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TRANSPORT/360
A Computer Program for Designing
Charged Particle Beam Transport Systems

Karl L. Brown

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SLAC Report No. 91

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STANFORD LINEAR ACCELERATOR CENTER
Stanford University • Stanford, California

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SLAC-91
UC-28
(EXPI) & (ACC)

TRANSPORT/360
A COMPUTER PROGRAM FOR DESIGNING CHARGED PARTICLE
BEAM TRANSPORT SYSTEMS

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FOREWORD

This report (SLAC-91) represents the completion of an extensive documentation of the first- and second-order theory of Beam Transport Optics and of the associated computer program (TRANSPORT) that has evolved at SLAC over the past several years. This report contains the following:

- 1) A Users Manual describing how to prepare data sets for a TRANSPORT computation.
- 2) Specific Examples of simple and complex TRANSPORT computations to illustrate first-order fitting techniques.
- 3) A set of Flow Charts for the benefit of the beginning user who needs to know what the program does.
- 4) An Appendix containing the underlying optics theory behind the program that is not adequately covered by other existing references. This also serves to introduce the user to TRANSPORT notation.

In addition to this report, other pertinent documents that may be useful to TRANSPORT users are the following:

- 5) SLAC-75: A First- and Second-Order Matrix Theory for the Design of Beam Transport Systems and Charged Particle Spectrometers.
- 6) A Matrix Element Test Deck (obtained from the SLAC program librarian). This has been prepared for the new user and is designed to check all of the first- and second-order matrix elements contained in the TRANSPORT program.
- 7) A TRANSPORT listing (also obtained from the SLAC program librarian).

For NEW users of TRANSPORT who desire a copy of the program and the appropriate backup documentation, please send your requests to:

Linda Lorenzetti
Program Librarian
SLAC
P.O. Box 4349
Stanford, California 94305

If all you require is the TRANSPORT manual (SLAC Report No. 91) send your requests to:

Reports Distribution Office
SLAC
P.O. Box 4349
Stanford, California 94305

TABLE OF CONTENTS

	PAGE
INTRODUCTION	0-1
Mathematical Formulation of TRANSPORT	0-3
Input Format for TRANSPORT/360	0-9
An Example of a TRANSPORT/360 Input Deck	0-12
Table of Data Input Parameters for TRANSPORT/360	0-14
Title Card	0-16
Indicator Card	0-17
Comment Cards	0-18
Type Code 1.0 Input Beam	1-1
The Phase Ellipse Beam Matrix	1-3
Example of a Two Dimensional Phase Ellipse	1-6
rms Addition to the Beam	1-7
Type Code 2.0 Fringing-Fields and Pole Face Rotations	2-1
Type Code 3.0 A Drift Distance	3-1
Type Code 4.0 Wedge Bending Magnets	4-1
Type Code 5.0 Quadrupoles	5-1
Type Code 6.0.1. Transform 1 Update	6-1
Type Code 6.0.2. Auxiliary Transformation Matrix	6-2
Type Code 7.0 Shift in the Beam Centroid	7-1
Type Code 8.0 Magnet Alignment Tolerances	8-1
Type Code 9.0 Repetition	9-1
Type Code 10. First-Order Vary Codes and Fitting Constraints	10-1
Coupled Vary Codes	10-3
R1 Matrix Fitting Constraints	10-4
R2 Matrix Fitting Constraints	10-5
BEAM Matrix Fitting Constraints	10-6
System Length Constraint	10-8
AGS Machine Constraint	10-9
First Moment Constraint	10-11
Internal Constraints	10-12
Type Code 11.0 Acceleration	11-1
Type Code 12.0 Beam (Rotated Ellipse)	12-1

Type Code 13.0	Output Print Controls	13-1
Type Code 14.0	Arbitrary Transformation Matrix Input	14-1
Type Code 15.0	Input-Output Units	15-1
Type Code 16.0	Special Input Parameters	16-1
	Tilt-To-Focal Plane	16-6
Type Code 17.0	Second-Order Calculation	17-1
Type Code 18.0	Sextupole	18-1
Type Code 19.0	Solenoid	19-1
Type Code 20.0	Beam Rotation	20-1
Type Code 21.0	Stray Magnetic Field	21-1
	Sentinel Card	21-2
	References	21-3
	Examples	E-1A
	Flow Charts	
	Transport	F-1
	Subroutine BKKIN	F-2
	Subroutine BKKPAK	F-9
	Subroutine BKKFX	F-10
	Subroutine BKKFL	F-12
	Subroutine SCANT	F-15
	Subroutine FITTIN	F-16
	Outfit Subprogram	F-22
	Element Subroutine	F-26
	Solve Subprogram	F-38
	Form Subroutine	F-40
	Alter Subprogram	F-41
	Partials Subroutine	F-44
	Subroutine CHEK	F-51
	Summary	F-52
	Appendix	
	Introduction	A-0
	1 - Beam Transport Optics (a set of lectures given to the SLAC technical staff)	
	Part I	A-1
	Geometric Light Optics vs. Magnetic Optics	A-1
	Introduction of Momentum Dispersion into the Matrix Formalism	A-8
	Second-Order Matrix Formalism	A-12
	Transformations Involving Many Trajectories	A-13

Part II	A-16
First-Order 3×3 Matrix Transformation	A-16
Example of a Beam Switchyard	A-17
TRANSPORT Notation	A-22
Second-Order Matrix	A-23
2 - The First-Order R Matrix and Sigma Matrix (Beam Ellipsoid)	
Formulation of TRANSPORT	A-26
First-Order Dispersion	A-29
First-Order Path Length	A-29
Achromaticity	A-30
Isochronicity	A-30
First-Order Imaging Conditions	A-30
Focal Lengths	A-31
Zero Dispersion	A-32
Magnification	A-32
First-Order Momentum Resolution	A-32
First-Order Phase Ellipse Formalism of TRANSPORT	A-35
Description of the Sigma BEAM Matrix	A-37
The Phase Ellipse Matrix used by TRANSPORT	A-39
Physical Interpretation of Various Projections of the	
2-dimensional BEAM Ellipse	A-41
The Upright Ellipse	A-44
Relationship between a Waist and Parallel-to-Point Image	A-45
Relationship between a Waist and Point-to-Point Image	A-46
Relationship between a Waist and the Smallest Spot Size	
Achievable at a Fixed Target Position	A-48
Imaging from an Erect Ellipse to an Erect Ellipse	A-51
Relationship between a First-Order Point-to-Point Image and	
the Minimum Spot Size Achievable at a Fixed Target Position	A-54
Orientation of the Major Axes of a Phase Space Ellipse	A-55
3 - Second-Order Calculations and Corrections	A-58
Second-Order Phase Ellipsoid Formalism	A-59
A Systematic Procedure for Designing High Resolving Power	
Beam Transport Systems or Charged Particle Spectrometers	A-60
I Introduction	A-62
II Theory	A-63
Multipole Strengths for Pure Multipole Field	A-65
Multipole Strengths for a Non-Uniform Field	
Expansion	A-66
Multipole Strengths for a Contoured Entrance or	
Exit Boundary of a Magnet	A-66

	The Description of the Trajectories as a Taylor's Expansion	A-67
III	Solution of the Equation of Motion	A-69
IV	Interpretation and use of the Coupling Coefficient	A-71
V	A Systematic Procedure for Designing High Resolution Systems	A-73
	First-Order Considerations	A-73
	First-Order Resolving Power	A-73
	Dispersion	A-75
	The Selection of the Optical Mode	A-75
	Aberrations and their Correction	A-75
	References	A-76
4 -	Theory of Magnet Alignment Tolerances	A-77
5 -	First-Order Parameter Optimization and Covariance	A-80

INTRODUCTION

TRANSPORT/360 is a computer program for the design of static-magnetic beam transport systems.

This report (SLAC-91) supercedes the original TRANSPORT instruction manual issued in October, 1963 by C. H. Moore, S. K. Howry, and H. S. Butler. Since the issuance of the original manual, several important modifications and additions have been made to the TRANSPORT program, among them are:

a) The second-order portion of the program has been extended to include the fringing-fields of bending magnets, second-order curvatures on the entrance and exit faces of bending magnets, and the second-order aberrations of solenoids. All of the second-order portion of the program has been thoroughly de-bugged using appropriate magnetic optical theorems, direct comparisons with hand calculations, and comparisons with results from ray-tracing programs.

b) Fringing-field corrections to the first-order transverse-optics of bending magnets have been added to the program via the Type Code 16. elements.

c) The program has been translated from the original Balgol version to Fortran. (We hereby wish to acknowledge the important contribution made by Stan Kowalski of M.I.T. in effecting this translation).

d) A complete free-field format for data input has been added to the Fortran version thereby considerably simplifying the use of the program.

e) A relativistically-correct first-order matrix for a traveling-wave linear accelerator section has been added to the program via Type Code 11.0.

f) Provision has been made to introduce second-order corrections (curvatures) to the input and output faces of bending magnets via Type Code 16. elements to facilitate correcting for various second-order aberrations.

g) Provision has also been made, via the Type Code 16. elements, to enable the user to rotate to the focal plane of a system and print-out the second-order aberrations along this plane.

For the benefit of the new users of TRANSPORT, we shall now review briefly the mathematical basis of the program. From the outset, it is important to emphasize that TRANSPORT is not a ray tracing program in the usual sense, but rather is based upon a first- and second-order matrix theory of beam transport optics(1,2).

Mathematical Formulation of TRANSPORT(*)

The following of a charged particle through a system of magnetic lenses may be reduced to a process of matrix multiplication. At any specified position in the system an arbitrary charged particle is represented by a vector (single column matrix), X , whose components are the positions, angles, and momentum of the particle with respect to a specified reference trajectory.

$$\text{i. e. } X = \begin{bmatrix} x \\ \theta \\ y \\ \varphi \\ \ell \\ \delta \end{bmatrix}$$

Definitions:

x = the radial displacement of the arbitrary ray with respect to the assumed central trajectory.

θ = the angle this ray makes in the radial plane with respect to the assumed central trajectory.

y = the transverse displacement of the ray with respect to the assumed central trajectory.

φ = the transverse angle of the ray with respect to the assumed central trajectory.

(*) For a more complete description of the mathematical basis of TRANSPORT, refer to SLAC Report 75, the Appendix of this report and to other References listed at the end of this report.

ℓ = the path length difference between the arbitrary ray and the central trajectory

$\delta = \Delta P/P$ is the fractional momentum deviation of the ray from the assumed central trajectory.

The magnetic lens is represented by a square matrix, R , which describes the action of the magnet on the particle coordinates. Thus the passage of a charged particle through the system may be represented by the equation:

$$X[1] = R X[0] \quad (1)$$

where $X[0]$ is the initial coordinate vector and $X[1]$ is the final coordinate vector of the particle under consideration; R is the transformation matrix for all such particles traversing the system (one particle differing from another only by its initial coordinate vector $X[0]$).

The traversing of several magnets and interspersing drift spaces is described by the same basic equation but with R now being the product matrix $R = R(n) \dots R(3)R(2)R(1)$ of the individual matrices of the system elements. The following of a charged particle via TRANSPORT through a system of magnets is thus analogous to tracing rays through a system of optical lenses except that TRANSPORT is a matrix calculation which truncates the problem to either first or second-order in a Taylor's expansion about a central trajectory. For studying beam optics to greater precision than a second-order TRANSPORT calculation permits,

ray-tracing programs which directly integrate the basic differential equation of motion are recommended(6).

In accelerator and beam transport systems, the behavior of an individual particle is often of less concern than is the behavior of a bundle of particles (the BEAM) of which an individual particle is a member. An extension of the matrix algebra of equation(1) provides a convenient means for defining and manipulating this BEAM. TRANSPORT assumes that the bundle of rays constituting a BEAM may correctly be represented in coordinate phase-space by an ellipsoid whose coordinates are the position, angle, and momentum coordinates of the arbitrary rays in the beam about an assumed central trajectory. Particles in a BEAM are assumed to lie within the boundaries of the ellipsoid with each point within the ellipsoid representing a possible ray. The sum total of all phase points, the phase space volume, is commonly referred to as the "phase space" occupied by the BEAM. The validity and interpretation of this phase ellipse formalism must be ascertained for each system being designed. However, in general, for charged particle beams in, or emanating, from accelerators, the first-order phase ellipse formalism of TRANSPORT is a reasonable representation of physical reality; but for other applications, such as charged particle spectrometers, caution is in order in its use and interpretation.

The equation of an n-dimensional ellipsoid may be written in matrix form as follows:

$$X[0]^T \sigma[0]^{-1} X[0] = 1 \quad (2)$$

where $X[0]^T$ is the transpose of the coordinate vector $X[0]$, and $\sigma[0]$ is a real, positive definite, symmetric matrix.

The volume of the n-dimensional ellipsoid defined by sigma is $\frac{\pi^{n/2}}{\Gamma(\frac{n}{2} + 1)} (\det \sigma)^{1/2}$, the area of the projection in one plane is $A = \pi (\det \sigma)^{1/2}$. This is the "phase space" occupied by the beam.

As a particle passes through a system of magnets, it undergoes the matrix transformation of equation(1). Combining this transformation with the equation of the initial ellipsoid, and using the identity $RR^{-1} = I$ (the unity matrix), it follows that:

$$X[0]^T (R^T R^{T^{-1}}) \sigma[0]^{-1} (R^{-1} R) X[0] = 1$$

from which:

$$(RX[0])^T (R \sigma[0] R^T)^{-1} (RX[0]) = 1 \quad (3)$$

The equation of the new ellipsoid after the transformation becomes:

$$X[1]^T \sigma[1]^{-1} X[1] = 1 \quad (4)$$

where:

$$\sigma[1] = R \sigma[0] R^T \quad (5)$$

It can readily be shown that the square roots of the diagonal terms of the sigma matrix are a measure of the "beam size" and the off-diagonal terms are a measure of the orientation of the ellipsoid in n-dimensional space (for TRANSPORT n = 6).* Thus, we may specify the beam at any point in the system via equation(5), given the initial "phase space" represented by the matrix elements of $\sigma[0]$.

Several types of physical elements have been incorporated in the program to facilitate the design of very general beam transport systems, included are: an arbitrary drift distance, bending magnets, quadrupoles, sextupoles, solenoids, and an accelerator section (to first-order only). Provision is made in the program to vary some of the physical parameters of the elements comprising the system and to impose first-order fitting conditions upon either the TRANSFORM (R) matrix representing the transformation of an arbitrary ray through the system and/or to impose first-order fitting conditions upon the phase ellipse (Sigma) matrix representing the transformation of a bundle of rays through the system. Thus, in principle, the program is capable of searching for and finding the first-order solution to any physically-realizable problem. In practice, life is not quite so

* See the appendix of this report for a derivation of these statements.

simple, the user will find that an adequate knowledge of geometric magnetic optics principles is a necessary prerequisite to the successful use of TRANSPORT. In other words, the program is superb at solving the mathematics of the problem but not the physics. The user must provide a reasonable physical input if he expects complete satisfaction from the program. For this reason a brief review of magnetic optics along with a list of pertinent reprints and references and some representative TRANSPORT calculations are given in the appendix of this manual to provide assistance to the inexperienced as well as the experienced user.

A thorough understanding of the first-order matrix algebra of beam transport optics and of the physical interpretation of the various matrix elements is an essential prerequisite to the successful use of this program.

INPUT FORMAT FOR TRANSPORT/360

The input format for TRANSPORT/360 is quite similar to that of the previous versions of the program. The input DATA SET consists of three kinds of cards: the TITLE card, the INDICATOR card, and the DATA cards.

The TITLE card has single quotes punched in any two columns on the card. Any information punched between the quotes will be used as a heading in the output of the TRANSPORT run.

The second card of the input is the INDICATOR card. If the data which follows describes a new problem, a zero(0) is punched in any column on the card. If the data which follows describes changes to be made in the previous problem, a one(1) is punched in any column on the card.

The rest of the cards in a deck contain the DATA describing a beam and the physical elements of the magnetic system. Each data set must be terminated by the word SENTINEL; the word SENTINEL need not be punched on a separate card. The input format of these cards is "free-field" which is described below. Each element uses a type code number which identifies the element; a field which indicates which physical parameters of the element are to be varied if there is to be any fitting; the parameters necessary to describe the physical element, such as length, magnetic field strength, etc. and, optionally, a one-to-four character label (enclosed between single quotes). The type code numbers and their interpretations are summarized in TABLE I. If the type code number is negative, the element will be ignored in the calculation which follows, but may be used in a succeeding calculation.

The vary field is punched immediately to the right of the decimal point of the type code number. The meaning is exactly the same as it was in the 7090 version of the TRANSPORT program. See section under Type Code 10. for an explanation of the use of vary codes.

The data parameters describing the element are punched in the order given in TABLE I, in the same order as has been required for previous versions of TRANSPORT. Each data entry must contain a decimal point unless it is equal to zero.

The label, if present, is enclosed by single quotes. It may contain up to four characters. It will be printed in the output during the calculation and can be useful in problems with many elements and/or when sequential fitting is used. Labels are essential if the data associated with the element is to be changed in succeeding problems.

Provision has been made in the program to allow the user to introduce comments before any type code entry in the data deck. This is accomplished by enclosing the comments made on each card with single parentheses.

The data associated with each type-code entry must be followed by a semi-colon. If the program encounters a semi-colon before the expected number of parameters has been read in, the rest of the parameters are set to zero if the indicator card was a zero(0). If the indicator card was a one(1) then the numbers on the card are substituted for the numbers from the previous solution, but the remaining numbers are unchanged.

The "free-field" input format of the data cards make it considerably easier to prepare input than the standard fixed-field formats of

FORTTRAN. Numbers may be punched anywhere on the card. They must be separated by one or more blanks. A single number must be all on one card; it may not continue from one card to the next. The program is limited to 500 data number entries and 200 type code entries.

A floating-point number e.g.(2.47) may be represented in any of the following ways:

2.47
.00247+3
.0247E+02
247.E-2
247000.-5

The sample problem input which is included below causes TRANSPORT to do a first order calculation with fitting and then to do a second order calculation with the data that is the result of the fitting. The type ten element which specifies the fitting condition is labeled FIT1. It is active for the first order calculation but is turned off for the second order calculation. The vary codes for elements DR1 are set to zero for the second-order problem. The second order element, SEC1, is ineffective during the fitting, but causes the program to compute the second-order matrices in the second calculation. The word SENTINEL need not be punched on a separate card.

An Example of a TRANSPORT/360 Input Deck

'FORTRAN H CHECK ON BETA FIT'

0

1. .5 1. .5 1. .5 1. 1. ;

-17. 'SEC1' ;

3.3 2.745 'DR1' ;

2. 0. ; 4. 9.879 10. .5 ; 2. 0. ;

3.3 2.745 'DR1' ;

13. 4. ;

10. -1. 1. 0. .0001 'FIT1' ;

SENTINEL

'SECOND ORDER'

1

17. 'SEC1' ;

3.0 'DR1' ;

-10. 'FIT1' ;

SENTINEL

The figure below shows the 360 TRANSPORT deck setup. If the data are punched on an 026 keypunch instead of an 029 keypunch, one should precede the title card with a card on which the characters KEY90 are punched in any column. No imbedded blanks are allowed in this control statement.

Note that as many problems as one wishes may be stacked in one job.

TABLE I

PHYSICAL ELEMENT	TYPE CODE	2nd ENTRY	3rd ENTRY	4th ENTRY	5th ENTRY	6th ENTRY	7th ENTRY	8th ENTRY	9th ENTRY	
BEAM	1.vvvvvv0	x (cm)	θ (mr)	y (cm)	ϕ (mr)	l (cm)	δ (percent)	P _o		
R.M.S. ADDITION TO BEAM ENVELOPE	1.	Δx (cm)	$\Delta \theta$ (mr)	Δy (cm)	$\Delta \phi$ (mr)	Δl (cm)	$\Delta \delta$ (percent)	ΔP_o (GeV/c)	0	
POLE FACE ROTATION	2.v	ANGLE OF ROTATION (Degrees)								
DRIFT	3.v	LENGTH (meters)								
BENDING MAGNET	4.0vv	LENGTH (meters)	FIELD (kg)	FIELD GRADIENT (n-value)						
QUADRUPOLE	5.vv0	LENGTH (meters)	FIELD (kg)	HALF-APERTURE (cm)						
TRANSFORM 1 UPDATE	6.	0.	1.							
TRANSFORM 2 UPDATE	6.	0.	2.							
BEAM CENTROID SHIFT	7.	SHIFT(x)(cm)	SHIFT(θ)(mr)	SHIFT(y)(cm)	SHIFT(ϕ)(mr)	SHIFT(l)(cm)	SHIFT(δ)(percent)			
ALIGNMENT TOLERANCE	8.vvvvvv0	DISPLACEMENT(x)cm	ROTATION(θ)mr	DISPLACEMENT(y)cm	ROTATION(ϕ)mr	DISPLACEMENT(l)cm	ROTATION(δ)mr	CODE NUMBER		
REPEAT CONTROL	9.	NUMBER OF REPEATS								
FITTING CONSTRAINTS	10.	$\pm I$	J	DESIRED VALUE OF (I,J) MATRIX ELEMENTS	ACCURACY OF FIT					
Note: + I is used for fitting a beam (σ) matrix element. - I is used for fitting an R1 matrix element. - (I + 20) is used for fitting an R2 matrix element.										
ACCELERATOR	11.	LENGTH (meters)	E (energy gain) (GeV)	ϕ (phase lag) (Degrees)	(WAVELENGTH) (cm)					
BEAM (Rotated Ellipse)	12.	THE FIFTEEN CORRELATIONS AMONG THE SIX ELEMENTS (This entry must be preceded by a Type Code 1. entry).								
INPUT/OUTPUT OPTIONS	13.	CONTROL CODE NUMBER								
ARBITRARY R MATRIX	14.vvvvvv0	R(J,1)	R(J,2)	R(J,3)	R(J,4)	R(J,5)	R(J,6)	J		
UNITS CONTROL (Transport Dimensions)	15.	CODE	LABEL	SCALE FACTOR (if required)						
			$\epsilon(1) = \beta \left(\frac{x}{\rho} \right)^2$ (dimensionless) x = 1							
QUADRATIC TERM OF BENDING FIELD	16.	1.								
MASS OF PARTICLES IN BEAM	16.	3.	M/m (dimensionless)							
HALF-APERTURE OF BENDING MAG. in x PLANE	16.	4.	w/2 (cm)							
HALF-APERTURE OF BENDING MAG. in y PLANE (gap)	16.	5.	g/2 (cm)							
LENGTH OF SYSTEM	16.	6.	L. (meters)							
FRINGE FIELD CORRECTION COEFFICIENT	16.	7.	K1 (dimensionless)							
FRINGE FIELD CORRECTION COEFFICIENT	16.	8.	K2 (dimensionless)							
CURVATURE OF ENTRANCE FACE OF BENDING MAGNET	16.	12.	(1/R1) (1/meters)							
CURVATURE OF EXIT FACE OF BENDING MAGNET	16.	13.	(1/R2) (1/meters)							
FOCAL PLANE ROTATION	16.	15.	angle of focal plane rotation (Degrees) See section 16 for details.							
SECOND ORDER CALCULATIONS	17.									
SEXTUPOLE	18.	LENGTH (meters)	FIELD (kg)	HALF-APERTURE (cm)						
SOLENOID	19.vv	LENGTH (meters)	FIELD (kg)							
BEAM ROTATION	20.v	ANGLE OF ROTATION (Degrees)								
STRAY FIELD	21.	See later section of report.								
Note: The v's following the type codes indicate the parameters which may be varied. See section under type code 10.0 for a detailed explanation of Vary Codes. The units are standard TRANSPORT units (as shown) unless changed via type code 15.0 entries.										

For Stanford TRANSPORT Users:

TRANSPORT is stored on disc in the program libraries of both the SLAC 360/91 and the Stanford Campus 360/67 computers. The call decks for these cases are listed below:

For batch or CRBE runs on the 360/91, use the following JCL if you wish the output to be printed on the line printers:

```
//jobname JOB   userid,bin#,CLASS=E1
//JOB LIB DD DSNAME=SYS1.USERLIB,DISP=SHR
//STEP1 EXEC PGM=TRANS
//FT04FO01 DD SYSOUT=B
//FT06FO01 DD SYSOUT=A
//FT05FO01 DD *
```

If you wish the output to be returned to the terminal for a CRBE run change SYSOUT=A to SYSOUT=R on line 5 of JCL.

If you desire to use the Campus 360/67 for either a Batch or Wylbur run, the JCL is as follows:

```
//jobname JOB   (acct#,bin#,1.0,05,001,,1,,X), 'your name',MSGLEVEL=1
//JOB LIB DD DSNAME=SYS2.PRGLIB,DISP=(OLD,PASS)
//STEP1 EXEC PGM=TRANSPORT
//FT06FO01 DD SYSOUT=A
//FT04FO01 DD UNIT=SYSCP,DCB=(RECFM=F,BLKSIZE=80)
//FT05FO01 DD *
```

The job terminator card for the 360/67 has a /* in columns 1,2.

-
- 1) ,TIME=(m,ss) should be included if your run will take over 30 seconds of CPU time. This will rarely be necessary, even if a large number of problems are included in the run. This JCL is valid for MVT, Release 18, HASP.

TITLE CARD

The title card is the first card in every TRANSPORT data set. The Title Card is always required and must be followed by a 0 or 1 card (see next section) to indicate whether the data to follow is new (0 card) or a continuation of a previous data set (1 card).

The title must be enclosed within single quotation marks on a single card. For TRANSPORT/360, the quotes may be entered in any column (Free field format).

Example of a DATA SET:

1st card 'SLAC 20/GeV/c SPECTROMETER'

2nd card 0

data

Last card SENTINEL

INDICATOR CARD (0 or 1)

The second card of the input is the indicator card. If the data which follow describe a new problem, a zero(0) is punched in any column on the card. If the data which follow describe changes to be made in the previous problem, a one(1) is punched in any column on the card. No other entries are allowed on this card.

The sample problem input which is included below causes TRANSPORT to do a first order calculation with fitting (0 indicator card) and then to do a second order calculation (1 indicator card) with the data that is the result of the fitting.

'FORTRAN H CHECK ON BETA FIT'

0 (Indicator Card)

1. .5 1. .5 1. .5 1. 1. ;

-17. 'SEC1' ;

3.3 2.745 'DR1' ;

2. 0. ; 4. 9.879 10. .5 ; 2. 0. ;

3.3 2.745 'DR1' ;

13. 4. ;

10. -1. 2. 0. .0001 'FIT1' ;

SENTINEL

'SECOND ORDER'

1 (Indicator Card)

17. 'SEC1' ;

3.0 'DR1' ;

-10. 'FIT1' ;

SENTINEL

COMMENT CARDS

Comment cards may be introduced in the data deck before any type code entry by inclosing the comments made on each card by single parentheses. No parentheses or quotation marks are allowed within the parentheses of any comment card.

Example of the use of comment cards in a data set:

'Title Card'

0

(THIS IS A TEST PROBLEM TO ILLUSTRATE THE)

(USE OF COMMENT CARDS)

data

(COMMENTS MAY ALSO BE MADE BETWEEN)

(TYPE CODE ENTRIES OF THE DATA SET)

data

SENTINEL

INPUT BEAM Type Code 1.0

This element specifies the phase space and the average momentum of the input beam for a TRANSPORT calculation. The input is specified by the semi-axes of a 6-dimensional erect* beam ellipsoid representing the phase space variables x, θ, y, φ, l , and δ . Each of these six parameters is entered as a positive quantity, but should be thought of as $\pm x, \pm \theta$, etc; hence, the total beam width is $2x$, the total horizontal beam divergence is 2θ and so forth.

Normally, the BEAM card is the third card in the deck if standard TRANSPORT units are to be used; otherwise it should immediately follow any special units cards (Type Code 15.) chosen for the calculation to be made. Standard TRANSPORT units for x, θ, y, φ, l , and δ are cms, mr, cms, mr, cms, and percent. The standard unit for the momentum $P(0)$ is GeV/c.

There are eight entries (all positive) to be made on the BEAM card.

- 1- The Type Code 1.0 (Specifies a BEAM entry follows)
- 2- One-half the horizontal beam extent (x) (cms in standard units)
- 3- One-half " " " divergence (θ) (mr).
- 4- One-half the vertical beam extent (y) (cms).
- 5- One-half " " " divergence (φ) (mr).
- 6- One-half the longitudinal beam extent (l) (cms).
- 7- One-half the momentum spread (δ) (in units of percent $\Delta P/P$)
- 8- The momentum of the Central Trajectory ($P(0)$) (GeV/c).

*For a rotated (non-erect) phase ellipsoid input, see Type Code 12.

All eight entries must be made even if they are zero(0) and the last entry must be followed by a semicolon. Thus a typical BEAM entry might be.

1.0 0.5 2.0 1.3 2.5 0. 1.5 10.0 ;

meaning, $x = \pm 0.5$ cms, $\theta = \pm 2.0$ mr, $y = \pm 1.3$ cms, $\varphi = \pm 2.5$ mr, $l = \pm 0.0$ cms, $\delta = \pm 1.5$ percent $\Delta P/P$, and the central momentum $P(0) = 10.0$ GeV/c.

The units of the tabulated matrix elements in either the 1st order R or sigma matrix or second order T matrix of a TRANSPORT printout will correspond to the units chosen for the BEAM card. For the above example, the $R(12) = (x/\theta)$ matrix element will have the dimensions of cms/mr; and the $T(236) = (\theta/y\delta)$ matrix element will have the dimensions mr per cm per percent $\Delta P/P$ and so forth.

The longitudinal extent l is useful for pulsed beams. It indicates the spread in length of particles in a pulse. It does not interact with any other component and may be set to zero if the pulse length is not important.

The phase ellipse (sigma matrix) beam parameters are normally printed as output after each data entry unless suppressed by a (13. 2. ;) type code entry. The projection of the semi-axes of the ellipsoid upon each of its six coordinate axes is printed in a vertical array and the correlations among these components indicating the phase ellipse orientations are printed in a triangular array (see the following pages).

THE PHASE ELLIPSE BEAM MATRIX

The beam matrix carried in the computer has the following construction:

	x	θ	y	φ	l	δ
x	$\sigma(11)$					
θ	$\sigma(21)$	$\sigma(22)$				
y	$\sigma(31)$	$\sigma(32)$	$\sigma(33)$			
φ	$\sigma(41)$	$\sigma(42)$	$\sigma(43)$	$\sigma(44)$		
l	$\sigma(51)$	$\sigma(52)$	$\sigma(53)$	$\sigma(54)$	$\sigma(55)$	
δ	$\sigma(61)$	$\sigma(62)$	$\sigma(63)$	$\sigma(64)$	$\sigma(65)$	$\sigma(66)$

The matrix is symmetric so that only a triangle of elements is needed.

In the printed output this matrix has a somewhat different format for ease of interpretation:

			x	θ	y	φ	l
x	$\sqrt{\sigma(11)}$	CM					
θ	$\sqrt{\sigma(22)}$	MR	r(21)				
y	$\sqrt{\sigma(33)}$	CM	r(31)	r(32)			
φ	$\sqrt{\sigma(44)}$	MR	r(41)	r(42)	r(43)		
l	$\sqrt{\sigma(55)}$	CM	r(51)	r(52)	r(53)	r(54)	
δ	$\sqrt{\sigma(66)}$	PC	r(61)	r(62)	r(63)	r(64)	r(65)

$$\text{where: } r(ij) = \frac{\sigma(ij)}{[\sigma(ii) \sigma(jj)]^{1/2}}$$

As a result of the fact that the σ matrix is positive definite, the $r(ij)$'s satisfy the relation

$$|r(ij)| \leq 1$$

The full significance of the $\sigma(ij)$'s and the $r(ij)$'s are discussed in detail in Appendix I ("Description of Beam Matrix"). The units are always printed with the matrix.

In brief, the meaning of the $\sqrt{\sigma(ii)}$'s is as follows:

$\sqrt{\sigma(11)} = x_{\max}$ = The maximum (half)-width of the beam envelope in the $x(\text{bend})$ plane at the point of the printout.

$\sqrt{\sigma(22)} = \theta_{\max}$ = The maximum (half)-angular divergence of the beam envelope in the $x(\text{bend})$ plane.

$\sqrt{\sigma(33)} = y_{\max}$ = The maximum (half)-height of the beam envelope.

$\sqrt{\sigma(44)} = \varphi_{\max}$ = The maximum (half)-angular divergence of the beam envelope in the $y(\text{non-bend})$ -plane.

$\sqrt{\sigma(55)} = l_{\max}$ = 1/2 the longitudinal extent of the bunch of particles.

$\sqrt{\sigma(66)} = \delta$ = The half-width 1/2 ($\Delta P/P$) of the momentum interval being transmitted by the system.

The units appearing next to the $\sqrt{\sigma(ii)}$'s in the TRANSPORT printout sheet are the units chosen for coordinates x, θ, y, φ, l and $\delta = \Delta P/P$ respectively.

To the immediate left of the listing of the beam envelope size in a TRANSPORT printout, there appears a column of numbers whose values will normally be zero. These numbers are the coordinates of the centroid of the beam phase ellipse (with respect to the initially assumed central trajectory of the system). They may become non-zero under one of three circumstances:

- 1) When the misalignment (Type Code 8.) is used.
- 2) When a Beam Centroid shift (Type Code 7.) is used.
- or 3) When a 2nd-order calculation (Type Code 17.) is used.

To aid in the interpretation of the phase ellipse parameters listed above, an example of an (x, θ) plane ellipse is illustrated below. For further details the reader should refer to the appendix of this report.

A Two Dimensional BEAM Phase Ellipse

The area of the ellipse is given by:

$$A = \pi (\det \sigma)^{1/2} = \pi x_{\max} \theta_{\text{int}} = \pi x_{\text{int}} \theta_{\max}$$

rms ADDITION TO THE BEAM Type Code 1.0

To allow for physical phenomena such as multiple-scattering, provision has been made in the program to permit an rms addition to the beam envelop to be made. There are nine entries to be included:

- 1 - Type Code 1.0 (specifying a BEAM entry follows)
- 2 - one-half the rms addition to the horizontal beam extent (Δx)(cms).
- 3 - one-half the rms addition to the horizontal beam divergence ($\Delta \theta$)(mr).
- 4 - one-half the rms addition to the vertical beam extent (Δy)(cms).
- 5 - one-half the rms addition to the vertical beam divergence ($\Delta \phi$)(mr).
- 6 - one-half the rms longitudinal beam extent (Δl)(cms).
- 7 - one-half the rms momentum spread ($\Delta \delta$)
(in percent $\Delta P/P$).
- 8 - the momentum change in the central trajectory ($\Delta P(0)$) in (GeV/c).
- 9 - the code digit 0. indicating an rms addition to to the beam is being made.

The units for the r.m.s. addition are the same as those selected for a regular beam Type Code 1.0 entry. Thus a typical r.m.s. addition to the beam would appear as follows:

1.0 0.1 0.2 0.15 0.3 0. 0.13 -0.1 0.0 ;

where the last entry (0.0) preceding the semicolon signifies an r.m.s. addition to the beam is being made and the next to the last entry indicates a central momentum change of -0.1 GeV/c .

FRINGING FIELDS and POLE-FACE ROTATIONS Type Code 2.0

To provide for fringing fields and/or pole-face rotations on bending magnets, the Type Code 2.0 element is used.

There are two parameters:

1 - Type Code 2.

2 - angle of pole-face rotation (degrees).

The Type Code 2. element must either immediately precede a bending magnet (Type Code 4.) element (in which case it indicates an entrance fringing field and pole-face rotation) or immediately follow a Type Code 4. element (exit fringing field and pole-face rotation) with no other data entries between.* A positive sign of the angle on either entrance or exit pole-faces corresponds to a transverse focusing action and radial defocusing action.

For example, a symmetrically oriented rectangular bending magnet whose total bend is 10 degrees would be represented by the three entries: 2. 5. ; 4. --- ; 2. 5. ;

The angle of rotation may be varied. For example, the element 2.1 5. ; would allow the angle to vary from an initial guess of 5 degrees to a final value which would, say, satisfy a vertical focus constraint imposed upon the system. See the Type Code 10. section for a complete discussion of vary codes.

Even if the pole-face rotation angle is zero, 2. 0. ; entries must be included in the data set before and after a Type Code 4.0 entry if fringing-field effects are to be calculated.

* It is extremely important that no data entries be made between a Type Code 2. and a Type Code 4. entry. If this occurs, it may result in an incorrect matrix multiplication in the program and hence an incorrect physical answer.

A single Type Code 2. entry that follows one bending magnet and precedes another will be associated with the latter.

Should it be desired to misalign such a magnet, an update must be forced prior to the first type 2. code entry and the convention appropriate to misalignment of a set of elements applied, since, indeed, three separate transformations are involved. See section under Type Code 8. for a discussion of misalignment calculations.

The type code signifying a rotated pole-face is 2.0. The input format is:

2.0 β . 'R0' ;

Label (if desired)

The units for β are degrees.

POLE-FACE ROTATION MATRIX

The first-order R matrix for a pole-face rotation used in a TRANSPORT calculation is as follows:

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{\tan \beta}{\rho} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\tan(\beta-\psi)}{\rho} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Definitions: β = angle of rotation of pole-face (see figure on following page for sign convention of β)

ρ = bending radius of central trajectory

g = total gap of magnet

ψ = correction term resulting from finite extent of fringing fields.**

where

$$\psi = K_1 \left(\frac{g}{\rho} \right) \left(\frac{1 + \sin^2 \beta}{\cos \beta} \right) \left[1 - K_1 K_2 \left(\frac{g}{\rho} \right) \tan \beta \right]^*$$

*See Type Code 16. for input formats for g , $K(1)$, and $K(2)$ TRANSPORT entries.

** See SLAC-Report No. 75 (page 74) for a discussion of ψ .

FIELD BOUNDARIES FOR BENDING MAGNETS

The TRANSPORT sign conventions for x , β , R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β 's imply transverse focusing. Positive R 's (convex curvatures) represent negative sextupole components of strength $S = (-\frac{h}{2R}) \sec^3 \beta$. (See SLAC-75, page 71).

DRIFT Type Code 3.0

A drift space is a field-free region through which the beam passes.

There are two parameters:

- 1 - Type Code 3. (specifying a drift length).
- 2 - (effective) drift length (meters).

Typical Input Format for a DRIFT

Label (if desired) (not to
exceed 4 spaces between
quotes)

3.0 6.0 'D1' ;

DRIFT SPACE MATRIX

The first-order R matrix for a DRIFT space is as follows:

$$\begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where

L = the length of the drift space.

The dimensions of L. are those chosen for longitudinal length via a

(Type Code 1.) card. If no 15. 8. entry is made, the units of L. will automatically be in METERS (Standard TRANSPORT/360 units).

WEDGE BENDING MAGNET Type Code 4.0

A wedge bending magnet implies that the central trajectory of the beam enters and exits perpendicularly to the pole-face boundaries (to include fringing-field effects and non-perpendicular entrance or exit boundaries--see Type Codes 2.0 and 16.0).

There are four first-order parameters to be specified for the wedge magnet via Type Code 4.0**

- 1 - Type Code 4.0 (specifying a wedge bending magnet)
- 2 - The (effective) length L of the central trajectory.
(meters)
- 3 - The central field strength $B(0)$ in KG,
 $B(0) = 33.356 (P/\rho)$

where P is the momentum in GeV/c and ρ is the
bending radius of the central trajectory in meters.

- 4 - The field gradient (n-value) dimensionless; where n is
defined by the equation $B_y(x,0,t) = B_y(0, 0, t)[1-nhx + \dots]$,

where

$$h = \frac{1}{\rho_0} . \text{ See SLAC-75 (page 31).}$$

$B(0)$, and n may be varied for 1st order fitting (see
section 10.0 for a discussion of vary codes).

A typical first-order TRANSPORT/360 input for a wedge magnet is:

4.0 L. B. n. ' ' ;
 ↓ Label (not to exceed 4 spaces)

** See Type Code 16. for special parameter entries affecting
fringing field and/or second-order calculations for bending
magnets.

If fringing field effects are to be included, a Type Code 2. entry must precede and follow the pertinent Type Code 4. entry (even if there are no pole-face rotations.)** Thus a typical TRANSPORT/360 input for a bending magnet including fringing fields might be:

Labels (not to exceed 4 spaces) if desired

```

2. 0. ' ' ;
4. L. B(0). n. ' ' ;
2. 0. ' ' ;

```

For non-zero pole-face rotations a typical data input might be:

```

2. 10. ; 4. L. B(0). n. ; 2. 20. ;

```

note that the use of labels is optional and that all data entries may be made on one line if desired.

The sign conventions for bending magnet entries are illustrated in the following figure (748A15). For TRANSPORT/360, a positive bend is to the right looking in the direction of particle travel. To represent a bend in another sense, the coordinate rotation matrix (Type Code 20.) should be used as follows:

A bend up is represented by rotating the (x, y) coordinates by - 90.0 degrees about the (z) beam axis as follows:

Labels (not to exceed 4 spaces) if desired

```

20.  -90.  '  '  ;
2.0  β(1).  '  '  ;
4.0  L.   B.   n.  '  '  ;
2.0  β(2).  '  '  ;
20.  +90.  '  '  ; (returns coordinates to their initial
                    orientation)

```

A bend down is accomplished via:

```

20.  +90.  '  '  ;
2.
4.
2.
20.  -90.  '  '  ;

```


A bend to the left (looking in the direction of beam travel) is accomplished by rotating the x, y coordinates by 180 degrees, e.g.

20. 180. ' ' ;

2.

4.

2.

20. -180 ' ' ;

FIELD BOUNDARIES FOR BENDING MAGNETS

The TRANSPORT sign conventions for x , β , R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β 's imply transverse focusing. Positive R 's (convex curvatures) represent negative sextupole components of strength $S = (-\frac{h}{2R}) \sec^3 \beta$. (See SLAC-75, page 71).

1st ORDER
WEDGE BENDING MAGNET MATRIX

$\cos k_x L$	$\frac{1}{k_x} \sin k_x L$	0	0	0	$\frac{h}{k_x^2} [1 - \cos k_x L]$
$-k_x \sin k_x L$	$\cos k_x L$	0	0	0	$(\frac{h}{k_x}) \sin k_x L$
0	0	$\cos k_y L$	$\frac{1}{k_y} \sin k_y L$	0	0
0	0	$-k_y \sin k_y L$	$\cos k_y L$	0	0
$-\frac{h}{k_x} \sin k_x L$	$-\frac{h}{k_x^2} [1 - \cos k_x L]$	0	0	1	$-\frac{h^2}{k_x^3} [k_x L - \sin k_x L]$
0	0	0	0	0	1

Definitions: $h = \frac{1}{\rho_0}$, $k_x^2 = (1-n)h^2$, $k_y^2 = nh^2$

$\alpha = hL =$ The angle of bend

$L =$ path length of the central trajectory

The field expansion for the midplane of a bending magnet is taken from eq(18) page 31 of SLAC-75 thereby defining the dimensionless quantities n and β as follows:

$$B_y(x,0,t) = B_y(0,0,t) [1 - nhx + \beta h^2 x^2 + \gamma h^3 x^3 + \dots] \quad (18)$$

The type code signifying a BEND is 4.0. The input format for a TRANSPORT/360 calculation is:

4.0 L. B. n. ' ' ;

Label (not to exceed 4 spaces)

If n is not included in the data entry, the program assumes it to be zero. A β entry for a second order calculation is made via the 16. 1. element. (Do Not confuse this β with a pole-face rotation)

The standard units for L. and B. are Meters and KG. If desired, these units may be changed by 15. 8. and 15. 9. type code entries preceding the BEAM Card.

748A10

ILLUSTRATION OF THE MAGNETIC MIDPLANE (x AXIS) FOR DIPOLE, QUADRUPOLE AND SEXTUPOLE ELEMENTS. THE MAGNET POLARITIES INDICATE MULTIPOLE ELEMENTS THAT ARE POSITIVE WITH RESPECT TO EACH OTHER.

QUADRUPOLE Type Code 5.0

A quadrupole provides focusing in one transverse plane and defocusing in the other.

There are four parameters to be specified for a TRANSPORT calculation:

- 1 - Type Code 5. (specifying a quadrupole).
- 2 - (effective) magnet length L (in meters).
- 3 - Field at pole tip B (in KG). A positive field implies horizontal focusing; a negative field, vertical focusing.
- 4 - Half-aperture a (in cm). Radius of the circle tangent to the pole tips.

The strength of the quadrupole is computed from its field, aperture and length. The horizontal focal length is printed as output; a positive focal length indicates horizontal focusing and a negative focal length indicates horizontal defocusing.

The type code for a QUAD is 5.0. The input format for a typical data set is:

└── Label (if desired) not to exceed 4 spaces
 between quotes

5.0 L. B. a. ' ' ;

The data may be entered in FREE FIELD format. The standard TRANSPORT/360 units for L. B. and a. are Meters, KG, and cms respectively. If other units are desired they must be chosen via the appropriate 15. type code entries preceding the BEAM (Type Code 1.) card.

1st ORDER
QUADRUPOLE MATRIX

$\cos k_q L$	$\frac{1}{k_q} \sin k_q L$	0	0	0	0
$-k_q \sin k_q L$	$\cos k_q L$	0	0	0	0
0	0	$\cosh k_q L$	$\frac{1}{k_q} \sinh k_q L$	0	0
0	0	$k_q \sinh k_q L$	$\cosh k_q L$	0	0
0	0	0	0	1	0
0	0	0	0	0	1

These elements are for a quadrupole which focuses in the horizontal (x) plane (B positive). A vertically (y plane) focusing quadrupole (B negative) has the two partial matrices interchanged.

Definitions: L = the effective length of the quadrupole

a = the radius of the aperture

B_o = the field at radius a

$$k_q^2 = \left(\frac{B_o}{a} \right) \left(\frac{1}{B\rho} \right)$$

where (Bρ) = the magnetic rigidity (momentum) of the central trajectory.

ILLUSTRATION OF THE MAGNETIC MIDPLANE (x AXIS) FOR DIPOLE, QUADRUPOLE AND SEXTUPOLE ELEMENTS. THE MAGNET POLARITIES INDICATE MULTIPOLE ELEMENTS THAT ARE POSITIVE WITH RESPECT TO EACH OTHER.

TRANSFORM 1 UPDATE* Type Code 6. 0. 1.

To provide a facility for updating TRANSFORM 1 (the product of the R matrices, R1) use Type Code 6.0. Thus a (6. 0. 1. ;) forces an update of the R1 matrix and initiates the accumulation of a new product matrix initialized from the point of the update. This facility is useful in conjunction with misaligning a set of magnets or fitting only a portion of a system.

The complete list of elements which update TRANSFORM 1 is:

- 1) The (6. 0. 1. ;) entry
- 2) Any fitting constraint imposed upon the beam (sigma) matrix
(See Type Code 10. for a discussion of this)
- 3) A misalignment Type Code 8.0 entry
- 4) A beam Type Code 1.0 entry

A TRANSFORM 1 matrix will be printed at any position in the data set where a (13. 4. ;) entry is inserted.

See the following section for the introduction of an auxiliary transformation matrix R2 (TRANSFORM 2) to circumvent the problems arising from TRANSFORM 1 updates.

* By "updating" we mean initiating a new starting point for the accumulation (multiplication) of the R matrix. i.e. at the point of update, the R matrix is assumed to be reset to the unity matrix and all previous accumulations are forgotten.

AUXILIARY TRANSFORMATION MATRIX (R2) Type Code 6. 0. 2.

To circumvent the difficulties caused by frequent beam updates and consequent redefinition of the cumulative transformation matrix, R1, provision has been made for introducing an independent transformation matrix, the R2 matrix.

This matrix is not normally accumulated, and must be introduced by defining its starting point with a (6. 0. 2. ;) card. This variant of the Type 6.0 code also serves to redefine the starting point of the R2 matrix (update it) when desired.

R2 has no effect upon the BEAM (sigma) matrix as does R1 nor do any constraints imposed upon the BEAM (sigma) matrix react upon it. It is solely a convenience for the user. It may be printed by a (13. 24. ;) entry. Any components of R2 may be constrained by a Type Card 10. entry with code digits: $-(i + 20.)$ and j .

For example, 10. -1. 2. 0. .01 ; and 10. -21. 2. 0. .01 ; are identical constraints applied to the R1 and R2 matrices, respectively.

R2 is updated by a beam (1.) element, provided R2 has been introduced, but by no other elements. No provision has been made for suppressing R2 once it is introduced.

SHIFT IN THE BEAM CENTROID Type Code 7.0

Sometimes it is convenient to redefine the BEAM centroid** such that it does not coincide with the TRANSPORT reference trajectory. Provision has been made for this possibility via Type Code 7.0. Seven parameters are required:

1 - Type Code 7.

