circ}$ |  | A | 5.75 | 54.25 |
| :--- | :--- | :--- | :--- | :--- |
|  | B | 5.75 | 54.25 |  |
|  | C | 3.50 |  | 80.50 |
|  | D | 3.50 |  | 80.50 |
|  | E | 3.50 |  | 80.50 |
|  | G | 3.50 | 80.50 |  |
|  | H | 3.50 |  | 80.50 |
| $350^{\circ}$ |  |  |  |  |
|  | A | 5.75 | 54.25 |  |
|  | B | 5.75 |  |  |
|  | C | 3.50 | 80.50 |  |
|  | D | 3.50 |  | 80.50 |
|  | E | 3.50 | 80.50 |  |
|  | G | 3.50 | 80.50 |  |
|  | H | 3.50 | 80.50 | $* * *$ |

* The figures in parenthesis indicate the power of ten by which the numbers to the left are to be multiplied.
- 16 -

Line source calculations were used rather than volume source calculations (10) for the reason that programming for the IBM- 650 calculating machine is simplified by the use of line sources.
B. Orientation of Source and Detection Points

The calculations break down into five different types according to the orientation of the line source with respect to the shield and with respect to the point on the upper surface of the shield. Since the shield is the operating floor, its orientation is always horizontal. The five orientations of the source with respect to the point of detection are shown in Figure 3 through 7. In the first two, the line source is vertical, and in the latter three the line source is horizontal. The dose at $P$ for each of these orientations must be calculated by means of a separate formula. Equations for the dose at any point on the operating floor for any given orientation are developed in Appendices D through H .

Case 1-Vertical line source. Detection point P lies directly above (or within $5^{\circ}$ of being directly above) the line source. (Figure 3)

Case 2 - Vertical line source. Detection point $P$ lies anywhere except directly above line source. (Figure 4)

Case 3 - Horizontal line source. A perpendịcular line drawn from the detection point $P$ to the source intersects the source at some point between the ends. (Figure 5)

Case 4 - Horizontal line source., A perpendicular line drawn from $P$ intersects the source only at one end. (Figure 6)

Case 5 - Horizontal line source. A perpendicular line drawn from $P$ does not intersect the source but intersects an extension of it. (Figure 7)
C. Total Dose at Point P

The machine calculations are so set up that at every point the dose rates due to each separate source are added together to give the total dose rate at the point. The results of the machined calculations shown in Appendix A are essentially only semi-processed data, and must be modified according to the pattern of correct steel and concrete thicknesses to obtain the true total expected dose rates. These modifications were made by hand calculations using the formulas given in Appendix B.

In actuality, the volume source formula such as given in Rockwell, "Reactor Shielding Design Manual," TID-7004, are line sources with a built-in mechanism for taking into account build-up and self-absorption in the source material.


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Figure 6

HORIZONTAL SOURCE, - CONFIGURATION 2


## Appendix A

## RAW DATA - THE MACHINE CALCULATION RESULTS

The results of the machine calculations are shown in Table A-l. These values are for a 5 -ft shield floor which consists entirely of $6-3 / 4$ inches of steel and 53-1/4 inches of concrete. (This is the largest single thickness configuration of concrete and steel used). Corrections to these values which are reported in Table II were obtained by application of the formulas shown in Appendix B.

Dose rates in $\mathrm{mr} / \mathrm{hr}$ on the operating floor of the reactor building (corresponding to the grid points as shown in Fig. 2) from $\mathrm{Na}^{24}$ at equilibrium activity, system full, with the reactor operating at an assumed 500 Mw .

$$
\begin{gathered}
\text { Conditions: } \begin{array}{c}
\text { Concrete thickness }-53-1 / 4 \text { inches } \\
\text { Steel thickness }
\end{array} \text { - } 6-3 / 4 \text { inches }
\end{gathered}
$$