(2 to 7)- the coordinates x, θ, y, φ, l , and δ defining the initial conditions of the new beam centroid with respect to the reference trajectory. The units for $x, \theta, y, \varphi, l, \delta$ are the same as those chosen for the beam (Type Code 1.0 entry), normally cm, mr, cm, mr, cm, and percent.

Only the beam (σ) matrix is affected by this code. The transformation matrices R1 and R2 are unaffected.

In order for this code to function properly, the initial BEAM entry (Type Code 1.0) must have a non-zero phase space volume.

e.g. a

1. 0 0 0 0 0 0 P(0). ;

beam entry is not permissible when calculating a shift in the beam centroid; whereas a

1.0 1. 1. 1. 1. 1. 1. P(0). ;

entry (finite phase volume) will work.

** By "Beam Centroid" we mean the center of the beam ellipsoid.

MAGNET ALIGNMENT TOLERANCES Type Code 8.0

The misalignment of a magnet to first order affects only the centroid of the beam. Two situations are commonly encountered: the magnet is displaced and/or rotated by a known amount; or the actual position of the magnet is uncertain within a given tolerance. Both these effects may be simulated through use of the 'align' element.

There are eight parameters to be specified:

- 1 - Type Code 8. (specifying alignment).
- 2 - the magnet displacement in the horizontal direction (cm).
- 3 - a rotation about the horizontal axis (mr).
- 4 - a displacement in the vertical direction (cm).
- 5 - a rotation about the vertical axis (mr).
- 6 - a displacement in the beam direction (cm).
- 7 - a rotation about the beam direction (mr).
- 8 - a three digit code number (defined below)
specifying the type of misalignment.

The coordinate system employed is that to which the beam is referred at the point it enters the magnet. For example, a rotation of a bending magnet about the beam direction (parameter 7 above) is referred to the direction of the beam where it enters the magnet. The units employed are the standard TRANSPORT units shown above unless redefined by Type Code 15. entries. If the units are changed, the units of the misalignment displacements are those determined by the 15. 1. type code entry; the units for the misalignment rotations are those determined by the 15. 2. type code entry.

Only the misalignments of bending magnets or quadrupoles may be simulated. The Type Code 8.0 align element(s) must directly follow the magnetic element(s) to be misaligned.

When the actual position of the magnet(s) is uncertain within a given tolerance, the BEAM card entry at the beginning of the system should correspond to a zero phase space input, i.e., the BEAM card should appear as follows:

```
1.0  0.  0.  0.  0.  0.  0.  P(0)  ;
```

The phase ellipse printouts following the misalignment will then possess a finite phase volume which represents the uncertainty in the position of the beam centroid resulting from the uncertainty in the knowledge of the positioning of the magnet(s).

When the misalignment is a known amount, the BEAM card entry at the beginning of the system should possess a non-zero phase volume. (e.g. a

```
1.0  1.  1.  1.  1.  1.  1.  P(0).  ;
```

is a permissible Type Code 1.0 entry.)

The beam centroid printouts following the misalignment will then show a definite shift in the position of the beam centroid resulting from the misalignment of the magnet(s).

An align element updates the BEAM (sigma) matrix and the R1 matrix but not the R2 matrix.

The following three digit code numbers provide several alternative misalignment situations:

A. The units position distinguishes between a single type code entry and a set of entries:

CODE NUMBER	INTERPRETATION
XX0.	In this case, the 0 means that the information contained on the align card refers only to the <u>single</u> magnet (type code entry) immediately preceding it.
XX1.	With this option the 1 means that all of the type code elements since the last R1 matrix update (a BEAM CARD entry, a constraint upon the BEAM (sigma) matrix or a 6. 0. 1. ; type code entry) are treated as a unit and the misalignment information on the card is applied to the unit as a whole. This option should be used for the misalignment of a bending magnet when fringing-fields or pole face rotations (Type Code 2.0) are included. See Examples 1 and 2 below for an illustration of this. Only <u>one</u> bending magnet may be included in the array. Another example of the use of this option is in the misalignment of an array of quadrupoles. By successive application of align elements, for example, the elements of a quadrupole doublet could be misaligned relative to each other and then the doublet as a whole could be misaligned.

XX2. | With this option the 2 means that all of the type code elements since the last R2 matrix update (a 6. 0. 2. ; type code entry) are treated as a unit. This code digit, for example, is necessary for studying misalignment of quadrupole triplets and other combinations involving more than two quadrupole magnets. It makes use of the fact that the R2 matrix remains unaffected by the usual update codes.

B. The tens position defines the system axis along which the succeeding magnets are positioned:

X0X. | The 0 means they are positioned along the same axis as if the magnet were not displaced.

X1X. | The 1 means they are positioned along the axis defined by the magnetic axis of the displaced magnet.

For instance, if a quadrupole is rotated, the remainder of the system may be left alone (X0X.) or rotated with the quadrupole X1X.

C. The hundreds position distinguishes between an uncertainty in position (0XX.) or a known displacement (1XX.).

Any combination of digits may be used to define the exact circumstances intended. Thus, code 111. (= 1. + 10. + 100.) indicates the deliberate displacement of a set of magnets and the remainder of the system (referred to the point the beam enters the set). Code 101. would leave the remainder of the system alone.

Typical code numbers for magnet alignment tolerances might be:

Code Number	Interpretation
000.	The reference trajectory is unchanged. Only the preceding magnet is misaligned and by an uncertain amount (but within the specified tolerances).
001.	The reference trajectory is unchanged. All magnets (type code entries) between the last R1 matrix update and the Type Code 8. entry are misaligned by an uncertain amount (but within the specified tolerances).
002.	Same as above except replace R1 with R2 in the statement
100.	The reference trajectory is unchanged. Only the preceding magnet is misaligned by the specified amount.
101.	The reference trajectory is unchanged. All magnets between the last R1 matrix update and the type code 8 entry are misaligned by the specified amount.
102.	Same as above except replace R1 with R2 in the statement

111. | The axis of the misaligned magnet(s) is the
 | new reference trajectory for all subsequent
 | magnets. All magnets between the last R1 matrix
 | update and the type code 8 alignment entry are
 | misaligned by the specified amount

The tolerances may be varied. Thus, type-vary code 8.111111 permits any of the six parameters (2 through 7 above) to be adjusted to satisfy whatever BEAM constraints may follow. (See the section under Type Code 10. for a discussion of the use of vary-codes.)

Example No. 1: A misaligned bending magnet

A bending magnet (including fringing-fields) misaligned by a known amount might be represented as follows:

```
3.0  L(1).  ;  
6.   0.   1.  ;  
2.0  0.0   ; 4.0  L.  B.  n.  ; 2.0  0.0   ;  
8.0  0.0  0.0  0.0  0.0  0.0  2.0  101.  ;  
3.0  L(2).  ;
```

This represents a known rotation of the bending magnet about the incoming beam direction (z-axis) by 2.0 mr. The result of this misalignment will be a definite shift in the beam centroid. The 6. 0. 1. ; transform 1 update is necessary because of the use of the 2.0 0.0 ; entries before and after the bending magnet. The code number

XX1. is required because the magnetic array (bending magnet + fringing-fields) consists of three type code elements instead of one.

Example No. 2: A misaligned bending magnet

A bending magnet having an uncertainty of 2 mr in its angular positioning about the incoming beam (z-axis) would be represented as follows:

```
3.0 L(1) ;
6. 0. 1. ;
2.0 0.0 ; 4.0 L. B. n. ; 2.0 0.0 ;
8.0 0.0 0.0 0.0 0.0 0.0 2.0 001. ;
3.0 L(2) ;
```

The result of this misalignment will be an uncertainty in the knowledge of the outgoing beam centroid - hence to observe this, the input Beam Card should have zero phase space volume as follows:

```
1.0 0. 0. 0. 0. 0. 0. P(0) ;
```

Example No. 3: A misaligned quadrupole triplet

One typical use of the R2 matrix is to permit the misalignment of a triplet. For example, an uncertainty in the position of the following triplet

```
5.  1.  -8.  10. ;  
5.  2.  +7.  10. ;  
5.  1.  -8.  10. ;
```

may be induced by appropriate 8. elements as noted.

```
6.  0.  2. ;  
5.  1. -8.  10. ;  
6.  0.  1. ;  
5.  2. +7.  10. ;  
5.  1. -8.  10. ;  
8.  --- --- --- --- --- --- 000. ;  
8.  --- --- --- --- --- --- 001. ;  
8.  --- --- --- --- --- --- 002. ;
```

The first 8. card in the list refers to misalignment of the third magnet only. The second 8. refers to misalignment of the second and third magnets as a unit via the R1 matrix update (the 6. 0. 1. ; entry). The last 8. refers to misalignment of the whole triplet as a unit via the R2 matrix update (the 6. 0. 2. ; entry).

The BEAM card (Type Code 1.0 entry) should have zero phase space volume as shown in Example 2 above.

REPETITION Type Code 9.0

Many systems include a set of elements that are repeated several times. To minimize the chore of input preparation, a 'repeat' facility has been added.

There are two parameters:

- 1 - Type Code 9.
- 2 - code digit. If non-zero, it states the number of repetitions desired from the point it appears. If zero it marks the end of a repeating unit.

For example, a total bend of 12 degrees composed of four 3-degree bending magnets each separated by 0.5 meters could be represented by 9.0 4. ; 4. --- ; 3. .5 ; 9. 0. ; Those elements (in this case a bend and drift) between the 9. 4. ; and 9. 0. ; would be employed four times.

There is no indication of the 9. cards in the printed TRANSPORT output, save the repeated listing of the elements they control. Vary codes may be used within a repeating unit in the usual fashion.

Repeat cards may be nested four deep. By "nesting" we mean a repeat within a repeat. An example is given below.

Example of Nesting:

The total length of this sequence is:

$$2*(10. + 3*(20 + 4* 50) + 1.5) = 1343.$$

FIRST-ORDER VARY CODES AND FITTING CONSTRAINTS

Type Code 10.

TRANSPORT/360 has the capability to vary some (not all) of the physical parameters of the elements comprising a beam transport system in order to fit selected matrix elements of either the R1 or R2 transformation matrices or the BEAM (sigma) matrix to prescribed values. Special constraints are also available. The physical parameters to be varied are selected via 'Vary Codes' attached to the Type Codes of the elements comprising the system; and the fitting constraints imposed upon the R and/or σ matrix elements are selected via Type Code 10. entries made at the location in the system where the constraint is to be imposed.

Vary Codes

Associated with the physical elements comprising a system is a vary code which specifies which physical parameters of the element may be varied. This code occupies the fraction portion of the type code specifying the element. It has one digit for each parameter, the digits having the same order in the code as the physical parameters have on the card. A '0' indicates the parameter may not be varied; a '1' that it may be. For instance, 3.0 is the combined type(3) and vary code(0) for a drift length which is to remain fixed; 3.1 indicates a drift length that may be varied (by the virtue of the .1). The type code 4.010 indicates a bending magnet with a variable magnetic field. In punching the code 3.0, the zero need not be punched. In punching the 4.010 code, the first zero must be punched but the second zero need not be.

The following parameters may be varied (0 or 1 may be placed in positions marked v; only 0 may be placed in positions marked 0):

BEAM.... 1.vvvvvv - all components of the input beam may be varied.

ROTAT... 2.v - the pole-face angle of a bending magnet may be varied.

DRIFT... 3.v - the drift length may be varied.

BEND.... 4.0vv - the length may not be varied; the field (first v) and/or the n-value (second v) may be varied.

QUAD.... 5.vv0 - the length may be varied; the field may be, the aperture may not be.

ALIGN.... 8.vvvvvv0 - any of the alignment parameters may be varied.

MATRIX....14.vvvvvv0 - any of the matrix elements may be varied.

SOLENOID.... 19.vv - the length and/or field may be varied.

BEAM ROTATION.... 20.v - the angle of rotation may be varied.

The use of the permissive 'may' rather than the imperative 'will' in discussing variables is meaningful. The program will choose the parameters it will vary from among those that it may vary. In general it chooses to vary those parameters that have the greatest influence upon the conditions to be fit.

Coupled Vary Codes

It is possible to apply the same correction to each of several variables. This may be done by replacing the digit 1 in the vary code with one of the digits 2 through 9. All variables whose vary digit has one of these values (and appears in the same position within the vary code) will receive the same correction. For example, the three type-vary codes (5.02, 5.01, 5.02) might represent a symmetric triplet. The same correction will be made to the first and third quadrupoles, guaranteeing that the triplet will remain symmetric. Variables whose vary digits appear in different positions will not be tied together (the vary digit positions are completely independent). Thus, the codes (8.666666, 3.6) apply the same correction to the drift length as to the horizontal misalignment, but do not restrict the six misalignment parameters with respect to each other. (Such a combination would never arise in practice.)

If the vary digit 9 is used, the correction associated with the vary digit 4 will be subtracted rather than added to this variable. Thus, the type-vary codes (3.4, 5.01, 3.9) will slide the quadrupole back and forth within a prescribed drift length, but will not change the total drift length. The vary digits (8 and 3), and (7 and 2) are coupled in a similar manner as are (9 and 4). This option may not be used with Type Code elements 1. or 8.

The total number of independent variables in a run is limited to 10 by reasons of the mathematical method of fitting. So far as this limit is concerned, variables that are tied together count as one variable. Also note that variables within repeat elements (Type Code 9.) count only once.

R1 MATRIX FITTING CONSTRAINTS

There are five parameters to be specified when imposing a constraint upon the (i, j) element of an R1 matrix.

- 1 - Type Code 10. (specifying that a fitting constraint follows)
- 2 - code digit (-i).
- 3 - code digit (j).
- 4 - desired value of the (i, j) matrix element.
- 5 - desired accuracy of fit (standard deviation).

Some typical R1 matrix constraints are as follows:

Desired Optical Condition	Typical Fitting Constraint
<u>point to point focus:</u>	
Radial plane R(12) = 0	10. -1. 2. 0. .0001 'F1' ;
Transverse plane R(34) = 0	10. -3. 4. 0. .0001 'F2' ;
<u>'parallel' to point focus:</u>	
Radial plane R(11) = 0	10. -1. 1. 0. .0001 'F3' ;
Transverse plane R(33) = 0	10. -3. 3. 0. .0001 'F4' ;
<u>achromatic beam:</u>	
Radial plane R(16) = R(26) = 0	10. -1. 6. 0. .0001 'F5' ;
	10. -2. 6. 0. .0001 'F6' ;

Note that any fitting constraint on an R1 matrix element is from the preceding update of the R1 matrix. An R1 matrix is updated by a (6. 0. 1. ;) entry or by any fitting constraint imposed upon the σ (beam) matrix.

R2 MATRIX FITTING CONSTRAINTS

There are five parameters to be specified when imposing a constraint upon the (i, j) element of an R2 matrix.

- 1 - Type Code 10.
- 2 - Code digit $-(20 + i)$.
- 3 - Code digit (j).
- 4 - desired value of the (i, j) matrix element.
- 5 - desired accuracy of fit (standard deviation).

Some typical R2 matrix constraints are as follows:

Desired Optical Condition	Typical Fitting Constraint
<u>point to point focus:</u>	
Radial plane $R(12) = 0$	10. -21. 2. 0. .0001 'F1' ;
Transverse plane $R(34) = 0$	10. -23. 4. 0. .0001 'F2' ;
<u>'parallel' to point focus:</u>	
Radial plane $R(11) = 0$	10. -21. 1. 0. .0001 'F1' ;
Transverse plane $R(33) = 0$	10. -23. 3. 0. .0001 'F2' ;
<u>achromatic beam:</u>	
Radial plane $R(16) = R(26) = 0$	10. -21. 6. 0. .0001 'F3' ;
	10. -22. 6. 0. .0001 'F4' ;

Note that the R2 matrix is updated by a (6. 0. 2. ;) type code entry only.

σ (BEAM) MATRIX FITTING CONSTRAINTS

There are five parameters to be specified when imposing a constraint upon the (i, j) element of a σ (BEAM) matrix.

- 1 - Type Code 10.n
- 2 - Code digit (1). ($i \geq j$)
- 3 - Code digit (j).
- 4 - desired value of the (i, j) matrix element.
- 5 - desired accuracy of fit (standard deviation).

(n) is normally zero or blank. If $n = 1$, then entry 4 is taken to be a lower limit on the matrix element. If $n = 2$, entry 4 is taken to be an upper limit. If $i = j$, then the value inserted in entry 4 is the desired beam size $(\sigma(ii))^{1/2}$ e.g. $X(\max) = (\sigma(11))^{1/2}$ etc.

Note:

Any constraint imposed upon the σ (BEAM) matrix has an important side effect, it always updates the R1 matrix (but not the R2 matrix) at the location where the constraint is imposed.

Some typical σ matrix constraints are as follows:

Desired Optical Condition	Typical Fitting Constraint
<u>Beam Waist</u>	
Radial Waist $\sigma(21) = 0$	10. 2. 1. 0. .001 'F1' ;
Transverse Waist $\sigma(43) = 0$	10. 4. 3. 0. .001 'F2' ;
Fit Beam size to $X(\max) = 1\text{cm}$	10. 1. 1. 1. .001 'F3' ;
Fit Beam size to $Y(\max) = 2\text{cm}$	10. 3. 3. 2. .001 'F4' ;
Limit Max beam size to $X = 2\text{cm}$	10.2 1. 1. 2. .01 'F5' ;
Limit Min beam size to $Y = 1\text{cm}$	10.1 3. 3. 1. .01 'F6' ;

In general, it will be found that achieving a satisfactory 'beam' fit with TRANSPORT is more difficult than achieving an R matrix fit. When difficulties are encountered, it is suggested that the user 'help' the program by employing sequential (step by step) fitting procedures when setting up his data set. More often than not a "failure to fit" is caused by the user requesting the program to find a physically unrealizable solution.

System Length Constraint

A running total of the lengths of the various elements encountered is kept by the program and may be fit. The code digits are $i = 0.$, $j = 0.$

Thus the element (10. 0. 0. 150. 5. ;) would make the length of the system prior to this element equal to 150 ± 5 meters. Presumably there would be a variable drift length somewhere in the system. By redefining the cumulative length via the (16. 6. L. ;) element, partial system lengths may be accumulated and fit.

AGS Machine Constraint*

Provision has been made in the program for fitting the betatron phase shift angle, μ , associated with usual AGS treatment of magnet systems.

In the horizontal plane: use code digits $i = -11.$, $j = 2.$ and specify:

$$\Delta = \frac{1}{2\pi} \cos^{-1} \left[0.5 (R_{11} + R_{22}) \right] = \frac{\mu}{2\pi} \text{ (horiz)} \\ = \text{freq}/(\text{no. of periods}).$$

In the vertical plane: $i = -13.$, $j = 4.$, and

$$\Delta = \frac{1}{2\pi} \cos^{-1} \left[0.5 (R_{33} + R_{44}) \right] = \frac{\mu}{2\pi} \text{ (vert)}$$

For example, if there are 16 identical sectors to a proposed AGS machine and the betatron frequencies per revolution are to be 3.04 and 2.14 for the horizontal and vertical planes respectively, then the last element of the sector should be followed by the constraints:

10. -11. 2. .190 .0001 ;

10. -13. 4. .134 .0001 ;

$$\text{i.e. } \frac{3.04}{16} = 0.190 \text{ and } \frac{2.14}{16} = 0.134$$

* See Courant and Snyder "Theory of the Alternating-gradient synchrotron", Annals of Physics 3; pp1-48 (1958).

For example: A typical data listing might be:

9.0 16.0 ;

5.01 --- ;

3.0 --- ;

5.01 --- ;

10. -11. 2. 0.190 .0001 ;

10. -13. 4. 0.134 .0001 ;

9.0 0.0 ;

First Moment Constraint

Misalignments and second order effects cause the center (centroid) of the phase ellipsoid to be shifted from the reference trajectory, i.e., they cause the beam to have a non-zero first moment. The first moments appear in a vertical array to the left of the vertical array which give the $\sqrt{\sigma_{ii}}$'s. The units of the corresponding quantities are the same.

It is perhaps helpful to emphasize that the origin always lies on the reference trajectory. First moments refer to this origin. However, the ellipsoid is defined with respect to its center, so the covariance matrix, as printed, defines the second moment about the mean.

First moments may be fitted. The code digits are $i = 7.$ and j , where j is the index of the quantity being fit. Thus 10. 7. 1. .1 .01 ; constrains the horizontal (1.) displacement of the ellipsoid to be 0.1 ± 0.01 cm.

This constraint is useful in deriving the alignment tolerances of a system or in warning the system designer to offset the element in order to accommodate a centroid shift.

Internal Constraints

A set of upper and lower bounds on the value of each type of parameter is in the memory of the program. If a correction is computed for a parameter which would take its value outside this range, it is reset to the limit of the range, and a constraint constructed to keep it within range. The current limits are:

Type Code	Limits
3.0	$0.1 < \text{drift} < 1000 \text{ (m)}$
5.0	$.01 < \text{quad length} < 10 \text{ (m)}$
5.0	$-14 < \text{quad field} < 14 \text{ (kg)}$
4.0	$-20 < \text{bend field} < 20 \text{ (kg)}$
4.0	$-500 < \text{bend gradient} < 500$
2.0	$-60 < \text{pole-face rotation} < 60 \text{ (deg)}$
8.0	$-1 < \text{alignment displacement} < 25 \text{ (cm)}$
8.0	$-50 < \text{alignment rotation} < 50 \text{ (mr)}$
1.0	$0.01 < \text{input beam} < 1000 \text{ (for all parameters)}$
20.0	$-360 < \text{beam rotation} < 360 \text{ (deg)}$

These limits apply only when a parameter is being varied. Fixed values that exceed this range may be used as desired.

These constraints were included to avoid physically meaningless solutions. However, they are rather ineffective since systems that require values outside these limits usually have some basic design difficulty.

The limits are not adjusted for different input units. So if drift lengths are expressed in inches, the effective limits are $0.1 < \text{drift} < 1000 \text{ (in.)}$ etc.

ACCELERATION Type Code 11.0

An energy gain is reflected in both the divergence and the width of the beam. This element provides a simulation of a traveling wave linear accelerator energy gain over a field free drift length. (i.e. no externally applied magnetic field)

There are five parameters:

- 1 - Type Code 11.
- 2 - accelerator length (meters).
- 3 - energy gain (GeV).
- 4 - ϕ (phase lag in degrees).
- 5 - λ (wavelength in cms).

The new beam energy is printed as output.

The energy of the reference trajectory is assumed to increase linearly over the entire accelerator length. If this is not the case, an appropriate model may be constructed by combining separate 11. elements. An 11. element with a zero energy gain is identical to a drift length.

None of the parameters may be varied.

Second order matrix elements have not been incorporated in the program for the accelerator section.

ACCELERATOR SECTION MATRIX

11-2

$$\begin{bmatrix}
 1 & \left[L \frac{E_0}{\Delta E \cos \varphi} \ln \left(1 + \frac{\Delta E \cos \varphi}{E_0} \right) \right] & 0 & 0 & 0 & 0 \\
 0 & \frac{E_0}{E_0 + \Delta E \cos \varphi} & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & \left[L \frac{E_0}{\Delta E \cos \varphi} \ln \left(1 + \frac{\Delta E \cos \varphi}{E_0} \right) \right] & 0 & 0 \\
 0 & 0 & 0 & \frac{E_0}{E_0 + \Delta E \cos \varphi} & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & \left(\frac{\Delta E \sin \varphi}{E_0 + \Delta E \cos \varphi} \right) \left(\frac{2\pi}{\lambda} \right) & \frac{E_0}{E_0 + \Delta E \cos \varphi}
 \end{bmatrix}$$

Definitions:

L = effective length of accelerator sector

E_0 = particle energy at start of sector

ΔE = energy gain over sector length

φ = phase lag of the reference particle behind the crest of the accelerating wave. i.e., if φ is positive then for some $z > 0$ the particles having this value are riding the crest of the wave. The units of φ are degrees.

λ = wave length of accelerating wave. The units of λ are those of z (normally cm).

This matrix element assumes that $E_c \gg m_0 c^2$ (fully relativistic).

BEAM (ROTATED ELLIPSE) Type Code 12.0

To allow the output beam from some point in a system to become the input beam of some succeeding system, provision has been made for re-entering the correlation matrix which appears as a triangular matrix in the beam output. (See section under Type Code 1.0 and/or the appendix for definitions.)

There are 16 parameters:

1 - Type Code 12.

2 to 16 - the 15 correlations ($r(ij)$'s) among the 6 beam components - in the order printed (by rows).

Several cards may be used to insert the 15 correlations, if necessary.

Since this element is solely an extension of the beam input, a 12. element must be preceded by a 1. (beam) element entry.

OUTPUT PRINT CONTROL INSTRUCTIONS Type Code 13.0

A number of control codes which transmit output print instructions to the program have been consolidated into a single type code:

There are two parameters:

- 1 - Type Code 13.
- 2 - code number.

The effects of the various code numbers will be described below (not in numerical order).

PRINTED OUTPUT CONTROLS 1., 2., 3., 4., 24.

Several codes are available to control various aspects of the printed output. Most type codes produce a line of output that advertises their existence. Those that do not, usually have an obvious effect upon the remainder of the output and thus make their presence clear.

(13. 2. ;): This code instructs the program to suppress the printing of the beam (σ) matrix.

(13. 1. ;): This code temporarily overrides the (13. 2. ;) suppress code. It causes a single beam matrix to be printed at that point in the system.

(13. 3. ;): This code permanently overrides the (13. 2. ;) suppress code and restores the normal code in which every beam matrix is printed.

(13. 4. ;): This code instructs the program to print the current transformation matrix R1 (TRANSFORM 1). If the program is computing a second order matrix, this second order transformation matrix will be included in the print out. This matrix is cumulative from the last R1 (TRANSFORM 1) update. The units of the elements of the printed matrix

are consistent with the input units associated with the Type Code 1. (BEAM) entry.

(13. 24. ;) : This code causes the TRANSFORM 2 matrix, R2, to be printed. The format and units of R2 are identical with those of R1 which is printed by the (13. 4. ;) code. The calculation of the R2 matrix is started by a 6. 0. 2. ; entry (see Type Code 6.).

The units of the tabulated matrix elements in either the 1st order R or sigma matrix or second order T matrix of a TRANSPORT printout will correspond to the units chosen for the BEAM card. For example, the $R(12) = (x/\theta)$ matrix element will have the dimensions of cms/mr; and the $T(236) = (\theta/y\delta)$ matrix element will have the dimensions mr per cm per percent $\Delta P/P$ and so forth.

PUNCHED OUTPUT CONTROLS 29, 30, 31, 32, 33, 34, 35, 36

If the control is equal to 29, all of the terms in the first order matrix and the x and y terms of the second order matrix are punched.

If the control is equal to 30, all of the terms of the first order matrix and all second order matrix elements are punched out.

If the control, n, is greater than 30, all of the first order terms are punched and the second order matrix elements which correspond to (n-30.) i.e., if n = 32, the second order theta matrix elements are punched out. If n = 31, the second order X matrix elements are punched and so forth.

ARBITRARY TRANSFORMATION INPUT Type Code 14.0

To allow for the use of empirically determined fringing fields and other specific (perhaps non-phase-space-conserving) transformations, provision has been made for reading in an arbitrary transformation matrix. The first-order 6 X 6 matrix is read in row by row.

There are eight parameters for each row of a first-order matrix entry:

- 1 - Type Code 14.
- 2 to 7 - the six numbers comprising the row. The units must be those used to print the transfer matrix; in other words, consistent with the BEAM input/output.
- 8 - row number (1. to 6.)

A complete matrix must be read and applied one row at a time. Rows that do not differ from the unit transformation need not be read.

For example, (14. -.1 .9 0. 0. 0. 0. 2. ;) introduces a transformation matrix whose second row is given but which is otherwise a unit matrix. Note that this transformation does not conserve phase space because $R(22) = 0.9$ i.e. the determinant of $R \neq 1$.

Any of the components of a row may be varied; however, there are several restrictions.

Type Code 14. elements that immediately follow one another will all be used to form a single transformation matrix. If distinct matrices are desired, another element must be inserted to separate the Type Code 14. cards. Several do-nothing elements are available; for example, a zero length drift (3. 0. ;) is a convenient one.

Note that

$$\begin{pmatrix} 1 & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ 0 & 1 \end{pmatrix} \neq \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Hence, a matrix formed by successive 14. (3. 0. ;), 14. - elements is not always equal to the one formed by leaving out the (3. 0. ;) element.

If components of a 14. card are to be varied it must be the last 14. card in its matrix. This will force a matrix to be split into factors if more than one row has variable components.

If it is desired to read in the second-order matrix coefficients for the *i*th row, then the following 22 additional numbers may be read in:**

9 - continuation Code 0.

10 to 30 - the twenty-one coefficients:

T(i11) T(i12) T(i13) T(i14) T(i15) T(i16)

T(i22) T(i23) T(i24) T(i25) T(i26) T(i33)

T(i34) T(i35) T(i36) T(i44) T(i45) T(i46)

T(i55) T(i56) T(i66)

in that order, where *i* is the row number. It is necessary to read in the first order matrix row which corresponds to the second order matrix row being read in.

**This feature frees the user from making repetitive, expensive, second-order runs through a fixed portion of his system while experimenting with other magnets. This is done by reading the full matrix of this portion (obtained from a previous run) back into the machine as a single "arbitrary matrix."

As in the first-order case, full rows not different from the identity matrix (i.e., $R(ii) = 1$, all other $R(ij) = 0$, and all $T(ijk) = 0$) need not be read in.

INPUT - OUTPUT UNITS Type Code 15.0

TRANSPORT is designed with a standard set of units that have been used throughout this manual. However, to accommodate other units conveniently and to relieve certain parameter-fitting problems, provision has been made for redefining the units to be employed. This is accomplished with the following type code elements.

There are 4 parameters to be specified:

- 1 - Type Code 15.
- 2 - code digit
- 3 - the abbreviation of the unit. (see examples below)

This will be printed on the output listing. It must be enclosed in single quotes and is a maximum of 3 characters long (4 for energy).

- 4 - the scale factor (if needed) that multiplies a parameter expressed in the new units to convert it to the appropriate reference unit. The examples and tables below should clarify this point.

The various units that may be changed are:

Code Digit	Quantity	Standard Reference Unit	Symbols used in SLAC-75
1.	horizontal and vertical transverse dimensions	cm	x, y
2.	horizontal and vertical angles	mr	θ, φ
3.	vertical beam extent (only)	cm	y
4.	vertical beam divergence (only)	mr	φ
5.	pulsed beam length	cm	ℓ
6.	momentum spread	% (PC)	δ
7.	undefined		
8.	length (longitudinal)	meters (M)	t
9.	magnetic fields	kg	B
10.	mass	electron mass	m
11.	momentum	GeV/c	P(0)

Units are not normally restored at the end of a run. Once changed, they remain the same for all succeeding runs in an input deck, unless specifically changed. The units may be reset to standard units at any time by inserting a (15. ;) Type Code entry. The 15. elements are normally the first cards in a deck, (immediately following the title card and the 0 or 1 indicator card) or perhaps the last cards--in the event units are being restored to avoid disturbing succeeding runs.

Example: To change length to feet, width to inches, and momentum to MeV/c, add to the front of the deck the elements

15. 8. ' FT' 0.3048;

15. 1. ' IN' 2.54;

15. 11. 'MEV' 0.001;

The scale factor, 0.3048, multiplies a length expressed in the new unit, feet, to convert it to the reference unit, meters etc.

For the conventional units listed below, it is sufficient to stop with the label; (the conversion factor is automatically calculated by the program). If units other than those listed below are desired, then your label and the appropriate conversion factor must be included. If the automatic feature is used, there must be no blank spaces between the quotes and the labels.

INPUT-OUTPUT UNITS (Type Code 15.0)
CONVERSION FACTORS FOR DIMENSION CHANGES VS. CODE DIGIT AND LABEL

LABEL	CODE DIGIT										
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
'CM'	1.	---	1.	---	1.	---	---	.01	---	---	---
'M'	100.	---	100.	---	100.	---	---	1.	---	---	---
'IN'	2.54	---	2.54	---	2.54	---	---	.0254	---	---	---
'FT'	30.48	---	30.48	---	30.48	---	---	.3048	---	---	---
'MM'	.1	---	.1	---	.1	---	---	.001	---	---	---
'R'	---	1000.	---	1000.	---	---	---	---	---	---	---
'MR'	---	1.	---	1.	---	---	---	---	---	---	---
'PC'	---	---	---	---	---	1.	---	---	---	---	---
'P/10'	---	---	---	---	---	.1	---	---	---	---	---
'N'	---	---	---	---	---	100.	---	---	---	---	---
'MEV'	---	---	---	---	---	---	---	---	---	---	.001
'GEV'	---	---	---	---	---	---	---	---	---	---	1.
'KG'	---	---	---	---	---	---	---	---	1.	---	---
'G'	---	---	---	---	---	---	---	---	.001	---	---

A number of constants are used by the program which do not appear as parameters on any of the other type code element cards. A special element has been provided to allow the designer to set their values. These special parameter entries must always precede the physical element(s) to which they apply. Once introduced, they apply to all succeeding elements in a data set unless reset to zero or to new values.

There are three parameters:

- 1 - Type Code 16.
- 2 - code digit
- 3 - value of constant

A number of such constants have been defined in this manner. All have a normal value that is initialized at the beginning of each run.

Code Digit

1. $\epsilon(1)$ - a second-order measure of magnetic field inhomogeneity in bending magnets. If

$$B(x) = B(0) \left[1 - n\left(\frac{x}{\rho}\right) + \beta\left(\frac{x}{\rho}\right)^2 - \dots \right]$$

is the field expansion in the median ($y = 0$) plane). Then $\epsilon(1)$ (dimensionless) is defined as

$$\epsilon(1) = \beta\left(\frac{x}{\rho_0}\right)^2$$

evaluated at $x = 1$ (in units of horizontal beam width - normally cm.). This parameter affects second-order calculations only. Normally the value is 0.

3. (M/m) - Mass of the particles comprising the beam, in units of the electron mass; normally 0. A non-zero mass introduces the dependence of pulse length on velocity, an important effect in low energy pulsed beams.

4. $W/2$ - Horizontal half-aperture of bending magnet, in the same units as horizontal beam width. normally 0. (i.e., effect of finite horizontal half aperture is ignored).
5. $g/2$ - Vertical half-aperture of bending magnet, in the same units as vertical beam height; this parameter must be inserted if the finite extent of the fringing fields upon transverse focusing is to be taken into account. (see Type Codes 2.0 and 4.0 as a cross reference) normally 0.
6. L - Cumulative length of system, in the same units as system length. It is set to zero initially, then increased by the length of each element, and finally printed at end of system. This element allows the cumulative length to be reset as desired.
12. $1/R1$ - Where $R1$ is the radius of curvature (in units of longitudinal length, normally meters) of the entrance face of bending magnets. (See Fig. 748A15 page 2-4)
13. $1/R2$ - Where $R2$ is the radius of curvature (in units of system length such as meters) of the exit face of bending magnets. (See Fig. 748A15, page 2-4)

FIELD BOUNDARIES FOR BENDING MAGNETS

The TRANSPORT sign conventions for x , β , R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β 's imply transverse focusing. Positive R 's (convex curvatures) represent negative sextupole components of strength $S = \left(-\frac{h}{2R}\right) \sec^3 \beta$. (See SLAC-75, page 71).

R1 and R2 affect the system only in second-order, creating an effective sextupole component in the neighborhood of the magnet. If the parameters are not specified, they are assumed to be zero, i.e., no curvature and hence no sextupole component.

Programming for Fringe-Field Corrections to the Transverse Focal Length

Insert: 16. 5. $g/2$ element, then

16. 7. $k(1)$. $\left[k(1) = 1/2 \text{ if } 16. 7. \text{ element is not inserted but } 16. 5. \text{ element is inserted.} \right]$

16. 8. $k(2)$. $\left[k(2) = 0 \text{ if } 16. 8. \text{ element is not inserted.} \right]$

where

$$\frac{1}{f_y} = -\frac{1}{\rho} \tan(\beta - \psi)$$

and

$$\psi = \frac{1 + \sin^2 \beta}{\cos \beta} \left[k_1 \left(\frac{g}{\rho} \right) - k_2 \left(\frac{k_1 g}{\rho} \right)^2 \tan \beta \right]^*$$

Fringing-field corrections will only be made where the pole-face rotation element (2. β . ;) has been inserted in conjunction with a bending magnet (4. element.)

* See SLAC-75, page 74 for a discussion of and a definition of $k(1)$.

Typical values of k_1 and k_2 are given below for three types of fringing field boundaries:

- a) a linear drop-off of the field.
- b) a clamped "Rogowski" fringing-field.
- and c) a "square-edged" non-saturating magnet.

Model	Linear	Clamped Rogowski	Square Edged Magnet
k_1	1/6	0.3 to 0.4	0.45
k_2	3.8	4.4	2.8

Tilt-to-Focal Plane (16. 15. α . ;) element

Very often it is desired to have a listing of the second order aberrations along the focal plane of a system rather than perpendicular to the optic axis i.e. along the x coordinate. If the focal plane makes an angle α with respect to the x axis (measured clockwise) then provision has been made to rotate to this focal plane and print out the second-order aberrations. This is achieved by the following procedures:

Alpha is the focal-plane tilt angle (in degrees) measured from the perpendicular to the optic axis (α is normally zero).

The programming procedure for a tilt in the x (bend)-plane (rotation about y axis) is:

```
16. 15.  $\alpha$ . ;  
3. 0. ; (a necessary do-nothing element)  
13. 4. ;  
16. 15.  $-\alpha$ . ; (Rotate back to zero)  
3. 0. ; (a necessary do-nothing element)  
16. 15. 0. ; (to turn off rotation element)
```

The programming procedure for a tilt in the y-plane (rotation about x axis) is:

```
16. 15.  $\alpha$ . ;  
20. 90. ;  
3. 0. ;  
20. -90. ;  
13. 4. ;  
16. 15.  $-\alpha$ . ; (Rotate back to zero)  
3. 0. ;  
16. 15. 0. ; (to turn off rotation element)
```

SECOND ORDER CALCULATION Type Code 17.0

A second order calculation may be obtained provided neither parameter fitting nor alignment elements are employed. A special element instructs the program to calculate the second order matrix elements. It must be inserted immediately following the beam (1. element).

Only one parameter need be specified:

- 1 - Type Code 17. ; (signifying a 2nd order calculation is to be made)

The values of the BEAM (sigma) matrix components may be perturbed from their first order value by the second order aberrations. To print out the second order T matrix terms at a given location in the system, the (13. 4. ;) print control card is used. The update rules are the same as those for the R1 first-order matrix.

See SLAC-75 for definitions of subscripts in the second order T(ijk) matrix elements.

Second-order matrices are included in the program for quadrupole, bending magnets (including fringing-fields), sextupoles, and solenoids. They have not been calculated for the acceleration (Type Code 11.0) element.

SEXTUPOLE Type Code 18.0

Sextupole (Hexapole) magnets are used to modify second-order aberrations in beam transport systems. The action of a sextupole on beam particles is a second and higher order effect, so in first order runs (absence of the 17. card) this element will act as a drift space.

There are 4 parameters:

- 1 - Type Code 18.
- 2 - effective length (meters).
- 3 - field at pole tips (kg). Both positive and negative fields are possible (see figures below).
- 4 - Half-aperture (cm). Radius of circle tangent to pole tips.

Other orientations of the sextupole may be obtained using the beam rotation element (Type Code 20.). None of the parameters of the sextupole may be varied since it is a second order element.

See SLAC-75 for a tabulation of sextupole matrix elements.

The TRANSPORT input format for a typical data set is:

18. L. B. a. ' ' ;
 └─ Label if desired (not to exceed 4 spaces)

DIPOLE

QUADRUPOLE

SEXTUPOLE

748A10

ILLUSTRATION OF THE MAGNETIC MIDPLANE (x AXIS) FOR DIPOLE, QUADRUPOLE AND SEXTUPOLE ELEMENTS. THE MAGNET POLARITIES INDICATE MULTIPOLE ELEMENTS THAT ARE POSITIVE WITH RESPECT TO EACH OTHER.

SOLENOID Type Code 19.0

The solenoid is most often used as a focusing element in systems passing low energy particles. Particles in a solenoidal field travel helical trajectories. The solenoid fringing field effects necessary to produce the focusing are included.

There are three parameters:

- 1 - Type Code 19.0
- 2 - effective length of the solenoid (meters)
- 3 - the field (kg). A positive field by convention points in the direction of positive z for positively charged particles.

The length and the field may be varied by using 19.1 or 19.01 vary codes respectively. Both 1st and 2nd order matrix calculations are available for the solenoid.

A typical input format is:

19. L. B. ' ' ;
 └─ Label if desired (not to exceed 4 spaces)

1ST ORDER

SOLENOID MATRIX ELEMENTS

Solenoid R Matrix

Definitions: L = effective length of solenoid

$K = B(0)/(2B\rho)$ where $B(0)$ is the field inside the solenoid and $(B\rho)$ is the (momentum) of the Central Trajectory.

$$C = \cos KL$$

$$S = \sin KL$$

For a derivation of this transformation see SLAC-4 report by R. Helm.

Alternate Forms of Matrix Representation of the Solenoid.

$$R(\text{Solenoid}) = \begin{pmatrix} C^2 & \frac{1}{K}SC & SC & \frac{1}{K}S^2 & 0 & 0 \\ -KSC & C^2 & -KS^2 & SC & 0 & 0 \\ -SC & -\frac{1}{K}S^2 & C^2 & \frac{1}{K}SC & 0 & 0 \\ KS^2 & -SC & -KSC & C^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Rotating the transverse coordinates about the z axis by an angle = $-KL$ decouples the x and y 1st order Terms, i.e.

$$R(-KL) \cdot R(\text{Solenoid}) = \begin{pmatrix} C & \frac{1}{K}S & 0 & 0 & 0 & 0 \\ -KS & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & \frac{1}{K}S & 0 & 0 \\ 0 & 0 & -KS & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

BEAM ROTATION Type Code 20.0

The transverse beam coordinates x and y may be rotated through an angle α about the z axis (the axis tangent to the central trajectory at the point in question).** Thus a rotated bending magnet, quadrupole, or sextupole may be inserted into a beam transport system by preceding and following the element with the appropriate coordinate rotation. (See examples below). The positive sense of rotation is clockwise about the positive z axis.

There are two parameters to be specified for a Beam Rotation:

- 1 - Type Code 20. (signifying a beam coordinate rotation)
- 2 - the angle of rotation α . (degrees)

The angle of rotation may be varied in a 1st order fitting (see Vary Codes Section 10.)

Note

This transformation assumes that the units of (x and y) and (θ and φ) are the same. This is always true unless a 15. 3. or a 15. 4. type code has been used.

** See SLAC-75, page 45 and Fig. 4, page 12 for definitions of x , y , and z coordinates.

Examples:

For a bending magnet, the Beam Rotation matrix may be used to specify a rotated magnet.

Example No. 1

A bend up is represented by rotating the x, y coordinates by -90.0 degrees as follows:

Labels (not to exceed 4 spaces) if desired

```
20. -90. ' ' ;  
2.0  $\beta(1)$ . ' ' ;  
4.0 L. B. n. ' ' ;  
2.0  $\beta(2)$ . ' ' ;  
20. +90. ' ' ; (returns coordinates to their initial  
orientation)
```

A bend down is accomplished via a +90 degree rotation.

```
20. +90. ' ' ;  
2.  
4.  
2.  
20. -90. ' ' ;
```

A bend to the left (looking in the direction of beam travel) is accomplished by rotating the x, y coordinates by 180 degrees, e.g.,

```
20. 180. ' ' ;  
2.  
4.  
2.  
20. -180. ' ' ;
```

Example No. 2

A quadrupole rotated clockwise by 60 degrees about the positive z axis would be specified as follows:

20. 60. ' ' ;

5. L. B. a. ' ' ;

20. -60. ' ' ;

$$R = \begin{matrix} & \text{BEAM ROTATION MATRIX} \\ \begin{pmatrix} C & & S & & & \\ & C & & S & & \\ -S & & C & & & \\ & -S & & C & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix} \end{matrix}$$

Where $C = \cos \alpha$

$S = \sin \alpha$

α = angle of coordinate rotation about the beam axis.

Blank spaces are zeros.

e.g.; for $\alpha = +90$ degrees, this matrix interchanges rows 1 and 2 with 3 and 4 of the accumulated R matrix as follows:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} R(11) & R(12) & R(13) & R(14) \\ R(21) & R(22) & R(23) & R(24) \\ R(31) & R(32) & R(33) & R(34) \\ R(41) & R(42) & R(43) & R(44) \end{bmatrix} = \begin{bmatrix} R(31) & R(32) & R(33) & R(34) \\ R(41) & R(42) & R(43) & R(44) \\ -R(11) & -R(12) & -R(13) & -R(14) \\ -R(21) & -R(22) & -R(23) & -R(24) \end{bmatrix}$$

(The rest of the matrix is unchanged).

Type Code 21.0

- : uncertainty in $\int B dz$ - effects beam second moment.

Random number generator

16. 14. $(2n + 1.)$.

specified initial point in the r. n. sequence.

SENTINEL

Each data set must be terminated with the word SENTINEL. The word SENTINEL need not be on a separate card.

REFERENCES

1. K. L. Brown, "A First and Second-Order Matrix Theory for the Design of Beam Transport Systems and Charged Particle Spectrometers" SLAC Report No. 75 (revised), July 1967.
2. S. Penner, "Calculations of Properties of Magnetic Deflection Systems," Rev. Sci. Instr. 32, No. 2, 150-160 (February 1961).
3. A. P. Banford, The Transport of Charged Particle Beams, (E. and F. N. Spon, Ltd., London, 1966).
4. K. L. Brown (with R. Belbeoch and P. Bounin), "First- and Second- Order Magnetic Optics Matrix Equations for the Midplane of Uniform-Field Wedge Magnets, Rev. Sci. Instr. 35, 481 (1964).
5. K. G. Steffen, High Energy Beam Optics, Interscience Monographs and Tests in Physics and Astronomy, Vol. 17 (John Wiley and Sons, 1965).
6. Harold Enge and Stan Kowalski at M.I.T. have developed an effective Ray Tracing program using essentially the same terminology as TRANSPORT. Hence any experienced user of TRANSPORT should find it relatively easy to adapt to the M.I.T. program.

EXAMPLES

Table of Contents

	PAGE
A Quadrupole Triplet used to achieve Point-to-Point Imaging	E-1A
A Quadrupole Triplet used to achieve Waist-to-Waist Imaging	E-7A
A Quadrupole Triplet used to achieve a Specified Spot Size at a Target	E-13A
A SLAC Beam Switchyard (an attempt to fit all variable parameters in one step)	E-19A
A SLAC Beam Switchyard (using sequential fitting of the variable parameters to achieve the desired result)	E-37A
Misalignment of a Quadrupole Triplet	E-114A

'USE TRIPLET TO FOCUS BEAM FROM SOURCE ONTO TARGET'

0

(THIS PROBLEM HAS A UNITS CONVERSION FOR THE BEAM EXTENT FROM CM TO IN AND)
(FOR THE LENGTH FROM METERS TO FEET. THE OUTER TWO QUADRUPOLES OF THE QUAD)
(TRIPLET HAVE VAR CODES SUCH THAT THEIR FIELDS WILL VARY TOGETHER. THE FIELD)
(OF THE CENTER QUADRUPOLE IS ALLOWED TO VARY INDEPENDENTLY. THE FITTING)
(CONDITIONS SPECIFY A FOCUS IN BOTH DIMENSIONS ON THE TARGET.)

15.0 1.0 'IN' ;

15.0 8.0 'FT' ;

1.0 0.5 17.5 0.5 17.5 0.0 3.0 1.05 ;

3.0 12.0 ;

5.02 1.5 -1.0 4.0 ;

3.0 0.5 ;

5.01 3.0 1.0 4.0 ;

3.0 0.5 ;

5.02 1.50 -1.0 4.0 ;

3.0 7.0 ;

10.0 -1.0 2.0 0.0 .001 'FIT1' ;

10.0 -3.0 4.0 0.0 .001 'FIT2' ;

13.0 4.0 ;

-15.0 'UNT' ;

SENTINEL

USE TRIPLET TO FOCUS BEAM FROM SOURCE ONTO TARGET

0

THIS PROBLEM HAS A UNITS CONVERSION FOR THE BEAM EXTENT FROM CM TO IN AND
FOR THE LENGTH FROM METERS TO FEET. THE OUTER TWO QUADRUPOLES OF THE QUAD
TRIPLET HAVE VARY CODES SUCH THAT THEIR FIELDS WILL VARY TOGETHER. THE FIELD
OF THE CENTER QUADRUPOLE IS ALLOWED TO VARY INDEPENDENTLY. THE FITTING
CONDITIONS SPECIFY A FOCUS IN BOTH DIMENSIONS ON THE TARGET.

15.00000	1.00000	IN	2.54000				
15.00000	8.00000	FT	0.30480				
1.00000	0.50000	17.49998	0.50000	17.49998	0.0	3.00000	1.05000
3.00000	12.00000						
5.00000	1.50000	-1.00000	4.00000				
VARY CODE	020000						
3.00000	0.50000						
5.00000	3.00000	1.00000	4.00000				
VARY CODE	010000						
3.00000	0.50000						
5.00000	1.50000	-1.00000	4.00000				
VARY CODE	020000						
3.00000	7.00000						
10.00000	-1.00000	2.00000	0.0	0.00100			
LABEL =	FIT1						
10.00000	-3.00000	4.00000	0.0	0.00100			
LABEL =	FIT2						
13.00000	4.00000						
-15.00000	0.0		0.0				
LABEL =	UNT						

SENTINEL

USE TRIplet TO FOCUS BEAM FROM SOURCE ONTO TARGET

BEAM	1.000000	1.05 GEV	0.0 FT	0.0	0.500 IN						
				0.0	17.500 MR	0.0					
				0.0	0.500 IN	0.0	0.0				
				0.0	17.500 MR	0.0	0.0	0.0			
				0.0	0.0 CM	0.0	0.0	0.0	0.0		
				0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	
DRIFT	3.0	12.0000 FT	12.0 FT	0.0	2.569 IN						
				0.0	17.500 MR	0.981					
				0.0	2.569 IN	0.0	0.0				
				0.0	17.500 MR	0.0	0.0	0.981			
				0.0	0.0 CM	0.0	0.0	0.0	0.0		
				0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	
QUAD	5.00	1.50000 FT	-1.0000 KG	4.000 IN	(-25.287 FT)						
					VARY CODE = 020000						
			13.5 FT	0.0	2.958 IN						
				0.0	26.373 MR	0.994					
				0.0	2.801 IN	0.0	0.0				
				0.0	8.991 MR	0.0	0.0	0.938			
				0.0	0.0 CM	0.0	0.0	0.0	0.0		
				0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	
DRIFT	3.0	0.5000 FT	14.0 FT	0.0	3.115 IN						
				0.0	26.373 MR	0.994					
				0.0	2.851 IN	0.0	0.0				
				0.0	8.991 MR	0.0	0.0	0.940			
				0.0	0.0 CM	0.0	0.0	0.0	0.0		
				0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	
QUAD	5.00	3.00000 FT	1.0000 KG	4.000 IN	(13.282 FT)						
					VARY CODE = 010000						
			17.0 FT	0.0	3.665 IN						
				0.0	4.422 MR	0.842					
				0.0	3.511 IN	0.0	0.0				
				0.0	29.016 MR	0.0	0.0	0.996			
				0.0	0.0 CM	0.0	0.0	0.0	0.0		
				0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	
DRIFT	3.0	0.5000 FT	17.5 FT	0.0	3.687 IN						
				0.0	4.422 MR	0.844					
				0.0	3.685 IN	0.0	0.0				
				0.0	29.016 MR	0.0	0.0	0.997			

0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.50000 FT -1.0000 KG 4.000 IN (-25.287 FT)
VARY CODE = 020000

19.0 FT

0.0	3.864	IN					
0.0	16.179	MR	0.990				
0.0	4.093	IN	0.0	0.0			
0.0	16.330	MR	0.0	0.0	0.991		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 7.0000 FT

26.0 FT

0.0	5.213	IN					
0.0	16.179	MR	0.995				
0.0	5.455	IN	0.0	0.0			
0.0	16.330	MR	0.0	0.0	0.995		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

FIT 10.0 -1. 2. 0.0 / 0.001

0.297

LABEL = FIT1

FIT 10.0 -3. 4. 0.0 / 0.001

0.311

LABEL = FIT2

TRANSFORM 1

0.94920	0.29666	0.0	0.0	0.0	0.0
-0.41306	0.92442	0.0	0.0	0.0	0.0
0.0	0.0	0.95425	0.31054	0.0	0.0
0.0	0.0	-0.35259	0.93307	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 26.0000 FT

CORRECTIGNS

(3.0368E 02)	-18.939	17.940
(2.1660E 03)	2.929	-12.179
(7.1664E 02)	2.078	1.313
(1.5780E 02)	0.399	0.313

(6.2508E 00) 0.024 0.011
(1.6005E-02) 0.000 0.000
*COVARIANCE (FIT 0.0)
0.014
-0.869 0.004

USE TRIPLET TO FOCUS BEAM FROM SOURCE ONTO TARGET

BEAM 1.000000 1.05 GEV

0.0 FT	0.0	0.500 IN						
	0.0	17.500 MR	0.0					
	0.0	0.500 IN	0.0	0.0				
	0.0	17.500 MR	0.0	0.0	0.0			
	0.0	0.0 CM	0.0	0.0	0.0	0.0	0.0	
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 12.0000 FT

12.0 FT	0.0	2.569 IN						
	0.0	17.500 MR	0.981					
	0.0	2.569 IN	0.0	0.0				
	0.0	17.500 MR	0.0	0.0	0.981			
	0.0	0.0 CM	0.0	0.0	0.0	0.0	0.0	
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.50000 FT -14.5080 KG 4.000 IN (-1.533 FT)
VARY CODE = 020000

13.5 FT	0.0	4.099 IN						
	0.0	164.756 MR	1.000					
	0.0	1.818 IN	0.0	0.0				
	0.0	94.749 MR	0.0	0.0	-0.999			
	0.0	0.0 CM	0.0	0.0	0.0	0.0	0.0	
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 0.5000 FT

14.0 FT	0.0	5.088 IN						
	0.0	164.756 MR	1.000					
	0.0	1.250 IN	0.0	0.0				
	0.0	94.749 MR	0.0	0.0	-0.997			
	0.0	0.0 CM	0.0	0.0	0.0	0.0	0.0	
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 3.00000 FT 8.3975 KG 4.000 IN (2.166 FT)
VARY CODE = 010000

17.0 FT	0.0	5.005 IN						
	0.0	168.545 MR	-1.000					
	0.0	1.966 IN	0.0	0.0				
	0.0	111.721 MR	0.0	0.0	0.999			
	0.0	0.0 CM	0.0	0.0	0.0	0.0	0.0	
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 0.5000 FT

17.5 FT	0.0	3.994 IN						
	0.0	168.545 MR	-1.000					
	0.0	2.636 IN	0.0	0.0				
	0.0	111.721 MR	0.0	0.0	1.000			

0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.50000 FT -14.5080 KG 4.000 IN (-1.533 FT)
VARY CCDE = 020000

19.0 FT

0.0	2.338	IN					
0.0	28.627	MR	-0.991				
0.0	3.327	IN	0.0	0.0			
0.0	40.489	MR	0.0	0.0	-0.998		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 7.0000 FT

26.0 FT

0.0	0.318	IN					
0.0	28.627	MR	0.273				
0.0	0.231	IN	0.0	0.0			
0.0	40.489	MR	0.0	0.0	0.351		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

FIT 10.0 -1. 2. 0.0 / 0.001

-0.000

LABEL = FIT1

FIT 10.0 -3. 4. 0.0 / 0.001

0.000

LABEL = FIT2

TRANSFORM 1

-0.63548	-0.00000	0.0	0.0	0.0	0.0
-15.63603	-1.57364	0.0	0.0	0.0	0.0
0.0	0.0	0.46156	0.00000	0.0	0.0
0.0	0.0	28.41595	2.16653	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 26.0000 FT

'USE TRIPLET TO PUT WAIST ON TARGET'

1

(THIS PROBLEM DIFFERS FROM THE PRECEDING ONE IN THAT A WAIST ON THE TARGET)
(IS REQUESTED. NOTE THE USE OF THE 1 AS THE INDICATOR CARD SIGNIFYING)
(THAT THIS RUN IS A CONTINUATION OF THE PREVIOUS ONE.)

10.0 2.0 1.0 0.0 .001 'FIT1' ;

10.0 4.0 3.0 0.0 .001 'FIT2' ;

SENTINEL

USE TRIPLET TO PUT WAIST ON TARGET

1

THIS PROBLEM DIFFERS FROM THE PRECEDING ONE IN THAT A WAIST ON THE TARGET
IS REQUESTED. NOTE THE USE OF THE 1 AS THE INDICATOR CARD SIGNIFYING
THAT THIS RUN IS A CONTINUATION OF THE PREVIOUS ONE.

15.00000	1.00000	IN	2.54000		
15.00000	8.00000	FT	0.30480		
1.00000	0.50000	17.49998	0.50000	17.49998	0.0
3.00000	12.00000				
5.00000	1.50000	-14.50800	4.00000		
VARY CODE	020000				
3.00000	0.50000				
5.00000	3.00000	8.39750	4.00000		
VARY CODE	010000				
3.00000	0.50000				
5.00000	1.50000	-14.50800	4.00000		
VARY CODE	020000				
3.00000	7.00000				
10.00000	2.00000	1.00000	0.0	0.00100	
LABEL =	FIT1				
10.00000	4.00000	3.00000	0.0	0.00100	
LABEL =	FIT2				
13.00000	4.00000				
-15.00000	0.0		0.0		
LABEL =	UNT				

SENTINEL

USE TRIPLET TO PUT WAIST ON TARGET

BEAM 1.000000 1.05 GEV

0.0 FT	0.0	0.500 IN						
	0.0	17.500 MR	0.0					
	0.0	0.500 IN	0.0	0.0				
	0.0	17.500 MR	0.0	0.0	0.0			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 12.0000 FT

12.0 FT	0.0	2.569 IN						
	0.0	17.500 MR	0.981					
	0.0	2.569 IN	0.0	0.0				
	0.0	17.500 MR	0.0	0.0	0.981			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.50000 FT -14.5080 KG 4.000 IN (-1.533 FT)
VARY CODE = 020000

13.5 FT	0.0	4.099 IN						
	0.0	164.756 MR	1.000					
	0.0	1.818 IN	0.0	0.0				
	0.0	94.749 MR	0.0	0.0	-0.999			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 0.5000 FT

14.0 FT	0.0	5.088 IN						
	0.0	164.756 MR	1.000					
	0.0	1.250 IN	0.0	0.0				
	0.0	94.749 MR	0.0	0.0	-0.997			
	0.0	0.0 CM	0.0	0.0	0.0	0.0	0.0	
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 3.00000 FT 8.3975 KG 4.000 IN (2.166 FT)
VARY CODE = 010000

17.0 FT	0.0	5.005 IN						
	0.0	168.545 MR	-1.000					
	0.0	1.966 IN	0.0	0.0				
	0.0	111.721 MR	0.0	0.0	0.999			
	0.0	0.0 CM	0.0	0.0	0.0	0.0	0.0	
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 0.5000 FT

17.5 FT	0.0	3.994 IN						
	0.0	168.545 MR	-1.000					
	0.0	2.636 IN	0.0	0.0				
	0.0	111.721 MR	0.0	0.0	1.000			

0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.50000 FT -14.5080 KG 4.000 IN (-1.533 FT)
VARY CGDE = 020000

19.0 FT

0.0	2.338	IN					
0.0	28.627	MR	-0.991				
0.0	3.327	IN	0.0	0.0			
0.0	40.489	MR	0.0	0.0	-0.998		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 7.0000 FT

26.0 FT

0.0	0.318	IN					
0.0	28.627	MR	0.273				
0.0	0.231	IN	0.0	0.0			
0.0	40.489	MR	0.0	0.0	0.351		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

FIT 10.0 2. 1. 0.0 / 0.001

2.485

LABEL = FIT1

FIT 10.0 4. 3. 0.0 / 0.001

3.279

LABEL = FIT2

TRANSFORM 1

-0.63548	-0.00000	0.0	0.0	0.0	0.0
-15.63603	-1.57364	0.0	0.0	0.0	0.0
0.0	0.0	0.46156	0.00000	0.0	0.0
0.0	0.0	28.41595	2.16653	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 26.0000 FT

CORRECTIONS

(2.9092E C3) 0.063 -0.026

(1.1744E 02) 0.003 -0.001

(8.0844E-01) 0.000 -0.000

(3.0664E-01) -0.000 0.000

*COVARIANCE (FIT 0.3)
0.000
-0.786 0.000

USE TRIPLET TO PUT WAIST ON TARGET

BEAM 1.000000 1.05 GEV

0.0 FT	0.0	0.500 IN						
	0.0	17.500 MR	0.0					
	0.0	0.500 IN	0.0	0.0				
	0.0	17.500 MR	0.0	0.0	0.0			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

CRIFT 3.0 12.0000 FT

12.0 FT	0.0	2.569 IN						
	0.0	17.500 MR	0.981					
	0.0	2.569 IN	0.0	0.0				
	0.0	17.500 MR	0.0	0.0	0.981			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.50000 FT -14.4423 KG 4.000 IN (-1.541 FT)
VARY CODE = 020000

13.5 FT	0.0	4.093 IN						
	0.0	164.000 MR	1.000					
	0.0	1.822 IN	0.0	0.0				
	0.0	94.316 MR	0.0	0.0	-0.999			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

URIFT 3.0 0.5000 FT

14.0 FT	0.0	5.077 IN						
	0.0	164.000 MR	1.000					
	0.0	1.257 IN	0.0	0.0				
	0.0	94.316 MR	0.0	0.0	-0.997			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 3.00000 FT 8.3706 KG 4.000 IN (2.170 FT)
VARY CODE = 010000

17.0 FT	0.0	5.001 IN						
	0.0	167.517 MR	-1.000					
	0.0	1.931 IN	0.0	0.0				
	0.0	110.245 MR	0.0	0.0	0.999			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 0.5000 FT

17.5 FT	0.0	3.996 IN						
	0.0	167.517 MR	-1.000					
	0.0	2.592 IN	0.0	0.0				
	0.0	110.245 MR	0.0	0.0	1.000			

0.0	0.0	CM	0.0	0.0	0.0	0.0	0.0
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.5000 FT -14.4423 KG 4.000 IN (-1.541 FT)
 VARY CCDE = 020000

19.0 FT

0.0	2.355	IN					
0.0	27.784	MR	-0.991				
0.0	3.283	IN	0.0	0.0			
0.0	38.993	MR	0.0	0.0	-0.998		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 7.0000 FT

26.0 FT

0.0	0.315	IN					
0.0	27.784	MR	0.000				
0.0	0.224	IN	0.0	0.0			
0.0	38.993	MR	0.0	0.0	-0.000		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

FIT 10.0 2. 1. 0.0 / 0.001

0.000

LABEL = FIT1

FIT 10.0 4. 3. 0.0 / 0.001

-0.000

LABEL = FIT2

TRANSFORM 1

-0.60539	0.00497	0.0	0.0	0.0	0.0
-15.34218	-1.52592	0.0	0.0	0.0	0.0
0.0	0.0	0.41949	-0.00456	0.0	0.0
0.0	0.0	27.71928	2.08264	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 26.0000 FT

'USE TRIPLET TO FIT CERTAIN SIZE BEAM ONTO TARGET'

1

(THIS PROBLEM IS THE SAME AS THE TWO PRECEDING ONES EXCEPT A PARTICULAR)
(SIZE OF THE BEAM IS ASKED FOR. THE BLANK TYPE 15 CARD WHICH RESETS THE UNITS)
(IS TURNED ON.)

10.0 1.0 1.0 0.6 .001 'FIT1' ;

10.0 3.0 3.0 0.6 .001 'FIT2' ;

15.0 'UNT' ;

SENTINEL

USE TRIPLET TO FIT CERTAIN SIZE BEAM ONTO TARGET

1

THIS PROBLEM IS THE SAME AS THE TWO PRECEDING ONES EXCEPT A PARTICULAR
SIZE OF THE BEAM IS ASKED FOR. THE BLANK TYPE 15 CARD WHICH RESETS THE UNITS
IS TURNED ON.

15.00000	1.00000	IN	2.54000				
15.00000	8.00000	FT	0.30480				
1.00000	0.50000	17.49998	0.50000	17.49998	0.0	3.00000	1.05000
3.00000	12.00000						
5.00000	1.50000	-14.4423C	4.00000				
VARY CODE	020000						
3.00000	0.50000						
5.00000	3.00000	8.37058	4.00000				
VARY CODE	010000						
3.00000	0.50000						
5.00000	1.50000	-14.4423C	4.00000				
VARY CODE	020000						
3.00000	7.00000						
10.00000	1.00000	1.00000	0.60000	0.00100			
LABEL = FIT1							
10.00000	3.00000	3.00000	0.60000	0.00100			
LABEL = FIT2							
13.00000	4.00000						
15.00000	0.0		0.0				
LABEL = UNT							

SENTINEL

USE TRIPLET TO FIT CERTAIN SIZE BEAM ONTO TARGET

BEAM 1.000000 1.05 GEV

0.0 FT	0.0	0.500 IN						
	0.0	17.500 MR	0.0					
	0.0	0.500 IN	0.0	0.0				
	0.0	17.500 MR	0.0	0.0	0.0			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 12.0000 FT

12.0 FT	0.0	2.569 IN						
	0.0	17.500 MR	0.981					
	0.0	2.569 IN	0.0	0.0				
	0.0	17.500 MR	0.0	0.0	0.981			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.50000 FT -14.4423 KG 4.000 IN (-1.541 FT)
VARY CODE = 020000

13.5 FT	0.0	4.093 IN						
	0.0	164.000 MR	1.000					
	0.0	1.822 IN	0.0	0.0				
	0.0	94.316 MR	0.0	0.0	-0.999			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 0.5000 FT

14.0 FT	0.0	5.077 IN						
	0.0	164.000 MR	1.000					
	0.0	1.257 IN	0.0	0.0				
	0.0	94.316 MR	0.0	0.0	-0.997			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 3.00000 FT 8.3706 KG 4.000 IN (2.170 FT)
VARY CODE = 010000

17.0 FT	0.0	5.001 IN						
	0.0	167.517 MR	-1.000					
	0.0	1.931 IN	0.0	0.0				
	0.0	110.245 MR	0.0	0.0	0.999			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 0.5000 FT

17.5 FT	0.0	3.996 IN						
	0.0	167.517 MR	-1.000					
	0.0	2.592 IN	0.0	0.0				
	0.0	110.245 MR	0.0	0.0	1.000			

0.0	0.0	CM	0.0	0.0	0.0	0.0	0.0
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.50000 FT -14.4423 KG 4.000 IN (-1.541 FT)
VARY CODE = 020000

19.0 FT

0.0	2.355	IN					
0.0	27.784	MR	-0.991				
0.0	3.283	IN	0.0	0.0			
0.0	38.993	MR	0.0	0.0	-0.998		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 7.0000 FT

26.0 FT

0.0	0.315	IN					
0.0	27.784	MR	0.000				
0.0	0.224	IN	0.0	0.0			
0.0	38.993	MR	0.0	0.0	-0.000		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

FIT 10.0 1. 1. 0.600 / 0.001

0.315

LABEL = FIT1

FIT 10.0 3. 3. 0.600 / 0.001

0.224

LABEL = FIT2

TRANSFORM 1

-0.60539	0.00497	0.0	0.0	0.0	0.0
-15.34218	-1.52592	0.0	0.0	0.0	0.0
0.0	0.0	0.41949	-0.00456	0.0	0.0
0.0	0.0	27.71928	2.08254	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LABEL = UNT

LENGTH 7.9248 M

CORRECTIONS

(2.3856E 02)	5.266	-1.786
(1.7593E 03)	1.335	-0.401
(1.4466E 02)	0.115	-0.018
(1.4665E 01)	0.012	-0.001

(2.5497E-01) 0.000 0.000
*COVARIANCE (FIT 0.3)
0.001
-0.918 0.001

USE TRIPLET TO FIT CERTAIN SIZE BEAM ONTO TARGET

BEAM 1.000000 1.05 GEV

0.0 FT	0.0	0.500 IN						
	0.0	17.500 MR	0.0					
	0.0	0.500 IN	0.0	0.0				
	0.0	17.500 MR	0.0	0.0	0.0			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 12.0000 FT

12.0 FT	0.0	2.569 IN						
	0.0	17.500 MR	0.981					
	0.0	2.569 IN	0.0	0.0				
	0.0	17.500 MR	0.0	0.0	0.981			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.50000 FT -7.7144 KG 4.000 IN (-3.073 FT)
VARY CODE = 020000

13.5 FT	0.0	3.507 IN						
	0.0	90.977 MR	1.000					
	0.0	2.296 IN	0.0	0.0				
	0.0	46.559 MR	0.0	0.0	-0.997			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 0.5000 FT

14.0 FT	0.0	4.053 IN						
	0.0	90.977 MR	1.000					
	0.0	2.017 IN	0.0	0.0				
	0.0	46.559 MR	0.0	0.0	-0.996			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 3.00000 FT 6.1646 KG 4.000 IN (2.671 FT)
VARY CODE = 010000

17.0 FT	0.0	3.995 IN						
	0.0	93.798 MR	-1.000					
	0.0	1.574 IN	0.0	0.0				
	0.0	19.690 MR	0.0	0.0	0.959			
	0.0	0.0 CM	0.0	0.0	0.0	0.0		
	0.0	3.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 0.5000 FT

17.5 FT	0.0	3.432 IN						
	0.0	93.798 MR	-1.000					
	0.0	1.687 IN	0.0	0.0				
	0.0	19.690 MR	0.0	0.0	0.965			

0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 1.50000 FT -7.7144 KG 4.000 IN (-3.073 FT)
 VARY CODE = 020000

19.0 FT

0.0	2.422	IN					
0.0	22.958	MR	-0.988				
0.0	1.638	IN	0.0	0.0			
0.0	24.827	MR	0.0	0.0	-0.977		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 7.0000 FT

26.0 FT

0.0	0.600	IN					
0.0	22.958	MR	-0.772				
0.0	0.600	IN	0.0	0.0			
0.0	24.827	MR	0.0	0.0	0.809		
0.0	0.0	CM	0.0	0.0	0.0	0.0	
0.0	3.000	PC	0.0	0.0	0.0	0.0	0.0

FIT 10.0 1. 1. 0.600 / 0.001

0.600

LABEL = FIT1

FIT 10.0 3. 3. 0.600 / 0.001

0.600

LABEL = FIT2

TRANSFORM 1

-0.46318	0.03163	0.0	0.0	0.0	0.0
-13.21502	-1.25632	C.C	0.0	0.0	0.0
0.0	0.0	-0.86633	-0.02372	0.0	0.0
0.0	0.0	-8.82873	-1.39605	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LABEL = UNT

LENGTH 7.9248 M

'BEAM SWITCHYARD END STATION A ALL FITTING AT ONCE'
 (THIS SYSTEM WAS ONE OF THE DESIGNS FOR THE SWITCHYARD ELEMENTS WHICH)
 (TRANSPORT THE SLAC BEAM TO END STATION A. THE AUTOMATIC BEAM PRINTOUT IS)
 (TURNED OFF BY THE 13. 2. ELEMENT. THE MAGNETIC SYSTEM BEGINS WITH A BENDING)
 (MAGNET WHICH DIVERTS THE BEAM BY .5 DEGREES. THIS MAGNET IS FOLLOWED BY AN)
 (EIGHTY METER DRIFT TO ALLOW THE BEAM TO CLEAR SUFFICIENTLY THE ACCELERATOR)
 (CENTER LINE TO ALLOW THE INSTALLATION OF A QUADRUPLER DOUBLET. THE DOUBLET)
 (IS FOLLOWED BY FOUR BENDING MAGNETS WHICH ARE ENTERED WITH THE USE OF THE TYPE)
 (NINE REPEAT CODE. THE INITIAL QUAD DOUBLET,Q1, IS ALLOWED TO VARY SO THAT A)
 (FOCUS IS OBTAINED AT THE EXIT OF THE SYMMETRY QUADRUPOLE,Q2. BOTH THE)
 (POSITION AND THE FIELD OF Q2 ARE ALLOWED TO VARY SO AS TO GET AN ACHROMATIC)
 (CONDITION AT THE EXIT OF THE SECOND FOUR BENDING MAGNETS. A SECOND DOUBLET,Q3)
 (IS ALLOWED TO VARY TO PRODUCE A CERTAIN BEAM SIZE AT THE ENTRANCE TO THE FINAL)
 (DOUBLET,Q4. THIS DOUBLET IS ALLOWED TO VARY TO GET A WAIST AND A CERTAIN SPOT)
 (SIZE AT THE TARGET.)
 (THIS IS AN EXAMPLE OF A PROGRAM WHICH WILL NOT FIT ALL OF THE CONSTRAINTS)
 (AT ONCE. THE SUBSEQUENT EXAMPLES SHOW THE USE OF SEQUENTIAL FITTING AND THE)
 (USE OF THE TYPE ONE INDICATOR CARD.)

```

0
1.0 0.3 0.1 0.3 0.1 0.3 1.0 25.0 ;
13.0 2.0 ;
2.0 0.0 ;
4.0 5.0 1.45500 0.0 ;
2.0 0.5 ;
13.0 1.0 ;
3.0 80.0 ;
13.0 1.0 ;
5.01 2.0 -2.2 5.0 'Q1' ;
3.0 2.0 ;
5.01 2.0 2.4 5.0 'Q1' ;
13.0 1.0 ;
9.0 4.0 ;
3.0 1.0 ;
2.0 1.5 ;
4.0 3.0 14.55 0.0 ;
2.0 1.5 ;
9.0 0.0 ;
13.0 1.0 ;
3.9 30.0 'DR1' ;
5.01 2.0 5.0 15.0 'Q2' ;
13.0 1.0 ;
10.0 -1.0 2.0 0.0 0.001 'FIT1' ;
10.0 -3.0 4.0 0.0 0.001 'FIT1' ;
13.0 4.0 ;
3.4 20.0 'DR1' ;
13.0 1.0 ;
9.0 4.0 ;
2.0 1.5 ;
4.0 3.0 14.55 0.0 ;
2.0 1.5 ;
3.0 1.0 ;
9.0 0.0 ;
13.0 1.0 ;
10.0 -1.0 6.0 0.0 0.001 'FIT2' ;
10.0 -2.0 6.0 0.0 0.001 'FIT3' ;
13.0 4.0 ;
5.01 2.0 -1.9 5.0 'Q3' ;
3.0 2.0 ;
5.01 2.0 1.85 5.0 'Q3' ;

```


3.0 200.0 ;
13.0 1.0 ;
10.2 1.0 1.0 4.0 0.1 'FIT4' ;
10.2 3.0 3.0 1.5 0.1 'FIT4' ;
13.0 4.0 ;
13.0 1.0 ;
5.01 2.0 4.5 5.0 'Q4' ;
3.00 2.0 ;
13.0 1.0 ;
5.01 2.0 -4.2 5.0 'Q4' ;
13.0 1.0 ;
3.0 15.0 ;
13.0 1.0 ;
10.0 2.0 1.0 0.0 0.001 'FIT5' ;
10.0 4.0 3.0 0.0 0.001 'FIT6' ;
10.2 1.0 1.0 0.02 0.01 'FIT7' ;
10.2 3.0 3.0 0.02 0.01 'FIT7' ;
13.0 4.0 ;
SENTINEL

BEAM SWITCHYARD END STATION A ALL FITTING AT ONCE
 THIS SYSTEM WAS ONE OF THE DESIGNS FOR THE SWITCHYARD ELEMENTS WHICH
 TRANSPORT THE SLAC BEAM TO END STATION A. THE AUTOMATIC BEAM PRINTOUT IS
 TURNED OFF BY THE 13. 2. ELEMENT. THE MAGNETIC SYSTEM BEGINS WITH A BENDING
 MAGNET WHICH DIVERTS THE BEAM BY .5 DEGREES. THIS MAGNET IS FOLLOWED BY AN
 EIGHTY METER DRIFT TO ALLOW THE BEAM TO CLEAR SUFFICIENTLY THE ACCELERATOR
 CENTER LINE TO ALLOW THE INSTALLATION OF A QUADRUPOLE DOUBLET. THE DOUBLET
 IS FOLLOWED BY FOUR BENDING MAGNETS WHICH ARE ENTERED WITH THE USE OF THE TYPE
 NINE REPEAT CODE. THE INITIAL QUAD DOUBLET,Q1, IS ALLOWED TO VARY SO THAT A
 FOCUS IS OBTAINED AT THE EXIT OF THE SYMMETRY QUADRUPOLE,Q2. BOTH THE
 POSITION AND THE FIELD OF Q2 ARE ALLOWED TO VARY SO AS TO GET AN ACHROMATIC
 CONDITION AT THE EXIT OF THE SECOND FOUR BENDING MAGNETS. A SECOND DOUBLET,Q3
 IS ALLOWED TO VARY TO PRODUCE A CERTAIN BEAM SIZE AT THE ENTRANCE TO THE FINAL
 DOUBLET,Q4. THIS DOUBLET IS ALLOWED TO VARY TO GET A WAIST AND A CERTAIN SPOT
 SIZE AT THE TARGET.
 THIS IS AN EXAMPLE OF A PROGRAM WHICH WILL NOT FIT ALL OF THE CONSTRAINTS
 AT ONCE. THE SUBSEQUENT EXAMPLES SHOW THE USE OF SEQUENTIAL FITTING AND THE
 USE OF THE TYPE ONE INDICATOR CARD.

```

0
1.00000  C.3000C  G.10000  0.30000  0.1000C  0.30000  1.00000  25.00000
13.00000  2.000C0
2.0C000  0.0
4.0C000  5.000C0  1.455C0  0.C
2.0C000  0.50000
13.00000  1.00000
3.CC000  80.00000
13.0C000  1.00000
5.0C000  2.000C0  -2.200C0  5.00000
VARY CODE  010000
LABEL = Q1
3.CC000  2.00000

```

5.00000	2.00000	2.40000	5.00000	
VARY CODE	010000			
LABEL = Q1				
13.00000	1.00000			
9.00000	4.00000			
3.00000	1.00000			
2.00000	1.50000			
4.00000	3.00000	14.55000	0.0	
2.00000	1.50000			
9.00000	0.0			
13.00000	1.00000			
3.00000	30.00000			
VARY CODE	900000			
LABEL = DR1				
5.00000	2.00000	5.00000	15.00000	
VARY CODE	010000			
LABEL = Q2				
13.00000	1.00000			
10.00000	-1.00000	2.00000	0.0	0.00100
LABEL = FIT1				
10.00000	-3.00000	4.00000	0.0	0.00100
LABEL = FIT1				
13.00000	4.00000			
3.00000	20.00000			
VARY CODE	400000			
LABEL = DR1				
13.00000	1.00000			
9.00000	4.00000			
2.00000	1.50000			
4.00000	3.00000	14.55000	0.0	
2.00000	1.50000			
3.00000	1.00000			
9.00000	0.0			
13.00000	1.00000			
10.00000	-1.00000	6.00000	0.0	0.00100
LABEL = FIT2				

10.00000	-2.00000	5.00000	0.0	0.00100
LABEL = FIT3				
13.00000	4.00000			
5.00000	2.00000	-1.90000	5.00000	
VARY CODE 010000				
LABEL = Q3				
3.00000	2.00000			
5.00000	2.00000	1.85000	5.00000	
VARY CODE 010000				
LABEL = Q3				
3.00000	200.00000			
13.00000	1.00000			
10.00000	1.00000	1.00000	4.00000	0.10000
VARY CODE 200000				
LABEL = FIT4				
10.00000	3.00000	3.00000	1.50000	0.10000
VARY CODE 200000				
LABEL = FIT4				
13.00000	4.00000			
13.00000	1.00000			
5.00000	2.00000	4.50000	5.00000	
VARY CODE 010000				
LABEL = Q4				
3.00000	2.00000			
13.00000	1.00000			
5.00000	2.00000	-4.20000	5.00000	
VARY CODE 010000				
LABEL = Q4				
13.00000	1.00000			
3.00000	15.00000			
13.00000	1.00000			
10.00000	2.00000	1.00000	0.0	0.00100
LABEL = FIT5				
10.00000	4.00000	3.00000	0.0	0.00100
LABEL = FIT6				
10.00000	1.00000	1.00000	0.02000	0.01000
VARY CODE 200000				
LABEL = FIT7				
10.00000	3.00000	3.00000	0.02000	0.01000

V1 .00E 200000
LABEL = F117
13.00000 4.00000
SENTINEL

BEAM SWITCHYARD END STATION A ALL FITTING AT ONCE

BEAM 1.000000 25.00 GEV

0.0 M	0.0	0.300 CM					
	0.0	0.100 MR	0.0				
	0.0	0.300 CM	0.0	0.0			
	0.0	0.100 MR	0.0	0.0	0.0		
	0.0	0.300 CM	0.0	0.0	0.0	0.0	
	0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0

ROTAT 2.0 0.0 D

BEND 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M	0.0	0.305 CM					
	0.0	0.133 MR	0.171				
	0.0	0.304 CM	0.0	0.0			
	0.0	0.100 MR	0.0	0.0	0.164		
	0.0	0.300 CM	-0.009	-0.001	0.0	0.0	
	0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000

DRIFT 3.0 80.0000 M

85.0 M	0.0	1.153 CM					
	0.0	0.133 MR	0.965				
	0.0	0.901 CM	0.0	0.0			
	0.0	0.100 MR	0.0	0.0	0.943		
	0.0	0.300 CM	-0.003	-0.001	0.0	0.0	
	0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000

QUAD 5.00 2.00000 M -2.2000 KG 5.000 CM (-9.151 M)
 VARY CODE = 010000
 LABEL = Q1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.4000 KG 5.000 CM (9.029 M)
 VARY CODE = 010000
 LABEL = Q1

91.0 M	0.0	1.675 CM					
	0.0	0.511 MR	-0.999				
	0.0	0.564 CM	0.0	0.0			
	0.0	0.151 MR	0.0	0.0	-0.936		

0.0	0.300 CM	-0.003	0.003	0.0	0.0	
0.0	1.000 PC	0.626	-0.616	0.0	0.0	-0.000

DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			

107.0 M

0.0	2.218 CM				
0.0	1.824 MR	0.865			
0.0	0.336 CM	0.0	0.0		
0.0	0.165 MR	0.0	0.0	-0.841	
0.0	0.457 CM	-0.715	-0.497	0.0	0.0
0.0	1.000 PC	0.954	0.975	0.0	0.0
					-0.610

DRIFT 3.C 3C.0000 M
VARY CODE = 900000
LABEL = DRI

QUAD 5.00 2.00000 M 5.0000 KG 15.000 CM (12.348 M)
VARY CODE = 010000
LABEL = Q2

139.0 M	0.0	7.236 CM				
	0.0	4.163 MR	-C.998			
	0.0	0.329 CM	0.0	0.0		
	0.0	0.384 MR	0.0	0.0	0.971	
	0.0	0.457 CM	-0.573	0.605	0.0	0.0
	0.0	1.000 PC	0.997	-1.000	0.0	0.0 -0.610

FIT 10.0 -1. 2. 0.0 / 0.001
-5.150
LABEL = FIT1

FIT 10.0 -3. 4. 0.0 / 0.001
-1.010
LABEL = FIT1

TRANSFORM 1
-C.93476 -5.14981 0.C 0.C 0.0 7.21223
0.24797 C.29932 C.C 0.0 0.0 -4.16246
0.0 0.0 -1.04482 -1.01024 0.0 0.0
0.0 0.0 -1.08980 -2.01031 0.0 0.0
-C.21025 -1.92988 0.0 0.0 1.00000 -0.27894
0.0 0.0 0.0 0.C 0.0 1.00000

DRIFT 3.C 20.0000 M
VARY CODE = 400000
LABEL = DRI

159.0 M	0.0	1.210 CM				
	0.0	4.163 MR	0.915			
	0.0	1.090 CM	0.0	0.0		
	0.0	0.384 MR	0.0	0.0	0.997	
	0.0	0.457 CM	0.735	0.605	0.0	0.0
	0.0	1.000 PC	-0.920	-1.000	0.0	0.0 -0.610

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M

175.0 M

0.0	6.005 CM						
0.0	2.070 MR	0.996					
0.0	1.664 CM	0.0	0.0				
0.0	0.334 MR	0.0	0.0	0.999			
0.0	0.588 CM	-0.824	-0.843	0.0	0.0		
0.0	1.000 PC	-0.998	-0.999	0.0	0.0	0.838	

FIT 10.0 -1.6 0.0 / 0.001

-5.991

LABEL = FIT2

FIT 10.0 -2. 6. 0.0 / 0.001
-2.068
LABEL = FIT3

TRANSFORM 1
-0.04222 -4.08323 0.0 0.0 0.0 -5.99086
0.24796 0.29627 0.0 0.0 0.0 -2.06827
0.0 0.0 -4.85194 -8.06319 0.0 0.0
0.0 0.0 -0.94335 -1.77381 0.0 0.0
-0.15728 -1.02203 0.0 0.0 1.00000 0.49253
0.0 0.0 0.0 0.0 0.0 1.00000

QUAD 5.00 2.00000 M -1.9000 KG 5.000 CM (-10.646 M)
VARY CODE = 010000
LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 1.8500 KG 5.000 CM (11.609 M)
VARY CODE = 010000
LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M 0.0 6.365 CM
0.0 0.211 MR -0.543
0.0 0.620 CM 0.0 0.0
0.0 0.049 MR 0.0 0.0 -0.138
0.0 0.588 CM -0.854 0.545 0.0 0.0
0.0 1.000 PC -0.943 0.790 0.0 0.0 0.838

FIT 10.0 1. 1. 4.000 / 0.100
6.365
VARY CODE = 200000
LABEL = FIT4

FIT 10.0 3. 3. 1.500 / 0.100
0.620
VARY CODE = 200000
LABEL = FIT4

TRANSFORM 1
3.42558 18.46803 0.0 0.0 0.0 -6.00397
0.16690 1.19030 0.0 0.0 0.0 0.16643

0.0	0.0	-0.30513	-6.13656	0.0	0.0
0.0	0.0	0.16271	-0.00492	0.0	0.0
-0.15728	-1.02203	0.0	0.0	1.00000	0.49253
0.0	0.0	0.0	0.0	0.0	1.00000

381.0 M	0.0	6.365 CM				
	0.0	0.211 MR	-0.543			
	0.0	0.620 CM	0.0	0.0		
	0.0	0.049 MR	0.0	0.0	-0.138	
	0.0	0.588 CM	-0.854	0.545	0.0	0.0
	0.0	1.000 PC	-0.943	0.790	0.0	0.0
						0.838

QUAD 5.00 2.00000 M 4.5000 KG 5.000 CM (4.984 M)
 VARY CODE = 010000
 LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M	0.0	2.447 CM				
	0.0	12.863 MR	-0.999			
	0.0	1.044 CM	0.0	0.0		
	0.0	1.431 MR	0.0	0.0	1.000	
	0.0	0.588 CM	-0.851	0.855	0.0	0.0
	0.0	1.000 PC	-0.935	0.947	0.0	0.0
						0.838

QUAD 5.00 2.00000 M -4.2000 KG 5.000 CM (-4.645 M)
 VARY CODE = 010000
 LABEL = Q4

387.0 M	0.0	0.232 CM				
	0.0	10.281 MR	-0.881			
	0.0	1.108 CM	0.0	0.0		
	0.0	0.813 MR	0.0	0.0	-0.999	
	0.0	0.588 CM	-0.721	0.857	0.0	0.0
	0.0	1.000 PC	-0.696	0.953	0.0	0.0
						0.838

DRIFT 3.0 15.0000 M

402.0 M	0.0	15.217 CM				
	0.0	10.281 MR	1.000			
	0.0	0.119 CM	0.0	0.0		
	0.0	0.813 MR	0.0	0.0	0.950	
	0.0	0.588 CM	0.857	0.857	0.0	0.0
	0.0	1.000 PC	0.955	0.953	0.0	0.0
						0.838

FIT 10.0 2. 1. 3.0 / 0.001
 156.438
 LABEL = FIT5

FIT 10.C 4. 3. 0.0 / 0.001
0.092
LABEL = FIT6

FIT 10.C 1. 1. 0.020 / 0.010
15.217
VARY CODE = 2C0000
LABEL = FIT7

FIT 10.0 3. 3. 0.020 / 0.010
0.119
VARY CODE = 2C0000
LABEL = FIT7

TRANSFORM 1
-2.33281 3.132CC 0.C 0.C 0.0 0.0
-1.58390 1.65786 0.0 0.0 0.0 0.0
0.0 0.0 -0.17269 0.77960 0.0 0.0
0.0 0.0 -1.30944 0.12067 0.0 0.0
0.0 0.0 0.0 0.0 1.00000 0.0
0.0 0.0 0.0 0.0 0.0 1.00000

LENGTH 401.9595 M

CORRECTIONS

(1.8565E 05)	0.152	-0.247	0.0	0.0	0.0	0.050	-0.057	0.0
(4.7894E 04)	0.009	-0.008	0.0	0.0	-0.040	0.042	0.0	0.0
(1.3659E 04)	0.022	-0.047	0.0	-1.439	-0.099	0.131	0.0	0.0
(1.9515E 05)	-0.009	0.017	0.0	0.391	0.0	0.0	-0.046	0.0
(5.5914E 04)	0.005	-0.010	0.0	-0.402	0.0	0.369	0.194	0.0
(4.2568E 05)	0.003	-0.000	0.0	0.221	0.0	0.0	-0.007	0.085
(2.1816E 05)	-0.002	0.000	0.0	-0.226	0.0	-0.233	-0.029	0.012
(2.8178E 05)	-0.002	0.003	0.0	0.258	0.0	0.0	-0.016	0.0
(9.1541E 04)	0.002	-0.003	0.0	-0.247	0.0	-1.646	-0.454	-0.115
(9.1299E 05)	0.057	-0.038	0.0	-0.266	0.0	0.0	-0.003	0.003

FAILED

BEAM SWITCHYARD END STATION A ALL FITTING AT ONCE

BEAM 1.000000 25.00 GEV

0.0 M	0.0	0.300 CM						
	0.0	0.100 MR	0.0					
	0.0	0.300 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.0			
	0.0	0.300 CM	0.0	0.0	0.0	0.0		
	0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

ROTAT 2.0 0.0 D

BEND 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M	0.0	0.305 CM						
	0.0	0.133 MR	0.171					
	0.0	0.304 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.164			
	0.0	0.300 CM	-0.009	-0.001	0.0	0.0		
	0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000	

DRIFT 3.0 80.0000 M

85.0 M	0.0	1.153 CM						
	0.0	0.133 MR	0.965					
	0.0	0.901 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.943			
	0.0	0.300 CM	-0.003	-0.001	0.0	0.0		
	0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000	

QUAD 5.00 2.00000 M -1.9227 KG 5.000 CM (-10.517 M)
 VARY CODE = 010000
 LABEL = Q1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.0662 KG 5.000 CM (10.431 M)
 VARY CODE = 010000
 LABEL = Q1

91.0 M	0.0	1.625 CM						
	0.0	0.356 MR	-0.998					
	0.0	0.615 CM	0.0	0.0				
	0.0	0.094 MR	0.0	0.0	-0.856			

0.0	C.300 CM	-0.003	0.003	0.0	0.0	
0.0	1.000 PC	0.627	-0.611	0.0	0.0	-0.000

DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			

107.0 M

0.0	2.385 CM				
0.0	1.898 MR	0.878			
0.0	0.477 CM	0.0	0.0		
0.0	0.111 MR	0.0	0.0	-0.824	
0.0	0.470 CM	-0.737	-0.542	0.0	0.0
0.0	1.000 PC	0.939	0.989	0.0	0.0
					-0.617

DRIFT 3.0 30.0000 M
VARY CODE = 900000
LABEL = DR1

QUAD 5.00 2.00000 M 3.2909 KG 15.000 CM (19.342 M)
VARY CODE = 010000
LABEL = Q2

139.0 M	0.0	7.830 CM				
	0.0	2.304 MR	-0.993			
	0.0	0.287 CM	0.0	0.0		
	0.0	0.168 MR	0.0	0.0	0.783	
	0.0	0.470 CM	-0.612	0.663	0.0	0.0
	0.0	1.000 PC	1.000	-0.994	0.0	0.0
						-0.617

FIT 10.0 -1. 2. 0.0 / 0.001
-0.405
LABEL = FIT1

FIT 10.0 -3. 4. 0.0 / 0.001
2.223
LABEL = FIT1

TRANSFCRM 1
-0.42470 -0.40498 0.0 0.0 0.0 7.82867
-0.16137 -2.50845 0.0 0.0 0.0 -2.29023
0.0 0.0 -0.60594 2.22344 0.0 0.0
0.0 0.0 -0.54673 0.35586 0.0 0.0
-0.22360 -2.05654 0.0 0.0 1.00000 -0.28970
0.0 0.0 0.0 0.0 0.0 1.00000

DRIFT 3.0 20.0000 M
VARY CODE = 400000
LABEL = DR1

159.0 M	0.0	3.301 CM				
	0.0	2.304 MR	-0.959			
	0.0	0.588 CM	0.0	0.0		
	0.0	0.168 MR	0.0	0.0	0.953	
	0.0	0.470 CM	-0.525	0.663	0.0	0.0
	0.0	1.000 PC	0.984	-0.994	0.0	0.0
						-0.617

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M

175.0 M

0.0	1.686 CM					
0.0	0.322 MR	-0.030				
0.0	0.827 CM	0.0	0.0			
0.0	0.143 MR	0.0	0.0	0.967		
0.0	0.787 CM	-0.706	0.616	0.0	0.0	
0.0	1.000 PC	0.809	-0.609	0.0	0.0	-0.922

FIT 10.0 -1.6. 0.0 / 0.001

1.365

LABEL = FIT2

FIT 10.0 -2. 6. 0.0 / 0.001

-0.196

LABEL = FIT3

TRANSFORM 1

-1.00555	-9.43406	0.0	0.0	0.0	1.36466
-0.16138	-2.50852	0.0	0.0	0.0	-0.19597
0.0	0.0	-2.51335	3.40842	0.0	0.0
0.0	0.0	-0.47030	0.23991	0.0	0.0
-0.04173	-0.52722	0.0	0.0	1.00000	-0.72547
0.0	0.0	0.0	0.0	0.0	1.00000

QUAD 5.00 2.00000 M -2.0393 KG 5.000 CM (-9.897 M)

VARY CODE = 010000

LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 0.5624 KG 5.000 CM (37.404 M)

VARY CODE = 010000

LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M

0.0	24.547 CM				
0.0	1.106 MR	1.000			
0.0	9.523 CM	0.0	0.0		
0.0	0.502 MR	0.0	0.0	1.000	
0.0	0.787 CM	-0.528	-0.512	0.0	0.0
0.0	1.000 PC	0.630	0.615	0.0	0.0

-0.922

FIT 10.0 1. 1. 4.000 / C.100

24.547

VARY CODE = 200000

LABEL = FIT4

FIT 10.0 3. 3. 1.500 / C.100

9.523

VARY CODE = 200000

LABEL = FIT4

TRANSFORM 1

-17.43910	-183.25438	0.0	0.0	0.0	15.47083
-0.79310	-8.39143	0.0	0.0	0.0	0.67966

C.0	0.C	27.77695	-46.C9175	0.0	0.0
0.0	0.C	1.47038	-2.40389	0.0	0.0
-0.04173	-0.52722	0.C	0.0	1.00000	-0.72547
0.0	0.C	0.0	0.0	0.0	1.00000

381.0 M	0.0	24.547 CM				
	0.0	1.106 MR	1.000			
	0.0	9.523 CM	0.0	0.0		
	0.0	0.502 MR	0.0	0.0	1.000	
	0.0	0.787 CM	-0.528	-0.512	0.0	0.0
	0.0	1.000 PC	0.630	0.615	0.0	0.0 -0.922

QUAD 5.00 2.00000 M 4.0428 KG 5.000 CM (5.506 M)
 VARY CODE = 010000
 LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M	0.0	11.409 CM				
	0.0	43.685 MR	-1.000			
	0.0	15.594 CM	0.0	0.0		
	0.0	20.287 MR	0.0	0.0	1.000	
	0.0	0.787 CM	-0.527	0.528	0.0	0.0
	0.0	1.000 PC	0.630	-0.631	0.0	0.0 -0.922

QUAD 5.00 2.00000 M -4.2154 KG 5.000 CM (-4.627 M)
 VARY CODE = 010000
 LABEL = Q4

387.0 M	0.0	4.456 CM				
	0.0	28.165 MR	-1.000			
	0.0	16.335 CM	0.0	0.0		
	0.0	13.128 MR	0.0	0.0	-1.000	
	0.0	0.787 CM	-0.525	0.529	0.0	0.0
	0.0	1.000 PC	0.628	-0.631	0.0	0.0 -0.922

DRIFT 3.0 15.0000 M

402.0 M	0.0	37.791 CM				
	0.0	28.165 MR	1.000			
	0.0	3.356 CM	0.0	0.0		
	0.0	13.128 MR	0.0	0.0	1.000	
	0.0	0.787 CM	0.529	0.529	0.0	0.0
	0.0	1.000 PC	-0.632	-0.631	0.0	0.0 -0.922

FIT 10.0 2. 1. 0.0 / 0.001

1064.355
 LABEL = FIT5

FIT 10.0 4. 3. 0.0 / 0.001

44.064
LABEL = FIT6

FIT 10.0 1. 1. 0.020 / 0.010

37.791

VARY CODE = 200000
LABEL = FIT7

FIT 10.0 3. 3. 0.020 / 0.010

3.356

VARY CODE = 200000
LABEL = FIT7

TRANSFORM 1

-1.68385	3.20353	0.0	0.0	0.0	0.0
-1.22568	1.73799	0.0	0.0	0.0	0.0
0.0	0.0	-0.39221	0.75345	0.0	0.0
0.0	0.0	-1.38436	0.10974	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.9995 M

```

'BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A 1'
0
(THE FIRST STEP TO FITTING IS TO ALLOW ONLY THE POSITION AND FIELD OF THE)
(SYMMETRY QUAD,Q2, TO VARY IN ORDER TO GET AN ACHROMATIC CONDITION BETWEEN THE)
(FIRST SET OF BENDING MAGNETS AND THE SECOND QUAD DOUBLET,Q3.)
1.0 0.3 0.1 0.3 0.1 0.3 1.0 25.0 ;
13.0 2.0 ;
4.0 5.0 1.45500 0.0 ;
2.0 0.5 ;
13.0 1.0 ;
3.0 80.0 ;
13.0 1.0 ;
5.0 2.0 -2.2 5.0 'Q1' ;
3.0 2.0 ;
5.0 2.0 2.4 5.0 'Q1' ;
13.0 1.0 ;
9.0 4.0 ;
3.0 1.0 ;
2.0 1.5 ;
4.0 3.0 14.55000 0.0 ;
2.0 1.5 ;
9.0 0.0 ;
13.0 1.0 ;
3.9 30.0 'DR1' ;
5.01 2.0 5.0 15.0 'Q2' ;
13.0 1.0 ;
-10.0 -1.0 2.0 0.0 0.001 'FIT1' ;
-10.0 -3.0 4.0 0.0 0.001 'FIT1' ;
13.0 4.0 ;
3.4 20.0 'DR1' ;
13.0 1.0 ;
9.0 4.0 ;
2.0 1.50 ;
4.0 3.0 14.55000 0.0 ;
2.0 1.5 ;
3.0 1.0 ;
9.0 0.0 ;
13.0 1.0 ;
10.0 -1.0 6.0 0.0 0.001 'FIT2' ;
10.0 -2.0 6.0 0.0 0.001 'FIT3' ;
13.0 4.0 ;
5.0 2.0 -1.9 5.0 'Q3' ;
3.0 2.0 ;
5.0 2.0 1.85000 5.0 'Q3' ;
3.0 200.0 ;
13.0 1.0 ;
-10.2 1.0 1.0 4.0 0.1 'FIT4' ;
-10.2 3.0 3.0 1.5 0.1 'FIT4' ;
13.0 4.0 ;
13.0 1.0 ;
5.0 2.0 4.5 5.0 'Q4' ;
3.0 2.0 ;
13.0 1.0 ;
5.0 2.0 -4.2 5.0 'Q4' ;
13.0 1.0 ;
3.0 15.0 ;
13.0 1.0 ;
-10.0 2.0 1.0 0.0 0.001 'FIT5' ;
-10.0 4.0 3.0 0.0 0.001 'FIT6' ;
-10.2 1.0 1.0 0.02000 0.01000 'FIT7' ;
-10.2 3.0 3.0 0.02000 0.01000 'FIT7' ;
13.0 4.0 ;
SENTINEL

```

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A I

0

THE FIRST STEP TO FITTING IS TO ALLOW ONLY THE POSITION AND FIELD OF THE
SYMMETRY QUAD,Q2, TO VARY IN ORDER TO GET AN ACHROMATIC CONDITION BETWEEN THE
FIRST SET OF BENDING MAGNETS AND THE SECOND QUAD DOUBLET,Q3.