| Degrees | A | B | c | D | E | F | G | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $6.07(-1)$ | 5.46(-1) | $3.24(-1)$ | 1.88(-1) | 88(-1) |  | 3.61(-1) | $4.31(-1)$ |
| 10 | 3.68 (-1) | 4.90 (-1) | $5.79(-1)$ | $6.11(-1)$ | $6.11(-1)$ |  | $6.11(-1)$ | 6.10(-1) |
| 20 | $1.54(-1)$ | 2.34 (-1) | 3.18(-1) | 4.02(-1) | 4.38(-1) |  | $4.55(-1)$ | $4.51(-1)$ |
| 30 | 7.17(-2) | $1.16(-1)$ | 1.72(-1) | $2.22(-1)$ |  |  | $2.11(-1)$ | 2.22(-1) |
| 40 | 9.11(-2) | 1.51 (-1) | 2.31(-1) | 2.88(-1) |  |  | 2.98(-1) | $1.23(-1)$ |
| 50 | $2.27(-1)$ | $3.13(-1)$ | 3.63(-1) | 3.60(01) |  |  | $4.14(-1)$ | $4.46(-1)$ |
| 60 | $4.03(-1)$ | $3.06(-1)$ | 6.59(-2) | 9.64(-2) | 4.20(-1) |  | 5.30(-1) | $5.83(-1)$ |
| 70 | 5.90(-1) | $4.93(-1)$ |  |  | 6.77(-1) |  | $8.91(-1)$ | 9.35(-1) |
| 80 | 7.88(-1) | $5.57(-1)$ | $4.85(-1)$ | 2.26(-1) | $9.19(-1)$ | $1.15(0)$ | $1.29(0)$ | 1.24(0) |
| 90 | 5.06 (-1) | $4.35(-1)$ |  |  | 1.65 (0) | $1.57(0)$ | $1.15(0)$ | $1.09(0)$ $9.02(-1)$ |
| 100 | $4.57(-1)$ $6.26(-1)$ | 2,21(-1) | $2.97(-1)$ $8.04(-1)$ | $5.45(-1)$ $8.90(-1)$ | $1.54(0)$ $1.30(0)$ | $1.75(0)$ $1.65(0)$ | $7.61(-1)$ $1.43(0)$ | $9.02(-1)$ $1.40(0)$ |
| 120 | $6.06(-1)$ | $7.82(-1)$ | 8.68(-1) | $9.57(-1)$ | 1.22 ( 0) | $1.30(0)$ | $1.12(0)$ | 1.01 ( 0) |
| 130 | $4041(-1)$ | $1.75(-1)$ | 2.13(-1) | 4.04(-1) | 1.08 (0) | 1.24 ( 0) | 3.49(-1) | 3.70(-1) |
| 140 | 5.17(-1) | $4.09(-1)$ |  |  | 1.31 ( 0) | $1.30(0)$ | $1.02(0)$ | 9.22(-1) |
| 150 | 8.42(-1) | $6.12(-1)$ | $4.59(-1)$ | 2.14(-1) | 7.37(-1) | 1.07 ( 0) | 1.03 ( 0) | $1.07(0)$ |
| 160 | $5.87(-1)$ | $4.86(-1)$ | $5.84(-3)$ |  | 7.83(-1) |  | $9.35(-1)$ | $9.23(-1)$ |
| 170 | 4.64(-1) | $3.86(-1)$ | 2.20(-1) | $2.144(-1)$ |  |  | $5 \cdot 13(-1)$ | $5.44(-1)$ |
| 180 | 2.96 (-1) | $4.37(-1)$ | 5.30(-1) | $4.94(-1)$ |  |  | $5.61(-1)$ | $4.84(-1)$ |
| 190 | 1.30(-1) | 2.62(-1) | 3.15 (-1) | $3.74(-1)$ |  |  | $5.20(-1)$ | $3.57(-1)$ |
| 200 | $6.13(-2)$ | $7.76(-2)$ | 7.09 (-1) | 1.04(-1) | 4.66 (-1) |  | $4.75(-1)$ | 1.03(-1) |
| 270 | $2.63(-2)$ $1.88(-2)$ | $4.88(-2)$ $3.60(-2)$ | $7.19(-2)$ $6.31(-2)$ | $1.39(-1)$ $1.00(-1)$ | $2.27(-1)$ $1.48(-1)$ | $3.01(-1)$ $1.76(-1)$ | $4.64(-1)$ $1.82(-1)$ | $5.40(-1)$ $1.10(-1)$ |
| 220 | $1.88(-2)$ $5.33(-2)$ | $3.60(-2)$ $9.63(-2)$ | $6.31(-2)$ $1.53(-1)$ | 1.00(-1) | 1.48(-1) | $1.76(-1)$ $3.38(-1)$ | $1.82(-1)$ $3.51(-1)$ | $1.10(-1)$ $1.96(-1)$ |
| 240 | 1.71(-1) | $2.88(-1)$ | $4.08(-1)$ | 5.29)-1) | $5.77(-1)$ | 5.82(-1) | 5.48(-1) | 5.30(-1) |
| 250 | $4.15(-1)$ | $4.97(-1)$ | $4.26(-1)$ | $4.17(-1)$ | $6.62(-1)$ | 7.60(-1) | $5.50(-1)$ | $5.83(-1)$ |
| 260 | $3.82(-1)$ | $8.09(-2)$ | 7.73(-2) | $3.34(-1)$ | 1.14 ( 0 ) | $1.30(0)$ | $8.34(-2)$ | $2.39(-1)$ |
| 270 | 5.06 (-1) | $4.07(-1)$ |  |  |  | 1.40( 0) | 9.72(-1) | $8.30(-1)$ $1.15(0)$ |
| 280 | 8.72(-1) | $6.20(-1)$ | $4.90(-1)$ | 2.77(-1) | $7.62(-1)$ | 9.35(-1) | 1.02( 0 ) | $1.15(0)$ $7.78(-1)$ |
| 290 300 | 6.44 $5.07(-1)$ | $5.45(-1)$ $3.99(-1)$ | $2.21(-1)$ $2.71(-1)$ | $2.800^{\circ}(-1)$ | $5.85(-1)$ $4.51(-1)$ |  | $7.95(-1)$ $5.76(-1)$ | $7.78(-1)$ $4.22(-1)$ |
| 310 | $4.90(-1)$ | 6.90 (-1) | $8.35(-1)$ | $9.04(-1)$ |  |  | 7.69(-1) | $4.68(-1)$ |
| 320 | 6.12(-1) | $7.31(-1)$ | 7.33(-1) | $6.77(-1)$ |  | t | $6.40(-1)$ | $3.77(-1)$ |
| 330 | $5.78(-1)$ | 2.92(-1) | 8.99(-2) | $6.55(-2)$ |  |  | 1.29(-2) | $3.08(-2)$ |
| 340 350 | $3.28(-1)$ $4.27(-1)$ | $\begin{aligned} & 2.03(-2) \\ & 7.36(-2) \end{aligned}$ | $2.85(-4)$ |  | 1.11(-5) |  | $3.77(-4)$ $2.78(-2)$ | $9.85(-3)$ $9.72(-2)$ |

* The figures in parenthesis indicate the power of ten by which the numbers to the left are to be multiplied.


## Appendix B

CORRECTIONS TO OTHER THICKNESSES

Once the dose at each point for a given thickness of each material is known, it is a simple matter to correct these values for the proper (i.e. design) thickness in each location.

There are two (2) cases to consider. The first is where the correct amounts of concrete and steel are simply different from the amounts used in the machine calculations. The other case is where the concrete changes to serpentine concrete or dry -packed aggregate in certain areas.

Case $I$ - Consider the dose rates $D_{1}$ and $D_{2}$ respectively. Let $D_{1}$ be the value of the dose rate resulting from the machine calculations using the fixed thicknesses $t_{l s}$ and $t_{l c} ;$ and let $D_{2}$ be the dose rate desired for thicknesses $t_{2 s}$ and t2c. $\quad$. The subscripts $s$ and $c$ refer to steel and concrete respectively. Let $S$ be the source term, $G$ be the geometric attenuation factor from the source to the detection point, and $B$ be the buildup through the shield. (See Appendix C for symbols).