1.00000	0.30000	0.10000	0.30000	0.10000	0.30000	1.00000	25.00000
13.00000	2.00000						
4.00000	5.00000	1.45500	0.0				
2.00000	0.50000						
13.00000	1.00000						
3.00000	80.00000						
13.00000	1.00000						
5.00000	2.00000	-2.20000	5.00000				
LABEL = Q1							
3.00000	2.00000						
5.00000	2.00000	2.40000	5.00000				
LABEL = Q1							
13.00000	1.00000						
9.00000	4.00000						
3.00000	1.00000						
2.00000	1.50000						
4.00000	3.00000	14.55000	0.0				
2.00000	1.50000						
9.00000	0.0						
13.00000	1.00000						
3.00000	30.00000						
VARY CODE 900000							
LABEL = OR1							
5.00000	2.00000	5.00000	15.00000				
VARY CODE 010000							
LABEL = Q2							
13.00000	1.00000						
-10.00000	-1.00000	2.00000	0.0	0.00100			
LABEL = FIT1							

-10.00000	-3.00000	4.00000	0.0	0.00100
LABEL = FIT1				
13.00000	4.00000			
3.00000	20.00000			
VARY CODE 400000				
LABEL = DR1				
13.00000	1.00000			
9.00000	4.00000			
2.00000	1.50000			
4.00000	3.00000	14.55000	0.0	
2.00000	1.50000			
3.00000	1.00000			
9.00000	0.0			
13.00000	1.00000			
10.00000	-1.00000	0.00000	0.0	0.00100
LABEL = FIT2				
10.00000	-2.00000	0.00000	0.0	0.00100
LABEL = FIT3				
13.00000	4.00000			
5.00000	2.00000	-1.90000	5.00000	
LABEL = Q3				
3.00000	2.00000			
5.00000	2.00000	1.35000	5.00000	
LABEL = C3				
3.00000	200.00000			
13.00000	1.00000			
-10.00000	1.00000	1.00000	4.00000	0.10000
VARY CODE 200000				
LABEL = FIT4				
-10.00000	3.00000	3.00000	1.50000	0.10000
VARY CODE 200000				
LABEL = FIT4				
13.00000	4.00000			
13.00000	1.00000			
5.00000	2.00000	4.50000	5.00000	
LABEL = C4				

3.00000	2.00000			
13.00000	1.00000			
5.00000	2.00000	-4.20000	5.00000	
LABEL = G4				
13.00000	1.00000			
3.00000	15.00000			
13.00000	1.00000			
-10.00000	2.00000	1.00000	0.0	0.00100
LABEL = FIT5				
-10.00000	4.00000	3.00000	0.0	0.00100
LABEL = FIT6				
-10.00000	1.00000	1.00000	0.02000	0.01000
VARY CODE 200000				
LABEL = FIT7				
-10.00000	3.00000	3.00000	0.02000	0.01000
VARY CODE 200000				
LABEL = FIT7				
13.00000	4.00000			
SENTINEL				

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A I

BEAM 1.000000 25.00 GEV

0.0 M	0.0	0.300 CM						
	0.0	0.100 MR	0.0					
	0.0	0.300 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.0			
	0.0	0.300 CM	0.0	0.0	0.0	0.0		
	0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

BEND 4.000 5.03000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M	0.0	0.305 CM						
	0.0	0.133 MR	0.171					
	0.0	0.304 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.164			
	0.0	0.300 CM	-0.009	-0.001	0.0	0.0		
	0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000	

DRIFT 3.0 80.0000 M

85.0 M	0.0	1.153 CM						
	0.0	0.133 MR	0.965					
	0.0	0.901 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.943			
	0.0	0.300 CM	-0.003	-0.001	0.0	0.0		
	0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000	

QUAD 5.00 2.00000 M -2.2000 KG 5.000 CM (-9.151 M)
LABEL = G1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.4000 KG 5.000 CM (9.029 M)
LABEL = G1

91.0 M	0.0	1.675 CM						
	0.0	0.511 MR	-0.999					
	0.0	0.564 CM	0.0	0.0				
	0.0	0.151 MR	0.0	0.0	-0.936			
	0.0	0.300 CM	-0.003	0.003	0.0	0.0		
	0.0	1.000 PC	0.626	-0.616	0.0	0.0	-0.000	

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D

107.0 M

0.0	2.218 CM					
0.0	1.824 MR	0.865				
0.0	0.336 CM	0.0	0.0			
0.0	0.165 MR	0.0	0.0	-0.841		
0.0	0.457 CM	-0.715	-0.497	0.0	0.0	
0.0	1.000 PC	0.954	0.975	0.0	0.0	-0.610

DRIFT 3.0 30.0000 M
 VARY CODE = 900000
 LABEL = DR1

QUAD 5.00 2.00000 M 5.0000 KG 15.000 CM (12.848 M)
 VARY CODE = 010000
 LABEL = Q2

139.C M	0.0	7.236 CM						
	0.0	4.163 MR	-0.998					
	0.0	0.329 CM	0.0	0.0				
	0.0	0.384 MR	0.0	0.0	0.971			
	0.0	0.457 CM	-0.573	0.605	0.0	0.0		
	0.0	1.000 PC	0.997	-1.000	0.0	0.0	-0.610	

TRANSFORM 1

-0.93476	-5.14981	0.0	0.0	0.0	7.21223
0.24797	0.29632	0.0	0.0	0.0	-4.16246
0.0	0.0	-1.04482	-1.01024	0.0	0.0
0.0	0.0	-1.08580	-2.01081	0.0	0.0
-0.21025	-1.92988	0.0	0.0	1.00000	-0.27894
0.0	0.0	0.0	0.0	0.0	1.00000

DRIFT 3.0 20.0000 M
 VARY CODE = 400000
 LABEL = DR1

159.0 M	0.0	1.210 CM						
	0.0	4.163 MR	0.915					
	0.0	1.090 CM	0.0	0.0				
	0.0	0.384 MR	0.0	0.0	0.997			
	0.0	0.457 CM	0.735	0.605	0.0	0.0		
	0.0	1.000 PC	-0.920	-1.000	0.0	0.0	-0.610	

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

```

*ROTAT* 2.0 1.50 D
*BEND* 4.000 3.00000 M 14.550 KG 0.0000 ( 2.999 D )
*ROTAT* 2.0 1.50 D
*DRIFT* 3.0 1.0000 M
*ROTAT* 2.0 1.50 D
*BEND* 4.000 3.00000 M 14.550 KG 0.0000 ( 2.999 D )
*ROTAT* 2.0 1.50 D
*DRIFT* 3.0 1.0000 M

```

```

175.0 M 0.0 6.005 CM
0.0 2.070 MR 0.996
0.0 1.664 CM 0.0 0.0
0.0 0.334 MR 0.0 0.0 0.999
0.0 0.588 CM -0.824 -0.843 0.0 0.0
0.0 1.000 PC -0.998 -0.999 0.0 0.0 0.838

```

```

*FIT* 10.0 -1. 6. 0.0 / 0.001

```

```

-5.991

```

```

LABEL = FIT2

```

```

*FIT* 10.0 -2. 6. 0.0 / 0.001

```

```

-2.068

```

```

LABEL = FIT3

```

```

*TRANSFORM* 1

```

```

-0.04222 -4.08323 0.0 0.0 0.0 -5.99086
0.24796 0.29627 0.0 0.0 0.0 -2.06827
0.0 0.0 -4.85194 -8.06319 0.0 0.0
0.0 0.0 -0.94335 -1.77381 0.0 0.0
-0.15728 -1.02203 0.0 0.0 1.00000 0.49253
0.0 0.0 0.0 0.0 0.0 1.00000

```

```

*QUAD* 5.00 2.00000 M -1.9000 KG 5.000 CM ( -10.646 M )

```

LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 1.8500 KG 5.000 CM (11.609 M)
 LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M	0.0	6.365 CM					
	0.0	0.211 MR	-0.543				
	0.0	0.620 CM	0.0	0.0			
	0.0	0.049 MR	0.0	0.0	-0.138		
	0.0	0.588 CM	-0.854	0.545	0.0	0.0	
	0.0	1.000 PC	-0.943	0.790	0.0	0.0	0.838

TRANSFCRM 1

3.42958	18.46803	0.0	0.0	0.0	-6.00397
0.16690	1.19030	0.0	0.0	0.0	0.16643
0.0	0.0	-0.30513	-6.13656	0.0	0.0
0.0	0.0	0.16271	-0.00492	0.0	0.0
-0.15728	-1.02203	0.0	0.0	1.00000	0.49253
0.0	0.0	0.0	0.0	0.0	1.00000

381.0 M	0.0	6.365 CM					
	0.0	0.211 MR	-0.543				
	0.0	0.620 CM	0.0	0.0			
	0.0	0.049 MR	0.0	0.0	-0.138		
	0.0	0.588 CM	-0.854	0.545	0.0	0.0	
	0.0	1.000 PC	-0.943	0.790	0.0	0.0	0.838

QUAD 5.00 2.00000 M 4.5000 KG 5.000 CM (4.984 M)
 LABEL = C4

DRIFT 3.0 2.0000 M

385.0 M	0.0	2.447 CM					
	0.0	12.863 MR	-0.999				
	0.0	1.044 CM	0.0	0.0			
	0.0	1.431 MR	0.0	0.0	1.000		
	0.0	0.588 CM	-0.851	0.855	0.0	0.0	
	0.0	1.000 PC	-0.935	0.947	0.0	0.0	0.838

QUAD 5.00 2.00000 M -4.2000 KG 5.000 CM (-4.645 M)
 LABEL = C4

387.0 M	0.0	0.232 CM				
	0.0	10.281 MP	-0.881			

0.0	1.108 CM	0.0	0.0			
0.0	0.813 MR	0.0	0.0	-0.999		
0.0	0.588 CM	-0.721	0.857	0.0	0.0	
0.0	1.000 PC	-0.696	0.953	0.0	0.0	0.838

DRIFT 3.0 15.0000 M

402.0 M

0.0	15.217 CM					
0.0	10.281 MR	1.000				
0.0	0.119 CM	0.0	0.0			
0.0	0.813 MR	0.0	0.0	0.950		
0.0	0.588 CM	0.857	0.857	0.0	0.0	
0.0	1.000 PC	0.955	0.953	0.0	0.0	0.838

TRANSFORM 1

-7.47783	-39.35443	0.0	0.0	0.0	14.52741
-5.14875	-27.23064	0.0	0.0	0.0	9.79230
0.0	0.0	0.17954	1.05592	0.0	0.0
0.0	0.0	0.41918	8.03487	0.0	0.0
-0.15728	-1.02203	0.0	0.0	1.00000	0.49253
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.9995 M

CORRECTIONS

(4.4815E 03)	2.801	-1.453
(1.5177E 02)	1.457	0.043
(1.5083E 01)	0.037	0.005
(1.2714E-02)	-0.000	0.000

*COVARIANCE (FIT 0.0)

0.011
0.480 0.000

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A I

BEAM 1.000000 25.00 GEV

0.0 M

0.0	0.300 CM						
0.0	0.100 MR	0.0					
0.0	0.300 CM	0.0	0.0				
0.0	0.100 MR	0.0	0.0	0.0			
0.0	0.300 CM	0.0	0.0	0.0	0.0		
0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0	

BEND 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M

0.0	0.305 CM						
0.0	0.133 MR	0.171					
0.0	0.304 CM	0.0	0.0				
0.0	0.100 MR	0.0	0.0	0.164			
0.0	0.300 CM	-0.009	-0.001	0.0	0.0		
0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000	

DRIFT 3.0 80.0000 M

85.0 M

0.0	1.153 CM						
0.0	0.133 MR	0.965					
0.0	0.901 CM	0.0	0.0				
0.0	0.100 MR	0.0	0.0	0.943			
0.0	0.300 CM	-0.003	-0.001	0.0	0.0		
0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000	

QUAD 5.00 2.00000 M -2.2000 KG 5.000 CM (-9.151 M)
LABEL = Q1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.4000 KG 5.000 CM (9.029 M)
LABEL = Q1

91.0 M

0.0	1.675 CM						
0.0	0.511 MR	-0.999					
0.0	0.564 CM	0.0	0.0				
0.0	0.151 MR	0.0	0.0	-0.936			
0.0	0.300 CM	-0.003	0.003	0.0	0.0		
0.0	1.000 PC	0.626	-0.616	0.0	0.0	-0.000	

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D

107.0 M

0.0	2.218 CM					
0.0	1.824 MR	0.865				
0.0	0.336 CM	0.0	0.0			
0.0	0.165 MR	0.0	0.0	-0.841		
0.0	0.457 CM	-0.715	-0.497	0.0	0.0	
0.0	1.000 PC	0.954	0.975	0.0	0.0	-0.610

DRIFT 3.0 25.7057 M
 VARY CCDE = 900000
 LABEL = CRI

QUAD 5.00 2.00000 M 3.5952 KG 15.000 CM (17.734 M)
 VARY CODE = 010000
 LABEL = Q2

134.7 M	0.0	6.672 CM				
	0.0	2.101 MR	-0.989			
	0.0	0.266 CM	0.0	0.0		
	0.0	0.279 MR	0.0	0.0	0.915	
	0.0	0.457 CM	-0.580	0.644	0.0	0.0
	0.0	1.000 PC	0.998	-0.997	0.0	0.0 -0.610

TRANSFORM 1

-0.75331	-3.74802	0.0	0.0	0.0	6.65790
-0.07884	-1.71568	0.0	0.0	0.0	-2.09424
0.0	0.0	-0.87796	-0.38176	0.0	0.0
0.0	0.0	-0.78899	-1.48206	0.0	0.0
-0.21025	-1.92988	0.0	0.0	1.00000	-0.27894
0.0	0.0	0.0	0.0	0.0	1.00000

DRIFT 3.0 24.2942 M
 VARY CODE = 400000
 LABEL = DR1

159.0 M	0.0	1.782 CM				
	0.0	2.101 MR	-0.840			
	0.0	0.928 CM	0.0	0.0		
	0.0	0.279 MR	0.0	0.0	0.993	
	0.0	0.457 CM	-0.328	0.644	0.0	0.0
	0.0	1.000 PC	0.881	-0.997	0.0	0.0 -0.610

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M


```

*ROTAT*    2.0    1.50 D

*BEND*     4.000  3.00000 M   14.550 KG   0.0000 (   2.999 D )

*ROTAT*    2.0    1.50 D

*DRIFT*     3.0    1.0000 M

*ROTAT*    2.0    1.50 D

*BEND*     4.000  3.00000 M   14.550 KG   0.0000 (   2.999 D )

*ROTAT*    2.0    1.50 D

*DRIFT*     3.0    1.0000 M

```

```

                                175.0 M
                                0.0    1.115 CM
                                0.0    0.174 MR    0.988
                                0.0    1.340 CM    0.0    0.0
                                0.0    0.238 MR    0.0    0.0    0.996
                                0.0    0.495 CM   -0.000 -0.000 0.0    0.0
                                0.0    1.000 PC    0.000 0.000 0.0    0.0    -0.796

```

```

*FIT*      10.0   -1. 6.   0.0   / 0.001
                                0.000
      LABEL = FIT2

```

```

*FIT*      10.0   -2. 6.   0.0   / 0.001
                                0.000
      LABEL = FIT3

```

```

*TRANSFORM* 1
      -1.C7094 -10.67648   0.0    0.0    0.0    0.00001
      -C.C7885 -1.71978   0.0    0.0    0.0    0.00000
      0.0      0.0      -3.95881 -6.20807 0.0    0.0
      0.0      0.0      -0.66530 -1.29747 0.0    0.0
      -0.00000 -0.00001 0.0      0.0    1.00000 -0.39405
      0.0      0.0      0.0      0.0    0.0    1.00000

```

```

*QUAD*     5.00  2.00000 M   -1.9000 KG   5.000 CM ( -10.646 M )

```

LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 1.8500 KG 5.000 CM (11.609 M)
LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M	0.0	1.742 CM					
	0.0	0.167 MR	0.995				
	0.0	0.824 CM	0.0	0.0			
	0.0	0.082 MR	0.0	0.0	0.895		
	0.0	0.495 CM	0.000	0.000	0.0	0.0	
	0.0	1.000 PC	-0.000	-0.000	0.0	0.0	-0.796

TRANSFORM 1

2.77343	15.30110	0.0	0.0	0.0	-0.00001
0.21337	1.53770	0.0	0.0	0.0	-0.00000
0.0	0.0	2.57533	-2.86091	0.0	0.0
0.0	0.0	0.27088	0.08736	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.39405
0.0	0.0	0.0	0.0	0.0	1.00000

381.0 M	0.0	1.742 CM					
	0.0	0.167 MR	0.995				
	0.0	0.824 CM	0.0	0.0			
	0.0	0.082 MR	0.0	0.0	0.895		
	0.0	0.495 CM	0.000	0.000	0.0	0.0	
	0.0	1.000 PC	-0.000	-0.000	0.0	0.0	-0.796

QUAD 5.00 2.00000 M 4.5000 KG 5.000 CM (4.984 M)
LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M	0.0	0.737 CM					
	0.0	3.364 MR	-1.000				
	0.0	1.424 CM	0.0	0.0			
	0.0	1.999 MR	0.0	0.0	1.000		
	0.0	0.495 CM	0.000	-0.000	0.0	0.0	
	0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.796

QUAD 5.00 2.00000 M -4.2000 KG 5.000 CM (-4.645 M)
LABEL = Q4

387.0 M	0.0	0.172 CM					
	0.0	2.478 MR	-0.998				

0.0	1.520 CM	0.0	0.0			
0.0	1.070 MR	0.0	0.0	-1.000		
0.0	0.495 CM	0.000	-0.000	0.0	0.0	
0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.796

DRIFT 3.0 15.0000 M

402.0 M

0.0	3.545 CM					
0.0	2.478 MR	1.000				
0.0	0.090 CM	0.0	0.0			
0.0	1.070 MR	0.0	0.0	0.950		
0.0	0.495 CM	-0.000	-0.000	0.0	0.0	
0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.796

TRANSFORM 1

-5.80162	-30.87849	0.0	0.0	0.0	0.00003
-4.03058	-21.62466	0.0	0.0	0.0	0.00002
0.0	0.0	-0.23357	0.56217	0.0	0.0
0.0	0.0	-3.33957	3.75673	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.39405
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.9993 M

'BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A II' \

1

(THE SECOND STEP TO FITTING IS TO ALLOW THE FIELDS OF THE FIRST QUAD)
(DOUBLET,Q1, TO VARY AS WELL AS THE SYMMETRY QUAD IN ORDER TO GET AN IMAGE AT)
(THE EXIT OF THE SYMMETRY QUAD,Q2, AS WELL AS THE PREVIOUSLY MENTIONED)
(ACHROMATIC CONDITION.)

5.01 'Q1' ;

10.0 'FIT1' ;

SENTINEL

1

THE SECOND STEP TO FITTING IS TO ALLOW THE FIELDS OF THE FIRST QUAD
 DCUBLET,Q1, TO VARY AS WELL AS THE SYMMETRY QUAD IN ORDER TO GET AN IMAGE AT
 THE EXIT OF THE SYMMETRY QUAD,Q2, AS WELL AS THE PREVIOUSLY MENTIONED
 ACHROMATIC CCNDITION.

1.00000	0.30000	0.10000	0.30000	0.10000	0.30000	1.00000	25.00000
13.00000	2.00000						
4.00000	5.00000	1.45500	0.0				
2.00000	0.50000						
13.00000	1.00000						
3.00000	80.00000						
13.00000	1.00000						
5.00000	2.00000	-2.20000	5.00000				
VARY CODE	010000						
LABEL = Q1							
3.00000	2.00000						
5.00000	2.00000	2.40000	5.00000				
VARY CODE	010000						
LABEL = Q1							
13.00000	1.00000						
9.00000	4.00000						
3.00000	1.00000						
2.00000	1.50000						
4.00000	3.00000	14.55000	0.0				
2.00000	1.50000						
9.00000	0.0						
13.00000	1.00000						
3.00000	25.70569						
VARY CODE	900000						
LABEL = DR1							
5.00000	2.00000	3.59519	15.00000				
VARY CODE	010000						
LABEL = Q2							

13.00000	1.00000			
10.00000	-1.00000	2.00000	0.0	0.00100
LABEL = FIT1				
10.00000	-3.00000	4.00000	0.0	0.00100
LABEL = FIT1				
13.00000	4.00000			
3.00000	24.29425			
VARY CODE 400000				
LABEL = CR1				
13.00000	1.00000			
9.00000	4.00000			
2.00000	1.50000			
4.00000	3.00000	14.55000	0.0	
2.00000	1.50000			
3.00000	1.00000			
9.00000	0.0			
13.00000	1.00000			
10.00000	-1.00000	6.00000	0.0	0.00100
LABEL = FIT2				
10.00000	-2.00000	6.00000	0.0	0.00100
LABEL = FIT3				
13.00000	4.00000			
5.00000	2.00000	-1.90000	5.00000	
LABEL = Q3				
3.00000	2.00000			
5.00000	2.00000	1.85000	5.00000	
LABEL = Q3				
3.00000	200.00000			
13.00000	1.00000			
-10.00000	1.00000	1.00000	4.00000	0.10000
VARY CODE 200000				
LABEL = FIT4				
-10.00000	3.00000	3.00000	1.50000	0.10000
VARY CODE 200000				
LABEL = FIT4				
13.00000	4.00000			

13.00000	1.00000			
5.00000	2.00000	4.50000	5.00000	
LABEL = Q4				
3.00000	2.00000			
13.00000	1.00000			
5.00000	2.00000	-4.20000	5.00000	
LABEL = Q4				
13.00000	1.00000			
3.00000	15.00000			
13.00000	1.00000			
-10.00000	2.00000	1.00000	0.0	0.00100
LABEL = FIT5				
-10.00000	4.00000	3.00000	0.0	0.00100
LABEL = FIT6				
-10.00000	1.00000	1.00000	0.02000	0.01000
VARY CODE 200000				
LABEL = FIT7				
-10.00000	3.00000	3.00000	0.02000	0.01000
VARY CODE 200000				
LABEL = FIT7				
13.00000	4.00000			
SENTINEL				

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A II

BEAM 1.000000 25.00 GEV

0.0 M	0.0	0.300 CM						
	0.0	0.100 MR	0.0					
	0.0	0.300 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.0			
	0.0	0.300 CM	0.0	0.0	0.0	0.0		
	0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

BEND 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M	0.0	0.305 CM						
	0.0	0.133 MR	0.171					
	0.0	0.304 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.164			
	0.0	0.300 CM	-0.009	-0.001	0.0	0.0		
	0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000	

DRIFT 3.0 80.0000 M

85.0 M	0.0	1.153 CM						
	0.0	0.133 MR	0.965					
	0.0	0.901 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.943			
	0.0	0.300 CM	-0.003	-0.001	0.0	0.0		
	0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000	

QUAD 5.00 2.00000 M -2.2000 KG 5.000 CM (-9.151 M)
VARY CODE = 010000
LABEL = Q1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.4000 KG 5.000 CM (9.029 M)
VARY CODE = 010000
LABEL = Q1

91.0 M	0.0	1.675 CM						
	0.0	0.511 MR	-0.999					
	0.0	0.564 CM	0.0	0.0				
	0.0	0.151 MR	0.0	0.0	-0.936			
	0.0	0.300 CM	-0.003	0.003	0.0	0.0		
	0.0	1.000 PC	0.626	-0.616	0.0	0.0	-0.000	

DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			

107.0 M	0.0	2.218 CM				
	0.0	1.824 MR	0.865			
	0.0	0.336 CM	0.0	0.0		
	0.0	0.165 MR	0.0	0.0	-0.841	
	0.0	0.457 CM	-0.715	-0.497	0.0	0.0
	0.0	1.000 PC	0.954	0.975	0.0	0.0 -0.610

DRIFT	3.0	25.7057 M
---------	-----	-----------

VARY CODE = 900000
 LABEL = DR1

QUAD 5.00 2.00000 M 3.5952 KG 15.000 CM (17.734 M)
 VARY CODE = 010000
 LABEL = Q2

134.7 M	0.0	6.672 CM					
	0.0	2.101 MR	-0.989				
	0.0	0.266 CM	0.0	0.0			
	0.0	0.279 MR	0.0	0.0	0.915		
	0.0	0.457 CM	-0.580	0.644	0.0	0.0	
	0.0	1.000 PC	0.998	-0.997	0.0	0.0	-0.610

FIT 10.0 -1. 2. 0.0 / 0.001
 -3.748
 LABEL = FIT1

FIT 10.0 -3. 4. 0.0 / 0.001
 -0.382
 LABEL = FIT1

TRANSFORM 1
 -0.75331 -3.74802 0.0 0.0 0.0 6.65790
 -0.07884 -1.71963 0.0 0.0 0.0 -2.09424
 0.0 0.0 -0.87796 -0.38176 0.0 0.0
 0.0 0.0 -0.78399 -1.48206 0.0 0.0
 -0.21025 -1.92983 0.0 0.0 1.00000 -0.27894
 0.0 0.0 0.0 0.0 0.0 1.00000

DRIFT 3.0 24.2942 M
 VARY CODE = 400000
 LABEL = DR1

159.0 M	0.0	1.782 CM					
	0.0	2.101 MR	-0.840				
	0.0	0.928 CM	0.0	0.0			
	0.0	0.279 MR	0.0	0.0	0.993		
	0.0	0.457 CM	-0.328	0.644	0.0	0.0	
	0.0	1.000 PC	0.881	-0.997	0.0	0.0	-0.610

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

```

*ROTAI*  2.0    1.50 D
*DRIFT*   3.0    1.0000 M
*ROTAT*   2.0    1.50 D
*BEND*    4.000  3.00000 M  14.550 KG  0.0000 ( 2.999 D )
*ROTAT*   2.0    1.50 D
*DRIFT*   3.0    1.0000 M
*ROTAT*   2.0    1.50 D
*BEND*    4.000  3.00300 M  14.550 KG  0.0000 ( 2.999 D )
*ROTAT*   2.0    1.50 D
*DRIFT*   3.0    1.0000 M
*ROTAT*   2.0    1.50 D
*BEND*    4.000  3.00000 M  14.550 KG  0.0000 ( 2.999 D )
*ROTAT*   2.0    1.50 D
*DRIFT*   3.0    1.0000 M

```

175.0 M	0.0	1.115 CM						
	0.0	0.174 MR	0.988					
	0.0	1.340 CM	0.0	0.0				
	0.0	0.238 MR	0.0	0.0	0.996			
	0.0	0.495 CM	-0.000	-0.000	0.0	0.0		
	0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.796	

```

*FIT*    10.0   -1. 6.  0.0  / 0.001

```

```

0.000

```

```

LABEL = FIT2

```

FIT 10.0 -2.6 0.0 / 0.001

0.000
LABEL = FIT3

TRANSFORM 1
-1.07094 -10.67648 0.0 0.0 0.0 0.00001
-0.07885 -1.71978 0.0 0.0 0.0 0.00000
0.0 0.0 -3.95881 -6.20807 0.0 0.0
0.0 0.0 -0.66630 -1.25747 0.0 0.0
-0.00000 -0.00001 0.0 0.0 1.00000 -0.39405
0.0 0.0 0.0 0.0 0.0 1.00000

QUAD 5.00 2.00000 M -1.9000 KG 5.000 CM (-10.646 M)
LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 1.8500 KG 5.000 CM (11.609 M)
LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M 0.0 1.742 CM
0.0 0.167 MR 0.995
0.0 0.824 CM 0.0 0.0
0.0 0.082 MR 0.0 0.0 0.895
0.0 0.495 CM 0.000 0.000 0.0 0.0
0.0 1.000 PC -0.000 -0.000 0.0 0.0 -0.796

TRANSFORM 1
2.77343 15.30110 0.0 0.0 0.0 -0.00001
0.21337 1.53770 0.0 0.0 0.0 -0.00000
0.0 0.0 2.57533 -2.86091 0.0 0.0
0.0 0.0 0.27088 0.08736 0.0 0.0
-0.00000 -0.00001 0.0 0.0 1.00000 -0.39405
0.0 0.0 0.0 0.0 0.0 1.00000

381.0 M 0.0 1.742 CM
0.0 0.167 MR 0.995
0.0 0.824 CM 0.0 0.0
0.0 0.082 MR 0.0 0.0 0.895
0.0 0.495 CM 0.000 0.000 0.0 0.0
0.0 1.000 PC -0.000 -0.000 0.0 0.0 -0.796

QUAD 5.00 2.00000 M 4.5000 KG 5.000 CM (4.984 M)
LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M	0.0	0.737 CM					
	0.0	3.364 MR	-1.000				
	0.0	1.424 CM	0.0	0.0			
	0.0	1.999 MR	0.0	0.0	1.000		
	0.0	0.495 CM	0.000	-0.000	0.0	0.0	
	0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.796

QUAD 5.00 2.0000 M -4.2000 KG 5.000 CM (-4.645 M)
 LABEL = Q4

387.0 M	0.0	0.172 CM					
	0.0	2.478 MR	-0.998				
	0.0	1.520 CM	0.0	0.0			
	0.0	1.070 MR	0.0	0.0	-1.000		
	0.0	0.495 CM	0.000	-0.000	0.0	0.0	
	0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.796

DRIFT 3.0 15.0000 M

402.0 M	0.0	3.545 CM					
	0.0	2.478 MR	1.000				
	0.0	0.090 CM	0.0	0.0			
	0.0	1.070 MR	0.0	0.0	0.950		
	0.0	0.495 CM	-0.000	-0.000	0.0	0.0	
	0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.796

TRANSFORM 1

-5.8C162	-30.87849	0.0	0.0	0.0	0.00003
-4.03058	-21.62466	0.0	0.0	0.0	0.00002
0.0	0.0	-0.23357	0.56217	0.0	0.0
0.0	0.0	-3.33957	3.75673	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.39405
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.9993 M

CORRECTIONS

(1.8837E C3)	0.104	-0.197	0.786	-0.019
(6.4767E 01)	0.006	-0.008	-0.001	0.002
(1.0700E-01)	0.000	-0.000	-0.000	-0.000

*COVARIANCE (FIT 0.1)

0.000			
-0.979	0.000		
-0.934	0.452	0.010	
-0.572	0.586	0.609	0.000

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A II

BEAM 1.000000 25.00 GEV

3.0 M

0.0	0.300 CM							
0.0	0.100 MR	0.0						
0.0	0.300 CM	0.0	0.0					
0.0	0.100 MR	0.0	0.0	0.0				
0.0	0.300 CM	0.0	0.0	0.0	0.0			
0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0		

BEND 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M

0.0	0.305 CM							
0.0	0.133 MR	0.171						
0.0	0.304 CM	0.0	0.0					
0.0	0.100 MR	0.0	0.0	0.164				
0.0	0.300 CM	-0.009	-0.001	0.0	0.0			
0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000		

DRIFT 3.0 80.0000 M

85.0 M

0.0	1.153 CM							
0.0	0.133 MR	0.965						
0.0	0.901 CM	0.0	0.0					
0.0	0.100 MR	0.0	0.0	0.943				
0.0	0.300 CM	-0.003	-0.001	0.0	0.0			
0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000		

QUAD 5.00 2.00000 M -2.0358 KG 5.000 CM (-9.650 M)
VARY CODE = 010000
LABEL = Q1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1943 KG 5.000 CM (9.843 M)
VARY CODE = 010000
LABEL = Q1

91.0 M

0.0	1.661 CM							
0.0	0.392 MR	-0.998						
0.0	0.582 CM	0.0	0.0					
0.0	0.147 MR	0.0	0.0	-0.937				
0.0	0.300 CM	-0.003	0.003	0.0	0.0			
0.0	1.000 PC	0.627	-0.613	0.0	0.0	-0.000		

DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			
DRIFT	3.0	1.0000 M			
ROTAT	2.0	1.50 D			
BEND	4.000	3.00000 M	14.550 KG	0.0000 (	2.999 D)
ROTAT	2.0	1.50 D			

107.0 M

0.0	2.366 CM				
0.0	1.880 MR	0.873			
0.0	0.357 CM	0.0	0.0		
0.0	0.163 MR	0.0	0.0	-0.856	
0.0	0.471 CM	-0.737	-0.533	0.0	0.0
0.0	1.000 PC	0.941	0.986	0.0	0.0 -0.617

DRIFT 3.0 24.9203 M

VARY CODE = 900000
 LABEL = CR1

QUAD 5.00 2.00000 M 3.5785 KG 15.000 CM (17.815 M)
 VARY CODE = 010000
 LABEL = Q2

133.9 M	0.0	6.823 CM					
	0.0	2.117 MR	-0.989				
	0.0	0.239 CM	0.0	0.0			
	0.0	0.254 MR	0.0	0.0	0.870		
	0.0	0.471 CM	-0.615	0.678	0.0	0.0	
	0.0	1.000 PC	1.000	-0.989	0.0	0.0	-0.617

FIT 10.0 -1.2 0.0 / 0.001
 -0.000
 LABEL = FIT1

FIT 10.0 -3.4 0.0 / 0.001
 -0.000
 LABEL = FIT1

TRANSFORM 1
 -0.33045 -0.30002 0.0 0.0 0.0 6.82236
 -0.22846 -3.02617 0.0 0.0 0.0 -2.09424
 0.0 0.0 -0.79757 -0.00002 0.0 0.0
 0.0 0.0 -0.73609 -1.25380 0.0 0.0
 -0.22507 -2.06457 0.0 0.0 1.00000 -0.29037
 0.0 0.0 0.0 0.0 0.0 1.00000

DRIFT 3.0 25.0795 M
 VARY CODE = 400000
 LABEL = DR1

159.0 M	0.0	1.765 CM					
	0.0	2.117 MR	-0.814				
	0.0	0.853 CM	0.0	0.0			
	0.0	0.254 MR	0.0	0.0	0.990		
	0.0	0.471 CM	-0.338	0.678	0.0	0.0	
	0.0	1.000 PC	0.890	-0.989	0.0	0.0	-0.617

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)


```

*ROTAT*    2.0    1.50 D
*DRIFT*    3.0    1.0000 M
*ROTAT*    2.0    1.50 D
*BEND*     4.000  3.00000 M  14.550 KG  0.0000 ( 2.999 D )
*ROTAT*    2.0    1.50 D
*DRIFT*    3.0    1.0000 M
*ROTAT*    2.0    1.50 D
*BEND*     4.000  3.00000 M  14.550 KG  0.0000 ( 2.999 D )
*ROTAT*    2.0    1.50 D
*DRIFT*    3.0    1.0000 M
*ROTAT*    2.0    1.50 D
*BEND*     4.000  3.00000 M  14.550 KG  0.0000 ( 2.999 D )
*ROTAT*    2.0    1.50 D
*DRIFT*    3.0    1.0000 M

```

175.0 M

0.0	1.300 CM						
0.0	0.310 MR	0.997					
0.0	1.227 CM	0.0	0.0				
0.0	0.216 MR	0.0	0.0	0.994			
0.0	0.504 CM	-0.000	0.000	0.0	0.0		
0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.804	

```

*FIT*      10.0   -1. 6.   0.0   / 0.001

```

0.000

LABEL = FIT2

FIT 10.0 -2.6. 0.0 / 0.001

0.000

LABEL = FIT3

TRANSFORM 1

-1.26882	-12.42980	0.0	0.0	0.0	0.00001
-0.22847	-3.02623	0.0	0.0	0.0	0.00000
0.0	0.0	-3.72852	-5.03437	0.0	0.0
0.0	0.0	-0.62031	-1.10576	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

QUAD 5.00 2.00000 M -1.9000 KG 5.000 CM (-10.646 M)

LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 1.8500 KG 5.000 CM (11.609 M)

LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M

0.0	0.620 CM				
0.0	0.125 MR	0.922			
0.0	0.873 CM	0.0	0.0		
0.0	0.079 MR	0.0	0.0	0.902	
0.0	0.504 CM	0.000	0.000	0.0	0.0
0.0	1.000 PC	-0.000	-0.000	0.0	0.0 -0.804

TRANSFORM 1

1.49989	4.27131	0.0	0.0	0.0	-0.00001
0.16754	1.14380	0.0	0.0	0.0	-0.00000
0.0	0.0	2.52306	-3.78386	0.0	0.0
0.0	0.0	0.26479	-0.00074	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

381.0 M

0.0	0.620 CM				
0.0	0.125 MR	0.922			
0.0	0.873 CM	0.0	0.0		
0.0	0.079 MR	0.0	0.0	0.902	
0.0	0.504 CM	0.000	0.000	0.0	0.0
0.0	1.000 PC	-0.000	-0.000	0.0	0.0 -0.804

QUAD 5.00 2.00000 M 4.5000 KG 5.000 CM (4.984 M)

LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M	0.0	0.282 CM						
	0.0	1.154 MR	-0.996					
	0.0	1.506 CM	0.0	0.0				
	0.0	2.111 MR	0.0	0.0	1.000			
	0.0	0.504 CM	0.000	-0.000	0.0	0.0		
	0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.804	

QUAD 5.00 2.00000 M -4.2000 KG 5.000 CM (-4.645 M)
 LABEL = Q4

387.0 M	0.0	0.098 CM						
	0.0	0.791 MR	-0.923					
	0.0	1.607 CM	0.0	0.0				
	0.0	1.135 MR	0.0	0.0	-1.000			
	0.0	0.504 CM	0.000	-0.000	0.0	0.0		
	0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.804	

DRIFT 3.0 15.0000 M

402.0 M	0.0	1.097 CM						
	0.0	0.791 MR	0.999					
	0.0	0.099 CM	0.0	0.0				
	0.0	1.135 MR	0.0	0.0	0.963			
	0.0	0.504 CM	-0.000	-0.000	0.0	0.0		
	0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.804	

TRANSFORM 1

-2.97423	-6.38180	0.0	0.0	0.0	0.00002
-2.09122	-4.82334	0.0	0.0	0.0	0.00001
0.0	0.0	-0.24656	0.65288	0.0	0.0
0.0	0.0	-3.40280	4.55466	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.9993 M

'BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A 111A'

1

{IN THE THIRD STEP THE FIRST QUAD DOUBLET,Q1, AND THE SYMMETRY QUAD,Q2,}

(ARE NOT ALLOWED TO VARY. THE FIELDS OF THE SECOND QUAD DOUBLET,Q3, ARE)

(ALLOWED TO VARY TO GET A CERTAIN BEAM SIZE AT THE ENTRANCE TO THE THIRD QUAD)

(DOUBLET,Q4.)

5.0 'Q1' ;

5.0 'Q2' ;

3.0 'DR1' ;

-10.0 'FIT1' ;

-10.0 'FIT2' ;

-10.0 'FIT3' ;

5.01 'Q3' ;

10.2 'FIT4' ;

SENTINEL

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A IIIA

1

IN THE THIRD STEP THE FIRST QUAD DOUBLET,Q1, AND THE SYMMETRY QUAD,Q2,
ARE NOT ALLOWED TO VARY. THE FIELDS OF THE SECOND QUAD DOUBLET,Q3, ARE
ALLOWED TO VARY TO GET A CERTAIN BEAM SIZE AT THE ENTRANCE TO THE THIRD QUAD
DOUBLET,Q4.

1.00000	0.30000	0.10000	0.30000	0.10000	0.30000	1.00000	25.00000
13.00000	2.00000						
4.00000	5.00000	1.45500	0.0				
2.00000	0.50000						
13.00000	1.00000						
3.00000	80.00000						
13.00000	1.00000						
5.00000	2.00000	-2.08981	5.00000				
	LABEL = Q1						
3.00000	2.00000						
5.00000	2.00000	2.19428	5.00000				
	LABEL = Q1						
13.00000	1.00000						
9.00000	4.00000						
3.00000	1.00000						
2.00000	1.50000						
4.00000	3.00000	14.55000	0.0				
2.00000	1.50000						
9.00000	0.0						
13.00000	1.00000						
3.00000	24.92036						
	LABEL = DR1						
5.00000	2.00000	3.57851	15.00000				
	LABEL = Q2						
13.00000	1.00000						
-10.00000	-1.00000	2.00000	0.0	0.00100			
	LABEL = FIT1						

-10.00000	-3.00000	4.00000	C.0	0.00100
LABEL = FIT1				
13.00000	4.00000			
3.00000	25.07953			
LABEL = OR1				
13.00000	1.00000			
9.00000	4.00000			
2.00000	1.50000			
4.00000	3.00000	14.55000	C.0	
2.00000	1.50000			
3.00000	1.00000			
9.00000	C.0			
13.00000	1.00000			
-10.00000	-1.00000	6.00000	0.0	0.00100
LABEL = FIT2				
-10.00000	-2.00000	6.00000	0.0	0.00100
LABEL = FIT3				
13.00000	4.00000			
5.00000	2.00000	-1.90000	5.00000	
VARY CODE 010000				
LABEL = C3				
3.00000	2.00000			
5.00000	2.00000	1.85000	5.00000	
VARY CODE 010000				
LABEL = C3				
3.00000	200.00000			
13.00000	1.00000			
10.00000	1.00000	1.00000	4.00000	0.10000
VARY CODE 200000				
LABEL = FIT4				
10.00000	3.00000	3.00000	1.50000	0.10000
VARY CODE 200000				
LABEL = FIT4				
13.00000	4.00000			
13.00000	1.00000			
5.00000	2.00000	4.50000	5.00000	
LABEL = C4				

3.00000	2.00000			
13.00000	1.00000			
5.00000	2.00000	-4.20000	5.00000	
LABEL = Q4				
13.00000	1.00000			
3.00000	15.00000			
13.00000	1.00000			
-1C.00000	2.00000	1.00000	0.0	0.00100
LABEL = FIT5				
-1C.00000	4.00000	3.00000	0.0	0.00100
LABEL = FIT6				
-1C.00000	1.00000	1.00000	0.02000	0.01000
VARY CODE 200000				
LABEL = FIT7				
-1C.00000	3.00000	3.00000	0.02000	0.01000
VARY CODE 200000				
LABEL = FIT7				
13.00000	4.00000			
SENTINEL				

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A 111A

BEAM 1.000000 25.00 GEV

0.0 M	0.0	0.300 CM						
	0.0	0.100 MR	0.0					
	0.0	0.300 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.0			
	0.0	0.300 CM	0.0	0.0	0.0	0.0		
	0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

BEND 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M	0.0	0.305 CM						
	0.0	0.133 MR	0.171					
	0.0	0.304 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.164			
	0.0	0.300 CM	-0.009	-0.001	0.0	0.0		
	0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000	

DRIFT 3.0 80.0000 M

85.0 M	0.0	1.153 CM						
	0.0	0.133 MR	0.965					
	0.0	0.901 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.943			
	0.0	0.300 CM	-0.003	-0.001	0.0	0.0		
	0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000	

QUAD 5.00 2.00000 M -2.0898 KG 5.000 CM (-9.650 M)
LABEL = G1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1943 KG 5.000 CM (9.843 M)
LABEL = G1

91.0 M	0.0	1.661 CM						
	0.0	0.392 MR	-0.998					
	0.0	0.582 CM	0.0	0.0				
	0.0	0.147 MR	0.0	0.0	-0.937			
	0.0	0.300 CM	-0.003	0.003	0.0	0.0		
	0.0	1.000 PC	0.627	-0.613	0.0	0.0	-0.000	

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D

107.0 M	0.0	2.366 CM					
	0.0	1.880 MR	0.873				
	0.0	0.357 CM	0.0	0.0			
	0.0	0.163 MR	0.0	0.0	-0.856		
	0.0	0.471 CM	-0.737	-0.533	0.0	0.0	
	0.0	1.000 PC	0.941	0.986	0.0	0.0	-0.617

DRIFT 3.0 24.9203 M
 LABEL = DRI

QUAD 5.00 2.00000 M 3.5785 KG 15.000 CM (17.815 M)
 LABEL = Q2

133.9 M	0.0	6.823 CM				
	0.0	2.117 MR	-0.989			
	0.0	0.239 CM	0.0	0.0		
	0.0	0.254 MR	0.0	0.0	0.870	
	0.0	0.471 CM	-0.615	0.678	0.0	0.0
	0.0	1.000 PC	1.000	-0.989	0.0	0.0 -0.617

TRANSFORM 1

-0.33045	-0.00002	0.0	0.0	0.0	6.8223E
-0.22846	-3.02617	0.0	0.0	0.0	-2.09424
0.0	0.0	-0.79757	-0.00002	0.0	0.0
0.0	0.0	-0.73609	-1.25380	0.0	0.0
-0.22507	-2.06457	0.0	0.0	1.00000	-0.29037
0.0	0.0	0.0	0.0	0.0	1.00000

DRIFT 3.0 25.0795 M
 LABEL = DR1

159.0 M	0.0	1.765 CM			
	0.0	2.117 MR	-0.814		
	0.0	0.853 CM	0.0	0.0	
	0.0	0.254 MR	0.0	0.0	0.990
	0.0	0.471 CM	-0.338	0.678	0.0 0.0
	0.0	1.000 PC	0.890	-0.989	0.0 0.0 -0.617

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

/

175.0 M

0.0	1.300 CM				
0.0	0.310 MR	0.997			
0.0	1.227 CM	0.0	0.0		
0.0	0.216 MR	0.0	0.0	0.994	
0.0	0.504 CM	-0.000	0.000	0.0	0.0
0.0	1.000 PC	0.000	0.000	0.0	0.0
					-0.804

TRANSFORM 1

-1.26882	-12.42980	0.0	0.0	0.0	0.00001
-0.22847	-3.02628	0.0	0.0	0.0	0.00000
0.0	0.0	-3.72852	-5.03437	0.0	0.0
0.0	0.0	-0.62031	-1.10576	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

QUAD 5.00 2.00000 M -1.9000 KG 5.000 CM (-10.646 M)
 VARY CODE = 010000
 LABEL = Q3

DRIFT 3.0 2.0000 M

QUAC 5.00 2.00000 M -1.8500 KG 5.000 CM (11.609 M)
 VARY CODE = 010000
 LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M

0.0	0.620 CM	
0.0	0.125 MR	0.922

0.0	0.373	CM	0.0	0.0				
0.0	0.079	MR	0.0	0.0	0.902			
0.0	0.504	CM	0.000	0.000	0.0	0.0		
0.0	1.000	PC	-0.000	-0.000	0.0	0.0	-0.804	

FIT 10.0 1. 1. 4.000 / C.100

C.620

VARY CODE = 200000
LABEL = FIT4

FIT 10.0 3. 3. 1.500 / C.100

C.873

VARY CODE = 200000
LABEL = FIT4

TRANSFORM 1

1.49989	4.27131	0.0	0.0	0.0	-0.00001
C.16754	1.14380	C.C	0.0	0.0	-0.00000
C.0	0.0	2.62306	-3.78386	0.0	0.0
C.0	0.0	0.26479	-0.00074	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

381.0 M

0.0	0.620	CM						
0.0	0.125	MR	0.922					
0.0	0.873	CM	0.0	0.0				
0.0	0.079	MR	0.0	0.0	0.902			
0.0	0.504	CM	0.000	0.000	0.0	0.0		
0.0	1.000	PC	-0.000	-0.000	0.0	0.0	-0.804	

QUAD 5.00 2.00000 M 4.5000 KG 5.000 CM (4.984 M)
LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M

0.0	0.282	CM						
0.0	1.154	MR	-0.996					
0.0	1.506	CM	0.0	0.0				
0.0	2.111	MR	0.0	0.0	1.000			
0.0	0.504	CM	0.000	-0.000	0.0	0.0		
0.0	1.000	PC	-0.000	0.000	0.0	0.0	-0.804	

QUAD 5.00 2.00000 M -4.2000 KG 5.000 CM (-4.645 M)
LABEL = Q4

387.0 M

0.0	0.098	CM						
-----	-------	----	--	--	--	--	--	--

0.0	0.791 MR	-0.923				
0.0	1.607 CM	0.0	0.0			
0.0	1.135 MR	0.0	0.0	-1.000		
0.0	0.504 CM	0.000	-0.000	0.0	0.0	
0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.804

DRIFT 3.0 15.0000 M

402.0 M

0.0	1.097 CM					
0.0	0.791 MR	0.999				
0.0	0.099 CM	0.0	0.0			
0.0	1.135 MR	0.0	0.0	0.963		
0.0	0.504 CM	-0.000	-0.000	0.0	0.0	
0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.804

TRANSFORM 1

-2.33281	3.13200	0.0	0.0	0.0	0.0
-1.58390	1.65786	0.0	0.0	0.0	0.0
0.0	0.0	-0.17269	0.77960	0.0	0.0
0.0	0.0	-1.30944	0.12067	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.9993 M

CORRECTIONS

(1.4240E 01)	-0.790	1.228
(2.4036E 02)	0.308	-0.526
(6.1862E 01)	0.181	-0.277
(1.3562E 01)	0.073	-0.107
(1.7670E 00)	0.013	-0.019
(5.1883E-02)	0.000	-0.001

*COVARIANCE (FIT 0.1)

0.010
-0.805 0.009

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A IIIA

BEAM 1.000000 25.00 GEV

0.0 M

0.0	0.300 CM						
0.0	0.100 MR	0.0					
0.0	0.300 CM	0.0	0.0				
0.0	0.100 MR	0.0	0.0	0.0			
0.0	0.300 CM	0.0	0.0	0.0	0.0		
0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0	

BEND 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M

0.0	0.305 CM						
0.0	0.133 MR	0.171					
0.0	0.304 CM	0.0	0.0				
0.0	0.100 MR	0.0	0.0	0.164			
0.0	0.300 CM	-0.009	-0.001	0.0	0.0		
0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000	

DRIFT 3.0 80.0000 M

85.0 M

0.0	1.153 CM						
0.0	0.133 MR	0.965					
0.0	0.901 CM	0.0	0.0				
0.0	0.100 MR	0.0	0.0	0.943			
0.0	0.300 CM	-0.003	-0.001	0.0	0.0		
0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000	

QUAD 5.00 2.00000 M -2.0858 KG 5.000 CM (-9.650 M)
LABEL = G1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1943 KG 5.000 CM (9.843 M)
LABEL = G1

91.0 M

0.0	1.661 CM						
0.0	0.392 MR	-0.998					
0.0	0.582 CM	0.0	0.0				
0.0	0.147 MR	0.0	0.0	-0.937			
0.0	0.300 CM	-0.003	0.003	0.0	0.0		
0.0	1.000 PC	0.627	-0.613	0.0	0.0	-0.000	

DRIFT 3.0 1.0000 M

RDTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 RDTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 RDTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 RDTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 RDTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 RDTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 RDTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 RDTAT 2.0 1.50 D

107.0 M

0.0	2.366 CM					
0.0	1.880 MR	0.873				
0.0	0.357 CM	0.0	0.0			
0.0	0.163 MR	0.0	0.0	-0.856		
0.0	0.471 CM	-0.737	-0.533	0.0	0.0	
0.0	1.000 PC	0.941	0.986	0.0	0.0	-0.617

DRIFT 3.0 24.9203 M
 LABEL = DR1

QUAD 5.00 2.00000 M 3.5785 KG 15.000 CM (17.815 M)
 LABEL = C2

133.9 M	0.0	6.823 CM					
	0.0	2.117 MR	-0.989				
	0.0	0.239 CM	0.0	0.0			
	0.0	0.254 MR	0.0	0.0	0.870		
	0.0	0.471 CM	-0.615	0.678	0.0	0.0	
	0.0	1.000 FC	1.000	-0.989	0.0	0.0	-0.617

TRANSFORM 1

-0.33045	-0.00002	0.0	0.0	0.0	6.82236
-0.22846	-3.02617	0.0	0.0	0.0	-2.09424
0.0	0.0	-0.79757	-0.00002	0.0	0.0
0.0	0.0	-0.73639	-1.25380	0.0	0.0
-0.22507	-2.06457	0.0	0.0	1.00000	-0.29037
0.0	0.0	0.0	0.0	0.0	1.00000

DRIFT 3.0 25.0795 M
 LABEL = DR1

159.0 M	0.0	1.765 CM					
	0.0	2.117 MR	-0.814				
	0.0	0.853 CM	0.0	0.0			
	0.0	0.254 MR	0.0	0.0	0.990		
	0.0	0.471 CM	-0.338	0.678	0.0	0.0	
	0.0	1.000 PC	0.890	-0.989	0.0	0.0	-0.617

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

175.0 M

0.0	1.300 CM					
0.0	0.310 MR	0.997				
0.0	1.227 CM	0.0	0.0			
0.0	0.216 MR	0.0	0.0	0.994		
0.0	0.504 CM	-0.000	0.000	0.0	0.0	
0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.804

TRANSFORM 1

-1.26882	-12.42980	0.0	0.0	0.0	0.00001
-0.22847	-3.02628	0.0	0.0	0.0	0.00000
0.0	0.0	-3.72852	-5.03437	0.0	0.0
0.0	0.0	-0.62031	-1.10576	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

QUAD 5.00 2.00000 M -2.1150 KG 5.000 CM (-9.532 M)
VARY CODE = 010000
LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1487 KG 5.000 CM (10.044 M)
VARY CODE = 010000
LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M

0.0	4.000 CM	
0.0	0.299 MR	1.000

0.0	1.500 CM	0.0	0.0		
0.0	0.113 MR	0.0	0.0	0.984	
0.0	0.504 CM	0.000	0.000	0.0	0.0
0.0	1.000 PC	-0.000	-0.000	0.0	0.0 -0.804

FIT 10.0 1. 1. 4.000 / 0.100

4.000

VARY CODE = 200000
LABEL = FIT4

FIT 10.0 3. 3. 1.500 / 0.100

1.500

VARY CODE = 200000
LABEL = FIT4

TRANSFORM 1

4.77122	31.35149	0.0	0.0	0.0	-0.00003
0.33310	2.81727	0.0	0.0	0.0	-0.00000
0.0	0.0	4.59109	-0.89239	0.0	0.0
0.0	0.0	0.37533	0.13324	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

381.0 M

0.0	4.000 CM				
0.0	0.299 MR	1.000			
0.0	1.500 CM	0.0	0.0		
0.0	0.113 MR	0.0	0.0	0.984	
0.0	0.504 CM	0.000	0.000	0.0	0.0
0.0	1.000 PC	-0.000	-0.000	0.0	0.0 -0.804

QUAD 5.00 2.00000 M 4.5000 KG 5.000 CM (4.984 M)
LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M

0.0	1.665 CM				
0.0	7.789 MR	-1.000			
0.0	2.582 CM	0.0	0.0		
0.0	3.612 MR	0.0	0.0	1.000	
0.0	0.504 CM	0.000	-0.000	0.0	0.0
0.0	1.000 PC	-0.000	0.000	0.0	0.0 -0.804

QUAD 5.00 2.00000 M -4.2000 KG 5.000 CM (-4.645 M)
LABEL = Q4

387.0 M

0.0 0.347 CM

0.0	5.828	MR	-1.000					
0.0	2.754	CM	0.0	0.0				
0.0	1.951	MR	0.0	0.0	-1.000			
0.0	0.504	CM	0.000	-0.000	0.0	0.0		
0.0	1.000	PC	-0.000	0.000	0.0	0.0	-0.804	

DRIFT 3.0 15.0000 M

402.0 M

0.0	8.395	CM						
0.0	5.828	MR	1.000					
0.0	0.173	CM	0.0	0.0				
0.0	1.951	MR	0.0	0.0	0.996			
0.0	0.504	CM	-0.000	-0.000	0.0	0.0		
0.0	1.000	PC	0.000	0.000	0.0	0.0	-0.804	

TRANSFORM 1

-2.33281	3.13200	0.0	0.0	0.0	0.0
-1.58390	1.65786	0.0	0.0	0.0	0.0
0.0	0.0	-0.17269	0.77960	0.0	0.0
0.0	0.0	-1.30944	0.12067	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.9993 M

'BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A 111B'

1

(IN THIS CASE THE VARY CODES FOR THE QUAD DOUBLET, Q3, ARE TURNED OFF, AND)
(THE THIRD QUAD DOUBLET, Q4, IS ALLOWED TO VARY TO GET A WAIST WITH A CERTAIN)
(SPOT SIZE AT THE TARGET.)

5.0 'Q3' ;
5.01 'Q4' ;
10.0 'FIT5' ;
10.0 'FIT6' ;
10.2 'FIT7' ;
SENTINEL

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A IIB

1

IN THIS CASE THE VARY CODES FOR THE QJAD DOUBLET,Q3, ARE TURNED OFF, AND
THE THIRD QUAD DOUBLET,Q4, IS ALLOWED TO VARY TO GET A WAIST WITH A CERTAIN
SPOT SIZE AT THE TARGET.

1.00000	0.30000	0.10000	0.30000	0.10000	0.30000	1.00000	25.00000
13.00000	2.00000						
4.00000	5.00000	1.45500	0.0				
2.00000	0.50000						
13.00000	1.00000						
3.00000	80.00000						
13.00000	1.00000						
5.00000	2.00000	-2.08981	5.00000				
LABEL = C1							
3.00000	2.00000						
5.00000	2.00000	2.19428	5.00000				
LABEL = C1							
13.00000	1.00000						
9.00000	4.00000						
3.00000	1.00000						
2.00000	1.50000						
4.00000	3.00000	14.55000	0.0				
2.00000	1.50000						
9.00000	0.0						
13.00000	1.00000						
3.00000	24.92036						
LABEL = DR1							
5.00000	2.00000	3.57851	15.00000				
LABEL = Q2							
13.00000	1.00000						
-10.00000	-1.00000	2.00000	0.0	0.00100			
LABEL = FIT1							
-10.00000	-3.00000	4.00000	0.0	0.00100			

LABEL = FIT1				
13.00000	4.00000			
3.00000	25.07953			
LABEL = CR1				
13.00000	1.00000			
9.00000	4.00000			
2.00000	1.50000			
4.00000	3.00000	14.55000	0.0	
2.00000	1.50000			
3.00000	1.00000			
9.00000	0.0			
13.00000	1.00000			
-10.00000	-1.00000	5.00000	0.0	0.00100
LABEL = FIT2				
-10.00000	-2.00000	5.00000	0.0	0.00100
LABEL = FIT3				
13.00000	4.00000			
5.00000	2.00000	-2.11497	5.00000	
LABEL = C3				
3.00000	2.00000			
5.00000	2.00000	2.14873	5.00000	
LABEL = C3				
3.00000	200.00000			
13.00000	1.00000			
10.00000	1.00000	1.00000	4.00000	0.10000
VARY CODE 200000				
LABEL = FIT4				
10.00000	3.00000	3.00000	1.50000	0.10000
VARY CODE 200000				
LABEL = FIT4				
13.00000	4.00000			
13.00000	1.00000			
5.00000	2.00000	4.50000	5.00000	
VARY CODE 010000				
LABEL = C4				
3.00000	2.00000			

13.00	1.00000			
5.00000	2.00000	-4.20000	5.00000	
VARY CODE	010000			
LABEL = Q4				
13.00000	1.00000			
3.00000	15.00000			
13.00000	1.00000			
10.00000	2.00000	1.00000	0.0	0.00100
LABEL = FIT5				
10.00000	4.00000	3.00000	0.0	0.00100
LABEL = FIT6				
10.00000	1.00000	1.00000	0.02000	0.01000
VARY CODE	200000			
LABEL = FIT7				
10.00000	3.00000	3.00000	0.02000	0.01000
VARY CODE	200000			
LABEL = FIT7				
13.00000	4.00000			
SENTINEL				

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A IIIB.

BEAM 1.000000 25.00 GEV

0.0 M	0.0	0.300 CM						
	0.0	0.100 MR	0.0					
	0.0	0.300 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.0			
	0.0	0.300 CM	0.0	0.0	0.0	0.0		
	0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

BEND 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 C

5.0 M	0.0	0.305 CM						
	0.0	0.133 MR	0.171					
	0.0	0.304 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.164			
	0.0	0.300 CM	-0.009	-0.001	0.0	0.0		
	0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000	

DRIFT 3.0 80.0000 M

85.0 M	0.0	1.153 CM						
	0.0	0.133 MR	0.965					
	0.0	0.901 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.943			
	0.0	0.300 CM	-0.003	-0.001	0.0	0.0		
	0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000	

QUAD 5.00 2.00000 M -2.0898 KG 5.000 CM (-9.650 M)
LABEL = Q1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1943 KG 5.000 CM (9.843 M)
LABEL = Q1

91.0 M	0.0	1.661 CM						
	0.0	0.392 MR	-0.998					
	0.0	0.582 CM	0.0	0.0				
	0.0	0.147 MR	0.0	0.0	-0.937			
	0.0	0.300 CM	-0.003	0.003	0.0	0.0		
	0.0	1.000 PC	0.627	-0.613	0.0	0.0	-0.000	

DRIFT 3.0 1.0000 M


```

*ROTAT*   2.0    1.50 D

*END*     4.000  3.00000 M 14.550 KG  0.0000 ( 2.999 D )

*ROTAT*   2.0    1.50 D

*DRIFT*   3.0    1.0000 M

*ROTAT*   2.0    1.50 D

*END*     4.000  3.00000 M 14.550 KG  0.0000 ( 2.999 D )

*ROTAT*   2.0    1.50 D

*DRIFT*   3.0    1.0000 M

*ROTAT*   2.0    1.50 D

*END*     4.000  3.00000 M 14.550 KG  0.0000 ( 2.999 D )

*ROTAT*   2.0    1.50 D

*DRIFT*   3.0    1.0000 M

*ROTAT*   2.0    1.50 D

*END*     4.000  3.00000 M 14.550 KG  0.0000 ( 2.999 D )

*ROTAT*   2.0    1.50 D

```

107.0 M

0.0	2.366 CM					
0.0	1.880 MR	0.873				
0.0	0.357 CM	0.0	0.0			
0.0	0.163 MR	0.0	0.0	-0.356		
0.0	0.471 CM	-0.737	-0.533	0.0	0.0	
0.0	1.000 PC	0.941	0.986	0.0	0.0	-0.617

```

*DRIFT*   3.0    24.9203 M
          LABEL = DRI

```

QUAD 5.00 2.00000 M 3.5785 KG 15.000 CM (17.815 M)
 LABEL = C2

133.9 M	0.0	6.823 CM				
	0.0	2.117 MR	-0.989			
	0.0	0.239 CM	0.0	0.0		
	0.0	0.254 MR	0.0	0.0	0.870	
	0.0	0.471 CM	-0.615	0.678	0.0	0.0
	0.0	1.000 FC	1.000	-0.989	0.0	0.0 -0.617

TRANSFORM 1

-0.33045	-0.00002	0.0	0.0	0.0	6.82236
-0.22846	-3.02617	0.0	0.0	0.0	-2.09424
0.0	0.0	-0.79757	-0.00002	0.0	0.0
0.0	0.0	-0.73609	-1.25380	0.0	0.0
-0.22507	-2.06457	0.0	0.0	1.00000	-0.29037
0.0	0.0	0.0	0.0	0.0	1.00000

DRIFT 3.0 25.0795 M
 LABEL = DR1

159.0 M	0.0	1.765 CM				
	0.0	2.117 MR	-0.814			
	0.0	0.853 CM	0.0	0.0		
	0.0	0.254 MR	0.0	0.0	0.990	
	0.0	0.471 CM	-0.338	0.678	0.0	0.0
	0.0	1.000 FC	0.890	-0.989	0.0	0.0 -0.617

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

175.0 M	0.0	1.300 CM					
	0.0	0.310 MR	0.997				
	0.0	1.227 CM	0.0	0.0			
	0.0	0.216 MR	0.0	0.0	0.994		
	0.0	0.504 CM	-0.000	0.000	0.0	0.0	
	0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.804

TRANSFORM 1

-1.26882	-12.42980	0.0	0.0	0.0	0.00001
-0.22847	-3.02628	0.0	0.0	0.0	0.00000
0.0	0.0	-3.72852	-5.03437	0.0	0.0
0.0	0.0	-0.62031	-1.10576	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

QUAD 5.00 2.00000 M -2.1150 KG 5.000 CM (-9.532 M)
LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1487 KG 5.000 CM (10.044 M)
LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M	0.0	4.000 CM			
	0.0	0.299 MR	1.000		
	0.0	1.500 CM	0.0	0.0	
	0.0	0.113 MR	0.0	0.0	0.984

0.0	0.504 CM	0.000	0.000	0.0	0.0	
0.0	1.000 PC	-0.000	-0.000	0.0	0.0	-0.804

FIT 10.0 1. 1. 4.000 / 0.100

4.000

VARY CODE = 200C00
LABEL = FIT4

FIT 10.0 3. 3. 1.500 / 0.100

1.500

VARY CODE = 200C00
LABEL = FIT4

TRANSFORM 1

4.77122	37.35149	0.0	0.0	0.0	-0.00003
0.33310	2.81727	0.0	0.0	0.0	-0.00000
0.0	0.0	4.99109	-0.89239	0.0	0.0
0.0	0.0	0.37533	0.13324	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

381.0 M

0.0	4.000 CM				
0.0	0.299 MR	1.000			
0.0	1.500 CM	0.0	0.0		
0.0	0.113 MR	0.0	0.0	0.984	
0.0	0.504 CM	0.000	0.000	0.0	0.0
0.0	1.000 PC	-0.000	-0.000	0.0	0.0

QUAD 5.00 2.00000 M 4.5000 KG 5.000 CM (4.984 M)

VARY CODE = 010000
LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M

0.0	1.665 CM				
0.0	7.789 MR	-1.000			
0.0	2.582 CM	0.0	0.0		
0.0	3.612 MR	0.0	0.0	1.000	
0.0	0.504 CM	0.000	-0.000	0.0	0.0
0.0	1.000 PC	-0.000	0.000	0.0	0.0

QUAD 5.00 2.00000 M -4.2000 KG 5.000 CM (-4.645 M)

VARY CODE = 010000
LABEL = Q4

387.0 M

0.0	0.347 CM
-----	----------

0.0	5.828	MR	-1.000				
0.0	2.754	CM	0.0	0.0			
0.0	1.951	MR	0.0	0.0	-1.000		
0.0	0.504	CM	0.000	-0.000	0.0	0.0	
0.0	1.000	PC	-0.000	0.000	0.0	0.0	-0.804

DRIFT 3.0 15.000C M

402.0 M

0.0	8.395	CM					
0.0	5.828	MR	1.000				
0.0	0.173	CM	0.0	0.0			
0.0	1.951	MR	0.0	0.0	0.996		
0.0	0.504	CM	-0.000	-0.000	0.0	0.0	
0.0	1.000	PC	0.000	0.000	0.0	0.0	-0.804

FIT 10.0 2. 1. 0.0 / 0.001

48.931

LABEL = FIT5

FIT 10.0 4. 3. 0.0 / 0.001

0.336

LABEL = FIT6

FIT 10.0 1. 1. 0.020 / 0.010

8.395

VARY CODE = 20000C

LABEL = FIT7

FIT 10.0 3. 3. 0.020 / 0.010

0.173

VARY CODE = 20000C

LABEL = FIT7

TRANSFORM 1

-2.33281	3.13200	0.0	0.0	0.0	0.0
-1.58390	1.65786	0.0	0.0	0.0	0.0
0.0	0.0	-0.17259	0.77960	0.0	0.0
0.0	0.0	-1.30944	0.12067	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.5993 M

CORRECTIONS

(7.4658E 04)	-0.780	0.213
(1.9087E 04)	-0.431	0.237
(5.0201E 03)	-0.240	0.163
(1.3456E 03)	-0.131	0.095
(3.7784E 02)	-0.068	0.045
(1.0482E 02)	-0.032	0.018
(1.6754E 01)	-0.007	0.003
(3.2649E-01)	-0.000	0.000

*COVARIANCE (FIT 0.3)
 0.000
 -0.793 0.000

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A IIIB

BEAM 1.000000 25.00 GEV

0.0 M	0.0	0.300 CM						
	0.0	0.100 MR	0.0					
	0.0	0.300 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.0			
	0.0	0.300 CM	0.0	0.0	0.0	0.0		
	0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

3END 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M	0.0	0.305 CM						
	0.0	0.133 MR	0.171					
	0.0	0.304 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.164			
	0.0	0.300 CM	-0.009	-0.001	0.0	0.0		
	0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000	

DRIFT 3.0 80.0000 M

85.0 M	0.0	1.153 CM						
	0.0	0.133 MR	0.965					
	0.0	0.901 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.943			
	0.0	0.300 CM	-0.003	-0.001	0.0	0.0		
	0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000	

QUAD 5.00 2.00000 M -2.0858 KG 5.000 CM (-9.650 M)
LABEL = Q1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1943 KG 5.000 CM (9.843 M)
LABEL = Q1

91.0 M	0.0	1.661 CM						
	0.0	0.392 MR	-0.998					
	0.0	0.582 CM	0.0	0.0				
	0.0	0.147 MR	0.0	0.0	-0.937			
	0.0	0.300 CM	-0.003	0.003	0.0	0.0		
	0.0	1.000 PC	0.627	-0.613	0.0	0.0	-0.000	

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D

107.0 M

0.0	2.366 CM					
0.0	1.880 MR	0.873				
0.0	0.357 CM	0.0	0.0			
0.0	0.163 MR	0.0	0.0	-0.856		
0.0	0.471 CM	-0.737	-0.533	0.0	0.0	
0.0	1.000 PC	0.941	0.986	0.0	0.0	-0.617

DRIFT 3.0 24.9203 M
 LABEL = DR1

QUAD 5.00 2.00000 M 3.5785 KG 15.000 CM (17.815 M)
 LABEL = C2

133.9 M	0.0	6.823 CM					
	0.0	2.117 MR	-0.989				
	0.0	0.239 CM	0.0	0.0			
	0.0	0.254 MR	0.0	0.0	0.870		
	0.0	0.471 CM	-0.615	0.678	0.0	0.0	
	0.0	1.000 PC	1.000	-0.989	0.0	0.0	-0.617

TRANSFORM 1

-C.33045	-0.00002	0.0	0.0	0.0	6.82235
-0.22846	-3.02617	C.C	0.0	0.0	-2.09424
0.0	0.0	-C.79757	-0.00002	0.0	0.0
0.0	0.0	-0.73609	-1.25380	C.0	0.0
-0.22507	-2.06457	0.0	0.0	1.00000	-0.29037
C.0	0.0	C.C	0.0	0.0	1.00000

DRIFT 3.0 25.0795 M
 LABEL = DR1

159.0 M	0.0	1.765 CM					
	0.0	2.117 MR	-0.814				
	0.0	0.853 CM	0.0	0.0			
	0.0	0.254 MR	0.0	0.0	0.990		
	0.0	0.471 CM	-0.338	0.678	0.0	0.0	
	0.0	1.000 PC	0.890	-0.989	0.0	0.0	-0.617

ROTAT 2.0 1.50 D

3END 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

3END 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

175.0 M

0.0	1.300 CM				
0.0	0.310 MR	0.997			
0.0	1.227 CM	0.0	0.0		
0.0	0.216 MR	0.0	0.0	0.994	
0.0	0.504 CM	-0.000	0.000	0.0	0.0
0.0	1.000 PC	0.000	0.000	0.0	0.0 -0.804

TRANSFORM 1

-1.26882	-12.42980	0.0	0.0	0.0	0.00001
-0.22847	-3.02628	0.0	0.0	0.0	0.00000
0.0	0.0	-3.72852	-5.03437	0.0	0.0
0.0	0.0	-0.62031	-1.10576	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

QUAD 5.00 2.00000 M -2.1150 KG 5.000 CM (-9.532 M)
LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1487 KG 5.000 CM (10.044 M)
LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M

0.0	4.000 CM			
0.0	0.299 MR	1.000		
0.0	1.500 CM	0.0	0.0	
0.0	0.113 MR	0.0	0.0	0.984

0.0	0.504 CM	0.000	0.000	0.0	0.0	
0.0	1.000 PC	-0.000	-0.000	0.0	0.0	-0.804

FIT 10.0 1. 1. 4.000 / 0.100
4.000

VARY CODE = 200000
LABEL = FIT4

FIT 10.0 3. 3. 1.500 / 0.100
1.500

VARY CODE = 200000
LABEL = FIT4

TRANSFORM 1
4.77122 37.35149 0.0 0.0 0.0 -0.00003
0.33310 2.81727 0.0 0.0 0.0 -0.00000
0.0 0.0 4.59109 -0.89239 0.0 0.0
0.0 0.0 0.37533 0.13324 0.0 0.0
-0.00000 -0.00001 0.0 0.0 1.00000 -0.40547
0.0 0.0 0.0 0.0 0.0 1.00000

381.0 M	0.0	4.000 CM				
	0.0	0.299 MR	1.000			
	0.0	1.500 CM	0.0	0.0		
	0.0	0.113 MR	0.0	0.0	0.984	
	0.0	0.504 CM	0.000	0.000	0.0	0.0
	0.0	1.000 PC	-0.000	-0.000	0.0	0.0 -0.804

QUAD 5.00 2.00000 M 2.8115 KG 5.000 CM (7.759 M)
VARY CODE = C10000
LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M	0.0	2.551 CM				
	0.0	4.896 MR	-1.000			
	0.0	2.179 CM	0.0	0.0		
	0.0	2.242 MR	0.0	0.0	1.000	
	0.0	0.504 CM	0.000	-0.000	0.0	0.0
	0.0	1.000 PC	-0.000	0.000	0.0	0.0 -0.804

QUAD 5.00 2.00000 M -3.4253 KG 5.000 CM (-5.765 M)
VARY CODE = 010000
LABEL = G4

387.0 M	0.0	1.948 CM
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0.0	-1.298	MR	-1.000				
0.0	2.255	CM	0.0	0.0			
0.0	1.503	MR	0.0	0.0	-1.000		
0.0	0.504	CM	0.000	-0.000	0.0	0.0	
0.0	1.000	PC	-0.000	0.000	0.0	0.0	-0.804

DRIFT 3.0 15.000C M

402.0 M

0.0	0.023	CM					
0.0	1.298	MR	0.000				
0.0	0.020	CM	0.0	0.0			
0.0	1.503	MR	0.0	0.0	-0.000		
0.0	0.504	CM	-0.000	-0.000	0.0	0.0	
0.0	1.000	PC	0.000	0.000	0.0	0.0	-0.804

FIT 10.0 2. 1. 0.0 / 0.001

0.000

LABEL = FIT5

FIT 10.0 4. 3. 0.0 / 0.001

-0.000

LABEL = FIT6

FIT 10.0 1. 1. 0.020 / 0.010

0.023

VARY CODE = 200000

LABEL = FIT7

FIT 10.0 3. 3. 0.020 / 0.010

0.020

VARY CODE = 200000

LABEL = FIT7

TRANSFORM 1

-0.23010	3.08075	0.0	0.0	0.0	0.0
-0.44764	1.64740	0.0	0.0	0.0	0.0
0.0	0.0	-0.07421	0.99798	0.0	0.0
0.0	0.0	-1.02264	0.27718	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.9993 M

'BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A IIIC'

2

(IN THE FOURTH STEP THE LAST TWO QUAD DOUBLETS ARE BOTH ALLOWED TO VARY)
(FOR A FINAL ADJUSTMENT TO GET THE WAIST AND THE SPOT SIZE AT THE TARGET.)
5.01 'Q3' ;
SENTINEL

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A IIIC

1

IN THE FOURTH STEP THE LAST TWO QUAD DOUBLETS ARE BOTH ALLOWED TO VARY
FOR A FINAL ADJUSTMENT TO GET THE WAIST AND THE SPOT SIZE AT THE TARGET.

1.00000	0.30000	0.10000	0.30000	0.10000	0.30000	1.00000	25.00000
13.00000	2.00000						
4.00000	5.00000	1.45500	0.0				
2.00000	0.50000						
13.00000	1.00000						
3.00000	80.00000						
13.00000	1.00000						
5.00000	2.00000	-2.08981	5.00000				
LABEL = Q1							
3.00000	2.00000						
5.00000	2.00000	2.19428	5.00000				
LABEL = Q1							
13.00000	1.00000						
9.00000	4.00000						
3.00000	1.00000						
2.00000	1.50000						
4.00000	3.00000	14.55000	0.0				
2.00000	1.50000						
9.00000	0.0						
13.00000	1.00000						
3.00000	24.92038						
LABEL = CR1							
5.00000	2.00000	3.57851	15.00000				
LABEL = C2							
13.00000	1.00000						
-10.00000	-1.00000	2.00000	0.0	0.00100			
LABEL = FIT1							
-10.00000	-3.00000	4.00000	0.0	0.00100			
LABEL = FIT1							

13.00000	4.00000			
3.00000	25.07953			
LABEL = DR1				
13.00000	1.00000			
9.00000	4.00000			
2.00000	1.50000			
4.00000	3.00000	14.55000	0.0	
2.00000	1.50000			
3.00000	1.00000			
9.00000	0.0			
13.00000	1.00000			
-10.00000	-1.00000	6.00000	0.0	0.00100
LABEL = FIT2				
-10.00000	-2.00000	6.00000	0.0	0.00100
LABEL = FIT3				
13.00000	4.00000			
5.00000	2.00000	-2.11497	5.00000	
VARY CODE 010000				
LABEL = Q3				
3.00000	2.00000			
5.00000	2.00000	2.14873	5.00000	
VARY CODE 010000				
LABEL = Q3				
3.00000	200.00000			
13.00000	1.00000			
10.00000	1.00000	1.00000	4.00000	0.10000
VARY CODE 200000				
LABEL = FIT4				
10.00000	3.00000	3.00000	1.50000	0.10000
VARY CODE 200000				
LABEL = FIT4				
13.00000	4.00000			
13.00000	1.00000			
5.00000	2.00000	2.81145	5.00000	
VARY CODE 010000				
LABEL = Q4				
3.00000	2.00000			

13.00000 1.00000
 5.00000 2.00000 -3.42531 5.00000
 VARY CODE 010000
 LABEL = 54
 13.00000 1.00000
 3.00000 15.00000
 13.00000 1.00000
 10.00000 2.00000 1.00000 0.0 0.00100
 LABEL = FIT5
 10.00000 4.00000 3.00000 0.0 0.00100
 LABEL = FIT6
 10.00000 1.00000 1.00000 0.02000 0.01000
 VARY CODE 200000
 LABEL = FIT7
 10.00000 3.00000 3.00000 0.02000 0.01000
 VARY CODE 200000
 LABEL = FIT7
 13.00000 4.00000
 SENTINEL

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A IIIC

BEAM 1.000000 25.00 GEV

0.0 M	0.0	0.300 CM						
	0.0	0.100 MR	0.0					
	0.0	0.300 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.0			
	0.0	0.300 CM	0.0	0.0	0.0	0.0		
	0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

BEND 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M	0.0	0.305 CM						
	0.0	0.133 MR	0.171					
	0.0	0.304 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.164			
	0.0	0.300 CM	-0.009	-0.001	0.0	0.0		
	0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000	

DRIFT 3.0 80.0000 M

85.0 M	0.0	1.153 CM						
	0.0	0.133 MR	0.965					
	0.0	0.901 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.943			
	0.0	0.300 CM	-0.003	-0.001	0.0	0.0		
	0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000	

QUAD 5.00 2.00000 M -2.0898 KG 5.000 CM (-9.650 M)
LABEL = G1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1943 KG 5.000 CM (9.843 M)
LABEL = G1

91.0 M	0.0	1.661 CM						
	0.0	0.392 MR	-0.998					
	0.0	0.582 CM	0.0	0.0				
	0.0	0.147 MR	0.0	0.0	-0.937			
	0.0	0.300 CM	-0.003	0.003	0.0	0.0		
	0.0	1.000 PC	0.627	-0.613	0.0	0.0	-0.000	

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0030 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D

107.0 M

0.0	2.366 CM						
0.0	1.880 MR	0.873					
0.0	0.357 CM	0.0	0.0				
0.0	0.163 MR	0.0	0.0	-0.856			
0.0	0.471 CM	-0.737	-0.533	0.0	0.0		
0.0	1.000 PC	0.941	0.986	0.0	0.0	-0.617	

DRIFT 3.0 24.9203 M
 LABEL = DRI

QUAD 5.00 2.00000 M 3.57E5 KG 15.000 CM (17.815 M)
 LABEL = Q2

133.9 M

0.0	6.823 CM					
0.0	2.117 MR	-0.989				
0.0	0.239 CM	0.0	0.0			
0.0	0.254 MR	0.0	0.0	0.870		
0.0	0.471 CM	-0.615	0.678	0.0	0.0	
0.0	1.000 PC	1.000	-0.989	0.0	0.0	-0.617

TRANSFORM 1

-0.33045	-0.00002	0.0	0.0	0.0	6.82236
-0.22846	-3.02617	0.0	0.0	0.0	-2.09424
0.0	0.0	-0.79757	-0.00002	0.0	0.0
0.0	0.0	-0.73609	-1.25380	0.0	0.0
-0.22507	-2.06457	0.0	0.0	1.00000	-0.29037
0.0	0.0	0.0	0.0	0.0	1.00000

DRIFT 3.0 25.0795 M
 LABEL = DR1

159.0 M

0.0	1.765 CM					
0.0	2.117 MR	-0.814				
0.0	0.853 CM	0.0	0.0			
0.0	0.254 MR	0.0	0.0	0.990		
0.0	0.471 CM	-0.338	0.678	0.0	0.0	
0.0	1.000 PC	0.890	-0.989	0.0	0.0	-0.617

ROTAT 2.0 1.50 D

EEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

EEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M

175.0 M

0.0	1.300 CM						
0.0	0.310 MR	0.997					
0.0	1.227 CM	0.0	0.0				
0.0	0.216 MR	0.0	0.0	0.994			
0.0	0.504 CM	-0.000	0.000	0.0	0.0		
0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.804	

TRANSFORM 1
 -1.26882 -12.42980 0.0 0.0 0.0 0.00001
 -0.22847 -3.02628 0.0 0.0 0.0 0.00000
 0.0 0.0 -3.72852 -5.03437 0.0 0.0
 0.0 0.0 -0.62031 -1.10576 0.0 0.0
 -0.00000 -0.00001 0.0 0.0 1.00000 -0.40547
 0.0 0.0 0.0 0.0 0.0 1.00000

QUAD 5.00 2.00000 M -2.1150 KG 5.000 CM (-9.532 M)
 VARY CCCE = C10000
 LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1487 KG 5.000 CM (10.044 M)
 VARY CCCE = C10000
 LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M

0.0	4.000 CM	
0.0	0.299 MR	1.000

0.0	1.500 CM	0.0	0.0			
0.0	0.113 MR	0.0	0.0	0.984		
0.0	0.504 CM	0.000	0.000	0.0	0.0	
0.0	1.000 PC	-0.000	-0.000	0.0	0.0	-0.804

FIT 10.0 1. 1. 4.000 / 0.100

4.000

VARY CODE = 200000
LABEL = FIT4

FIT 10.0 3. 3. 1.500 / 0.100

1.500

VARY CODE = 200000
LABEL = FIT4

TRANSFORM 1

4.77122	37.35149	0.0	0.0	0.0	-0.00003
0.33310	2.81727	0.0	0.0	0.0	-0.00000
0.0	0.0	4.99109	-0.89239	0.0	0.0
0.0	0.0	0.37533	0.13324	0.0	0.0
-0.00000	-0.00001	0.0	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

381.0 M

0.0	4.000 CM					
0.0	0.299 MR	1.000				
0.0	1.500 CM	0.0	0.0			
0.0	0.113 MR	0.0	0.0	0.984		
0.0	0.504 CM	0.000	0.000	0.0	0.0	
0.0	1.000 PC	-0.000	-0.000	0.0	0.0	-0.804

QUAD 5.00 2.00000 M 2.8115 KG 5.000 CM (7.759 M)

VARY CODE = 010000
LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M

0.0	2.551 CM					
0.0	4.896 MR	-1.000				
0.0	2.179 CM	0.0	0.0			
0.0	2.242 MR	0.0	0.0	1.000		
0.0	0.504 CM	0.000	-0.000	0.0	0.0	
0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.804

QUAD 5.00 2.00000 M -3.4253 KG 5.000 CM (-5.765 M)

VARY CODE = 010000
LABEL = Q4

387.0 M	0.0	1.948 CM						
	0.0	1.298 MR	-1.000					
	0.0	2.255 CM	0.0	0.0				
	0.0	1.503 MR	0.0	0.0	-1.000			
	0.0	0.504 CM	0.000	-0.000	0.0	0.0		
	0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.804	

DRIFT 3.0 15.0000 M

402.0 M	0.0	0.023 CM						
	0.0	1.298 MR	0.000					
	0.0	0.020 CM	0.0	0.0				
	0.0	1.503 MR	0.0	0.0	-0.000			
	0.0	0.504 CM	-0.000	-0.000	0.0	0.0		
	0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.804	

FIT 10.0 2. 1. 0.0 / 0.001

0.000

LABEL = FIT5

FIT 10.0 4. 3. 0.0 / 0.001

-0.000

LABEL = FIT6

FIT 10.0 1. 1. 0.020 / 0.010

0.023

VARY CODE = 200000

LABEL = FIT7

FIT 10.0 3. 3. 0.020 / 0.010

0.020

VARY CODE = 200000

LABEL = FIT7

TRANSFORM 1

-0.23010	3.0E075	0.0	0.0	0.0	0.0
-0.44764	1.64740	0.0	0.0	0.0	0.0
0.0	0.0	-0.07421	0.99798	0.0	0.0
0.0	0.0	-1.02264	0.27718	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.9993 M

CORRECTIONS

(1.3621E-01) 0.0 0.000 0.000 -0.000

*COVARIANCE (FIT 0.1)

0.0
0.0 0.001
0.0 -0.003 0.000
0.0 -0.753 -0.007 0.000

BEAM TRANSPORT SYSTEM FOR SLAC BEAM SWITCHYARD END STATION A IIIC

BEAM 1.000000 25.00 GEV

0.0 M	0.0	0.300 CM						
	0.0	0.100 MR	0.0					
	0.0	0.300 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.0			
	0.0	0.300 CM	0.0	0.0	0.0	0.0		
	0.0	1.000 PC	0.0	0.0	0.0	0.0	0.0	0.0

BEND 4.000 5.00000 M 1.455 KG 0.0000 (0.500 D)

ROTAT 2.0 0.50 D

5.0 M	0.0	0.305 CM						
	0.0	0.133 MR	0.171					
	0.0	0.304 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.164			
	0.0	0.300 CM	-0.009	-0.001	0.0	0.0		
	0.0	1.000 PC	0.071	0.657	0.0	0.0	-0.000	

DRIFT 3.0 80.0000 M

85.0 M	0.0	1.153 CM						
	0.0	0.133 MR	0.965					
	0.0	0.901 CM	0.0	0.0				
	0.0	0.100 MR	0.0	0.0	0.943			
	0.0	0.300 CM	-0.003	-0.001	0.0	0.0		
	0.0	1.000 PC	0.624	0.657	0.0	0.0	-0.000	

QUAD 5.00 2.00000 M -2.0858 KG 5.000 CM (-9.650 M)
LABEL = C1

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1943 KG 5.000 CM (9.843 M)
LABEL = C1

91.0 M	0.0	1.661 CM						
	0.0	0.392 MR	-0.998					
	0.0	0.582 CM	0.0	0.0				
	0.0	0.147 MR	0.0	0.0	-0.937			
	0.0	0.300 CM	-0.003	0.003	0.0	0.0		
	0.0	1.000 PC	0.627	-0.613	0.0	0.0	-0.000	

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D
 DRIFT 3.0 1.0000 M
 ROTAT 2.0 1.50 D
 BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)
 ROTAT 2.0 1.50 D

107.0 M

0.0	2.366 CM						
0.0	1.880 MR	0.873					
0.0	0.357 CM	0.0	0.0				
0.0	0.163 MR	0.0	0.0	-0.856			
0.0	0.471 CM	-0.737	-0.533	0.0	0.0		
0.0	1.000 PC	0.941	0.986	0.0	0.0	-0.617	

DRIFT 3.0 24.9203 M
 LABEL = DRI

QUAD 5.00 2.00000 M 3.5785 KG 15.000 CM (17.815 M)
 LABEL = C2

133.9 M

0.0	6.823 CM						
0.0	2.117 MR	-0.989					
0.0	0.239 CM	0.0	0.0				
0.0	0.254 MR	0.0	0.0	0.870			
0.0	0.471 CM	-0.615	0.678	0.0	0.0		
0.0	1.000 PC	1.000	-0.989	0.0	0.0	-0.617	

TRANSFORM 1

-0.33045	-0.00002	0.0	0.0	0.0	6.82236
-0.22846	-3.02617	0.0	0.0	0.0	-2.09424
0.0	0.0	-0.79757	-0.00002	0.0	0.0
0.0	0.0	-0.73609	-1.25380	0.0	0.0
-0.22507	-2.06457	0.0	0.0	1.00000	-0.29037
0.0	0.0	0.0	0.0	0.0	1.00000

DRIFT 3.0 25.0795 M
 LABEL = DR1

159.0 M

0.0	1.765 CM						
0.0	2.117 MR	-0.814					
0.0	0.853 CM	0.0	0.0				
0.0	0.254 MR	0.0	0.0	0.990			
0.0	0.471 CM	-0.338	0.678	0.0	0.0		
0.0	1.000 PC	0.890	-0.989	0.0	0.0	-0.617	

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

ROTAT 2.0 1.50 D

BEND 4.000 3.00000 M 14.550 KG 0.0000 (2.999 D)

ROTAT 2.0 1.50 D

DRIFT 3.0 1.0000 M

175.0 M

0.0	1.300 CM				
0.0	0.310 MR	0.997			
0.0	1.227 CM	0.0	0.0		
0.0	0.216 MR	0.0	0.0	0.994	
0.0	0.504 CM	-0.000	0.000	0.0	0.0
0.0	1.000 PC	0.000	0.000	0.0	0.0
					-0.804

TRANSFORM 1

-1.26882	-12.42980	0.C	0.0	0.0	0.00001
-0.22847	-3.02628	0.C	0.0	0.0	0.00000
0.0	0.0	-3.72852	-5.03437	0.0	0.0
0.0	0.0	-0.62031	-1.10576	0.0	0.0
-0.00000	-0.00001	0.C	0.0	1.00000	-0.40547
0.0	0.0	0.C	0.0	0.0	1.00000

QUAD 5.00 2.00000 M -2.1150 KG 5.000 CM (-9.532 M)

VARY CODE = 010000
LABEL = Q3

DRIFT 3.0 2.0000 M

QUAD 5.00 2.00000 M 2.1487 KG 5.000 CM (10.044 M)

VARY CODE = 010000
LABEL = Q3

DRIFT 3.0 200.0000 M

381.0 M

0.0	4.000 CM
0.0	0.299 MR
	1.000

0.0	1.500 CM	0.0	0.0			
0.0	0.113 MR	0.0	0.0	0.984		
0.0	0.504 CM	0.000	0.000	0.0	0.0	
0.0	1.000 PC	-0.000	-0.000	0.0	0.0	-0.804

FIT 10.0 1. 1. 4.000 / 0.100

4.000
VARY CODE = 200000
LABEL = FIT4

FIT 10.0 3. 3. 1.500 / 0.100

1.500
VARY CODE = 200000
LABEL = FIT4

TRANSFORM 1

4.77126	37.35184	C.C	0.0	0.0	-0.00003
0.33310	2.81729	0.C	0.0	0.0	-0.00000
0.0	0.0	4.59107	-0.89241	0.0	0.0
0.0	0.0	0.37533	0.13324	0.0	0.0
-0.00000	-0.00001	0.C	0.0	1.00000	-0.40547
0.0	0.0	0.0	0.0	0.0	1.00000

381.0 M

0.0	4.000 CM					
0.0	0.299 MR	1.000				
0.0	1.500 CM	0.0	0.0			
0.0	0.113 MR	0.0	0.0	0.984		
0.0	0.504 CM	0.000	0.000	0.0	0.0	
0.0	1.000 PC	-0.000	-0.000	0.0	0.0	-0.804

QUAD 5.00 2.00000 M 2.8115 KG 5.000 CM (7.759 M)
VARY CODE = 010000
LABEL = Q4

DRIFT 3.0 2.0000 M

385.0 M

0.0	2.551 CM					
0.0	4.896 MR	-1.000				
0.0	2.179 CM	0.0	0.0			
0.0	2.242 MR	0.0	0.0	1.000		
0.0	0.504 CM	0.000	-0.000	0.0	0.0	
0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.804

QUAD 5.00 2.00000 M -3.4253 KG 5.000 CM (-5.765 M)
VARY CODE = 010000
LABEL = Q4

387.0 M

0.0	1.948 CM						
0.0	1.298 MR	-1.000					
0.0	2.255 CM	0.0	0.0				
0.0	1.503 MR	0.0	0.0	-1.000			
0.0	0.504 CM	0.000	-0.000	0.0	0.0		
0.0	1.000 PC	-0.000	0.000	0.0	0.0	-0.804	

DRIFT 3.0 15.0000 M

402.0 M

0.0	0.023 CM						
0.0	1.298 MR	-0.000					
0.0	0.020 CM	0.0	0.0				
0.0	1.503 MR	0.0	0.0	0.000			
0.0	0.504 CM	-0.000	-0.000	0.0	0.0		
0.0	1.000 PC	0.000	0.000	0.0	0.0	-0.804	

FIT 10.0 2. 1. 0.0 / 0.001
-0.000
LABEL = FIT5

FIT 10.0 4. 3. 0.0 / 0.001
0.000
LABEL = FIT6

FIT 10.0 1. 1. 0.020 / 0.010
0.023
VARY CGDE = 200000
LABEL = FIT7

FIT 10.0 3. 3. 0.020 / 0.010
0.020
VARY CODE = 200000
LABEL = FIT7

TRANSFORM 1

-0.23010	3.08075	0.0	0.0	0.0	0.0
-0.44764	1.64740	0.0	0.0	0.0	0.0
0.0	0.0	-0.07421	0.99798	0.0	0.0
0.0	0.0	-1.02264	0.27718	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 401.9993 M

'UNCERTAINTY IN MISALIGNMENT OF MAGNETIC TRIPLET'

0

(THIS IS AN EXAMPLE OF AN UNCERTAINTY IN THE MISALIGNMENT OF A TRIPLET.)
(ZERO PHASE SPACE IS USED. IT IS SIMILAR TO EXAMPLE THREE IN SECTION EIGHT OF)
(SLAC 91. THE UNCERTAINTIES APPEAR IN THE PHASE SPACE ELLIPSE PRINTOUT.)