Then:

$$
\begin{aligned}
& D_{1}=S G\left[B\left(u_{s} t_{1 s}, u_{c} t_{1 c}\right)\right] e^{-\left(u_{s} t_{1 s}+u_{c} t_{1 c}\right)} \\
& D_{2}=S G\left[B\left(u_{s} t_{2 s}, u_{c} t_{2 c}\right)\right] e^{-\left(u_{s} t_{2 s}+u_{c} t_{2 c}\right)}
\end{aligned}
$$

Dividing $A_{1}$ by $D_{2}$ we have:

$$
\begin{equation*}
\frac{D_{I}}{D_{2}}=\frac{B\left(u_{s} t_{l}, u_{c} t_{l}\right) e^{-\left(u_{s} t_{s}+u_{c} t_{1 c}\right)}}{B\left(u_{s} t_{2 s}, u_{c} t_{2 c}\right)} \tag{B. 3}
\end{equation*}
$$

Now, we assume the buildups can be expressed as products in each case, or:

$$
\begin{equation*}
\frac{B\left(u_{s} t_{2 s}, u_{c} t_{2 c}\right)}{B\left(u_{s} t_{1 s}, u_{c} t_{1 c}\right)}=\frac{\left(u_{s} t_{2 s}\right)\left(u_{c} t_{2 c}\right)}{\left(u_{s} t_{1 s}\right)\left(u_{c} t_{2 c}\right)}=\left(\frac{t_{2 s}}{t_{1 s}}\right)\left(\frac{t_{2 c}}{t_{1 c}}\right) \tag{B. 4}
\end{equation*}
$$

Therefore, $D_{2}$ can be expressed in terms of $D_{1}$, thusly:

$$
\begin{equation*}
D_{2}=D_{1}\left(\frac{t_{2 s}}{t_{1 s}}\right)\left(\frac{t_{2 c}}{t_{1 c}}\right) e^{* u s\left(t_{1 s}-t_{2 s}\right)+u_{c}\left(t_{1 c}-t_{2 c}\right)} \tag{B. 5}
\end{equation*}
$$

Case II - When concrete is replaced byserpentine, the correction becomes slightly more complex. Let the subscript sp refer to the serpentine area. In this case, $D_{1}$ is the same as given in Eq . (B.1) but $\mathrm{D}_{3}$ becomes:

$$
03-25 \quad-25-
$$

$$
\begin{equation*}
D_{3}=S G\left[B\left(u_{s} t_{3 s}, u_{s p} t_{s p}\right)\right] e-\left(u_{s} t_{3 s}+u_{s p} t_{s p}\right) . \tag{B. 6}
\end{equation*}
$$

Then the ratio $D_{3} / D_{1}$ is:

$$
\frac{D_{3}}{D_{1}}=\frac{\left.p\left(u_{s} t_{3 s}, u_{s p} t_{s p}\right) e^{-\left(u_{s} t_{3} s^{+} u_{s p} s p\right.}\right)}{\left.B\left(u_{s} t_{l s}, u_{c} t_{l c}\right) e^{-\left(u_{s} t_{l s}+u_{c} t_{l c}\right.}\right)}
$$

The ratio of the buildups becomes for this case:

$$
\frac{B\left(u_{s} t_{3 s}, u_{s p} t_{s p}\right)}{B\left(u_{s} t_{l s}, u_{c} t_{l c}\right)}=\frac{\left(u_{s} t_{3 s}\right)\left(u_{s p} t_{s p}\right)}{\left(u_{s} t_{l s}\right)\left(u_{c} t_{l c}\right)}=\left|\frac{t_{3 s}}{t_{l s}}\right|,\left|\frac{u_{s p} t_{s p}}{u_{l c} t_{l c}}\right| . \quad \text { B. } 8
$$

and therefore, the value of $D$ can be found by:

$$
\begin{equation*}
D_{3}=D_{1}\left|\frac{t_{3 s}}{t_{l s}}\right|\left|\frac{u_{s p} t_{s p}}{u_{c} t_{l c}}\right|\left[e^{-u_{s}}\left(t_{l s}-t_{2 s}\right)-u_{s p} t_{s p}+u_{c} t_{c}\right] \tag{B. 9}
\end{equation*}
$$

## Appendix C

## LIST OF SYMBOLS

The following list of symbols applies to all equations given either in the text or in the appendices.

1. Prime Symbols

A = Factor converting $\boldsymbol{\gamma}$-ray flux to dose rate
$=4.00 \times 10^{-6}$ for $2.76-\mathrm{Mev} \mathrm{Na}-248$-rays
$b_{1}=$ Number of mean free paths through floor shield
B = Buildup factor
c. = Effective position of line source within source material
$D_{p}=\gamma$-ray dose rate at point $P$
$\mathrm{f}=$ Self absorption factor in source material
$\mathrm{L}=$ Length of line sauce
$r=$ Radius of source container
$\mathrm{p}=$ Detection point
$p^{\prime}=$ Projection of detection point $P$ onto plane containing horizontal source, or onto horizonal plane containing lower end of vertical source
$S_{L}=$ Line source strength, photons/cm-sec
$S_{v}=$ Volume source strength, photons/ cm ${ }^{3}$-sec
$t_{1}=$ Steel thickness in shield floor
$t_{2}=$ Concrete thickness in shield floor
$t_{s}=$ Steel thickness of sodium tanks or pipes (sources)
$t_{s p}=$ Thickness of serpentine concrete in operating floor
$u_{c}=\gamma$-ray linear absorption coefficient for concrete
$u_{\text {na }}=\gamma$-ray linear absorption coefficient for sodium
$u_{s}=\gamma$-ray linear absorption coefficient for steel
$u_{s p}=\gamma$-ray linear absorption coefficient for serpentine concrete
2. Geometric Quantities
a) Horizontal line sources
$a=$ Distance from $p^{\prime}$ to near end of source
$b=$ Distance from $p^{\prime}$ to far end of source
$d=$ Perpendicular distance from $p^{\prime}$ to source (or to source extension)
$\mathrm{g}=$ Perpendicular distance from $P$ to source (or to source extension)
$j=$ Length of extension of source line from the near end of source to line $d$.
$m=D_{\text {istance }}$ from $P$ to near end of source
$n=$ Distance from $P$ to far end of source
$p=$ Distance from $P^{\prime}$ to midpoint of source
$\mathrm{q}=$ Distance from P to midpoint of source
$\mathbf{y}=$ Perpendicular distance from underside of floor shield to $\mathbf{P}^{\prime}$
$s=$ Distance between $P$ and $P^{\prime}$ (See Sec. C-3)
$\theta_{2}=$ Angle formed by lines $g$ and $n ; \theta_{1} \Leftarrow \theta_{2}$
$\theta_{1}=$ Angle formed by lines $g$ and $m ; \theta_{2}>\theta_{1}$
$\mathcal{\beta}=$ Angle formed by lines $P P^{\prime}$ and $q$ or $g$
b) Vertical line source
$d=$ Horizontal distance from $P^{\prime}$ to lower end of line source
$u=$ Distance from midpoint of source to $P$
$\mathrm{v}=$ Distance from lower end of source to $P$
$x=$ Distance from upper end of source to $P$
$\mathrm{y}=$ Perpendicular distance from upper end of line source to underside of floor shield
$\beta=$ Angle formed by lines PP' and $u$ $\theta_{1}=$ Angle formed by lines PP' and x $\theta_{2}=$ Angle formed by lines $P P^{\prime}$ and $v$
3. Derived Quantities