1.0 0.0 0.0 0.0 0.0 0.0 0.0 1.0 ;
3.0 3.66 ;
6.0 0.0 2.0 ;
5.0 0.45730 -14.50300 10.15 ;
3.0 0.15250 ;
6.0 0.0 1.0 ;
5.0 0.91400 8.39750 10.15 ;
3.0 0.15250 ;
5.0 0.45730 -14.50300 10.15 ;
8.0 0.1 0.1 0.1 0.1 0.1 0.1 000.0 ;
8.0 0.1 0.1 0.1 0.1 0.1 0.1 001.0 ;
8.0 0.1 0.1 0.1 0.1 0.1 0.1 002.0 ;
3.0 2.13500 ;
13.0 4.0 ;
SENTINEL

UNCERTAINTY IN MISALIGNMENT OF MAGNETIC TRIPLET

0

THIS IS AN EXAMPLE OF AN UNCERTAINTY IN THE MISALIGNMENT OF A TRIPLET.

ZERO PHASE SPACE IS USED. IT IS SIMILAR TO EXAMPLE THREE IN SECTION EIGHT OF

SLAC 91. THE UNCERTAINTIES APPEAR IN THE PHASE SPACE ELLIPSE PRINTOUT.

1.00000	0.0	0.0	0.0	0.0	0.0	0.0	1.00000
3.00000	3.66000						
6.00000	0.0	2.00000					
5.00000	0.45730	-14.50800	10.15000				
3.00000	0.15250						
6.00000	0.0	1.00000					
5.00000	0.91400	8.35750	10.15000				
3.00000	0.15250						
5.00000	0.45730	-14.50800	10.15000				
8.00000	0.10000	0.10000	0.10000	0.10000	0.10000	0.10000	0.0
8.00000	0.10000	0.10000	0.10000	0.10000	0.10000	0.10000	1.00000
8.00000	0.10000	0.10000	0.10000	0.10000	0.10000	0.10000	2.00000
3.00000	2.13500						
13.00000	4.00000						

SENT INEL

UNCERTAINTY IN MISALIGNMENT OF MAGNETIC TRIPLET

BEAM	1.000000	1.00 GEV	0.0 M	0.0	0.0	CM													
				0.0	0.0	MR	0.0												
				0.0	0.0	CM	0.0	0.0											
				0.0	0.0	MR	0.0	0.0	0.0										
				0.0	0.0	CM	0.0	0.0	0.0	0.0									
				0.0	0.0	PC	0.0	0.0	0.0	0.0	0.0								
DRIFT	3.0	3.6600 M	3.7 M	0.0	0.0	CM													
				0.0	0.0	MR	0.0												
				0.0	0.0	CM	0.0	0.0											
				0.0	0.0	MR	0.0	0.0	0.0										
				0.0	0.0	CM	0.0	0.0	0.0	0.0									
				0.0	0.0	PC	0.0	0.0	0.0	0.0	0.0								
UPDATE	6.00	(0.0)	2.00	3.7 M	0.0	0.0	CM												
					0.0	0.0	MR	0.0											
					0.0	0.0	CM	0.0	0.0										
					0.0	0.0	MR	0.0	0.0	0.0									
					0.0	0.0	CM	0.0	0.0	0.0	0.0								
					0.0	0.0	PC	0.0	0.0	0.0	0.0	0.0							
QUAD	5.00	0.45730 M -14.5080 KG 10.150 CM (-0.441 M)	4.1 M	0.0	0.0	CM													
				0.0	0.0	MR	0.0												
				0.0	0.0	CM	0.0	0.0											
				0.0	0.0	MR	0.0	0.0	0.0										
				0.0	0.0	CM	0.0	0.0	0.0	0.0									
				0.0	0.0	PC	0.0	0.0	0.0	0.0	0.0								
DRIFT	3.0	0.1925 M	4.3 M	0.0	0.0	CM													
				0.0	0.0	MR	0.0												
				0.0	0.0	CM	0.0	0.0											
				0.0	0.0	MR	0.0	0.0	0.0										
				0.0	0.0	CM	0.0	0.0	0.0	0.0									
				0.0	0.0	PC	0.0	0.0	0.0	0.0	0.0								
UPDATE	6.00	(0.0)	1.00	4.3 M	0.0	0.0	CM												
					0.0	0.0	MR	0.0											
					0.0	0.0	CM	0.0	0.0										
					0.0	0.0	MR	0.0	0.0	0.0									
					0.0	0.0	CM	0.0	0.0	0.0	0.0								
					0.0	0.0	PC	0.0	0.0	0.0	0.0	0.0							
QUAD	5.00	0.91400 M 8.3975 KG 10.150 CM (0.640 M)	5.2 M	0.0	0.0	CM													

[illegible]

0.0	0.0	CM	0.0	0.0	0.0	0.0	0.0
0.0	0.0	PC	3.0	0.0	0.0	0.0	0.0

TRANSFORM 1

1.00000	0.21350	C.C	0.0	0.0	0.0
0.0	1.00000	0.0	0.0	0.0	0.0
0.0	0.0	1.00000	0.21350	0.0	0.0
0.0	0.0	0.0	1.00000	0.0	0.0
0.0	0.0	0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0	0.0	1.00000

LENGTH 7.9286 M

'DELIBERATE DISPLACEMENT OF A TRIPLET'

0

(THIS IS AN EXAMPLE OF A KNOWN DISPLACEMENT IN A TRIPLET. A FINITE PHASE)

(SPACE IS ENTERED, AND THE DISPLACEMENT APPEARS IN THE BEAM CENTROID.)

1.0 0.1 0.2 0.3 0.4 0.50 0.6 7.0 ;

3.0 3.66 ;

6.0 0.0 2.0 ;

5.0 0.45730 -14.50800 10.15 ;

3.0 0.15250 ;

6.0 0.0 1.0 ;

5.0 0.91400 8.39750 10.15 ;

3.0 0.15250 ;

5.0 0.45730 -14.50800 10.15 ;

8.0 0.1 0.1 0.1 0.1 0.1 0.1 100.0 ;

8.0 0.1 0.1 0.1 0.1 0.1 0.1 101.0 ;

8.0 0.1 0.1 0.1 0.1 0.1 0.1 102.0 ;

3.0 2.13500 ;

15.0 4.0 ;

SENTINEL

DELIBERATE DISPLACEMENT OF A TRIPLET

0

THIS IS AN EXAMPLE OF A KNOWN DISPLACEMENT IN A TRIPLET. A FINITE PHASE

SPACE IS ENTERED, AND THE DISPLACEMENT APPEARS IN THE BEAM CENTROID.

1.00000	C.10000	0.20000	0.30000	0.40000	0.50000	0.60000	7.00000
3.00000	2.66000						
6.00000	0.0	2.00000					
5.00000	C.45730	-14.50800	10.15000				
3.00000	C.15250						
6.00000	0.0	1.00000					
5.00000	0.91400	8.35750	10.15000				
3.00000	C.15250						
5.00000	0.45730	-14.50800	10.15000				
8.00000	0.10000	0.10000	0.10000	0.10000	0.10000	0.10000	100.00000
8.00000	C.10000	0.10000	0.10000	0.10000	0.10000	0.10000	101.00000
8.00000	C.10000	0.10000	0.10000	0.10000	0.10000	0.10000	102.00000
3.00000	2.13500						
13.00000	4.00000						

SENTINEL

DELIBERATE DISPLACEMENT OF A TRIPLET

BEAM 1.000000 7.00 GEV

0.0 M	0.0	0.100 CM						
	0.0	0.200 MR	0.0					
	0.0	0.300 CM	0.0	0.0				
	0.0	0.400 MR	0.0	0.0	0.0			
	0.0	0.500 CM	0.0	0.0	0.0	0.0		
	0.0	0.600 PC	0.0	0.0	0.0	0.0	0.0	0.0

DRIFT 3.0 3.6600 M

3.7 M	0.0	0.124 CM						
	0.0	0.200 MR	0.591					
	0.0	0.334 CM	0.0	0.0				
	0.0	0.400 MR	0.0	0.0	0.439			
	0.0	0.500 CM	0.0	0.0	0.0	0.0		
	0.0	0.600 PC	0.0	0.0	0.0	0.0	0.0	0.0

UPDATE 6.00 (0.) 2.00

3.7 M	0.0	0.124 CM						
	0.0	0.200 MR	0.591					
	0.0	0.334 CM	0.0	0.0				
	0.0	0.400 MR	0.0	0.0	0.439			
	0.0	0.500 CM	0.0	0.0	0.0	0.0		
	0.0	0.600 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 0.45730 M -14.5080 KG 10.150 CM (-3.497 M)
4.1 M

0.0	0.138 CM							
0.0	0.510 MR	0.959						
0.0	0.321 CM	0.0	0.0					
0.0	0.822 MR	0.0	0.0	-0.891				
0.0	0.500 CM	0.0	0.0	0.0	0.0			
0.0	0.600 PC	0.0	0.0	0.0	0.0	0.0		

DRIFT 3.0 0.1525 M

4.3 M	0.0	0.145 CM						
	0.0	0.510 MR	0.963					
	0.0	0.310 CM	0.0	0.0				
	0.0	0.822 MR	0.0	0.0	-0.882			
	0.0	0.500 CM	0.0	0.0	0.0	0.0		
	0.0	0.600 PC	0.0	0.0	0.0	0.0	0.0	0.0

UPDATE 6.00 (0.) 1.00

4.3 M	0.0	0.145 CM						
	0.0	0.510 MR	0.963					
	0.0	0.310 CM	0.0	0.0				
	0.0	0.822 MR	0.0	0.0	-0.882			
	0.0	0.500 CM	0.0	0.0	0.0	0.0		
	0.0	0.600 PC	0.0	0.0	0.0	0.0	0.0	0.0

QUAD 5.00 0.91400 M 8.3975 KG 10.150 CM (3.246 M)
5.2 M

0.0	0.167 CM
-----	----------

E-120

0.0	0.500 CM	0.0	0.0	0.0	0.0	0.0
0.0	0.600 PC	0.0	0.0	0.0	0.0	0.0

```
*TRANSFORM* 1
1.00000 0.21350 C.0 0.0 0.0 0.0
0.0 1.00000 C.C 0.0 0.0 0.0
C.0 0.0 1.00000 0.21350 0.0 0.0
C.0 0.0 0.0 1.00000 0.0 0.0
0.0 0.0 0.0 0.0 1.00000 0.0
C.0 0.0 C.0 0.0 0.0 1.00000
```

LENGTH 7.9286 M

IHC217I FIOCS - END OF DATA SET ON UNIT 5

TRACEBACK FOLLOWS-	ROUTINE	ISN	REG. 14	REG. 15	REG. 0	REG. 1
	IBCOM		000755FC	0007D0E8	00000001	00065FD4
	8KKIN	0013	40066060	00075248	00000000	00065FD4
	TITLE	0029	40078D6A	00065F50	00000C00	0007B83C
	FITTIN	0046	4007856A	00078628	00000C0F	0007B414
	MAIN		00015C8A	0107B338	FD000C08	000ABFF8

ENTRY POINT= 0107B338

SUMMARY OF ERRORS FOR THIS JOB	ERROR NUMBER	NUMBER OF ERRORS
	217	1

TRANSPORT FLOW CHART

TABLE OF CONTENTS

TRANSPORT	F-1
SUBROUTINE BKKIN	F-2
SUBROUTINE BKKPAK	F-9
SUBROUTINE BKKFX	F-10
SUBROUTINE BKKFL	F-12
SUBROUTINE SCANT	F-15
SUBROUTINE FITTIN	F-16
OUTFIT SUBPROGRAM	F-22
ELEMENT SUBROUTINE	F-26
SOLVE SUBPROGRAM	F-38
FORM SUBROUTINE	F-40
ALTER SUBPROGRAM	F-41
PARTIALS SUBROUTINE	F-44
SUBROUTINE CHEK	F-51
SUMMARY	F-52

-SUBROUTINE BKIN(1)-

ENTRY TRANIN(DATA, VARY, KABLE, NSTART, NSTOP, NWORD, IR)

SET FLAG TO
INDICATE
ENTRY POINT

ENTRY BKIN (DATA, NSTART, NSTOP, NWORD, IR)

EXAMINE CHARACTER
IN COLUMN IW

SET FLOATING
POINT FLAG

-SUBROUTINE BKKIN (3)-

CHARACTER IS PART
OF NUMERIC FIELD

-SUBROUTINE BKKIN (4) -

FLOATING POINT
NUMBER

DATA

TYPE CODE

-SUBROUTINE BKKIN (5)-

NON-NUMERIC
FIELD

STRING ENCLOSED
IN QUOTES. STORE
IN DATA ARRAY

CHECK TO SEE IF
STRING WAS TYPE
15 ID OR LABEL

-SUBROUTINE BKIN (6)-

CHECK FOR
RESERVED WORDS

-SUBROUTINE BKKPAK-

-SUBROUTINE BKKFX (1)-

-SUBROUTINE BKKFX (2)-

CHARACTER FOUND
THAT WAS NOT A
NUMBER

-SUBROUTINE BKKFL(1)-

-SUBROUTINE BKKFL (2)-

LETTER E FOUND,
BUT NO SIGN

HIGHEST COLUMN
OF FRACTION

FIRST COLUMN
OF EXPONENT

PLUS SIGN

FIRST COLUMN
OF EXPONENT

MINUS SIGN

-SUBROUTINE BKKFL (3)-

EXPONENT TO
BE EVALUATED

EVALUATE
FRACTION
IN COLUMNS
NLOW TO
NHIGH

— SUBROUTINE SCANT —

TYPE CODE.
SEPARATE TYPE CODE
FROM VARY CODES

-SUBROUTINE FITTIN (1)-

-SUBROUTINE FITTIN (2)-

-SUBROUTINE FTTIN (3)-

FEWER WORDS READ IN

CHECK VARY CODES FOR
THIS ELEMENT AND SET UP
THE ARRAY

-SUBROUTINE FITTIN (4)-

CHECK BIND ARRAY

INCREMENT INDEX OF
F.W.A. OF ELEMENTS
IN DATA ARRAY

RETURN TO PROCESS
ANOTHER ELEMENT

SET N1 TO ZERO SO NO
FITTING WILL OCCUR
FIRST TIME THROUGH

-SUBROUTINE FITTIN (5)-

OUTFIT CALLS ELEMENTS
WHICH CALCULATES THE
R MATRICES FOR THE
ELEMENTS

DATAP = 1
READ IN THE CHANGES IN
THE DATA ARRAY FROM THE
PRECEDING PROBLEM

-SUBROUTINE FITTIN (6)-

-OUTFIT SUBPROGRAM-

-OUTFIT SUBPROGRAM (2)-

151082

-OUTFIT SUBPROGRAM(3)-

-OUTFIT SUBPROGRAM(4)-

-ELEMENT SUBROUTINE-

-ELEMENT SUBROUTINE (2)-

151083

-ELEMENT SUBROUTINE (3)-

151084

-ELEMENT SUBROUTINE (5)-

1510C5

-MISALIGNMENT-

NOTES: IN PROGRAM LISTING:

$\vec{m}$ IS DENOTED BY VM(
 A_0 IS " " CTO(
 A IS " " CT(,

-SUBROUTINE TFL USES
 ACCUMULATED MATRICES
 $\Theta(ir,.)$ AND VECTORS $XO(ir,.)$
 TO CALC $A(.,.)$.

-SUBROUTINE RESET (ir)
 INITIALIZES THESE ARRAYS.

-SUBROUTINE ADVANCE (ir)
 ACCUMULATES THESE ARRAYS.

-ON 13. 2lr. CARD, 1r = 1, 2, 3
 FOR R, RC, RC2 CASES
 RESPECTIVELY.

-IF TENS DIGIT OF TYT
 IS $xx1.$ THEN matrix A is
 taken = (0) (this is executed
 by subroutine TFL).

151086

-ELEMENT SUBROUTINE (6)-

151087

-ELEMENT SUBROUTINE (7)-

IF 'BLANK' TYPE
15 CARD, CHANGE
BACK TO STANDARD
UNITS

UNITO() SET OF STANDARD UNITS
(UNCHANGEABLE)
UNIT() SET OF USER-CHANGEABLE
UNITS, INITIALLY THE SAME
AS UNITO()

-ELEMENT SUBROUTINE (8)-

151089

-ELEMENT SUBROUTINE(9)-

-ELEMENT SUBROUTINE (10)-

1510810

-SOLVE SUBPROGRAM-

1510812

-SOLVE SUBPROGRAM (2)-

FORMAT OF ARRAY CA(,)

AFTER FORM, BUT BEFORE INQ

	1	2		(N+1)
1		0	0	0
2		MATRIX ON LEFT SIDE OF NORMAL EQNS		
...				
...				
...				
...				
(N+1)				

COLUMN VECTOR ON RIGHT
SIDE OF NORMAL EQUATIONS

AFTER INQ

	1	2		(N+1)
1	S			
2	Δ ₁	COVARIANCE MATRIX		
...	Δ ₂			
...	...			
...	...			
...	Δ _v			
(N+1)				

S = SUM OF SQUARES OF THE LEAST SQUARES RESIDUALS

Δ_i = CORRECTION FOUND TO ith PARAMETER

1510813

-FORM SUBROUTINE-

1510814

-ALTER SUBPROGRAM-

1510815

-ALTER SUBROUTINE (2)-

1510816

-ALTER SUBPROGRAM (3)-

1510817

-PARTIALS SUBROUTINE-

RCV(., N) CONTAINS $\frac{\partial [RC]}{\partial \lambda_N}$

WHERE: RC(.) IS ACCUMULATED
TRANSFORM MX..

R(.) IS MX. FROM
ELEMENTS

λ_N IS THE Nth PARAMETER

INDEX N RUNS THRU THE
PARAMETERS UP
TO N1 = TOT. NUM-
BER OF PARAM-
ETERS.

1510819

-PARTIALS SUBROUTINE (2)-

1510818

-SUBROUTINE PARTLS(3)-

JV=1 is illegal
L may not be varied

NOTE: Z IS THE 6x6 MATRIX AND $\vec{m}$ IS THE 6 VECTOR DESCRIBED ABOVE ON FLOW CHART FOR THE 8-TYPE BRANCH IN SUB-ROUTINE ELEMENTS m_{JV} REFERS TO THE JVth COMPONENT OF $\vec{m}$.

-PARTIALS SUBROUTINE(4)-

-PARTIALS SUBROUTINE (5)-

1510820

1510821

-PARTIALS SUBROUTINE (7)-

1510822

-SUBROUTINE CHEK(1)-

CHECK DATA FOLLOWING
POLE FACE ROTATION TO
SEE IF IS A BEND OR A
PRINTOUT REQUEST

NO BEND FOUND.
CHECK PRECEDING
DATA

IMPROPER
ELEMENT

-SUBROUTINE CHEK(2)-

FIELD

SET FAGS FOR POLE
FACE ROTATION BEFORE
BEND FOR 2ND ORDER
CALCULATION

-SUMMARY-

MATRIX FORMAT

SI:

	1	2	3	...	NV	(NV+1)
1	SI_{jk}					
2						
3						
.						
.						
.						
NV						
(NV+1)	CENTRAL TRAJECTORY					

NV=6 (FIRST ORDER RUN)
=42 (SECOND ORDER RUN)

FLAGS:

S1P:

CENTRAL TRAJECTORY
IS NOT IDENTICALLY=0

1510823

APPENDIX

Table of Contents

Introduction		A-0
1 -	Beam Transport Optics (a set of lectures given to the SLAC technical staff)	
	Part I	A-1
	Geometric Light Optics vs. Magnetic Optics	A-1
	Introduction of Momentum Dispersion into the Matrix Formalism	A-8
	Second-Order Matrix Formalism	A-12
	Transformations Involving Many Trajectories	A-13
	Part II	A-16
	First-Order 3×3 Matrix Transformation	A-16
	Example of a Beam Switchyard	A-17
	TRANSPORT Notation	A-22
	Second-Order Matrix	A-23
2 -	The First-Order R Matrix and Sigma Matrix (Beam Ellipsoid)	
	Formulation of TRANSPORT	A-26
	First-Order Dispersion	A-29
	First-Order Path Length	A-29
	Achromaticity	A-30
	Isochronicity	A-30
	First-Order Imaging Conditions	A-30
	Focal Lengths	A-31
	Zero Dispersion	A-32
	Magnification	A-32
	First-Order Momentum Resolution	A-32
	First-Order Phase Ellipse Formalism of TRANSPORT	A-35
	Description of the Sigma BEAM Matrix	A-37
	The Phase Ellipse Matrix used by TRANSPORT	A-39
	Physical Interpretation of Various Projections of the 2-dimensional BEAM Ellipse	A-41
	The Upright Ellipse	A-44
	Relationship between a Waist and a Parallel-to-Point Image	A-45
	Relationship between a Waist and Point-to-Point Image	A-46
	Relationship between a Waist and the Smallest Spot Size Achievable at a Fixed Target Position	A-48
	Imaging from an Erect Ellipse to an Erect Ellipse	A-51
	Relationship between a First-Order Point-to-Point Image and the Minimum Spot Size Achievable at a Fixed Target Position	A-54
	Orientation of the Major Axes of a Phase Space Ellipse	A-55

Table of Contents - (continued)

3 -	Second-Order Calculations and Corrections	A-58
	Second-Order Phase Ellipsoid Formalism	A-59
	A Systematic Procedure for Designing High Resolving Power Beam Transport Systems or Charged Particle Spectrometers	A-60
	I Introduction	A-62
	II Theory	A-63
	Multipole Strengths for Pure Multipole Fields	A-65
	Multipole Strengths for a Non-Uniform Field Expansion	A-66
	Multipole Strengths for a Contoured Entrance or Exit Boundary of a Magnet	A-66
	The Description of the Trajectories as a Taylor's Expansion	A-67
	III Solution of the Equation of Motion	A-69
	IV Interpretation and use of the Coupling Coefficient	A-71
	V A Systematic Procedure for Designing High Resolution Systems	A-73
	First-Order Considerations	A-73
	First-Order Resolving Power	A-73
	Dispersion	A-75
	The Selection of the Optical Mode	A-75
	Aberrations and their Correction	A-75
	References	A-76
4 -	Theory of Magnet Alignment Tolerances	A-77
5 -	First-Order Parameter Optimization and Covariance	A-80

APPENDIX

Introduction

This appendix has been included as an addition to the manual in an attempt to better acquaint the user with 'what TRANSPORT does', and with the notation and mathematical formalism used in a TRANSPORT calculation.

The first section (Beam Transport Optics - Part I and Part II) is a rewrite of two lectures given to members of the SLAC technical staff on the elementary matrix algebra of optics. We include them here for the benefit of the new user who may need a brief refresher course on charged particle optics and/or has a need to become familiar with TRANSPORT notation. The new user should also acquaint himself with the contents of the books and other publications listed under 'references' at the end of the manual. References 1 and 2 are essential if the user is to obtain the maximum value from TRANSPORT.

The second section of this appendix was written to introduce the mathematical formalism of the first-order R matrix and Sigma matrix (phase ellipsoid) beam optics used in a TRANSPORT calculation and to correlate this with the printed output.

Section three discusses second-order calculations and, in particular, a procedure for calculating the "Sextupole" strengths required to minimize and/or eliminate second-order aberrations in a beam transport system.

Section four is a brief derivation of the mathematical formalism used by TRANSPORT for calculating magnet alignment tolerances.

Section five deals with the first-order parameter optimization code of TRANSPORT and includes a brief explanation of the covariance matrix that is printed after each first-order fit routine.

BEAM TRANSPORT OPTICS

Section I

Beam Transport Optics - Part I

(K. L. Brown)

1. Introduction

A convenient starting point for this lecture is the equation relating the magnetic rigidity of a particle ($B\rho$) to the particle momentum P .

$$B\rho = \frac{10^2}{2.99793} P \quad \text{or} \quad B\rho = 33.356 P$$

where

B is in kilogauss

ρ is the bending radius in meters

P is the particle's momentum in BeV/c.

A note of caution: When using this equation for a TRANSPORT calculation, it is necessary to use at least 5 significant figures for the constant to avoid round-off errors in the readout.

2. Geometric Light Optics vs. Magnetic Optics

To relate geometrical light optics to charged particle optics, we begin with the thin lens. Figure 1 shows a thin lens with a ray leaving a focal point A at an angle θ_0 , impinging on the lens at x_0 . As the ray leaves the lens, it is at x_1 and going toward a focal point B at an angle of θ_1 .

Figure 1

Thin lens optics says that $1/p + 1/q = 1/f$. Using this equation it is readily verified that the matrix transformation for the lens action between principal planes is

$$\begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} x_o \\ \theta_o \end{bmatrix}$$

The transformation for a drift distance L is

Figure 2

$$\begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_o \\ \theta_o \end{bmatrix}$$

Note that the determinant of the matrix in both examples is equal to unity. This is always the case as will be proved formally later. That this is so is a manifestation of Liouville's theorem of conservation of phase space area.

Consider now a thick lens, as illustrated in Figure 3.

Figure 3

If L_1 is the object distance to the face of the lens and L_2 is the corresponding image distance, then, in general,

$$1/L_1 + 1/L_2 \neq 1/f .$$

If, however, we introduce two planes P_1 and P_2 located at a distance z_1 and z_2 from the entrance and exit faces of the lens, it is always possible to find a z_1 and a z_2 such that the equation

$$1/p + 1/q = 1/f \text{ is valid.}$$

where

$$\begin{aligned} p &= L_1 + z_1 \\ q &= L_2 + z_2 . \end{aligned}$$

When this is so, P_1 and P_2 are called the principal planes of the lens.

Now, relating the above statement to matrix formalism, the matrix transformation for a thick lens between the input and output faces of the lens has the general form:

$$\begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} x_0 \\ \theta_0 \end{bmatrix} \quad (1)$$

where as before the $\det R = 1$. For a general transformation, R_{12} is not necessarily equal to 0 and R_{11} and R_{22} are not necessarily equal to 1.

The principal planes may be located by the transformation

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & z_1 \\ 0 & 1 \end{bmatrix} \quad (2)$$

Using the relation

$$\begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \text{ (the unit matrix)}$$

the previous equation may be manipulated into the form

$$\begin{bmatrix} R_{11} - z_2 R_{21} & R_{12} - z_2 R_{22} \\ R_{21} & R_{22} \end{bmatrix} = \begin{bmatrix} 1 & -z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} 1 & -z_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \quad (3)$$

Solving for z_1 and z_2 , we find

$$z_1 = \frac{R_{22} - 1}{R_{21}}$$

$$z_2 = \frac{R_{11} - 1}{R_{21}}$$

where z_1 and z_2 are the location of the principal planes as shown in Fig. 3. The principal planes of any system may be determined by this method.

Note that $R_{21} = -1/f$ is not affected by the transformation and that the upper right hand matrix element is zero if $\det R = 1$. The principal planes may coincide, may be close together, be far apart; or in many systems, may be located external to all of the elements comprising the system. An example of the latter case is a quadrupole pair.

Some examples of principal plane locations for simple systems follow:

A quadrupole singlet:

Figure 4

The principal planes in a single quadrupole are located very close to each other and very near the center of the lens. As such, a quadrupole singlet may be considered as a thin lens if the object and image distances are measured to the center of the lens.

A simple uniform-field
wedge magnet:

Figure 5

If the optic axis enters and exits perpendicularly to the pole boundary, the principal planes are at the "center" of the magnet, as shown in Figure 5. From this, we conclude that a simple wedge bending magnet may be considered as a "thin" lens if the object and image distances are measured to the lens center O .

A quadrupole pair:

x plane

Figure 6

y plane

Figure 7

For a quadrupole pair, the principal planes are displaced toward and, usually, beyond the focusing element of the pair, as shown in Figures 6 and 7.

For any lens system, no matter how many elements are involved

Figure 8

$1/p + 1/q = 1/f$, if p and q are distances measured to the principal planes. Then the magnification between object and image planes is $M = q/p$.

Since the quadrupole pair is different in the two planes, (x) and (y), both situations must be examined. The interesting result turns out to be that in the x plane, the principal planes are to the right (Figure 6) and in the y plane, they are to the left (Figure 7). Therefore, in the y plane

the magnification is greater than 1, and in the x plane the magnification is less than 1. Typically, for a quadrupole pair the ratio of

$$M_y/M_x$$

may be as high as 20:1 and such cases can be disastrous if not recognized beforehand. This is a first-order image distortion. For example, if the source is a circular spot at A, the image at B will appear as a long thin line.

The situation is different for the

Quadrupole triplet:

In the symmetric triplet, as shown in Figures 9 and 10, the principal planes are located symmetrically about the center of the system, although $z_x > z_y$. This is, perhaps, the dominant reason why quadrupole triplets are used. The magnification is approximately equal in both planes; consequently, a circular spot can be imaged through the system with much less first-order image distortion than is the case for the doublet.

3. Introduction of Momentum Dispersion into the Matrix Formalism

The foregoing discussion and examples dealt only with monoenergetic first-order effects. First-order dispersion may be taken into account by introducing a 3×3 matrix as follows:

Consider two particles of momentum p_0 and $p_0 + \Delta p$ passing through the midplane of a static magnetic field, as illustrated in Figure 11.

Figure 11

Since the scalar momentum of a particle is constant in a static magnetic field, the transport equation from A to B may be expressed as:

$$\begin{bmatrix} x_1 \\ \theta_1 \\ \delta \end{bmatrix}_B = \begin{bmatrix} R_{11} & R_{12} & d \\ R_{21} & R_{22} & d' \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ \theta_0 \\ \delta \end{bmatrix}_A$$

where

$$\delta = \Delta p / p_0$$

d = the spatial momentum dispersion

d' = the derivative of the dispersion (the angular momentum dispersion)

and

1 = a carrying term to generate a square matrix and denote a constant momentum.

The determinant of the matrix $|R|$ is equal to 1 as for the 2×2 matrix. However, because of the zeros in the bottom row, the fact that $|R| = (R_{11} R_{22} - R_{12} R_{21}) = 1$ only checks the 2×2 matrix and not the terms containing d and d' .

Consider now a general system from an object point A to an image point B.

Figure 12

The above matrix equation is still valid for midplane trajectories. If A is a source point and if $R_{12} = 0$ (i.e., x is independent of θ_0), then B is an image point for monoenergetic particles. Under these circumstances:

$R_{11} = M_x$ is the x plane magnification

$R_{22} = + 1/M_x$ because $\det R = 1$

and $R_{21} = - 1/f_x$

In fact, $R_{21} = - 1/f_x$ for the system between A and B, even if A and B are not foci.

It is now convenient to develop a more general definition of the matrix elements R_{ij} and, at the same time, introduce the first-order matrix transformation for the y (non-bend) plane. Consider, again, a general system where the projection of the central trajectory is allowed to bend in the x plane but is a straight line in the y plane. The x plane and y plane matrix transformations may be written as follows:

For the x plane $x_1 = R_x x_0$

or

$$\begin{bmatrix} x_1 \\ \theta_1 \\ \delta \end{bmatrix} = \begin{bmatrix} c_x(t) & s_x(t) & d_x(t) \\ c'_x(t) & s'_x(t) & d'_x(t) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ \theta_0 \\ \delta \end{bmatrix}$$

Similarly, for the y plane, $y_1 = R_y y_0$

or

$$\begin{bmatrix} y_1 \\ \phi_1 \end{bmatrix} = \begin{bmatrix} c_y(t) & s_y(t) \\ c'_y(t) & s'_y(t) \end{bmatrix} \begin{bmatrix} y_0 \\ \phi_0 \end{bmatrix}$$

The c and s functions may be defined in terms of their initial conditions.

Let τ be the distance measured along the

Figure 13

central trajectory. Then:

$$s(0) = 0 \quad s'(0) = 1 \quad \text{where} \quad s'(\tau) = \frac{ds(\tau)}{d\tau}$$

$$c(0) = 1 \quad c'(0) = 0 \quad c'(\tau) = \frac{dc(\tau)}{d\tau} = -\frac{1}{f}$$

Within an "ideal" magnet, where the bending radius ρ_0 is constant, s and c are sine and cosine or else sinh and cosh functions. Because of

this, the terminology s = a sine-like function and c = a cosine-like function has been adopted for describing the general case where $\rho_o = \rho_o(\tau)$ is a function of τ .

By analogy with previous discussions, we observe that whenever $s(\tau) = 0$, we are at an image of point A. Also, $c(\tau)$ at the position where $s(\tau) = 0$ is the magnification of point A at that image.

$c'(t) = -1/f$ where f is the focal length of the system between A and B. The dispersion d_x may be derived from the general differential equations of motion of a charged particle in a static magnetic field.(1) The results may always be expressed as a function of s_x and c_x as follows:

$$d_x(t) = s_x(t) \int_0^t c_x(\tau) d\alpha - c_x(t) \int_0^t s_x(\tau) d\alpha$$

and

$$d'_x(t) = s'_x(t) \int_0^t c_x(\tau) d\alpha - c'_x(t) \int_0^t s_x(\tau) d\alpha$$

where

$$d\alpha = \frac{d\tau}{\rho_o(\tau)}$$

is the differential angle of bend of the central trajectory. At an image point $[s(t) = 0]$ note that

$$d_x(t) = -c(t) \int_0^t s(\tau) d\alpha$$

This approach to the problem may be generalized to include all of the second-order aberrations of a system. When this is done, it is always

(1) See SLAC-75 for a derivation of these equations.

possible to express these aberrations as functions of the first-order matrix elements: c_x , s_x , d_x , c_y and s_y .

Having developed the above physical concepts and mathematical tools, we are now in a position to study more complicated systems. As an example, we consider the general system shown in Figure 14.

Figure 14

L = drift elements

= magnetic elements

The matrix formalism states that in the x plane, the transformation from A to B is given by the following matrix equation.

$$\begin{bmatrix} x_1 \\ \theta_1 \\ \delta \end{bmatrix} = R_{L_4} R_{Q_1} R_{L_3} R_{M_2} R_{L_2} R_{M_1} R_{L_1} \begin{bmatrix} x_0 \\ \theta_0 \\ \delta \end{bmatrix}$$

As in all matrix calculations, the order of writing down the elements comprising the system is from right to left. The individual matrix elements must be derived from the solution of the equation of motion within each element. If this has been done, then the calculation for the total system is carried out in the fashion shown by the above equation.

4. Second-Order Matrix Formalism (1)

It is possible to extend the 3×3 matrix formalism to solve simultaneous sets of power series by generating a second-order matrix equation

as follows:

$$\begin{bmatrix} x_1 \\ \theta_1 \\ \delta \\ x_1^2 \\ x_1 \theta_1 \\ x_1 \delta \\ \vdots \end{bmatrix} = \begin{bmatrix} \begin{array}{c} 3 \times 3 \\ \text{first-order terms} \\ "R_1" \end{array} & \begin{array}{c} \text{second-order} \\ \text{terms} \end{array} \\ \hline \begin{array}{c} \text{all zero} \end{array} & \begin{array}{c} "R_1^2" \end{array} \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ \theta_0 \\ \delta \\ x_0^2 \\ x_0 \theta_0 \\ x_0 \delta \\ \vdots \end{bmatrix} \left. \vphantom{\begin{bmatrix} x_0 \\ \theta_0 \\ \delta \\ x_0^2 \\ x_0 \theta_0 \\ x_0 \delta \\ \vdots \end{bmatrix}} \right\} \begin{array}{l} \text{all other} \\ \text{order} \\ \text{terms} \end{array}$$

The " R_1^2 " term is obtained by squaring the upper left corner (3×3) matrix so as to obtain second-order equations for x_1^2 , $x_1 \theta_1$, $x_1 \delta$, etc., as functions of products of the initial first-order variables x_0 , θ_0 , and δ .

This is, then, a convenient mathematical formalism for keeping all the terms desired and dropping those undesired. In the above example, all first- and second-order terms are retained and all higher-order terms are automatically dropped by the matrix multiplication.

5. Transformations Involving Many Trajectories

All of the discussion to this point relates to the transformation of a single trajectory (in addition to the central trajectory) through a static magnetic system. We wish now to extend the discussion to include "bundles" of rays. To accomplish this, we take advantage of Liouville's Theorem, which states that the "phase space" is conserved through the system. While the Theorem is strictly true to all orders, a convenient mathematical transformation

has only been developed to first-order. A manifestation of Liouville's Theorem is the fact that $|R| = 1$.

Now, so long as there is no coupling mechanism between the x plane and y plane of a magnetic-optical system (which is the case if the midplane symmetry prevails throughout the system) then, the phase space area in a given plane is also conserved. Consider a bundle of rays represented by the parallelogram, shown in Figure 15(a), representing the phase space distribution of the rays at some initial position. If we now look at the phase space distribution of the same bundle after it has drifted down stream, we observe the $\theta_{\max}$ boundary and the x intercept x_i remain unchanged. In other words, the area of the parallelogram is the same or "phase space area has been conserved."

Figure 15

For mathematical convenience, the parallelogram is rather difficult to work with and, hence, a phase ellipse is usually used.

Figure 16

The phase ellipse transformation for a drift distance is illustrated in Figure 16. Figure 16(a) corresponds to a beam which is at its minimum width (a "waist") and Figure 16(b) shows the same beam after it has drifted downstream from the waist position. The physical meaning of this is that particles entering at $\theta = 0$ are parallel to the optic axis and, therefore, cannot change their relative positions with respect to the optic axis; that is, all particles on the x_0 axis act in this manner. Those that enter at a given angle continue at the same angle.

The phase ellipse transformation for a thin lens is illustrated in Figure 17. In passing through a thin lens, θ changes and the x dimension remains constant for a given trajectory.

Figure 17

Stated in other terms, $x_{\max}$ remains constant, as does θ_i (the θ intercept). It is apparent, in the given example, that the spot size now becomes, or can become, smaller at the new image because $\theta_{\max}$ is larger. This can be related to the physics of the system by saying that the x magnification is less than unity. This fact is observed directly by comparison of the x intercept of the ellipse before and after the lens action. It is interesting to observe that a particle initially at c is transformed to c' and that particles entering at $x = 0$ do not change their direction (θ_i is constant). If the particles are now allowed to drift, the ellipse rotates clockwise; when the ellipse is vertical, the spot size is at a minimum, namely, $x_{\max} = x_1$, as was illustrated in Figure 16.

1. Introduction

In Part I, the basic concepts of beam transport optics were established. Starting from the essentials of geometric optics, the methods of matrix algebra were introduced with the example of calculating the principal planes of a thick lens. The 3×3 matrix for the first-order beam transport calculations were introduced to take into account the particle momenta.

2. First Order Transformation Matrix

Figure 1 shows a general region containing a magnetic field.

Figure 1: General Magnetic Field Configuration

The matrix presents a convenient way of writing the family of equations which describe the transformation from surface A to surface B. If x_0 , θ_0 and δ represent the conditions of a ray entering the system at A, then the conditions of the ray at B are x_1 , θ_1 and δ . Here x_0 is the distance from the central ray to the ray C, θ_0 is the angle between C and the parallel to the central ray and δ is the ratio $\Delta p/p$ where Δp is the difference between the momentum of C and the momentum of the central ray.

The linear transformation equations are:

$$\begin{aligned}x_1 &= c_x x_0 + s_x \theta_0 + d_x \delta \\ \theta_1 &= c'_x x_0 + s'_x \theta_0 + d'_x \delta \\ \delta &= 0 + 0 + \delta\end{aligned}\tag{1}$$

Expressed as a matrix, Eq. (1) are:

$$\begin{bmatrix} x_1 \\ \theta_1 \\ \delta \end{bmatrix} = \begin{bmatrix} c_x & s_x & d_x \\ c'_x & s'_x & d'_x \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ \theta_0 \\ \delta \end{bmatrix}\tag{2}$$

The equation $\delta = \delta$ expresses the fact that the magnetic field cannot change the scalar momentum of the particle. The δ terms in the x and θ equations express the momentum dispersion of the system.

If it happens that A is an object point and B is an image point of the system, then x_1 is independent of θ_0 , thus $s_x = 0$. In this case, c_x is given by $c_x = x_1/x_0 = M_x$ = the magnification in x plane, (for $\delta = 0$). if $\theta_0 = \delta = 0$, then $\theta_1 = c'_x x_0 = -x_0/f$ or $c'_x = -1/f$. It must always be true that the determinant of the matrix $|R|$, is unity. Thus for this special case of $s_x = 0$, it follows that $s'_x = 1/M_x$.

3. Beam Switchyard

As an example of a system which can be calculated with the matrix method, we next consider the beam switchyard of the two-mile accelerator. Figure 2 shows the three essential elements, two bending magnets and a quadrupole lens. In common with many beam transport systems, this one is designed to be achromatic. Mathematically, this means that the matrix elements, d_x and d'_x should be zero, so that there is no x or θ dependence on the momentum of the particles.

Figure 2: The Essentials of the Beam Switchyard

As a preliminary step, we will find the matrix expression for a bending magnet when measured from the principal planes. The matrix for a bending magnet, when measured from the ends of the poles, is given by:

$$R_{\text{bend}} = \begin{bmatrix} c & s & l-c \\ -s & c & s \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

where $c = \cos \alpha$ and $s = \sin \alpha$ and α is the deflection angle of the central ray. This expression has been normalized by setting the bending radius equal to unity. To restore ordinary units it is only necessary to insert the bending radius wherever a length is needed dimensionally. In this case, the matrix then becomes

$$R_{\text{bend}} = \begin{bmatrix} c & \rho s & \rho(1-c) \\ -s/\rho & c & s \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

If the distance from the entrance plane to the first principal plane is z_1 and the distance from the second principal plane to the exit plane is z_2 , we can find the values z_1 and z_2 by solving the following matrix

equation:

$$\begin{bmatrix} 1 & -z_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c & s & 1-c \\ -s & c & s \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -z_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_x \\ -1/f & 1 & d'_x \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

The matrix multiplication need only be done for the 2×2 matrices as outlined. To illustrate matrix multiplication the indicated operations will be given below in natural stages as follows:

$$\begin{bmatrix} 1 & -z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c & -cz_1 + s \\ -s & +sz_1 + c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

$$\begin{bmatrix} c + z_2 s & -cz_1 + s \\ -s & +sz_1 + c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \quad (6)$$

Note that these transformations do not change the focal length expression, $-1/f = -s$. In order for two matrices to be equal, each individual element must be equal to its counterpart in the other matrix. Thus we have

$$\begin{aligned} c + z_2 s &= 1 \\ c + z_1 s &= 1 \end{aligned} \quad (7)$$

which when solved for z_1 and z_2 yield

$$z_2 = (1-c)/s \quad \text{and} \quad z_1 = (1-c)/s$$

If we substitute the trigonometric equivalents, and apply standard identities, we have $z_2 = z_1 = \tan(\alpha/2)$ which can be seen from Figure 3 to indicate that the two principal planes are coincident with the symmetry plane in the middle of the magnet.

Figure 3; The Principal Planes of a Simple Bending Magnet are Coincident with the Center Plane.

The simplified matrix for a bending magnet measured to the principal planes is then:

$$R_{\text{bend}} = \begin{bmatrix} 1 & 0 & 0 \\ -s & 1 & s \\ 0 & 0 & 1 \end{bmatrix} \quad (8)$$

To calculate the transformation matrix for the entire Beam Switchyard system as shown in Fig. 2, we write the matrices in opposite order from that in which the beam passes through the elements. That this must be true can be seen from the way in which one element alone is calculated by

$$\begin{bmatrix} x_1 \\ \theta_1 \\ \delta \end{bmatrix} = R_1 \begin{bmatrix} x_0 \\ \theta_0 \\ \delta \end{bmatrix} \quad (9)$$

Then for a second element we have

$$\begin{bmatrix} x_2 \\ \theta_2 \\ \delta \end{bmatrix} = R_2 \begin{bmatrix} x_1 \\ \theta_1 \\ \delta \end{bmatrix} = R_2 R_1 \begin{bmatrix} x_0 \\ \theta_0 \\ \delta \end{bmatrix} \quad (10)$$

and so forth.

If we allow the system to be symmetrical, i.e. $s_1 = s_2$ and $L_1 = L_2$; the complete series of matrices for Fig. 2 are

$$R_{BSY} = \begin{bmatrix} 1 & 0 & 0 \\ -s & 1 & s \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1/f & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -s & 1 & s \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

We will show the step-by-step multiplication of the matrices to get the d_x and d'_x terms.

$$R_{BSY} = \begin{bmatrix} 1 & L & 0 \\ -s & 1-sL & s \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1/f & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1-sL & L & Ls \\ -s & 1 & s \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{BSY} = \begin{bmatrix} (1-\frac{L}{f}) & L(2-\frac{L}{f}) & Ls(2-\frac{L}{f}) \\ -Ls(2-\frac{L}{f}) & (1-\frac{L}{f}) & s(1-Ls)(2-\frac{L}{f}) \\ 0 & 0 & 1 \end{bmatrix} \quad (12)$$

To obtain the required condition that $d_x = d'_x = 0$, we set

$$\left(2 - \frac{L}{f}\right) = 0 \quad \text{or} \quad f = \frac{L}{2}$$

then

$$R_{BSY} = \begin{bmatrix} -1 & 0 & 0 \\ -\frac{2}{L}(1-Ls) & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (13)$$

Thus the quadrupole acts as a lens to refocus rays from the center of the first bending magnet to the center of the last one. For the serious student, it is a worthwhile exercise to do the BSY problem without the simplification which resulted from introducing the principal planes.

4. TRANSPORT

As an aid to solving beam transport problems, a computer program TRANSPORT has been developed at SLAC which takes the greatest amount of labor out of this work. The program operates in about the way as the BSY example above was calculated, but with some important exceptions. Most importantly:

1. TRANSPORT has the ability to find the best first-order solution given a certain set of constraints;
2. TRANSPORT also calculates the transformation of a whole family of rays as found in a beam by means of the concept of "phase space" which was introduced in Part I;
3. TRANSPORT can, as an option, calculate the second order effects on the beam. By second order is meant, for example, terms which depend not linearly on the displacement x_0 , but on x_0^2 or $x_0 \theta_0$, etc.

To aid in the discussion of TRANSPORT and of the second order terms we now introduce an abbreviated notation. By writing out the complete equations for x and y , to second order, we will adequately have displayed the new notation.

$$\begin{aligned}
 x_1 = & (x|x_0)x_0 + (x|\theta_0)\theta_0 + (x|\delta)\delta \\
 & + (x|x_0^2)x_0^2 + (x|x_0\theta_0)x_0\theta_0 + (x|x_0\delta)x_0\delta \\
 & + (x|\theta_0^2)\theta_0^2 + (x|\theta_0\delta)\theta_0\delta + (x|\delta^2)\delta^2 \\
 & + (x|y_0^2)y_0^2 + (x|y_0\phi_0)y_0\phi_0 + (x|\phi_0^2)\phi_0^2
 \end{aligned} \tag{14}$$

$$\begin{aligned}
 y_1 = & (y|y_0) + (y|\phi_0)\phi_0 \\
 & + (y|x_0y_0)x_0y_0 + (y|x_0\phi_0)x_0\phi_0 + (y|\theta_0y_0)\theta_0y_0 \\
 & + (y|\theta_0\phi_0)\theta_0\phi_0 + (y|y_0\delta)y_0\delta + (y|\delta\phi_0)\delta\phi_0
 \end{aligned} \tag{15}$$

The absence of certain terms which might otherwise be expected in Eqs. (14) and (15) is due to the fact that horizontal mid-plane symmetry has been assumed in the derivation. That is, the field on the horizontal mid-plane is normal to the plane. Thus there can be no $(y|x)$ or $(y|\theta)$ term. Similarly, there can only be even powers of y and ϕ , such as $(x|y_0^2)$ and $(x|y_0\phi_0)$, in the x equation. Also, note that there is no $(y|\delta)$ or $(y|\delta^2)$ term if there is mid-plane symmetry.

TRANSPORT uses a numerical notation to signify the six basic coordinates:

$$\begin{array}{cccccc} x & \theta & y & \phi & \ell & \delta \\ 1 & 2 & 3 & 4 & 5 & 6 \end{array} \quad (16)$$

The ℓ term has not been introduced here before. Its significance is the preservation of the bunch length of a beam such as the SIAC electron beam.

The first order output from TRANSPORT is a 6×6 matrix printout of the R matrix where the labels are implied by row and column position of the elements. For example the element appearing at the intersection of row 3 and column 4 is the coefficient $(y|\phi_0)$ etc.

	x_0	θ_0	y_0	ϕ_0	ℓ_0	δ_0
x	-	-	-	-	-	-
θ	-	-	-	-	-	-
y	-	-	-	-	-	-
ϕ	-	-	-	-	-	-
ℓ	-	-	-	-	-	-
δ	-	-	-	-	-	-

The second order terms are labelled by the convention indicated in Eqs. (14), (15) and (16). For example, $(x|x_0^2)$ becomes 1 11 and $(x|\theta_0\delta)$ becomes 1 26.

5. Second Order Matrix

Normally the matrix method is expected only to apply to the solution

of linear i.e., first order, equations. However, the method has been extended to include second order terms in a 12×12 matrix as discussed in Part I and as shown above.

For a more extensive discussion of the second-order matrix formalism, the reader is referred to SLAC Report Number 75 by K.L. Brown.

x_1	$(x x_0)$	$(x \theta_0)$	$(x \delta)$	$(x x_0^2)$	$(x x_0\theta_0)$	etc.	-	-	-	-	-	-	x_0
θ_1	(θx_0)	$(\theta \theta_0)$	$(\theta \delta)$	(θx_0^2)	etc.	-	-	-	-	-	-	-	θ_0
δ	0	0	1	0	0	0	0	0	0	0	0	0	δ
x_1^2	All terms are zero			$(x x_0)^2$	$2(x x_0)(x \theta_0)$	-	-	-	-	-	-	-	x_0^2
$x_1\theta_1$													$x_0\theta_0$
$x_1\delta$													$x_0\delta$
θ_1^2													θ_0^2
$\theta_1\delta$													$\theta_0\delta$
δ^2													δ^2
y_1^2													y_0^2
$y_1\phi_1$													$y_0\phi_0$
ϕ_1^2													ϕ_0^2

For example, the x_1^2 terms come from squaring Eq. (14).

FIRST-ORDER R MATRIX FORMALISM FOR TRANSPORT

Section II

Beam transport optics may be reduced to a process of matrix multiplication(1,2). To first-order, this is represented by the matrix equation (using the notation of SLAC-75).

$$x_i(t) = \sum_{j=1}^6 R_{ij} x_j(0) \quad (1)$$

where

$$x_1=x \quad x_2=\theta \quad x_3=y \quad x_4=\varphi \quad x_5=l \quad \text{and} \quad x_6=\delta$$

The determinant $|R| = 1$. This is a direct consequence of the basic equation of motion for a charged particle in a static magnetic field and is a manifestation of Liouville's theorem of conservation of phase space volume. (See SLAC-75, page 41 for a proof that $|R| = 1$.)

For static magnetic systems possessing midplane symmetry, the six simultaneous linear equations represented by Eq. (1) may be expanded in matrix form as follows:

$$\begin{bmatrix} x(t) \\ \theta(t) \\ y(t) \\ \varphi(t) \\ l(t) \\ \delta(t) \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_o \\ \theta_o \\ y_o \\ \varphi_o \\ l_o \\ \delta_o \end{bmatrix} \quad (2)$$

where the transformation is from an initial position $\tau = 0$ to a final position $\tau = t$ measured along the assumed central reference trajectory.

Thus at any specified position in a system, an arbitrary charged particle is represented by a vector (single column matrix), X , whose components are the positions, angles, and momentum of the particle with respect to a specified reference trajectory.

i.e. $X =$

$$\begin{bmatrix} x \\ \theta \\ y \\ \varphi \\ l \\ \delta \end{bmatrix}$$

where:

x = the radial displacement of the arbitrary ray with respect to the assumed central trajectory.

θ = the angle this ray makes in the radial plane with respect to the assumed central trajectory.

y = the transverse displacement of the ray with respect to the assumed central trajectory.

φ = the transverse angle of the ray with respect to the assumed central trajectory.

l = the path length difference between the arbitrary ray and the central trajectory.

$\delta = \Delta P/P$ is the fractional momentum deviation of the ray from the assumed central trajectory.

The magnetic lens is represented by the square matrix, R , which describes the action of the magnet on the particle coordinates. Thus the passage of a charged particle through the system may be represented by the matrix equation:

$$X(1) = R X(0) \quad (3)$$

where $X(0)$ is the initial coordinate vector and $X(1)$ is the final coordinate vector of the particle under consideration; R is the transformation matrix for all such particles traversing the system (one particle differing from another only by its initial coordinate vector $X(0)$).

The traversing of several magnets and interspersing drift spaces is described by the same basic equation but with R now being the product matrix $R = R(n) \dots R(3)R(2)R(1)$ of the individual matrices of the system elements. TRANSPORT calculates and tabulates the product matrix R representing the system.

The zero elements $R_{13} = R_{14} = R_{23} = R_{24} = R_{31} = R_{32} = R_{41} = R_{42} = R_{36} = R_{46} = 0$ in the R matrix are a direct consequence of midplane symmetry. If midplane symmetry is destroyed, these elements will in general become non-zero. The zero elements in column five occur because the variables x , θ , y , φ , and δ are independent of the path length difference ℓ . The zero's in row six result from the fact that we have restricted the problem to static magnetic fields, i.e., the scalar momentum is a constant of the motion.

In SLAC report 75 (Ref. 1), a physical significance has been attached to the non-zero matrix elements in the first four rows in terms of their identification with characteristic first-order trajectories. We include figures showing these characteristic functions as a convenient reference.

We now wish to relate the elements appearing in column six and those in row five in terms of simple integrals of the characteristic first-order matrix elements: $c_x(t) = R_{11}$ and $s_x(t) = R_{12}$. In order to do this, we make use of the Green's function integral, Eq. (43), Section II of SLAC-75, and of the expression for the differential path length in curvilinear coordinates

17-2-A

FIG. 1--CURVILINEAR COORDINATE SYSTEM USED IN DERIVATION OF EQUATIONS OF MOTION.

748A10

FIG. 2--ILLUSTRATION OF THE MAGNETIC MIDPLANE (X AXIS) FOR DIPOLE, QUADRUPOLE AND SEXTUPOLE ELEMENTS. THE MAGNETIC POLARITIES INDICATE MULTIPOLE ELEMENTS THAT ARE POSITIVE IN RESPECT TO EACH OTHER.

$$C_x(0) = 1$$

$$C'_x(0) = \left. \frac{dC_x}{dt} \right|_{t=0} = 0$$

FIG. 3--COSINE-LIKE FUNCTION $c_x(t) = R_{11}$ IN MAGNETIC MIDPLANE. $c'_x(t) = R_{21}$.

748A7

FIG. 4--SINE-LIKE FUNCTION $s_x(t) = R_{12}$ IN MAGNETIC MIDPLANE. $s'_x(t) = R_{22}$.

$$d_x(0) = 0$$

$$d'_x(0) = \left. \frac{d(d_x)}{dt} \right|_{t=0} = 0$$

748A9

FIG. 5--DISPERSION FUNCTION $d_x(t) = R_{16}$ IN MAGNETIC MIDPLANE. $d'_x(t) = R_{26}$.

$$C_y(0) = 1$$

$$C'_y(0) = \left. \frac{dC_y}{dt} \right|_{t=0} = 0$$

748A6

FIG. 6--COSINE-LIKE FUNCTION $c_y(t) = R_{33}$ IN THE NON-BEND (y) PLANE. $c'_y(t) = R_{43}$.

FIG. 7--SINE-LIKE FUNCTION $s_y(t) = R_{34}$ IN NON-BEND (y) PLANE. $s'_y(t) = R_{44}$.

$$dT = \left[(dx)^2 + (dy)^2 + (1+hx)^2 (dt)^2 \right]^{1/2} \quad (4)$$

used in the derivation of the equation of motion.

First-Order Dispersion

The spatial dispersion $d_x(t) = R_{16}$ of a system at position t is derived using the Green's function integral, and the driving term $f(\tau) = h(\tau) = \frac{1}{\rho_0(t)}$ for the dispersion (see Table I of SLAC-75). The result is

$$d_x(t) = R_{16} = s_x(t) \int_0^t c_x(\tau) h(\tau) d\tau - c_x(t) \int_0^t s_x(\tau) h(\tau) d\tau \quad (5)$$

where τ is the variable of integration. Note that $h(\tau)d\tau = d\alpha$ is the differential angle of bend of the central trajectory at any point in the system. Thus first-order dispersion is generated only in regions where the central trajectory is deflected (i.e., in dipole elements.) The angular dispersion is obtained by direct differentiation of $d_x(t)$ with respect to t , the curvilinear distance along the central trajectory.

$$d'_x(t) = R_{26} = s'_x(t) \int_0^t c_x(\tau) h(\tau) d\tau - c'_x(t) \int_0^t s_x(\tau) h(\tau) d\tau \quad (6)$$

where

$$c'_x(t) = R_{21} \text{ and } s'_x(t) = R_{22}$$

First-Order Path Length

The first-order path length difference is obtained by expanding and integrating Eq. (4) and retaining only the first-order term, i.e.,

$$l - l_0 = (T - t) = \int_0^t x(\tau) h(\tau) d\tau + \text{higher order terms}$$

from which

$$\begin{aligned}
 l &= x_0 \int_0^t c_x(\tau) h(\tau) d\tau + \theta_0 \int_0^t s_x(\tau) h(\tau) d\tau + l_0 + \delta \int_0^t d_x(\tau) h(\tau) d\tau \\
 &= R_{51} x_0 + R_{52} \theta_0 + l_0 + R_{56} \delta
 \end{aligned} \tag{7}$$

Inspection of Eqs. (5), (6), and (7) yields the following useful theorems:

Achromaticity: A system is defined as being achromatic if $R_{16} = R_{26} = 0$, i.e., if $d'_x(t) = d''_x(t) = 0$. Therefore it follows from Eq's. (5) and (6) that the necessary and sufficient conditions for achromaticity are that

$$\int_0^t s_x(\tau) h(\tau) d\tau = \int_0^t c_x(\tau) h(\tau) d\tau = 0 \tag{8}$$

By comparing Eq. (7) with Eq. (8), we note that if a system is achromatic, all particles of the same momentum will have equal (first-order) path lengths through the system.

Isochronicity: It is somewhat unfortunate that this word has been used in the literature to mean equal path lengths since equal path lengths only imply equal transit times for highly relativistic particles. Nevertheless, from Eq. 7, the necessary and sufficient conditions that the first-order path length of all particles (independent of their initial momenta) will be the same through a system are that $R_{51} = R_{52} = R_{56} = 0$, i.e., if

$$\int_0^t c_x(\tau) h(\tau) d\tau = \int_0^t s_x(\tau) h(\tau) d\tau = \int_0^t d_x(\tau) h(\tau) d\tau = 0 \tag{9}$$

First-Order Imaging

First-order point-to-point imaging in the x plane occurs when $x(t)$ is independent of the initial angle θ_0 . This can only be so when

$$s_x(t) = R_{12} = 0. \tag{10}$$

Similarly first-order point-to-point imaging occurs in the y plane when

$$s_y(t) = R_{34} = 0. \quad (11)$$

First-order parallel-to-point imaging occurs in the x plane when $x(t)$ is independent of the initial particle position x_0 . This will occur only if

$$c_x(t) = R_{11} = 0. \quad (12)$$

and correspondingly in the y plane, parallel-to-point imaging occurs when

$$c_y(t) = R_{33} = 0. \quad (13)$$

A parallel ray entering a system exits parallel to the central trajectory if

$$c'_x(t) = R_{21} = 0. \quad (14)$$

in the x plane; and if

$$c'_y(t) = R_{43} = 0. \quad (15)$$

in the y plane.

Point-to-parallel imaging occurs in the x plane if

$$s'_x(t) = R_{22} = 0. \quad (16)$$

and in the y plane if

$$s'_y(t) = R_{44} = 0. \quad (17)$$

Focal Lengths

A simple ray diagram of a "thick" lens demonstrates that R_{21} and R_{43} have the following physical interpretations

$$c'_x(t) = R_{21} = -\frac{1}{f_x} \quad \text{and} \quad c'_y(t) = R_{43} = -\frac{1}{f_y} \quad (18)$$

where f_x and f_y are the system focal lengths in the x and y planes respectively.

Zero Dispersion

For point-to-point imaging, using Eq's. (5) and (10), the necessary and sufficient condition for zero dispersion at an image is

$$d_x(t) = R_{16} = \int_0^t s_x(\tau) h(\tau) d\tau = 0 \quad (19)$$

For parallel-to-point imaging, (i.e., $c_x(t) = 0$), the condition for zero dispersion at the image is

$$d_x(t) = R_{16} = \int_0^t c_x(\tau) h(\tau) d\tau = 0. \quad (20)$$

Magnification

For monoenergetic point-to-point imaging in the x-plane, the magnification is given by

$$M_x = \frac{x(t)}{x_0} = R_{11} = c_x(t)$$

and in the y plane by

$$M_y = R_{33} = c_y(t) \quad (21)$$

where a negative number means an inverted image.

First-Order Momentum Resolution

For point-to-point imaging the first-order momentum resolving power R_1 (not to be confused with the matrix R) is the ratio of the momentum dispersion to the total image size. Thus if $2x_0$ is the total source size then

$$R_1 = \frac{P}{\Delta P} = \left| \frac{R_{16}}{2x_0 R_{11}} \right| = \left| \frac{d_x(t)}{2x_0 c_x(t)} \right|$$

For point-to-point imaging $s_x(t) = 0$. Using Eq. (5), the dispersion at an image is

$$d_x(t) = -c_x(t) \int_0^t s_x(\tau) h(\tau) d\tau \quad (22)$$

from which the first-order momentum resolving power R_1 becomes

$$2x_o R_1 = \left| \frac{d_x(t)}{c_x(t)} \right| = \left| \int_0^t s_x(\tau) h(\tau) d\tau \right| = |R_{52}| \quad (23)$$

Equation (23) for the first-order resolving power of a system may be expressed in a number of useful forms. If we consider a ray (particle) originating at the source with $x_o = 0$ and $\delta = \frac{\Delta P}{P} = 0$ and lying in the midplane. (i.e., a mono-energetic point source), the first-order equation representing the midplane displacement, x , of this trajectory is

$$x(t) = s_x(t) \theta_o \quad (24)$$

We may then rewrite Equation (23) as follows:

$$2x_o R_1 = \int_0^t s_x(\tau) h(\tau) d\tau = \frac{1}{\theta_o} \int_0^t x(\tau) h(\tau) d\tau = \frac{(\ell - \ell_o)}{\theta_o} = |R_{52}| \quad (25)$$

or we may also write it in the form

$$R_1 = \frac{1}{2x_o \theta_o} \int_0^t B \frac{x(t) d\tau}{B \rho_o} = \left(\frac{1}{2x_o \theta_o} \right) \left(\frac{1}{B \rho_o} \right) \int_0^t B dA \quad (26)$$

where $\int B dA$ is the magnetic flux inclosed between the central trajectory and the ray represented by Eq. (24), and $B\rho$ is the magnetic rigidity (momentum) of the central trajectory. Please note, however, that if the ray crosses the central trajectory or the sign of B changes, this changes the sign of the integration.

Some important observations may be made from Eq's. (25) and (26).

1) Resolving particles of different momentum requires that a path length difference must exist between the central trajectory and the trajectory defined by Eq. (22). The greater the path length difference, the greater the resolving power.

2) From Eq. (24), we may define resolving power as the magnetic flux inclosed per unit phase space area per unit momentum ($B\rho$) of the central ray.

First-Order σ Matrix (Phase Ellipse) Formalism for TRANSPORT

In accelerators and beam transport systems, the behavior of an individual particle is often of less concern than is the behavior of a bundle of particles (the BEAM) of which an individual particle is a member. An extension of the matrix algebra of Eq. (3) provides a convenient means for defining and manipulating this BEAM. TRANSPORT assumes that the bundle of rays constituting a BEAM may correctly be represented in coordinate phase-space by an ellipsoid whose coordinates are the position, angle, and momentum coordinates of the arbitrary rays in the beam about an assumed central trajectory. TRANSPORT is a matrix calculation that truncates the problem to either first- or second-order in a Taylor's expansion about the central trajectory. Particles in a BEAM are assumed to lie within the boundaries of the ellipsoid with each point within the ellipsoid representing a possible ray. The sum total of all phase points, the phase space volume, is commonly referred to as the "phase space" occupied by the BEAM. The validity and interpretation of this phase ellipse formalism must be ascertained for each system being designed. However, in general, for charged particle beams in, or emanating, from accelerators, the first-order phase ellipse formalism of TRANSPORT is a reasonable representation of physical reality; but for other applications, such as charged particle spectrometers, caution is in order in its use and interpretation.

The equation of an n-dimensional ellipsoid may be written in matrix form as follows:

$$X(0)^T \sigma(0)^{-1} X(0) = 1 \quad (27)$$

where $X(0)^T$ is the transpose of the coordinate vector $X(0)$, and $\sigma(0)$ is a real, positive definite, symmetric matrix.

The volume of the n-dimensional ellipsoid defined by sigma is

$$\frac{\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2} + 1)} (\det \sigma)^{1/2}, \text{ the area of the projection in one plane is } A = \pi (\det \sigma)^{1/2}.$$

This is the "phase space" occupied by the beam.

As a particle passes through a system of magnets, it undergoes the matrix transformation of Eq. (3). Combining this transformation with the equation of the initial ellipsoid, and using the identity $RR^{-1} = I$ (the unity matrix), it follows that:

$$X(0)^T (R^T R^{-1}) \sigma(0)^{-1} (R^{-1} R) X(0) = 1$$

from which:

$$(RX(0))^T (R\sigma(0) R^T)^{-1} (RX(0)) = 1 \quad (28)$$

The equation of the ellipsoid representing the "BEAM" at the end of the system is thus:

$$X(1)^T \sigma(1)^{-1} X(1) = 1 \quad (29)$$

where the equation for the sigma matrix at the end may be related to that at the beginning by:

$$\sigma(1) = R \sigma(0) R^T \quad (30)$$

In addition to calculating the product matrix R, TRANSPORT also computes the sigma "BEAM" matrix at the end of each physical element via Eq. (30).

All of the important physical parameters of the BEAM ellipsoid may be expressed as functions of the matrix elements of the sigma matrix at the location

in question. In particular the square roots of the diagonal elements ($\sqrt{\sigma_{ii}}$'s) are the projection of the ellipse upon the coordinate axes and thus represent the maximum extent of the BEAM in the various coordinate directions. The correlation between components (the orientation of the ellipse) is determined by the off-diagonal terms (the σ_{ij} 's). An illustration of this is given below for a 2-dimensional ellipse.

Description of the Sigma BEAM Matrix

Consider a 2-dimensional (x, θ) plane projection of the general 6-dimensional ellipsoid. Let

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

be a real, positive definite, symmetric matrix; the inverse of which is

$$\sigma^{-1} = \frac{1}{\epsilon^2} \begin{bmatrix} \sigma_{22} & -\sigma_{21} \\ -\sigma_{21} & \sigma_{11} \end{bmatrix}$$

where ϵ^2 is the determinant of σ .

The 2-dimensional coordinate vector (column matrix) and its transpose are:

$$X = \begin{pmatrix} x \\ \theta \end{pmatrix} \text{ and } X^T = (x \ \theta)$$

The expansion of the matrix equation $X^T \sigma^{-1} X = 1$ is the equation of the ellipse

$$\sigma_{22} x^2 - 2\sigma_{21} x\theta + \sigma_{11} \theta^2 = \epsilon^2 = \det \sigma \quad (31)$$

The (x, θ) plane BEAM ellipse represented by Eq. (31) is shown in the following figure along with the physical meaning of the sigma matrix elements.

1358A1

The area of the ellipse is given by:

$$A = \pi(\det \sigma)^{1/2} = \pi x_{\max} \theta_{\text{int}} = \pi x_{\text{int}} \theta_{\max} \quad (32)$$

A Two Dimensional BEAM Phase Ellipse

The correlation between x and θ (the orientation of the ellipse) depends upon the off-diagonal term σ_{21} . This correlation is defined as

$$r_{21} = r_{12} = \frac{\sigma_{21}}{\sqrt{\sigma_{11} \sigma_{22}}}$$

So defined r always falls in the range

$$-1 \leq r \leq 1$$

The correlation, r , measures the tilt of the ellipse and the intersection of the ellipse with the coordinate axes.

Since the $\det R = 1$ for all static magnetic beam transport elements, it follows that the determinant of $\sigma(1)$ and $\sigma(0)$ are identical under the transformation of Eq. (30). Hence the "phase space" area is an invariant under the transformation of Eq. (30). This is a statement of Liouville's Theorem for the magnetostatic fields employed and results from the fact that the $\det R = 1$.

It is perhaps worthwhile noting that this 2-dimensional representation of the BEAM matrix has a one to one correspondence with the Courant-Snyder treatment of the theory of the Alternating Gradient Synchrotron* as follows:

$$\begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \epsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix} \quad (33)$$

The Phase Ellipse Beam Matrix used by TRANSPORT

For static magnetic systems possessing midplane symmetry, the (x, θ) plane and (y, φ) plane trajectories are decoupled in first-order, i.e. there is no mixing of phase space between the two planes. However for mathematical simplicity and to allow for the possibility of more general systems, the sigma BEAM matrix used in a TRANSPORT calculation has the following general 6-dimensional construction.

	x	θ	y	φ	ℓ	δ
x	$\sigma(11)$					
θ	$\sigma(21)$	$\sigma(22)$				
y	$\sigma(31)$	$\sigma(32)$	$\sigma(33)$			
φ	$\sigma(41)$	$\sigma(42)$	$\sigma(43)$	$\sigma(44)$		
ℓ	$\sigma(51)$	$\sigma(52)$	$\sigma(53)$	$\sigma(54)$	$\sigma(55)$	
δ	$\sigma(61)$	$\sigma(62)$	$\sigma(63)$	$\sigma(64)$	$\sigma(65)$	$\sigma(66)$

* E. D. Courant and H. S. Snyder, "Theory of the Alternating Gradient Synchrotron", Annals of Physics 3, pp 1-48 (1958).

The matrix is symmetric so that only a triangle of elements is needed.

In the printed output this matrix has a somewhat different format for ease of interpretation:

			x	θ	y	φ	ℓ
x	$\sqrt{\sigma(11)}$	CM					
θ	$\sqrt{\sigma(22)}$	MR	r(21)				
y	$\sqrt{\sigma(33)}$	CM	r(31)	r(32)			
φ	$\sqrt{\sigma(44)}$	MR	r(41)	r(42)	r(43)		
ℓ	$\sqrt{\sigma(55)}$	CM	r(51)	r(52)	r(53)	r(54)	
δ	$\sqrt{\sigma(66)}$	PC	r(61)	r(62)	r(63)	r(64)	r(65)

where:
$$r(ij) = \frac{\sigma(ij)}{[\sigma(ii) \sigma(jj)]^{1/2}} \quad (34)$$

As a result of the fact that the matrix is positive definite, the $r(ij)$'s satisfy the relation

$$|r(ij)| \leq 1 \quad (35)$$

The physical meaning of the $\sqrt{\sigma(ii)}$'s is as follows:

$\sqrt{\sigma(11)} = x_{\max}$ = The maximum (half)-width of the beam envelope in the x plane at the point of the printout.

$\sqrt{\sigma(22)} = \theta_{\max}$ = The maximum (half)-angular divergence of the beam envelope in the x plane.

$\sqrt{\sigma(33)} = y_{\max}$ = The maximum (half)-height of the beam envelope.

$\sqrt{\sigma(44)} = \varphi_{\max}$ = The maximum (half)-angular divergence of the beam envelope in the y plane.

$\sqrt{\sigma(55)} = \ell_{\max}$ = 1/2 the longitudinal extent of the bunch of particles.

$\sqrt{\sigma(66)} = \delta$ = The half-width 1/2 ($\Delta P/P$) of the momentum interval being transmitted by the system.

The units appearing next to the $\sqrt{\sigma(ii)}$'s in the TRANSPORT printout sheet are the units chosen for coordinates x , θ , y , ϕ , l and $\delta = \Delta P/P$ respectively.

To the immediate left of the listing of the beam envelope size in a TRANSPORT printout, there appears a column of numbers whose values will normally be zero. These numbers are the coordinates of the centroid of the beam phase ellipse (with respect to the initially assumed central trajectory of the system). They may become non-zero under one of three circumstances:

- 1) When the misalignment (Type Code 8.) is used.
- 2) When a Beam Centroid shift (Type Code 7.) is used.
- or 3) When a 2nd-order calculation (Type Code 17.) is used.

Physical Interpretation of Various Projections of the 2-dimensional BEAM Ellipse

Consider again Eq. (30) $\sigma(1) = R \sigma(0) R^T$ and expand it in it's most general form for the 2-dimensional (x, θ) plane case.

$$\sigma(1) = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} \sigma_{11}(0) & \sigma_{21}(0) \\ \sigma_{21}(0) & \sigma_{22}(0) \end{pmatrix} \begin{pmatrix} R_{11} & R_{21} \\ R_{12} & R_{22} \end{pmatrix} = \begin{pmatrix} \sigma_{11}(1) & \sigma_{21}(1) \\ \sigma_{21}(1) & \sigma_{22}(1) \end{pmatrix}$$

the result is:

$$\sigma(1) = \left[\begin{array}{c|c} R_{11}^2 \sigma_{11}(0) + 2R_{11}R_{12} \sigma_{21}(0) + R_{12}^2 \sigma_{22}(0) & R_{11}R_{21} \sigma_{11}(0) + (R_{11}R_{22} + R_{12}R_{21}) \sigma_{21}(0) + R_{12}R_{22} \sigma_{22}(0) \\ \hline R_{21}^2 \sigma_{11}(0) + 2R_{21}R_{22} \sigma_{21}(0) + R_{22}^2 \sigma_{22}(0) & \end{array} \right] \quad (36)$$

In the special case when the initial ellipse is erect i.e., $\sigma_{21}(0)=0$, $\sigma(1)$ reduces to:

$$\sigma(1) = \left[\begin{array}{c|c} R_{11}^2 \sigma_{11}(0) + R_{12}^2 \sigma_{22}(0) & R_{11}R_{21} \sigma_{11}(0) + R_{12}R_{22} \sigma_{22}(0) \\ \hline R_{21}^2 \sigma_{11}(0) + R_{22}^2 \sigma_{22}(0) & \end{array} \right] \quad (37)$$

Similar results are, of course, obtained for the (y, ϕ) plane.

If an arbitrary beam transport system is reduced to the most elementary first-order form of representing it as an initial drift distance, followed by a lens action between two principal planes, and a final drift distance; then we observe that for the 2-dimensional representation there are only two basic phase ellipse transformations of interest.

(1) An arbitrary DRIFT distance and

(2) A LENS action

Each of these elementary cases are illustrated on Fig. 8 for both a parallelogram as well as ellipse phase space transformations. Note that a DRIFT followed by a LENS action is not necessarily equal to a LENS action followed by a DRIFT; i.e., the matrices do not necessarily commute.

The phase ellipse transformations for a DRIFT and for a LENS action (between principal planes) as shown in Fig. (8) may be readily calculated using the results of Eq. (37).

The 2-dimensional R matrix representing a drift of distance L is:

$$R(\text{Drift}) = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \quad (38)$$

Substituting into Eq. (37) we find

$$\sigma(1) = \left[\begin{array}{c|c} \sigma_{11}(0) + L^2 \sigma_{22}(0) & L \sigma_{22}(0) \\ \hline L \sigma_{22}(0) & \sigma_{22}(0) \end{array} \right] = \left[\begin{array}{c|c} \sigma_{11}(1) & \sigma_{21}(1) \\ \hline \sigma_{21}(1) & \sigma_{22}(1) \end{array} \right] \quad (39)$$

Attaching the physical meaning to the matrix elements yields the following interpretations:

$$\sigma_{11}(1) = \sigma_{11}(0) + L^2 \sigma_{22}(0)$$

or

$$(\mathbf{x}_1^2)_{\max} = (\mathbf{x}_0^2)_{\max} + L^2 (\theta_0^2)_{\max} \quad (40)$$

1358B2

Fig. 8

similarly

$$\sigma_{22}(1) = \sigma_{22}(0)$$

or

$$(\theta_1^2)_{\max} = (\theta_0^2)_{\max} \quad (41)$$

Note that this transformation assumes that the initial phase ellipse is erect, i.e., $\sigma_{21}(0) = 0$.

The 2-dimensional R matrix for a lens actions (between principal planes)

is

$$R(\text{Lens}) = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \quad (42)$$

Substitution into Eq. (37) yields

$$\sigma(2) = \left[\begin{array}{c|c} \sigma_{11}(0) & -\frac{\sigma_{11}(0)}{f} \\ \hline -\frac{\sigma_{11}(0)}{f} & \frac{\sigma_{11}(0)}{f^2} + \sigma_{22}(0) \end{array} \right] = \left[\begin{array}{c|c} \sigma_{11}(2) & \sigma_{21}(2) \\ \hline \sigma_{21}(2) & \sigma_{22}(2) \end{array} \right] \quad (43)$$

Again attaching physical meaning to the matrix elements we have:

$$\sigma_{11}(2) = \sigma_{11}(0)$$

or

$$(x_2^2)_{\max} = (x_0^2)_{\max} \quad (44)$$

and

$$\sigma_{22}(2) = \frac{\sigma_{11}(0)}{f^2} + \sigma_{22}(0)$$

or

$$(\theta_2^2)_{\max} = \frac{1}{f^2} (x_0^2)_{\max} + (\theta_0^2)_{\max} \quad (45)$$

Note the change in sign of the σ_{21} elements for the Drift and the Lens actions indicating the different sense of orientation of the resulting ellipses as illustrated in Fig. 8.

The Upright Ellipse:

A case of particular interest in any 2-dimensional phase ellipse projection (e.g., the (x, θ) plane or the (y, ϕ) plane) is when the off-diagonal correlation matrix elements are equal to zero; i.e., an erect ellipse. In a field-free region this corresponds to a so-called "waist" in the BEAM as illustrated in Fig. 9.

It is important to understand correctly the meaning of a waist: For an existing beam, it is the location of the minimum beam size in a given region of the system. Although the waist is the minimum beam size in any given beam, the minimum beam size achievable at a fixed target position by varying the focal strength of the preceding lens system is not the same as the above defined waist. See Fig. 10. In a field-free region, the minimum beam spot size achievable at a fixed target position will occur when the preceding lens system is adjusted such that a waist precedes the target position. Only in the limit of zero phase space area do these quantities occur at the same location. A useful criterion that determines the physical proximity of these quantities is the following: If the system has been adjusted for the smallest spot size at a fixed position and if the size of the beam at the principal planes of the optical system is large compared to its size at the waist, or at the minimum spot size, then the location of these quantities, the waist and the minimum, will closely coincide; if, on the other hand, the size of the beam does not change substantially throughout the system, then the locations of a waist and the minimum beam size may (and usually will) differ substantially. The numerical proximity of these two quantities will be discussed in greater detail later in the report.

In a field free region (i.e., a Drift), the distance to a waist from any location may be readily calculated if the σ matrix at the location is known. Using Eq. (36) and the R matrix for a Drift (Eq. 38) we have for the (x, θ) plane:

$$\sigma_{21}(1) = \sigma_{21}(0) + L \sigma_{22}(0) = 0 \text{ (specifying that } \sigma(1) \text{ shall be at a waist)}$$

842A6

FIG. 9--BEAM WAIST.

$$\frac{1}{\sigma_{||}(\text{Waist})} = \frac{1}{\sigma_{||}(\text{Min})} + \frac{1}{\sigma_{||}(\text{lens})}$$

842A7

Fig. 10

or the distance to the waist is:

$$L = - \frac{\sigma_{21}(0)}{\sigma_{22}(0)} = - r_{21} \sqrt{\frac{\sigma_{11}}{\sigma_{22}}} \quad (46)$$

Similarly for the (y, φ) plane the distance to a waist is:

$$L = - \frac{\sigma_{43}(0)}{\sigma_{44}(0)} = - r_{43} \sqrt{\frac{\sigma_{33}}{\sigma_{44}}} \quad (47)$$

Relationship between a Waist and a Parallel-to-Point Image

A parallel-to-point image in the (x, θ) plane occurs when $R_{11} = 0$. The R matrix corresponding to this is

$$R = \left(\begin{array}{c|c} 0 & R_{12} \\ \hline R_{21} & R_{22} \end{array} \right) = \left(\begin{array}{c|c} 0 & s_x \\ \hline c_x & s_x \end{array} \right) = \left(\begin{array}{c|c} 0 & f_x \\ \hline -\frac{1}{f_x} & s_x \end{array} \right) \quad (48)$$

Since $|R| = 1$, $R_{12}R_{21} = -1$ for this situation.

If we assume an erect ellipse $\sigma(0)$ as the beginning of the system, the final beam matrix $\sigma(1)$ is given by substitution of Eq. (48) into Eq. (37) as follows:

$$\sigma(1) = \left[\begin{array}{c|c} R_{12}^2 \sigma_{22}(0) & R_{12} R_{22} \sigma_{22}(0) \\ \hline R_{12} R_{22} \sigma_{22}(0) & R_{21}^2 \sigma_{11}(0) + R_{22}^2 \sigma_{22}(0) \end{array} \right] \quad (49)$$

for parallel-to-point imaging.

Several conclusions may be extracted from this result: The first observation is that a waist and a parallel-to-point image will coincide if $R_{11} = R_{22} = 0$. This is equivalent to requiring that the object and image distances (measured to the principal planes) are both equal to the focal length f of the system.

The distance to a waist in this example is:

$$L = -\frac{\sigma_{21}(1)}{\sigma_{22}(1)} = -\frac{R_{22}R_{12}\sigma_{22}(0)}{R_{21}^2\sigma_{11}(0) + R_{22}^2\sigma_{22}(0)} = -\frac{s_x s'_x \sigma_{22}(0)}{(c'_x)^2\sigma_{11}(0) + (s'_x)^2\sigma_{22}(0)} \quad (50)$$

If $s_x s'_x = R_{12}R_{22} = 0$, a waist and a parallel-to-point image coincide. If $s_x s'_x = R_{12}R_{22} > 0$, the waist precedes the image; and if $s_x s'_x = R_{12}R_{22} < 0$, the waist follows the image; unless $\sigma_{22}(0) = 0$ (zero phase space area) in which case a waist and an image always coincide.

The size of the beam at the image is:

$$(x_1^2)_{\max} = \sigma_{11}(1) = R_{12}^2 \sigma_{22}(0) = f^2(\theta_0^2)_{\max} \quad (51)$$

independent of the source size x_0 and of the object distance.

The size of the beam at the waist is:

$$(\text{size at waist})^2 = \frac{|\sigma(0)|}{\sigma_{22}(1)} = \frac{\sigma_{11}(0)\sigma_{22}(0)}{R_{21}^2\sigma_{11}(0) + R_{22}^2\sigma_{22}(0)} \quad (52)$$

If $R_{22} = s'_x = 0$, the two sizes are equal as expected, otherwise the size at the waist is always smaller.

Relationship between a Waist and a Point-to-Point Image

A point-to-point first-order image in the (x, θ) plane occurs when $R_{12} = s_x = 0$. The R matrix representing this case is:

$$R = \left(\begin{array}{c|c} R_{11} & 0 \\ \hline R_{21} & R_{22} \end{array} \right) = \left(\begin{array}{c|c} c_x & 0 \\ \hline c'_x & s'_x \end{array} \right) = \left(\begin{array}{c|c} M & 0 \\ \hline -\frac{1}{f_x} & \frac{1}{M} \end{array} \right) \quad (53)$$

where $|R| = 1 = R_{11}R_{22}$, and M is the magnification.

If we again assume an erect ellipse $\sigma(0)$ as the beginning, the final beam matrix $\sigma(1)$ is given by Eq. (37) as:

$$\sigma(1) = \left[\begin{array}{c|c} R_{11}^2 \sigma_{11}(0) & R_{11} R_{21} \sigma_{11}(0) \\ \hline R_{11} R_{21} \sigma_{11}(0) & R_{21}^2 \sigma_{11}(0) + R_{22}^2 \sigma_{22}(0) \end{array} \right] \quad (54)$$

for point-to-point imaging.

Our first observation is that except for a zero source size, an image and a waist will coincide only if $R_{12} = R_{21} = 0$. Clearly this is not possible with a single lens; at least two lenses are needed. Such an optical situation is as follows:

1510A26

The distance to a waist is

$$L = - \frac{\sigma_{21}(1)}{\sigma_{22}(1)} = - \frac{R_{11} R_{21} \sigma_{11}(0)}{R_{21}^2 \sigma_{11}(0) + R_{22}^2 \sigma_{22}(0)} \quad (55)$$

So if $R_{11} R_{21} = c_x c'_x = 0$, a waist and a point-to-point image coincide.

If $c_x c'_x > 0$, the waist precedes the image and if $c_x c'_x < 0$, the waist follows the image.

The size of the beam at the image is

$$(\sigma_1^2)_{\max} = \sigma_{11}(1) = R_{11}^2 \sigma_{11}(0) = (Mx_0)^2 \quad (56)$$

and the size of the beam at the waist is:

$$(\text{size at waist})^2 = \frac{|\sigma(0)|}{\sigma_{22}(1)} = \frac{\sigma_{11}(0)\sigma_{22}(0)}{R_{21}^2 \sigma_{11}(0) + R_{22}^2 \sigma_{22}(0)} \quad (57)$$

Thus if $R_{21} = 0$, the two sizes are equal since $|R| = R_{11}R_{22} = 1$.

Otherwise the size at the waist is smaller than the image size.

Relationship between a Waist and the Smallest Spot Size Achievable at a Fixed Target Position

Consider the following general situation:

1510A27

Assume that the size of the beam $\sqrt{\sigma_{11}(\text{lens})}$ at the principal planes of the lens system being adjusted is held constant (i.e., that no other preceding optical elements of the system are being varied); and that the remainder of the system may be represented by a general matrix R which is also held constant. The focal length f is then varied until a minimum spot size $\sqrt{\sigma_{11}(\min)}$ is achieved at the target location. The sigma beam matrix at the target position then has the

following unique form independent of the orientation of the initial beam ellipse at the lens.

$$\sigma(\text{at target}) \text{ for a minimum spot size at target} = \left[\begin{array}{c|c} \frac{R_{12}^2 |\sigma|}{\sigma_{11}(\text{lens})} & \frac{R_{12} R_{22} |\sigma|}{\sigma_{11}(\text{lens})} \\ \hline \frac{\sigma_{11}(\text{lens})}{R_{12}^2} + \frac{R_{22}^2 |\sigma|}{\sigma_{11}(\text{lens})} \end{array} \right] = \left[\begin{array}{c|c} \sigma_{11}(\text{min}) & \sigma_{21}(\text{min}) \\ \hline \sigma_{22}(\text{min}) \end{array} \right] \quad (58)$$

thus

$$\sigma_{11}(\text{min}) = \frac{R_{12}^2 |\sigma|}{\sigma_{11}(\text{lens})}$$

or

$$x_{\text{min}} = \frac{R_{12} |\sigma|^{1/2}}{x(\text{lens})} \quad (59)$$

If the position of the waist and the minimum beam size both fall within the same field-free region, then the distance to the waist from the target is:

$$Z = - \frac{\sigma_{21}(\text{min})}{\sigma_{22}(\text{min})} = - \frac{R_{12} R_{22}}{R_{22}^2 + \frac{\sigma_{11}(\text{lens})}{\sigma_{11}(\text{min})}} = - R_{12} R_{22} \frac{\sigma_{11}(\text{waist})}{\sigma_{11}(\text{lens})} \quad (60)$$

So if $s_x s_x' = R_{12} R_{22} = 0$, the waist and the minimum spot size coincide. If $R_{12} R_{22} > 0$, the waist precedes the target; and if $R_{12} R_{22} < 0$, the waist occurs after the target position.

If the waist and the target positions fall within the same field-free region, the following simple relationship exists between the beam size at the lens $\sqrt{\sigma_{11}(\text{lens})}$, at the waist $\sqrt{\sigma_{11}(\text{waist})}$, and at the target $\sqrt{\sigma_{11}(\text{min})}$.

$$\frac{1}{\sigma_{11}(\text{waist})} = \frac{1}{\sigma_{11}(\text{min})} + \frac{R_{22}^2}{\sigma_{11}(\text{lens})} \quad (61)$$

If now the lens system is readjusted to form a waist at the target position as shown by the dotted lines in Fig. 10, the relative size of this waist and the minimum spot size achieved by the previous lens setting is:

$$\frac{\sigma_{11}(\text{min at target})}{\sigma_{11}(\text{waist at target})} = 1 - \frac{R_{22}^2 \sigma_{11}(\text{waist at target})}{\sigma_{11}(\text{lens})} \quad (62)$$

Again we observe that the two quantities approach each other if the size of the beam at the lens is large compared to the beam size at the target.

There are several cases of special interest that may be derived from the above equations:

1) If $R_{22} = 0$ at the target position, then a minimum spot size at the target is also a waist. This corresponds to point-to-parallel imaging from the principal planes of the variable lens system to the target position. Beyond the last lens in the field-free region preceding the target, $R_{12} = \text{a constant}$ if $R_{22} = 0$; thus we conclude from Eq. (59) that in this field-free region, the minimum spot size achievable at a target is a waist and is independent of the target position. Such a system is a "Zoom" lens.

2) If there are no lenses beyond the variable lens system, i.e., R is an entirely field-free region (a drift), then R is of the form:

$$R = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

In this situation $R_{12}R_{22} = L$ is a positive quantity, consequently the waist always precedes a minimum spot size at a target. A case of particular interest is when the minimum spot size achievable is equal to the initial beam size at the lens. It then follows from Eq. (60) that $Z = -L/2$, i.e., a waist occurs midway between the

lens and the target. From Eq. (61), the ratio of the size of the beam at the lens and at the waist is:

$$\frac{x(\text{lens})}{x(\text{waist})} = \sqrt{\frac{\sigma_{11}(\text{lens})}{\sigma_{11}(\text{waist})}} = \sqrt{2} \quad (63)$$

Combining this result with Eq. (59),

$$L = R_{12} = \frac{x(\text{lens})x(\text{min})}{|\sigma|^{1/2}} = \frac{x^2(\text{lens})}{x(\text{waist})\theta(\text{waist})}$$

or

$$L = \frac{2 x(\text{waist})}{\theta(\text{waist})} \quad (64)$$

where L is the longest distance a beam can drift without exceeding its initial size at the lens.

Imaging from an Erect Ellipse to an Erect Ellipse

The general sigma matrix for imaging from an erect ellipse to an erect ellipse may be derived by inspection from Eq. (36) by setting $\sigma_{21}(1) = \sigma_{21}(0) = 0$.

The result is:

$$\sigma(1) = \left[\begin{array}{c|c} R_{11}^2 \sigma_{11}(0) + R_{12}^2 \sigma_{22}(0) & 0 \\ \hline 0 & R_{21}^2 \sigma_{11}(0) + R_{22}^2 \sigma_{22}(0) \end{array} \right] = \left[\begin{array}{c|c} \sigma_{11}(1) & 0 \\ \hline 0 & \sigma_{22}(1) \end{array} \right] \quad (65)$$

For symmetric magnetic systems $R_{11} = R_{22}$. Using this property and the fact that $|R| = 1$, it follows that $R_{12}R_{21} = (R_{11}^2 - 1)$. So for symmetric magnetic systems Eq. (65) reduces to:

$$\sigma(1) = \left[\begin{array}{c|c} \left[-\frac{R_{12}|\sigma|}{R_{21}} \right]^{1/2} & 0 \\ \hline 0 & \left[-\frac{R_{21}|\sigma|}{R_{12}} \right]^{1/2} \end{array} \right] = \left[\begin{array}{c|c} \sigma_{11}(1) & 0 \\ \hline 0 & \sigma_{22}(1) \end{array} \right] \quad (66)$$

The above equations may be used to calculate the optimum design parameters for periodic beam transport systems.

Example No. 1:

Consider a unit-cell of a periodic focusing array consisting of focusing elements only as indicated below.

The R matrix for the unit-cell, i.e., from the principal planes of the first lens to the principal planes of the second lens, is

$$R = \begin{bmatrix} R_{11} & 2l \\ -\frac{1}{f} + \frac{l}{2f^2} & R_{11} \end{bmatrix} \quad (67)$$

If now we require that the beam envelope possess symmetry coincident with the lens symmetry i.e., that erect ellipses occur at the principal planes of each lens and a waist midway in between, and furthermore that the beam size at the second lens be kept to a minimum and equal to the beam size at the first lens: then substituting Eq.(67) into Eq. (66) and setting $\sigma_{11}(1)$ to be a minimum yields:

$$\begin{aligned} \sqrt{\sigma_{11}(1)} &= \sqrt{\sigma_{11}(0)} \\ f = l &= \frac{1}{2} \sqrt{\frac{\sigma_{11}(0)}{\sigma_{22}(0)}} = \sqrt{\frac{\sigma_{11}(w)}{\sigma_{22}(w)}} \end{aligned} \quad (68)$$

where $\sigma_{11}(0)$ and $\sigma_{22}(0)$ are measured at the principal planes of the first lens.

and finally

$$\frac{x(\max)}{x(\min)} = \sqrt{\frac{\sigma_{11}(0)}{\sigma_{11}(w)}} = \sqrt{2} \quad (69)$$

Note that the ratio of the maximum to the minimum beam size (Eq. 69) is independent of the phase space area and of the lens spacing.

Example No. 2:

If the unit cell is a FODO array as follows:

1510A29

The R matrix for the unit cell (from the principal planes of the first lens to the principal planes of the third lens) is:

$$R = \begin{bmatrix} R_{11} & \ell(2 + \frac{\ell}{f}) \\ -\frac{\ell}{4f^2}(2 - \frac{\ell}{f}) & R_{11} \end{bmatrix} \quad (70)$$

If we now impose the symmetry requirements that erect ellipses occur at the principal planes of each lens; and that the beam size $\sqrt{\sigma_{11}(1)}$ at lens 3 be a minimum and equal to the beam size $\sqrt{\sigma_{11}(0)}$ at lens 1, then it follows that:

$$\begin{aligned} \sqrt{\sigma_{11}(1)} &= \sqrt{\sigma_{11}(0)} \\ \frac{\ell}{f} &= \frac{4}{1+\sqrt{5}} \cong 1.236 \end{aligned} \quad (71)$$

$$\frac{\sigma_{11}(0)}{\sigma_{11}(w)} = -\frac{3 + \sqrt{5}}{1 - \sqrt{5}} \approx 4.23$$

or

$$\frac{x(\max)}{x(\min)} = \sqrt{\frac{\sigma_{11}(0)}{\sigma_{11}(w)}} \approx 2.055 \quad (72)$$

and finally

$$l \approx 0.3003 \sqrt{\frac{\sigma_{11}(0)}{\sigma_{22}(0)}} \quad (73)$$

where $\sigma_{11}(0)$ and $\sigma_{22}(0)$ are measured at the principal planes of the first lens in the FODO array.

For a FODO quadrupole array where the field strength is held constant for all elements rather than the focal lengths, the results are somewhat different than those above. This case may be readily calculated via TRANSPORT using the above results as initial guesses in the calculation.

Relationship between a First-Order Point-to-Point Image and the Minimum Spot Size Achievable at a Fixed Target Position

This problem is not as easy to explore as were the preceding ones because the question arises "the first-order image of what?" If, however, we restrict the discussion to a thick or thin lens system that does not have intermediate images between the source and the image under consideration, then the following comments are applicable.

The ratio of the minimum beam size to the size of a first-order image at a fixed target position may be calculated using Eq.'s (56) and (59). From Eq. (59) we have:

$$\sigma_{11}(\min) = \frac{L^2 |\sigma|}{\sigma_{11}(\text{lens})}$$

and from Eq. (56) the size of a first-order image at the target position is:

$$\sqrt{\sigma_{11}(\text{image})} = |M| \sqrt{\sigma_{11}(\text{object})} = \left(\frac{L}{p}\right) \sqrt{\sigma_{11}(\text{object})}$$

where M is the magnification of the first-order image, p is the object distance measured to the principal planes, and L is the distance to the target measured from the principal planes.

The ratio of sizes is

$$\frac{\sigma_{11}(\text{min})}{\sigma_{11}(\text{1st order image})} = \frac{p^2 |\sigma|}{\sigma_{11}(\text{object}) \sigma_{11}(\text{lens})} \quad (74)$$

Using Eq. (36), we may write

$$\sigma_{11}(\text{lens}) = \sigma_{11}(\text{object}) + 2p \sigma_{21}(\text{object}) + p^2 \sigma_{22}(\text{object}) \quad (75)$$

and since

$$|\sigma| = \sigma_{11}(\text{object}) \sigma_{22}(\text{object}) - \sigma_{21}^2(\text{object})$$

it follows that the first-order image will coincide with the smallest spot size only if the orientation of the initial beam ellipse at the object is such that

$$p \sigma_{21}(\text{object}) = -\sigma_{11}(\text{object}) \quad (76)$$

or if $\sigma_{11}(\text{object}) = 0$ i.e., for a point source.