$$
\begin{aligned}
& g=\left(d^{2}+s^{2}\right)^{\frac{1}{2}}=\left[d^{2}+\left(t_{1}+t_{2}\right)^{2}\right]^{\frac{1}{2}} \\
& h=1 / 2 L+j=1 / 2 L+\left(a^{2}-d^{2}\right)^{\frac{1}{2}} \\
& j=\left(a^{2}-d\right)^{\frac{1}{2}} \\
& k=\left(b^{2}-d^{2}\right)^{\frac{1}{2}} \\
& m=\left(a^{2}+s^{2}\right)^{\frac{1}{2}} \\
& n=\left(b^{2}+s^{2}\right)^{\frac{1}{2}} \\
& p=\left(h^{2}+d^{2}\right)^{\frac{1}{2}} \\
& q=\left(p^{2}+s_{s}^{2}\right)^{\frac{1}{2}} \\
& s=y+t_{1}+t_{2} \\
& K=1 / 4 S_{v r^{2}} A \\
& M=y+t_{1}+t_{2}+1 / 2 L \\
& N=y+t_{1}+t_{2}+L \\
& Q=\left(L^{2}+n^{2}\right)^{\frac{1}{2}} \\
& R=\left(L^{2}+n^{2}\right)^{\frac{1}{2}} \\
& V=s+L \\
& T_{0}=u_{n a} \\
& T_{1}=\left(u_{n a} c\right) \csc \beta \\
& T_{2}=u_{B} t_{1} \\
& T_{3}=\left(u_{s} t_{1}\right) \sec \beta \\
& T_{4}=u_{c} t_{2} \\
& T_{5}=\left(u_{c} t_{2}\right) \sec \beta \\
& =S 3
\end{aligned}
$$

$$
\begin{aligned}
& T_{6}=u_{s} t_{s} \\
& \boldsymbol{\omega}_{1}=1 / 2\left(\theta_{1}+\theta_{2}\right) \\
& \omega_{2}=1 / 2\left(\theta_{1}-\theta_{2}\right) \\
& \omega_{3}=1 / 6\left(\theta_{1}-\theta_{2}\right) \\
& \omega_{4}=1 / 6\left(\theta_{1}+\theta_{2}\right) \\
& \nu_{1}=\sec \theta_{1} \\
& \nu_{2}=\sec \theta_{2} \\
& \nu_{3}=\sec \left[1 / 2\left(\theta_{1}+\theta_{2}\right)\right] \\
& \nu_{4}=\sec \left[1 / 2\left(\theta_{1}-\theta_{2}\right)\right] \\
& \nu_{5}=\sec \left[1 / 6\left(\theta_{1}-\theta_{2}\right)\right] \\
& \nu_{6}=\left(\sec \theta_{1} \sec \theta_{2}\right)^{\frac{1}{2}} \\
& \sqrt{7}=\sec \boldsymbol{\beta} \\
& V_{8}=\sec \frac{\theta_{1}}{2} \\
& \tau_{0}=u_{s} t_{1}+u_{c} t_{2} \\
& \tau_{1}=\left(u_{s} t_{1}+u_{c} t_{2}\right) \sec B \\
& \tau_{2}=\left(u_{s} t_{s}+u_{c} t_{2}\right) \csc \beta \\
& \tau_{3}=u_{s} t_{s}+u_{N a} c \\
& \tau_{4}=\left(u_{s} t_{s}+u_{N a} c\right) \csc B \\
& \tau_{5}=u_{s}\left(t_{1}+t_{s}\right)
\end{aligned}
$$

$\tau_{6}=\mathrm{u}_{\mathrm{s}}\left(\mathrm{t} \mathrm{m}^{\left.\sec \beta+\mathrm{t}_{\mathrm{s}} \csc \beta\right)}\right.$

$$
\alpha=\left|\frac{T_{2}+T_{4}}{T_{2} \mathrm{~T}_{4}}\right|^{\frac{1}{2}}
$$

## Appendix D

## DERIVATION OF THE DOSE AT POINT P DIRECTLY ABOVE A VERTICAL LINE SOURCE

## 1. Basic derivation

Referring to Figure D-1 the dose rate at $P$ due to the radiation emitted from an element $d z$ on a line source of length $L$ is:


The meaning of each of the symbols is given in appendix C. The total dose is found by integration over $z_{\text {. }}$

The buildup B and the self-absorption factor $f$ may be handled either in the integration directly, or separate from the main integration. It is found convenient for the machine calculations to handle these factors separately, as the direct integration leads to solutions involving exponential integrals. Since a relatively few points have this geometry, it was felt that the time and expense involved in developing'a program involving the E-functions was not worth any increase in accuracy that is aftorded by their use. Trial hand calculations. by both methods indicate a difference of only $10 \%$.

Treating $f$ and $B$ separately, the total dose at $P$ is then given by

$$
\begin{align*}
D_{p} & =\frac{A B S_{\mathrm{L}} \mathrm{f}^{-\mathrm{T}_{0}}}{4} \int_{\mathrm{s}}^{\mathrm{s}+\mathrm{L}} \frac{\mathrm{dz}}{z^{2}} \\
& =\frac{A B S_{\mathrm{L}} f e^{-\tau_{0}}}{4 \pi}\left|\frac{1}{\mathrm{~s}}-\frac{1}{\mathrm{~s}+\mathrm{L}}\right| \tag{D.2}
\end{align*}
$$

Now the selfabsorption for gama-rays efitted from the end of a $Q$ line source can be derived as follows:

Geometry for self-absorption from end of line source. $\longrightarrow+{ }^{2}$

Consider a line element emitting $S_{I} \gamma / \mathrm{s} / \mathrm{cm}-\mathrm{sec}$. In any increment dz half of the gamma-rays are assumed to be emitted in an upward direction and half in a downward direction.

Therefore, the effective source at $Q$ is: $\bar{S}_{L}=\int_{0}^{I} I / 2_{L} e^{-u z} d z$

$$
\begin{equation*}
=-\left.\frac{S_{L}}{2 u L} e^{-u z}\right|_{0} ^{L}=\frac{S_{L}}{2 u L}\left(1-e^{-u L}\right) \tag{D.3}
\end{equation*}
$$

Pherefore, the self-absorption factor is:

$$
\begin{equation*}
f=\frac{\bar{S}_{L}}{S_{L}}=\frac{1-e^{-u L}}{2 u L} \tag{B,4}
\end{equation*}
$$

If $u L \geqslant 7$, then

$$
\begin{equation*}
f \cong \frac{1}{2 u L} . \tag{D.5}
\end{equation*}
$$

The buildup factor can likewise be represented.

The average buildup through the source may be represented by:

$$
\begin{equation*}
\overline{\mathrm{B}}=\frac{\int_{0}^{L} \mathrm{~B}(u z) \mathrm{dz}}{\int_{0}^{\mathrm{L}} \mathrm{dz}} \tag{D.6}
\end{equation*}
$$

where

$$
\begin{equation*}
B(u z)=1+u z \tag{0}
\end{equation*}
$$

and

$$
\begin{align*}
\bar{B} & =\frac{I}{L} \int_{0}^{I}(I+u z) d z \\
& =\frac{I}{L}\left(I+\frac{u L^{2}}{2}\right) \\
& =(1+1 / 2 u L) . \tag{D.8}
\end{align*}
$$

The total buildup through the source and the shield is given by:

$$
\begin{equation*}
B=\alpha \bar{B} T_{2} T_{4}=\alpha T_{2} T_{4}(1+1 / 2 u L) \tag{D.9}
\end{equation*}
$$

Therefore, the dose at $P$ is given by:

$$
\begin{align*}
D_{p} & =\frac{\alpha A S_{L}}{4 \pi} T_{2} \mathrm{~T}_{4} e^{-T_{0}(1+1 / 2 \mu L)}\left|\frac{1}{2 \pi L}\right|\left|\frac{1}{s} \frac{1}{s+1}\right| \\
& =\frac{\alpha A S_{L}}{8 \pi} T_{2} T_{4} e^{-T_{0}}\left|\frac{1}{u L}+\frac{1}{2}\right|\left|\frac{1}{s}-\frac{1}{s+\mathrm{E}}\right| \tag{D,10}
\end{align*}
$$

Now, in actuality, the specific activity $S_{V}$ will be known. Therefore, the relationship between the line source $S_{L}$ and the specific activity $S$ is:

$$
\begin{equation*}
S_{\mathbf{L}}=\pi r^{2} S_{S v} \tag{D.11}
\end{equation*}
$$

where $r$ is the radius of the container of the line source. Therefore, finally:

$$
\begin{align*}
& D_{p}=\frac{\alpha}{2} T_{2} T_{4} \frac{A S_{\mathbf{v}} r^{2}}{4} e^{-\gamma_{0}}\left(\frac{1}{2}+\frac{1}{u L}\right)\left|\frac{1}{s}-\frac{1}{s+L}\right| \\
&= \frac{\alpha}{2} K_{2} T_{4} e^{-\tau_{0}}\left|\frac{1}{2}+\frac{1}{u_{E}}\right|\left(\left.\frac{1}{s}-\frac{1}{s+L} \right\rvert\,\right.  \tag{D.12}\\
&-34: 3
\end{align*}
$$

## Appendix E

## DERIVATION OF THE DOSE AT A POINT P AT A SKEW ANGLE ABOVE A VERTICAL LINE SOURCE

1. The basic equation


Referring to Figure E-l, the dose at point $P$ from the element of line segment $d l$ is simply:

$$
d D_{p}=\frac{B S_{L} f A d l e^{-\sum u_{i} t_{i} \sec \theta}}{4 \pi \rho^{2}}
$$

$$
\text { E. } 1
$$

where

$$
\rho=a \csc \theta
$$

$$
\text { E. } 2
$$

and

$$
d \rho=-a \csc \theta \cot \theta d \theta \quad \text { E. } 3
$$

also,

$$
\rho^{2}=(1+y+t)^{2}+a^{2}
$$

$$
\text { E. } 4
$$

and

$$
\rho \mathrm{d} \rho=(l+y+t) \mathrm{dl} .
$$

$$
\text { E. } 5
$$

Solving for dl :

$$
\begin{aligned}
d I & =\frac{\rho d \rho}{(1+y+t)}=\frac{-a^{2} \csc ^{2} \theta \cot \theta d \theta}{\left(\rho^{2}-a^{2}\right)^{\frac{1}{2}}} \\
& =\frac{-a^{2} \csc ^{2} \theta \cot \theta d \theta}{a \cot \theta}=-a \csc ^{2} \theta d \theta
\end{aligned}
$$

After substituting in E. 1 and letting $b_{1}=\sum u_{i} t_{i}$ we have

$$
\begin{aligned}
\mathrm{dD}_{\mathrm{p}} & =-\frac{B A S_{L f}\left(a \csc ^{2} \theta\right) \mathrm{e}^{-b_{1} \sec \theta} d \theta}{4 \pi a^{2} \csc ^{2} \theta} \\
& =-\frac{\mathrm{BAS}_{\mathrm{L}^{f}} \mathrm{e}^{-b_{1} \sec \theta} \mathrm{~d} \mathrm{\theta}}{4 \pi \mathrm{a}}
\end{aligned}
$$

Again, we treat the buildup and self-absorption apart from the main integration. Thus:
$\left.D_{p}=-\frac{B A S_{L} f}{4 \pi a}\right)_{\theta_{1}}^{\theta_{2}} e^{-b_{1} \sec \theta} d \theta$
$=\frac{B A S_{L} f}{4 \pi a}\left|\int_{\theta_{1}}^{\infty} e^{-b_{1} \sec \theta} d \theta-\int_{\theta_{2}}^{\infty} e^{-b_{1} \sec \theta} d \theta\right|$,
Now

$$
\begin{equation*}
F(\theta, b) \equiv \int_{\theta}^{\infty} e^{-b \sec \theta} d \theta \tag{E. 9}
\end{equation*}
$$

Therefore, the dose at $P$ is given by :

$$
D_{p}=\frac{B A S_{L} f}{4 \pi a}\left[F\left(\theta_{1}, b_{1}\right)-F\left(\theta_{2}, b_{1}\right)\right]
$$

$$
\text { E. } 10
$$

Substituting for the volume source Sv (Appendix D, Eq., D.11) and noting that the distance $d$ shown in Figure $l$ is the same as distance a of Figure D-1, we have:

$$
D_{p}=\frac{K}{d} \operatorname{Bf}\left[F\left(\theta_{1}, b_{1}\right)-F\left(\theta_{2}, b_{1}\right)\right]
$$

The number of mean free paths $b_{1}$ is given by

$$
\begin{equation*}
 \tag{E. 12}
\end{equation*}
$$

and the angles $\theta_{1}, \theta_{2}$, and $\boldsymbol{\beta}$, are given in this geometry by:

$$
\begin{array}{ll}
\theta_{1}=\tan ^{-1} \mathrm{~d} / \mathrm{s} & \text { E. } 13 \\
\theta_{2}=\tan ^{-1} \mathrm{~d} / \mathrm{N} & \text { E. } 14
\end{array}
$$

Also $\operatorname{Sec} \theta_{1}$, and $\operatorname{Sec} \theta_{2}$ are given directly by

$$
\begin{align*}
& \sec \theta_{1}=V_{1}=\mathrm{g} / \mathrm{s}  \tag{E. 15}\\
& \operatorname{Sec} \theta_{2}=V_{2}=\left[1+\left(\frac{d}{N}\right)^{2}\right]^{\frac{1}{2}} \tag{E. 16}
\end{align*}
$$

$\operatorname{Sec} 1 / 2\left(\theta_{1}+\theta_{2}\right)=V_{3} \tilde{a} \operatorname{Sec} \boldsymbol{\beta}$ (See Fig. 4, text)
$\sec \beta=\left[1+\left(\frac{d}{M}\right)^{2}\right]^{\frac{1}{2}}$
$\csc \beta=\left[I+\binom{M}{d}^{2}\right]^{\frac{1}{2}}$.
The latter value is needed to evaluate $\tau_{2}$.
2. Numerical evaluation of the F-Function for computer use.

The F-functions can be approximated very nicely for machine computation by the use of Simpson's 3-Point Rule:

$$
\begin{equation*}
\int_{x_{0}}^{x_{2}} f(x) d x=\frac{h}{3} \cdot\left(f_{0}+4 f_{1}+f_{2}\right)-c \tag{E. 20}
\end{equation*}
$$

In the above formula $f_{0}, f_{1}$, and $f_{2}$ are evenly -spaced points evaluated by $X_{0}$, $\mathrm{X}_{1}$, and $\mathrm{X}_{2}$ on the curve to be integrated, and the spacing h is given by:

$$
\begin{equation*}
h=1 / 2\left(x_{2}-x_{0}\right) \tag{E. 21}
\end{equation*}
$$

The terms $f_{0}, f_{1}$, and $f_{2}$ become for our case:

$$
\begin{array}{ll}
f_{0}=e^{-b_{1}} \cdot \sec \theta_{1}=e^{-b_{1}} V_{1} & \text { E.22 } \\
f_{1}=e^{-b_{1}} \cdot \sec \frac{\theta_{1}+\theta_{2}}{2}=e^{-b_{1} V_{3}} & \text { E.23 } \\
f_{2}=e^{-b_{1}} \sec \theta=e^{-b_{1}} \not Z_{2} & \text { E.24 } \tag{E. 24}
\end{array}
$$

The error term $C$ is evaluated as:

$$
c=\frac{h^{5}}{90} \frac{d^{4} \xi}{d x^{4}}\left(x_{0}<\xi<x_{2}\right) . \quad \text { E. } 25
$$

In our case

$$
\begin{align*}
\int_{x_{0}}^{x_{2}} f(x) d x & =\int_{\theta_{1}}^{\theta_{2}} e^{-b_{1} \sec \theta} d \theta \\
& =\omega_{3}\left(e^{-b_{1} \nu_{1}}+4 e^{-b_{1} \nu_{3}}+e^{-b_{1} \nu_{2}}\right) \tag{E. 26}
\end{align*}
$$

The error term $C$ is ommited since the correction is less than $1 \%$.

## 3. The Buildup Factor B

The buildup factor is here assumed to beevaluated independently of the main integration. In the geometry shown in Figure E-1, the buildup will vary with the path from the source element to point $P_{1}$. It is found that the average path length from source to $P$ is not significantly different from that drawn from the center of the source to $P_{1}$. Accordimgly, the buildup factor is taken as:

$$
\begin{equation*}
B=\alpha T_{1} T_{5} T_{6} \tag{E. 27}
\end{equation*}
$$

4. The Self-Absorption Factor $f$

The self-absorption factor is taken over the average distance in the source material from the effective line source to one outside of the real source. This distance is given by' $\mathcal{L}_{4}$, therefore, the self-absorption factor $f$ is

$$
\begin{equation*}
f=e^{-\tau_{4}} \tag{E. 28}
\end{equation*}
$$

5. Final Equation for Computer Use

Collecting the above information, we have, finally:

$$
\begin{equation*}
\mathrm{D}_{\mathrm{p}}=\alpha_{K} \tau_{6} \mathrm{~T}_{1} \mathrm{~T}_{5} \omega_{3} e^{-\gamma_{4}}\left(\mathrm{e}^{-\mathrm{b}_{1} \nu_{1}}+4 \mathrm{e}^{-\mathrm{b}_{1} \nu_{3}}+\mathrm{e}^{-\mathrm{b}_{1} \nu_{2}}\right) . \tag{E. 29}
\end{equation*}
$$

## DERIVATION OF THE DOSE AT A POINT P ABOVE A HORIZONTAL LINE

## Case I

Perpendicular line drawn from $P$ to line source intersects source somewhere between the ends.


Figure F-1

1. Basic Derivation


Figure F-2

The configuration of this case is given in Figure $F-1 . P^{\prime}$ is a point in the plane of the course directly below $P$. A perpendicular line $g$ dropped from $P$ intersects line source $L$ at $Q$. The easily-measured quantities are $a, b$, and $d$, as well as $s=t+y$.

However, the integration over the line source takes place in the plane defined by the line source and lines $g, m$, and $n$. This plane is inclined at an angle $\beta$ from the vertical. The quantities $g, m$, and $n$ may be found by triangulation from the measured quantities $a, b$, $d$, and $s$, as given in Appendix $C$, Section 3.

Referring to Figure $\mathrm{F}-2$, the dose at P due to an element of line dx is given by:

$$
d D_{p}=A S_{L} B f d x_{e}-u t \sec \beta \sec \theta
$$

But $\mathbf{x}=\mathrm{g} \tan \theta$

$$
\begin{array}{ll}
\rho=g \sec \theta & \text { F. } 3 \\
d x=g \sec ^{2} \theta d \theta & \text { F. } 4
\end{array}
$$

Assuming that the evaluation of $B$ and $f$ for purposes of mechanical computing can be treated separately from the basic integration, we have, as before:

$$
\begin{aligned}
D_{p} & =\frac{A S_{L} B f}{4 \pi} \int_{\theta_{1}}^{\theta_{2}} \frac{d x}{\rho^{2}} e^{-b_{1} \sec \theta} \\
& =K B f \quad \int_{\theta_{1}}^{\theta_{2}} \frac{g^{2} e^{-b_{1}} \sec \theta}{\sec ^{2} \theta} \sec ^{2} \theta d \theta \\
& =\frac{K B f}{g}\left|\int_{0}^{\theta_{1}} e^{-b_{1} \sec \theta} d \theta+\int_{0}^{\theta_{2}} e^{-b_{1} \sec \theta} d \theta\right|
\end{aligned}
$$

$$
=\frac{K B f}{g}\left[F\left(\theta_{1}, b_{1}\right)+F\left(\theta_{2}, b_{1}\right)\right] .
$$

$$
F .5
$$

Here we have taken $b_{1}$ as:

$$
b_{1}=u t \sec \beta
$$

where ut is shorthand notation for:

$$
u t=u_{s} t_{1}+u_{c} t_{2}=\tau_{0}
$$

Therefore:

$$
\begin{equation*}
b_{1}=T_{1} \tag{F. 8}
\end{equation*}
$$

The various angles can be given in terms of the measured quantities as:

$$
\begin{array}{lll}
\theta_{1} & =\tan ^{-1} \mathrm{k} / \mathrm{g} & \mathrm{~F} .9 \\
\theta_{2} & =\tan ^{-1} \mathrm{j} / \mathrm{g} & \mathrm{~F} .10
\end{array}
$$

$$
\begin{array}{ll}
\sec \theta_{1}=\mathrm{n} / \mathrm{g} & \mathrm{~F} .11 \\
\sec \theta_{2}=\mathrm{m} / \mathrm{g} & \mathrm{~F} .12 \\
\sec \beta & =\mathrm{g} / \mathrm{s} .
\end{array} \mathrm{F} .13
$$

2. Numerical evaluation of the F-Functions.

The evaluation of the F -Function for this case is:

$$
\begin{equation*}
\int_{\theta_{1}}^{\theta_{2}} e^{-b_{1} \sec \theta} d \theta=\omega_{4}\left(f_{0}+L_{1} f_{1}+f_{2}\right) \tag{F. 14}
\end{equation*}
$$

where $f_{0}=e^{-b_{1}{ }_{1}}$

$$
\begin{align*}
& \left.f_{1}=e^{-b_{1}} \sec \frac{\theta_{1}-\theta_{2}}{2} \right\rvert\,=e^{-b_{1} \nu_{4}}  \tag{F. 16}\\
& f_{2}=e^{b_{1} \sec \theta_{2}}=e^{-b_{1} \nu_{2}}
\end{align*}
$$

or

$$
\begin{equation*}
\int_{\theta_{1}}^{\theta_{2}} e^{-b_{1} \sec \theta} d \theta=\omega_{4}\left(e^{-b_{1} \nu_{1}}+L^{-b_{1} \nu_{4}}+e^{-b_{1} \nu_{2}}\right) \tag{F. 18}
\end{equation*}
$$

Here $\omega_{4}=1 / 6\left(\theta_{1}+\theta_{2}\right)$
3. The buildup factor $B$

Here again, B varies with position. We have chosen to take the build up as:

$$
\begin{aligned}
B & =I / 2\left[\alpha\left(u_{s} t_{1} \sec \right)\left(u_{c} t_{2} \sec \right)+u_{s} t_{s}\right]\left(\sec \theta_{1} \sec \theta_{2}\right)^{\frac{1}{2}} \\
& =1 / 2\left(\alpha T_{3} T_{5}+T_{6}\right) \nu / 6
\end{aligned}
$$

This equation is easily arrived at by considering the mean of the average value of $\sec \theta$ on each side of point $\theta_{1}$.
4. Self-absorption factor $f$

The self-absorption factor for this case is given by:

$$
f=e^{-\frac{1}{2}}\left(u_{s} t_{s}+u_{N a} C\right)\left(\sec \theta_{1} \sec \theta_{2}\right)^{\frac{1}{2}}
$$

$$
=e^{-\frac{1}{2} \Upsilon_{3} \nu_{6}}
$$

5. General Equation for Computer Use

Putting all of the above together, we have, finally:

This equation is expressed in terms of the prime variables $t_{1}, t_{2}, a, b$, $\mathrm{d}, \mathrm{y}$, and L. (See Appendix C).


Figure G-I

1. Basic Derivation


Figure G-2

The configuration for this case is shown in Figure G-1. As in the case of Appendix $F$, the integration over $\theta$ takes place in the plane formed by the line source L , and the lines g and n .

The treatment of the equation is the same as in Appendix F. The dose rate at $P$ from a given element $d x$ on $L$ is:

$$
d D_{p}=\frac{A S L B f d x}{1 \pi \rho^{2}} e^{-u t \sec \beta \sec \theta}
$$

where $\rho$ and $x$ have the same meanings as in Appendix $F$. The total dose rate at $P$ is then given by integrating over the line source:

$$
D_{p}=\frac{A S_{L} B f}{4 \pi} \int_{0}^{\theta_{1}} e^{-b_{1} \sec \theta} d \theta
$$

$$
\begin{aligned}
& =K B f \quad \int_{0}^{\theta_{1}} e^{-b_{1} \sec \theta} d \theta \\
& =\frac{K B f}{g} \quad F \quad\left(\theta_{1}, b_{1}\right)
\end{aligned}
$$

In this case, ut is a shorthand way of writing:

$$
\begin{equation*}
b_{1}=\tau_{1}=\left(u_{s} t_{1}+u_{c} t_{s}\right) \sec \beta \tag{G. 3}
\end{equation*}
$$

and the angles are given by:

$$
\begin{align*}
& \theta_{1}=\tan ^{-1} \mathrm{~L} / \mathrm{g} \\
& \sec \theta_{1}=\mathrm{n} / \mathrm{g} \\
& \sec \frac{\theta 1}{2}=\left(\frac{2 \mathrm{n}}{\mathrm{n}+\mathrm{g}}\right)^{1 / 2} \\
& \sec \beta=\mathrm{g} / \mathrm{s}
\end{align*}
$$

2. Evaluation of the F-Function

Following the method outlined in Appendix E, the F-Function is numerically evaluated to be:

$$
F\left(\theta_{1}, b_{1}\right)=\int_{0}^{\theta_{1}} e^{-b_{1} \sec \theta} d \theta=\frac{\theta_{1}}{6}\left(e^{-b_{1}}+4 e^{-b_{1} \nu_{8}}+e^{-b_{1} \mathcal{L}_{1}}\right)_{G .8}
$$

3. The Buildup B

The buildup B is taken over the mean path length, which in this case turns out to be:

$$
\begin{align*}
B & =\left[\alpha\left(u_{s} t_{1} \sec \beta\right)\left(u_{c} t_{z} \sec \beta\right)+u_{s} t_{s}\right]\left(\sec \theta_{1}\right)^{1 / 2} \\
& =\left(\alpha T_{3} T_{5}+T_{6}\right) \nu_{1}^{1 / 2} \tag{G. 9}
\end{align*}
$$

4. The Self-Attenuation Factor $f$

$$
f=e^{-\gamma_{1}} 1^{\frac{1}{2} \tau_{3}}
$$

$$
\text { G. } 10
$$

5. Final Equation for Computer Use

Putting everything together, we have the final equation suitable for computer use:

$$
\begin{equation*}
D_{p}=\frac{\sqrt{1}_{1}^{\frac{2}{2}} K\left(\alpha T_{3} T_{5}+T_{6}\right) \theta_{1} e^{-\sqrt{2}} 1^{\frac{1}{2}} T_{3}}{6 g}\left(e^{-b_{1}}+4 e^{-b_{1} \nu_{8}}+e^{-b_{1} V_{1}}\right) . \tag{G. 11}
\end{equation*}
$$

$$
103 \quad 045 \quad-45=
$$

## Appendix H

DERIVATION OF THE DOSE AT A POINT P ABOVE A HORIZONTAL LINE SOURCE - CASE III. PERPENDIUCLAR LINE DRAWN FROM P TO LINE SOURCE INTERSECTS AN EXTENSION OF THE

## SOURCE



## 1. Basic Derivation

The derivation of the dose at P for this case follows quite closely those of appendices $F$ and $G$. The geometry is slightly different because the perpendicular line PO does not intersect $L$, but its extension.

The dose at $P$ from the line source $L$ in this case is:

$$
\begin{aligned}
D_{p} & =\frac{A S_{L} B f}{4 \pi} \int_{\theta_{1}}^{\theta_{2}} \frac{e^{-u t} \sec \beta \sec \theta d x}{\rho^{2}} \\
& =\frac{K B f}{g} \int_{\theta_{1}}^{\theta_{2}} e^{-b_{1} \sec \theta} d \theta=\frac{K B f}{g}\left[F\left(\theta_{1}, b_{1}\right)-F\left(\theta_{2}, b_{1}\right)\right] H \cdot I
\end{aligned}
$$

Here, as in Appendix $F, b_{1}$ is taken as:

$$
\begin{equation*}
b_{1}=\gamma_{1} . \tag{H. 2}
\end{equation*}
$$

For this case the angles are given as:

$$
\begin{array}{ll}
\theta_{1}=\tan ^{-1} \mathrm{k} / \mathrm{g} & \mathrm{H} .3 \\
\theta_{2}=\tan ^{-1} \mathrm{j} / \mathrm{g} & \mathrm{H} .4 \\
\sec \theta_{1}=\mathrm{n} / \mathrm{g} & \mathrm{H} .5 \\
\sec \theta_{2}=\mathrm{m} / \mathrm{g} & \mathrm{H} .6 \\
\sec \beta=\mathrm{q} / \mathrm{s} & \mathrm{H} .7
\end{array}
$$

2. Evaluation of the F-Function

The F-Function in this geometry has the value:

$$
\int_{\theta_{1}}^{\theta_{2}} e^{-b_{1} \sec \theta} d \theta=\omega_{3}\left(e^{-b_{1} \nu_{1}}+4 e^{-b_{1} \nu_{3}}+e^{-b_{1} V_{2}}\right) \cdot H .8
$$

## 3. The Buildup Factor B

The buildup factor is expressed exactly the same as in Appendix $F$

$$
B=1 / 2\left(\alpha T_{3} T_{5}+T_{6}\right) \cdot V_{6} .
$$

4. The Self-Absorption Factor $f$

Likewise, the self-absorption factor is expressed the same as in Appendix F. ie.

$$
\begin{equation*}
f=e^{-1 / 2} T_{3} V_{6} \tag{H. 10}
\end{equation*}
$$

5. The Final Equation for Computer Use

The final equation for computer use may now be expressed as:

$$
D_{p}=\frac{K \omega_{3} \nu_{3}\left(\alpha T_{3} T_{5}+T_{6}\right)}{2 g} e^{-1 / 2 T_{3} \nu_{6}\left(e^{-b_{1} \nu_{1}}+4 e^{-b_{1} \nu_{3}}+e^{\left.-b_{1} \nu_{2}\right)} H_{0} 11\right.}
$$


[^0]:    - 11 -