For an erect ellipse at the source and the lens adjusted to provide a minimum spot size at the target, it can be shown that the first-order image will always follow the target position (the minimum spot size) by a distance

$$Z = L|M| \frac{\sigma_{11}(\text{object})}{\sigma_{11}(\text{lens})} = L|M| \frac{x_o^2}{x^2(\text{lens})} \quad (77)$$

where L is the distance to the target position from the principal planes of the lens system, x_o is the source size, and M is the magnification of the first-order image. Again we observe that the ratio of the beam size at the source and the beam size at the "lens" is the criterion determining the proximity of these two quantities.

Orientation of the Major Axes of a Phase Space Ellipse

The matrix equation for a coordinate rotation as shown in Fig. 11 is

1510A31

Fig. 11

$$\begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} x_0 \\ \theta_0 \end{pmatrix} \quad (78)$$

or

$$X_1 = M X_0$$

The equation of an ellipse in either set of coordinates is

$$X^T \sigma^{-1} X = 1 \quad \text{where} \quad X = \begin{pmatrix} x \\ \theta \end{pmatrix}; \quad X^T = (x \ \theta)$$

and the transformation from $\sigma(0)$ to $\sigma(1)$ is

$$\sigma(1) = M \sigma(0) M^T \quad (79)$$

provided $|M| = 1$, which it does.

If we assume a general ellipse for $\sigma(0)$ and an "erect" ellipse for $\sigma(1)$,
i.e.,

$$\sigma(0) = \begin{bmatrix} \sigma_{11}(0) & \sigma_{21}(0) \\ \sigma_{21}(0) & \sigma_{22}(0) \end{bmatrix} \quad \text{and} \quad \sigma(1) = \begin{bmatrix} \sigma_{11}(1) & 0 \\ 0 & \sigma_{22}(1) \end{bmatrix}$$

It follows from Eq. (72) that:

$$\sigma_{21}(1) = 0 = M_{11}M_{21}\sigma_{11}(0) + (M_{11}M_{22} + M_{21}M_{12})\sigma_{12}(0) + M_{12}M_{22}\sigma_{22}(0)$$

from which

$$\tan 2\alpha = \frac{2\sigma_{21}(0)}{\sigma_{22}(0) - \sigma_{11}(0)} \quad (80)$$

or using the definition

$$r_{21} = \frac{\sigma_{21}}{\sqrt{\sigma_{11} \sigma_{22}}}$$

an alternate form of expressing the ellipse orientation is

$$\tan 2\alpha = \frac{2r_{21} \sqrt{\frac{\sigma_{11}}{\sigma_{22}}}}{\sigma_{22} - \sigma_{11}} = \frac{2r_{21}}{\sqrt{\frac{\sigma_{22}}{\sigma_{11}}} - \sqrt{\frac{\sigma_{11}}{\sigma_{22}}}} \quad (81)$$

Clearly α is dependent upon the units chosen for σ_{11} and σ_{22} except in the obvious case of $\alpha = 0$; i.e., an erect ellipse.

SECOND-ORDER ABERRATIONS

Section III

TRANSPORT has the capability of calculating the second-order matrix elements (aberration coefficients) of any static-magnetic beam transport system composed of combinations of bending magnets, quadrupoles, solenoids, sextupoles and interspersed drift spaces. It is assumed that mid-plane symmetry prevails for any given magnetic element in a system (except for solenoids) but not necessarily for the system as a whole. The notation used in a TRANSPORT printout is described in reference 1 (SLAC-75) beginning on page 46. The subscript notation is the same as that used for first-order where the subscript 1 means x, 2 means θ , 3 means y, 4 means φ , 5 means ℓ , and 6 means δ .

The symbol R_{ij} has been used to signify a first-order matrix element and the symbol T_{ijk} will be used to signify a second-order matrix element. Thus we may write the second-order Taylor expansion representing the deviation of an arbitrary trajectory from the central trajectory as:

$$x_i(t) = \sum_{j=1}^6 R_{ij} x_j(0) + \sum_{j=1}^6 \sum_{k=1}^6 T_{ijk} x_j(0) x_k(0)$$

where $x_1=x$ $x_2=\theta$ $x_3=y$ $x_4=\varphi$ $x_5=\ell$ and $x_6=\delta$ denotes the subscript notation. In an actual computer printout, the T_{ijk} 's are abbreviated as (i jk); for example $T_{126} = (x|\theta\delta)$ would appear in a printout as (1 26) followed by the computed value of the aberration coefficient for the system being designed.

In order to modify the magnitude of any given aberration coefficient, it is necessary to introduce multipole component(s) of the magnetic field of order equal to or less than the order of the aberration. Thus sextupole, quadrupole and dipole components of the field may all be used to modify any given second-order aberration. But, in practice, the second-order aberrations are usually minimized by only introducing sextupole components so as not to disturb the first-order optics of the system. It should always be kept in mind, however, that it may be

beneficial to go back and change the first-order solution (optical mode) so as to provide a more favorable situation for correcting aberrations; a wise selection of the first-order optical mode may in many instances be the deciding factor between the success or failure of a design.

For a fixed location of a sextupole component, the partial derivative of any second-order aberration coefficient T_{ijk} with respect to the strength S_2 of a given sextupole component is a constant. i.e.,

$$\frac{\partial T_{ijk}}{\partial S_2} = \text{a constant} = \text{the coupling coefficient of } S_2 \text{ to } T_{ijk}$$

Thus minimizing a selected group of aberrations is a straight forward problem of solving a set of simultaneous linear equations once the coupling coefficients are known.

The coupling coefficients may be derived directly either by using two successive TRANSPORT runs (with and without unity strength sextupoles) or by direct calculation via the coupling coefficient equations derived later in this section (see equation 19). Eventually TRANSPORT may have a built-in second-order fitting routine; but until this happens, each user will have to devise his own least squares program for second-order aberration minimization.

Second-Order Phase Ellipsoid Formalism

It will be noted by the user, that a second-order TRANSPORT calculation modifies the phase-ellipsoid printout. In a second-order run, TRANSPORT calculates and prints out the second-moments of the phase space distribution function in the $\sqrt{\sigma_{ii}}$ columns. In addition, it also calculates and prints out the new coordinates of the centroid (first-moment) of the phase space distribution function and tabulates this result to the left of the $\sqrt{\sigma_{ii}}$ columns in the same manner as it does for a magnet misalignment run.

Caution should be used in the use and interpretation of the second-order phase ellipsoid results especially if it is known or suspected that the phase space

distribution resulting from a second-order run is not symmetrical about the beam centroid. To be certain of the situation in any given design, it would be wise to calculate the actual distribution function by sampling of a large number of trajectories using the first- and second-order matrix elements of a representative TRANSPORT run as the basis for the calculation.

A Systematic Procedure for Designing High Resolving Power Beam Transport Systems or Charged Particle Spectrometers

The following is a report submitted to the Third International Magnet Symposium held in Hamburg, Germany - May 1970. It is a general description of a suggested procedure for designing systems to any order and includes the derivation of the coupling coefficient of an n th-order multipole to any n th-order aberration coefficient. The report also derives the multipole strengths for the three techniques for introducing multipole components into a system: namely, 1) pure multipole fields, 2) non-uniform fields, and 3) contoured entrance or exit boundaries of magnets.

The notation used in this report is identical to TRANSPORT notation except for the following:

Replace x' and y' in the report by θ and ϕ respectively to convert to TRANSPORT notation.

REPORT SUBMITTED TO THE THIRD INTERNATIONAL MAGNET SYMPOSIUM
HELD IN HAMBURG, GERMANY-May 1970

by

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Summary

By extrapolating the systematics of the general first- and second-order theory of beam transport optics (1,2,3) to include higher order multipole terms, it has been possible to evolve a simple, step by step, procedure for the design of high resolving power static-magnetic beam transport systems. The choice of the appropriate dipole and quadrupole elements for a given system may be determined once the resolving power, solid angle, momentum range and detector system of the instrument have been specified. The partial derivative of any nth-order aberration coefficient with respect to an nth-order multipole component located anywhere in the system has been derived. From this "coupling coefficient", the strength and the optimum location of multipole element(s) to correct or modify a given aberration or group of aberrations is uniquely determined.

I. Introduction

Within the last two decades, significant advances have been made in the understanding of charged particle optics. Perhaps the first major contribution was the development of the theory of the Alternating Gradient Synchrotron (A.G.S.) by Courant, Livingston, and Snyder(4) which led to the first-order matrix algebra formulation of beam-transport optics. Subsequent to this a second-order matrix algebra was developed by Brown, Belbeoch, and Bounin(5); followed by the development at SLAC of the digital computer program called TRANSPORT(6) that is widely used today in many laboratories for solving first- and second-order static-magnetic beam transport problems. In principle, the second-order matrix formalism may be extended to any order, but in practice this approach has proved to be too cumbersome. Thus beyond second-order it has been more efficient to use computer ray-tracing programs which integrate the basic differential equation of motion of the charged particles through the known or assumed magnetic fields. The fundamental difficulty with ray-tracing has been the required computational time to complete a design involving the minimization of many higher-order aberrations.

In this report, we will outline a systematic procedure for the design of high-resolution systems based upon the extrapolation of the first- and second-order theory (1,2,3) to include higher-order multipole components. A general equation has been derived for the coupling coefficient of an n th-order multipole to any given n th-order aberration coefficient. As will be shown later, these coupling coefficients are a function only of the characteristic first-order trajectories (matrix elements) introduced and defined in References 1 and 2.

Given this information, a systematic procedure for designing high resolution beam transport systems is as follows:

- 1) Find a satisfactory first-order solution to the problem using TRANSPORT or its equivalent.
- 2) Calculate and make the necessary corrections to the second-order aberrations by introducing sextupole components into the system. The "best" locations and strengths of the sextupole components required may be selected via the coupling coefficients for the aberrations to be minimized.
- 3) Calculate and make the necessary corrections (via ray-tracing) to the third-order aberrations by introducing octupole components into the system. (Note that an n th-order multipole couples with terms of order n or higher but not with terms of order lower than n . Thus an octupole component will not disturb the first- and second-order solutions already found from steps 1 and 2.)
- 4) Repeat the above procedure up to the multipole order desired or needed to achieve the design objectives.

If the design requires a solution to n th-order and m multipoles at each order are necessary to minimize the aberrations, the number of computer runs previously needed to complete a design was at least $(n+m)^2$. Having a knowledge of the coupling coefficients, after the first-order design has been selected, now (in principle) reduces the number of computer runs required to n . Since ray-tracing is very time consuming, this is indeed a significant saving.

II. Theory*

The following results are applicable to static-magnetic charged particle optical systems possessing median plane symmetry. As in Ref. 1, we shall use a right-handed curvilinear coordinate system (x, y, t) where x and y are the transverse coordinates. x is the outward normal distance in the median plane away from the central trajectory, y is the perpendicular distance from the median plane, t is the distance along the central trajectory, and $h=h(t)$ is the curvature of the central trajectory.

The existence of the median plane requires that the scalar potential Φ be an odd function of y , i.e., $\Phi(x, y, t) = -\Phi(x, -y, t)$. The most general form of Φ may therefore, be expressed as follows:

$$\Phi(x, y, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{2m+1, n} \frac{x^n}{n!} \frac{y^{2m+1}}{(2m+1)!} \quad (1)$$

where the coefficients $A_{2m+1, n}$ are functions of t .

In this coordinate system, the differential line element dT is given by

$$dT^2 = dx^2 + dy^2 + (1+hx)^2 dt^2$$

The Laplace equation has the form

$$\nabla^2 \Phi = \frac{1}{(1+hx)} \frac{\partial}{\partial x} \left[(1+hx) \frac{\partial \Phi}{\partial x} \right] + \frac{\partial^2 \Phi}{\partial y^2} + \frac{1}{(1+hx)} \frac{\partial}{\partial t} \left[\frac{1}{(1+hx)} \frac{\partial \Phi}{\partial t} \right] = 0 \quad (2)$$

Substitution of (1) into (2) gives the following recursion formula for the coefficients:

$$\begin{aligned} -A_{2m+3, n} &= A_{2m+1, n}'' + nhA_{2m+1, n-1}'' - nh'A_{2m+1, n-1}' + A_{2m+1, n+2} \\ &+ (3n+1)hA_{2m+1, n+1} + n(3n-1)h^2A_{2m+1, n} + n(n-1)h^3A_{2m+1, n-1} \\ &+ 3nhA_{2m+3, n-1} + 3n(n-1)h^2A_{2m+3, n-2} + n(n-1)(n-2)h^3A_{2m+3, n-3} \end{aligned} \quad (3)$$

where prime means $\frac{d}{dt}$, and where it is understood that all coefficients A with one or more negative subscripts are zero. This recursion formula expresses all

* The notation used in this report follows that used in Ref. 1 unless otherwise indicated.

the coefficients in terms of the midplane field $B_y(x, 0, t)$:

where

$$A_{1,n} = \left(\frac{\partial^n B_y}{\partial x^n} \right)_{\substack{x=0 \\ y=0}} = \text{functions of } t. \quad (4)$$

Since Φ is an odd function of y , on the median plane we have $B_x = B_t = 0$. The normal (in x direction) derivatives of B_y on the reference curve defines B_y over the entire median plane, hence the magnetic field $\vec{B}$ over the whole space. The components of the field are expressed in terms of Φ explicitly by $\vec{B} = \vec{\nabla}\Phi$ or

$$\begin{aligned} B_x &= \frac{\partial \Phi}{\partial x} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{2m+1, n+1} \frac{x^n}{n!} \frac{y^{2m+1}}{(2m+1)!} \\ B_y &= \frac{\partial \Phi}{\partial y} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{2m+1, n} \frac{x^n}{n!} \frac{y^{2m}}{(2m)!} \\ B_t &= \frac{1}{(1+hx)} \frac{\partial \Phi}{\partial t} = \frac{1}{(1+hx)} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A'_{2m+1, n} \frac{x^n}{n!} \frac{y^{2m+1}}{(2m+1)!} \end{aligned} \quad (5)$$

The expression for the magnetic field on the midplane is

$$B_y(x, 0, t) = \sum_{n=0}^{\infty} A_{1,n} \frac{x^n}{n!} \quad (6)$$

At this point we deviate from the notation and formalism of Ref. 1 and introduce $K_n(t)$, the multipole strength per unit length; and S_n , the total multipole strength of a static-magnetic field.

We rewrite equation (6) as

$$B_y(x, 0, t) = B\rho \sum_{n=0}^{\infty} K_n(t) x^n \quad (7)$$

Where $B\rho = \frac{B}{h} = \frac{P_0}{e}$ is the magnetic rigidity of a particle of momentum P_0 and charge e along the central trajectory; from which

$$K_n(t) = \left(\frac{1}{B\rho} \right) \left(\frac{1}{n!} \right) \left(A_{1,n}(t) \right) = \left(\frac{1}{B\rho} \right) \left(\frac{1}{n!} \right) \left(\frac{\partial^n B_y}{\partial x^n} \right)_{x=y=0} \quad (8)$$

We define S_n as

$$S_n = \int_0^L K_n(t) dt \quad (9)$$

S_n so defined is the strength of the n th-order multipole component of a field over the interval of integration.

Multipole Strengths for Pure Multipole Fields

Consider the scalar potential of an n th-order $[2(n+1)\text{pole}]$ pure multipole element:

$$\phi = \frac{B_o r^{n+1}}{(n+1)a^n} \left[\sin(n+1) \theta \right] \quad (10)$$

where

$$x = r \cos \theta \quad \text{and} \quad y = r \sin \theta$$

B_o is the field at the pole and a is the radial distance to the pole from the central trajectory.

Expanding ϕ as a function of x and y and differentiating, we have

$$B_y = \frac{\partial \phi}{\partial y} = \frac{B_o}{a^n} \left[x^n + \dots \dots \dots \right]$$

From which

$$K_n = \left(\frac{B_o}{a^n} \right) \left(\frac{1}{B\rho} \right)$$

and

$$S_n = \left(\frac{B_o}{a^n} \right) \left(\frac{L}{B\rho} \right) \quad (11)$$

Where L is the length of the multipole element.

For a dipole $n=0$ and the dipole strength is

$$S_0 = \frac{L}{\rho} = \alpha \text{ (The angle of bend of the central trajectory)}$$

For a quadrupole $n=1$ and

$$S_1 = \left(\frac{B_0}{a} \right) \left(\frac{L}{B\rho} \right)$$

For a sextupole $n=2$ and

$$S_2 = \left(\frac{B_0}{a^2} \right) \left(\frac{L}{B\rho} \right)$$

etc. for higher-order multipoles.

Multipole Strengths for a Non-Uniform Field Expansion

From the midplane field expansion of a non-uniform magnetic field

$$B_y(x, 0, t) = B_y(0, 0, t) \left[1 - nhx + \beta(hx)^2 + \gamma(hx)^3 + \dots \right] \quad (12)$$

the multipole strength factors are:

$$K_0 = h, \quad K_1 = -nh^2, \quad K_2 = \beta h^3, \quad \text{etc.}$$

and S_n evaluated over the length L of the central trajectory is:

$$S_0 = hL = \alpha \text{ as before,}$$

$$S_1 = -nh^2L, \quad \text{and} \quad S_2 = \beta h^3L, \quad \text{etc.}$$

Multipole Strengths for a Contoured Entrance or Exit Boundary of a Magnet

A third method of introducing multipole components is via a curved entrance or exit boundary of a magnet. To calculate the multipole strengths in this case, we integrate equation (7), holding x constant, as follows:

$$\int_0^L B_y(x, 0, t) dt = B\rho \sum x^n \int_0^L K_n(t) dt = B\rho \sum S_n x^n \quad (13)$$

To relate this to the field boundary, we assume B_y to be a constant inside the effective field boundary and zero outside (i.e., we ignore the finite extent of the fringing field). In this sharp-cutoff approximation, the field boundary $Z = Z(x)$ is:

$$Z = \frac{1}{B_y} \int_0^L B_y(x, 0, t) dt = \frac{1}{h} \sum S_n x^n = -x \tan \beta + \frac{S_2}{h} x^2 + \dots \quad (14)$$

where $h = \frac{1}{\rho}$ and β is the angle of rotation of the entrance or exit face of the magnet at $x=0$. A positive β implies radial (x) defocusing and transverse (y) focusing. We note that:

$$S_1 = -h \tan \beta = \text{The "quadrupole strength"}$$

The radius of curvature of the boundary is related to the sextupole strength as follows:

$$\frac{1}{R} = \frac{Z''}{(1+Z'^2)^{3/2}} = \frac{2 S_2}{h \sec^3 \beta}$$

or

$$S_2 = \frac{h \sec^3 \beta}{2R} = \text{The "sextupole strength"}$$

From equation (13), we note that a positive multipole component of the field increases the $\int B dt$ for a positive x ; thus a positive sextupole is represented by a concave surface of the entrance or exit boundary.

The Description of the Trajectories as a Taylor's Expansion

The deviation of an arbitrary trajectory from the central trajectory is described by expressing x and y as functions of t . The expressions will also contain x_0 , y_0 , x'_0 , y'_0 and δ , where the subscript 0 indicates that the quantity is evaluated at $t=0$. The prime (') denotes the derivative with respect to t , and $\delta = \frac{\Delta P}{P_0}$ is the fractional momentum deviation of the ray from that of the central trajectory. These five initial boundary values will have the value zero for the central trajectory itself. x and y are expressed as a five-fold Taylor expansion using these initial boundary values. The expansions are written:

$$\begin{aligned}
x(t) &= \sum (x|x_0^\kappa y_0^\lambda x_0'^\mu y_0'^\nu \delta^\chi) x_0^\kappa y_0^\lambda x_0'^\mu y_0'^\nu \delta^\chi \\
y(t) &= \sum (y|x_0^\kappa y_0^\lambda x_0'^\mu y_0'^\nu \delta^\chi) x_0^\kappa y_0^\lambda x_0'^\mu y_0'^\nu \delta^\chi
\end{aligned} \tag{15}$$

Here, the parentheses are symbols for the Taylor coefficients; the first part of the symbol identifies the coordinate represented by the expansion, and the second indicates the term in question. These coefficients are functions of t to be determined. The symbol $\sum$ indicates summation over zero and all positive integer values of the exponents $\kappa, \lambda, \mu, \nu, \chi$; . The constant term is zero, and the terms that would indicate a coupling between the coordinates x and y are also zero; this results from the midplane symmetry. Thus we have

$$\begin{aligned}
(x|1) &= (y|1) = 0 \\
(x|y_0) &= (y|x_0) = 0 \\
(x|y_0') &= (y|x_0') = 0
\end{aligned} \tag{16}$$

Here, the first line is a consequence of choosing the central trajectory as the reference axis, while the second and third lines follow directly from considerations of median plane symmetry.

Since they will appear often in the formalism, it is convenient to introduce the following abbreviations for the first-order Taylor coefficients:

$$\begin{aligned}
(x|x_0) &= c_x(t) & (x|x_0') &= s_x(t) & (x|\delta) &= d_x(t) \\
(y|y_0) &= c_y(t) & (y|y_0') &= s_y(t)
\end{aligned} \tag{17}$$

When the transverse position of an arbitrary trajectory at position t is written as a first-order Taylor's expansion as a function of the initial boundary conditions, the above five quantities are just the coefficients appearing in the expansion for the transverse coordinates x and y as follows:

$$x(t) = c_x(t) x_0 + s_x(t) x_0' + d_x(t) \delta + \text{higher-order terms}$$

and

$$y(t) = c_y(t) y_0 + s_y(t) y_0' + \text{higher-order terms.}$$

III. Solution of the Equations of Motion

The general differential equation of motion of a charged particle in a static-magnetic field valid to all orders in x and y and their derivatives as derived in Ref. 1, equation (5) is:

$$\begin{aligned} & \hat{x} \left\{ [x'' - h(1+hx)] - \frac{x'}{(T')^2} [x'x'' + y'y'' + (1+hx)(hx' + h'x)] \right\} \\ & + \hat{y} \left\{ y'' - \frac{y'}{(T')^2} [x'x'' + y'y'' + (1+hx)(hx' + h'x)] \right\} \\ & + \hat{t} \left\{ (2hx' + h'x) - \frac{(1+hx)}{(T')^2} [x'x'' + y'y'' + (1+hx)(hx' + h'x)] \right\} \\ & = \frac{e}{P} T' (\vec{T}' \times \vec{B}) = \frac{e}{P} T' \left\{ \hat{x} [y'B_t - (1+hx)B_y] + \hat{y} [(1+hx)B_x - x'B_t] \right. \\ & \quad \left. + \hat{t} [x'B_y - y'B_x] \right\} \end{aligned} \quad (18)$$

If this equation is solved to n th-order for the Taylor's coefficients of equation (15), it will be observed that the result has the remarkably simple form:

$$\begin{aligned} (x_i | x_o^{\kappa} y_o^{\lambda} x_o^{\mu} y_o^{\nu} t_o^X) &= \frac{1}{n!} \left[\frac{n!}{\kappa! \lambda! \mu! \nu! X!} \right] \int_0^t G_i(t, \tau) c_x^{\kappa}(\tau) c_y^{\lambda}(\tau) s_x^{\mu}(\tau) s_y^{\nu}(\tau) d_x^X(\tau) K_n(\tau) d\tau \\ &+ \text{Terms containing } K_o, \dots, K_{n-1} \end{aligned} \quad (19)$$

where the variable of integration is τ and $n = (\kappa + \lambda + \mu + \nu + X)$.

The x_i have the following meaning:

$$x_1 = x(t) \quad x_2 = x'(t) \quad x_3 = y(t) \quad x_4 = y'(t)$$

c_x , c_y , s_x , s_y , and d_x are defined by equation (17) and in general are functions of the variable of integration τ over the interval of integration. K_n is defined by equation (8) and in general is also a function of τ .

The G_i 's are Green's functions where:

$$\begin{aligned}
G_1(t, \tau) &= (x(t)|x'(\tau)) = s_x(t)c_x(\tau) - c_x(t)s_x(\tau) \\
G_2(t, \tau) &= (x'(t)|x'(\tau)) = s'_x(t)c_x(\tau) - c'_x(t)s_x(\tau) \\
G_3(t, \tau) &= (y(t)|y'(\tau)) = s_y(t)c_y(\tau) - c_y(t)s_y(\tau) \\
G_4(t, \tau) &= (y'(t)|y'(\tau)) = s'_y(t)c_y(\tau) - c'_y(t)s_y(\tau)
\end{aligned} \tag{20}$$

Note that the G_i 's are just first-order Taylor's coefficients measured from the location (τ) of the multipole component to the end of the system (t) .

Thus we see that the coupling coefficient to an n th-order multipole is a function only of the first-order matrix elements c_x , c_y , s_x , s_y , d_x and their derivatives with respect to t .

From median-plane symmetry considerations, the allowed aberrations are those with y and/or y' appearing an even number of times in the Taylor coefficient. For example $(x|x_0^2)$, $(x|y_0 y'_0)$ and $(y|y_0^2 y'_0)$ are allowed aberrations; whereas $(x|y_0)$, $(x|x_0^2 y'_0)$ or $(y|y_0^2)$ are not allowed and are therefore equal to zero.

The minus sign is used when y and/or y' appear 0, 4, 8, 12 $\dots$ times and the plus sign is used when y and/or y' appear 2, 6, 10 $\dots$ times. For example for the coefficients $(x|x_0^2)$ and $(y|y_0^3)$, the minus sign is applicable; whereas for the coefficients $(x|y_0^2)$ and $(y'|y_0^3 y'_0)^2$ the plus sign is applicable.

Equation (19) is derived by observing in the pattern of the solution of the differential equation that an n th-order aberration term containing the n th-order multipole strength factor K_n cannot include multipole strength factors of lower order than n ; or stated physically, an n th-order multipole cannot couple to aberrations (terms) of order lower than n . This fact allows the recursion formula equation (3) to be reduced to the simple form

$$A_{2m+3,n} = -A_{2m+1,n+2} \tag{21}$$

in so far as it applies to the derivation of n th-order terms containing only K_n . As a consequence, the scalar potential for deriving these terms assumes the simplified form

$$\varphi(x, y, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (-1)^m A_{1,2m+n} \frac{x^n}{n!} \frac{y^{2m+1}}{(2m+1)!} \tag{22}$$

From which, it follows that

$$B_x(x, y, t) = \left(\frac{P_0}{e}\right) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (-1)^m K_{2m+n+1} \frac{(2m+n+1)!}{n!(2m+1)!} x^n y^{2m+1}$$

and

$$B_y(x, y, t) = \left(\frac{P_0}{e}\right) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (-1)^m K_{2m+n} \frac{(2m+n)!}{n! 2m!} x^n y^{2m} \quad (23)$$

For terms containing only K_n , the basic differential equations assume the form:

$$\begin{aligned} x'' + \dots &= -\frac{e}{P} B_y \\ y'' + \dots &= \frac{e}{P} B_x \end{aligned} \quad (24)$$

Substituting the Taylor's expansion of equation (15) and solving for the n th-order terms using a conventional Green's function solution (see Ref. 1) yields equation (19) above.

IV. Interpretation and Use of Equation (19)

For most practical cases of interest, K_n will be a constant over the interval of integration. In this event we may define the coupling coefficient of an n th-order multipole to an n th-order aberration as the partial derivative of equation (19) with respect to the K_n in question as follows:

$$\frac{\partial(x_1 | x_0^{\kappa} y_0^{\lambda} x_0^{\mu} y_0^{\nu} \delta X)}{\partial K_n} = \pm \left[\frac{n!}{\kappa! \lambda! \mu! \nu! X!} \right] \int_0^L G_i c_x^{\kappa} c_y^{\lambda} s_x^{\mu} s_y^{\nu} d_x^X d\tau \quad (25)$$

where now the interval of integration is over the multipole length L represented by K_n . For a distributed multipole component (such as in a non-uniform field bending magnet), equation (25) is used.

In many cases where a curved entrance or exit pole contour is used or a short multipole magnet is used such that the characteristic first-order functions c_x , c_y , s_x , s_y and d_x are essentially constants over the interval of integration (the length of one multipole), then the coupling coefficient is best defined as the partial derivative of equation (19) with respect to S_n as follows:

$$\frac{\partial(x_i | x_o^{\kappa} y_o^{\lambda} x_o^{\mu} y_o^{\nu} \delta^X)}{\partial S_n} = \pm \left[\frac{n!}{\kappa! \lambda! \mu! \nu! X!} \right] G_i c_x^{\kappa} c_y^{\lambda} s_x^{\mu} s_y^{\nu} d_x^X \quad (26)$$

Examples

Assume a situation where the end of the system is a point-to-point image or the origin (i.e., $s_x(t)=0$), then using equation 26, the coupling coefficients of a sextupole of strength S_2 to various second order aberration coefficients are:

$$\frac{\partial(x | x_o' \delta)}{\partial S_2} = c_x(t) s_x^2 d_x$$

$$\frac{\partial(x | x_o'^2 \delta)}{\partial S_2} = c_x(t) s_x^3$$

$$\frac{\partial(x | y_o y_o' \delta)}{\partial S_2} = -2 c_x(t) c_y s_x s_y \quad (27)$$

etc. Where the Green's function used in these examples is

$$G_1 = s_x(t) c_x - c_x(t) s_x = -c_x(t) s_x \quad (\text{since } s_x(t) = 0 \text{ for point-to-point imaging})$$

The aberration and $c_x(t)$ are evaluated at the end of the system. $c_x(t)$ is equal to the magnification M_x in the examples given. The remaining coefficients c_y , s_x , s_y and d_x are evaluated at the location of the sextupole S_2 . The above results are in agreement with Table VII of Ref. 1.

To illustrate a more complex example, consider the fourth-order aberration coefficient $(y | y_o^2 y_o' \delta)$ and assume parallel-to-point imaging in the y coordinate (i.e., $c_y(t) = 0$). The appropriate Green's function is:

$$G_3 = s_y(t) c_y - c_y(t) s_y = s_y(t) c_y$$

and the coupling coefficient to a fourth-order multipole of strength S_4 is:

$$\frac{\partial(y | y_o^2 y_o' \delta)}{\partial S_4} = - \left(\frac{4!}{2!} \right) s_y(t) c_y^3 s_y d_x \quad (28)$$

where again the aberration coefficient $(y|y_o^2 y_o' \delta)$ and $s_y(t)$ are evaluated at the end of the optical system and c_y , s_y , and d_x are evaluated at the location of the fourth-order multipole S_4 .

V. A Systematic Procedure for Designing High Resolution Systems

First-Order Considerations

In many respects, the determination of a satisfactory first-order magnetic-optical design is more difficult to achieve than is the subsequent higher-order design. This is true not only because the basic equipment configuration is dominated by first-order optical considerations but also because the choice of the first-order optics affects the magnitude of all higher-order aberrations and the ease with which these aberrations may be minimized by introducing multipole components into the design.

The dominating design parameters that must be clearly specified in order to evolve a first-order design are the momentum resolving power; the spatial resolution of the particle detector system to be used (this determines the momentum dispersion required); the required phase space acceptance (the solid angle, the source size, and the momentum range) of the instrument, and the first-order imaging requirements in both the x and y coordinates.

Given the above specifications (assuming they are self-consistent), the optical mode and physical configuration of the instrument may be determined. Often, more than one theoretical solution exists; in which case the choice is usually resolved by practical or economic considerations. In other cases, no solution is evident and the basic specifications must be modified accordingly. In any event, the following equations and discussion are applicable to the solution of the problem.

1) First-Order Resolving Power

A general equation for the first-order resolving power has been derived in References (1,2, and 3). For point-to-point imaging the first-order momentum resolving power R_1 is defined as the ratio of the momentum dispersion at the image plane to the total image size. Thus if $2x_o$ is the total source size then from Reference 1 we have:

$$R_1 = \frac{P}{\Delta P} = \left| \frac{d_x(t)}{2x_o c_x(t)} \right| = \frac{1}{2x_o} \left| \int_0^t s_x(\tau) h(\tau) d\tau \right| \quad (29)$$

Note that $h(\tau) d\tau = d\alpha$ is the differential angle of bend of the central trajectory of the optical system.

Equation (29) may be expressed in a number of useful forms. If we consider a particle originating at the source with $x_0 = 0$ and $\delta = \frac{\Delta P}{P_0} = 0$ and lying in the midplane (i.e., a monoenergetic point source), the first-order equation of its trajectory is

$$x(\tau) = s_x(\tau)x'_0 \quad (30)$$

We may then rewrite equation (29) as follows:

$$R_1 = \frac{1}{2x_0 x'_0} \left| \int_0^t x(\tau)h(\tau)d\tau \right| = \frac{(l-l_0)}{2x_0 x'_0} \quad (31)$$

where $(l-l_0)$ is the path length difference between the trajectory described by equation (30) and the central trajectory. Or we may also write equation (31) in the form

$$R_1 = \frac{1}{2x_0 x'_0} \left| \int_0^t \frac{B x(\tau)d\tau}{B\rho} \right| = \left(\frac{1}{2x_0 x'_0} \right) \left(\frac{1}{B\rho} \right) \left| \int_0^t B dA \right| \quad (32)$$

where $\int B dA$ is the magnetic flux enclosed between the central trajectory and the trajectory described by equation (30), and $B\rho$ is the magnetic rigidity of the central trajectory. Please note, however, that if the trajectory of equation (30) crosses the central trajectory or the sign of B changes, this changes the sign of the integration. From equation (32) we may define resolving power as the magnetic flux enclosed per unit phase space area $(2x_0 x'_0)$, per unit momentum $(B\rho)$ of the central ray.

In any given design, one or more of the above equations may be used as a guide toward achieving the required resolving power. One of the design decisions that must be made is the appropriate choice of the dipole magnet parameters (width and length) to achieve the required $\int B dA$. From first-order considerations, this choice is dominated primarily by practical and economic factors. However, a study of the nature of the origin of aberrations (see for example Ref. 1) suggests that it is advisable to keep the amplitude of s_x small. In order to simultaneously satisfy this requirement and meet the required resolving power R_1 , we see from equation (29) that the total angle of bend α of the central trajectory should be chosen as large as is practical. Also, in general, the focal plane angle tends to be more normal to the optic axis for larger α - a property usually desired in most designs.

2) Dispersion

From Reference 1, 2, or 3; for point-to-point imaging ($s_x(t)=0$) the dispersion at the image plane is

$$d_x(t) = -c_x(t) \int_0^t s_x(\tau) h(\tau) d\tau \quad (33)$$

where $c_x(t)$ is the magnification at the image plane.

The dispersion and hence the magnification in the design of a spectrometer is dominated almost entirely by a compromise between the spatial resolution of the particle detectors used at the image plane and the momentum range to be covered by the instrument; or in the case of a momentum defining (analyzing) system, by the acceptable momentum-defining slit spacings.

3) The Selection of the Optical Mode

By optical mode, we mean the type of imaging (e.g., point-to-point or parallel-to-point, etc.) required at the image plane in both the x and y coordinates, and the number of intermediate images imposed between the source and image planes. The imaging requirements at the image plane are usually dominated by the physics to be performed by the instrument and the nature of the particle detectors used. However often (especially at low energies) the imaging in the y plane may be unimportant as far as the physics requirements are concerned which in turn provides some additional flexibility in the optics design.

A study of the coupling coefficients to the aberration coefficients (equation 19) shows the not surprising result, that multipoles located at intermediate images in a system do not couple to aberrations in the plane in which the intermediate image occurs. Hence it often proves beneficial to intentionally create an intermediate image in the y plane of an optical system so as to achieve some degree of "orthogonality" in the minimizing of x and y aberrations.

The considerations of 1), 2), and 3) above are the determining factors in the selection of the first-order solution of a system design.

The optical mode and dispersion of the system are determined to a great extent by the choice of the quadrupole components chosen to achieve the first-order imaging although it is clear that the dipole elements also influence the first-order imaging to a greater or lesser extent depending upon the total angle of bend of the system.

4) Aberrations and their Correction

A study of the source of second- and higher-order aberrations (see for example Ref. 1) suggests that it is advisable to maintain the characteristic

first-order functions c_x , s_x , d_x and c_y , s_y and their derivatives as small as is feasible through the magnetic elements of a system when choosing the first-order design. This procedure will tend to reduce the initial size of the aberrations and hence simplify the problem of minimizing them by the addition of multipole components to the system design.

The procedure for minimizing aberrations has already been outlined in the Introduction and as such will not be repeated here. The "key" to the minimization procedure is the coupling coefficient given by the integral expression in equation (19). The "best" location for the correcting multipole is where the coupling coefficient has its maximum value.

The preferred method of introducing the multipole components, i.e., via pure multipoles, contoured entrance or exit boundaries, or non-uniform fields is a combination of practical and economical considerations and, of course, personal taste and experience. All three methods have been used with pure multipoles dominating the situation for higher energy physics and the other two methods dominating medium and low-energy physics applications. All three techniques should be considered in any given design situation to be certain that an important economic or practical advantage has not been ignored.

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THEORY OF MAGNET ALIGNMENT TOLERANCES

Section IV

When a charged particle passes through a perfectly aligned magnet the transformation may be described by the matrix equation

$$X(1) = R X(0) \quad (1)$$

The ray $X(0) = 0$ is the central trajectory and passes undisturbed along the axis of the magnet. A misalignment causes some other ray to play the role of the central trajectory thru the magnet. Suppose that this ray relative to the magnet axis is described by a vector displacement $D(0)$ at the entrance face and by $D(1)$ at the exit face of the magnet (see figure below).

Beam displacement due to magnet misalignment

Then the transformation through the misaligned magnet is described to first-order by

$$X(1) - D(1) = R [X(0) - D(0)] \quad (2)$$

The problem now is to express the 6-vectors $D(0)$ and $D(1)$ in terms of numbers readily measured as misalignments.

A rigid magnet has six degrees of freedom, three translational and three rotational. These are conveniently represented by the six quantities

$$m = \begin{pmatrix} \delta x \\ \theta_x \\ \delta y \\ \theta_y \\ \delta z \\ \theta_z \end{pmatrix} \quad (3)$$

where δx , δy , δz are the displacements in the x, y, and z directions and θ_x , θ_y , θ_z are the rotations about the x, y, and z axes, respectively. The origin of the xyz coordinate system, called the pivot, is the point about which the misalignments are measured. Assume that the pivot is at some point C along the central ray $X = 0$ (see Figure (1) above). At C the x, y, z axes of the misalignment coordinate system coincide with the x, y TRANSPORT beam coordinates and the beam direction z respectively.

Since misalignment is a rigid transformation, the 6-vectors $D(0)$ and $D(1)$ are uniquely determined by m , along with the length and curvature of the circular arc ACB. For small $|m|$ the relationship is

$$\begin{aligned} D(0) &= A_0 m \\ D(1) &= A_1 m \end{aligned} \quad (4)$$

where

$$A = \begin{pmatrix} \cos \alpha & 0 & 0 & \rho \sin \alpha & \sin \alpha & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\rho \sin \alpha & 1 & 0 & 0 & -\rho(1 - \cos \alpha) \\ 0 & -\cos \alpha & 0 & 0 & 0 & -\sin \alpha \\ -\sin \alpha & 0 & 0 & -\rho(1 - \cos \alpha) & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

Define α_0 as the angle of bend from the pivot to the entrance face of the magnet and α_1 as the angle of bend from the pivot to the exit face of the magnet. ρ is the radius of curvature of arc ACB.

In TRANSPORT, the pivot is chosen as the magnet entrance face, so $\alpha_0 = 0$ and $\alpha_1 = \alpha$ the full angle of bend. In this case A_0 reduces to:

$$A_0 \text{ (for pivot at entrance face)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (6)$$

Matrices A_1 and A_0 for quadrupoles are obtained by going to the limits:

$$\alpha_0 = \alpha_1 = 0$$

and $\alpha_1 \rho = L$, the effective length of the quadrupole. For the pivot at the entrance face of the quadrupole, A_0 is the same as for a bending magnet and A_1 becomes:

$$A_1(\text{for quad with pivot at entrance face}) = \begin{pmatrix} 1 & 0 & 0 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -L & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (7)$$

We now can write the transformation of a ray through a misaligned magnet about the pivot point as:

$$X(1) = R X(0) + (A_1 - R A_0)m \quad (8)$$

This completes the model for a known misalignment m . Consider now an uncertainty in the magnet position and let the covariance matrix $\langle mm^T \rangle$ measure the resulting distribution of magnet positions, where m^T is the transpose of m . If the position of the reference particle upon entering the magnet is $\langle XX^T \rangle_0$ (due to, say, uncertainties introduced by misalignments of preceding magnets) and if there is no correlation of errors between any two magnets, then from Eq. (8) we conclude that:

$$\langle XX^T \rangle_1 = R \langle XX^T \rangle_0 R^T + (A - R A_0) \langle mm^T \rangle (A - R A_0)^T \quad (9)$$

This equation expresses the resultant distribution in uncertainty upon leaving the magnet. This model for misalignment has been implemented in TRANSPORT. It enables one to introduce a displacement uncertainty δx , of a magnet and trace its effect through the system. The computer can then be used to generate tables of misalignment effects of all the magnets as described in the TRANSPORT manual under Type Code 8.

Section V

To optimize the selected parameters, TRANSPORT uses the method of non-linear least squares, differential correction, a good description of which is found in Ref. (7) below pages, 390-393. A useful by-product of this method is the covariance matrix C, printed by the program at the successful conclusion of any run involving parameter fitting. In many applications C may be used to estimate tolerances on the fitted parameters subject to the specified tolerances (i.e., the standard deviations) of the constraints.

The covariance matrix C is symmetric. This admits a geometrical interpretation as an ellipsoid, and is printed in the same suggestive format as is the beam ellipsoid σ , except that in this case the dimension is equal to N, the number of parameters varied. The center of the ellipsoid is at

$$\vec{\lambda}^0 = (\lambda_1^0, \lambda_2^0 \dots \lambda_N^0) , \quad (1)$$

the N values found by TRANSPORT to be the best estimate of the varied parameters. The equation of the covariance ellipsoid is

$$(\vec{\lambda} - \vec{\lambda}^0) C^{-1} (\vec{\lambda} - \vec{\lambda}^0)^T \leq 1 \quad (2)$$

where $(C_{11})^{1/2}$, the first diagonal element printed, measures the maximum extent of the ellipsoid along the λ_1 axis (the first varied parameter) in the same sense that $(\sigma_{11})^{1/2}$ measures the maximum extent of the beam ellipsoid along the x axis. The off diagonal elements are normalized so that they are ≤ 1 in magnitude, in analogy with the r_{ij} of the beam matrix, and can be interpreted as measures of the orientation of the covariance ellipsoid.

(7) SOLMITZ Analysis of Experiment in Particle Physics, Ann. Rev. Nuc. Sci., Vol 14, 1964.

The best estimates (or optimized values) of the varied parameters λ are precisely those that minimize the quantity:

$$\chi^2 = \sum_{K=1}^M \left[\frac{\xi_K - f_K(\lambda_1 \dots \lambda_N)}{S_{KK}} \right]^2 \quad (3)$$

where:

M = number of constraints

f_K = a function selected by the code digits (i,j) on the constraint definition (Type Code 10.). For example, i = -1, j = 1 means that the transform matrix element R_{11} is to be constrained.

ξ_K = the desired value of the selected function.

S_{KK} = the desired accuracy of fit (i.e., the standard deviation).

In our notation this minimum is expressed by:

$$\chi^2_{\min} = \sum_{K=1}^M \left[\frac{\xi_K - f_K(\lambda_1^0 \dots \lambda_N^0)}{S_{KK}} \right]^2 \quad (4)$$

$\chi^2_{\min}$ is printed at the successful conclusion of any run involving parameter fitting. Whether or not the optimization $(\chi^2_{\min}, \vec{\lambda}^0)$ is 'acceptable' depends on each application and must be evaluated by the user. Values of $\chi^2_{\min} \sim (M-N)$ are sometimes (but not necessarily) regarded as 'good'. In particular if $M=N$, then an exact solution, $\chi^2_{\min} = 0$ should be found by TRANSPORT.

If the resulting fit is acceptable, then the following interpretation may be put on the covariance matrix C: Let the parameters be changed to values $\vec{\lambda}$ near the optimum $\vec{\lambda}^0$, such that they stay within the ellipsoid defined by:

$$(\vec{\lambda} - \vec{\lambda}^0)^T C^{-1} (\vec{\lambda} - \vec{\lambda}^0) \leq 1$$

Then the resulting deviation of the specified constrained quantities is bounded by:

$$\sum_{K=1}^M \left[\frac{\xi_K - f_K(\lambda)}{S_{KK}} \right]^2 \leq \chi^2_{\min} + 1 \quad (5)$$

This interpretation is strictly true if the constrained functions f_K are linear in the parameters $\vec{\lambda}$. In the non-linear case, it is an approximation valid only in some neighborhood of $\vec{\lambda}^0$.

Example:

On the following page is an example of a TRANSPORT data deck and the resulting covariance fit of a first-order run. We have ask for a point-to-point image in both the x and y planes by varying the fields of the quadrupole triplet. The following definitions and solution are applicable:

$$B_1 = \lambda_1, B_2 = \lambda_2, B_1^0 = \lambda_1^0, B_2^0 = \lambda_2^0, f_1 = R_{12}, f_2 = R_{34}, \xi_1 = 0, \xi_2 = 0$$

$$S_{11} = 0.005, S_{22} = 0.005 \text{ and } N=2.$$

From the data deck and the TRANSPORT printout* shown on the following page, we learn that:

$$\frac{\chi^2_{\min}}{N} = 0.8$$

$$B_1^0 = -7.4096 \text{ Kg}, B_2^0 = 6.1577 \text{ Kg}$$

$$\sqrt{C_{11}} = 0.079$$

$$r_{12} = -0.883$$

$$\sqrt{C_{22}} = 0.038$$

Covariance matrix information

The ellipsoid $(B - B^0) C^{-1} (B - B^0)^T \leq 1.0$ can then be constructed as

shown:

1510A30

* Note: the printout format is:

COVARIANCE (FIT $\chi^2_{\min}/N$)

$$\begin{array}{ccccccc} \sqrt{C_{11}} & & & & & & \\ r_{12} & & \sqrt{C_{22}} & & \dots & & \\ \vdots & & \vdots & & \dots & & \\ r_{1n} & \dots & r_{n,n-1} & \dots & \sqrt{C_{NN}} & & \end{array}$$

Example:

```

1. 'USE TRIPLET TO FORM POINT-TO-POINT IMAGE AT TARGET'
2. 0
3. 13. 2. ;
4. 15. 1. 'IN' ; 15. 8. 'FT' ;
5. 1. 0. 0. 0. 0. 0. 0. 1.05 ;
6. 3.0 12. ;
7. 5.02 1.5 -8.0 4.0 'Q1' ;
8. 3.0 0.5 ;
9. 5.01 3.0 7.0 4.0 'Q2' ;
10. 3.0 0.5 ;
11. 5.02 1.5 -8. 4.0 'Q1' ;
12. 3.0 7.0 ;
13. 10. -1. 2. 0. .005 'FIT1' ;
14. 10. -3. 4. 0. .005 'FIT2' ;
15. SENTINEL
16. /*

```

Data Deck for TRANSPORT Run

$$\left(\frac{\chi^2_{\min}}{N} \right)$$

```

*COVARIANCE ( FIT 0.8 )
0.079      } Covariance Matrix
-0.883 0.038 }
USE TRIPLET TO FORM POINT-TO-POINT IMAGE AT TARGET

*BEAM*      1.000000      1.05 GEV

*DRIFT*      3.0      12.0000 FT       $B_1^0$ 

*QUAD*      5.00      1.50000 FT       $\overbrace{-7.4096}^{B_2^0}$  KG      4.000 IN ( -3.208 FT )

*DRIFT*      3.0      0.5000 FT       $B_2^0$ 

*QUAD*      5.00      3.00000 FT       $\overbrace{6.1577}^{B_1^0}$  KG      4.000 IN ( 2.673 FT )

*DRIFT*      3.0      0.5000 FT       $B_1^0$ 

*QUAD*      5.00      1.50000 FT       $\overbrace{-7.4096}^{B_2^0}$  KG      4.000 IN ( -3.208 FT )

*DRIFT*      3.0      7.0000 FT       $\xi_1$ 

*FIT*      10.0      -1. 2.       $\overbrace{0.0}^{\xi_1}$  / 0.005      { Constraint on  $R_{12}$  }
                                -0.000

      LABEL = FIT1

*FIT*      10.0      -3. 4.       $\overbrace{0.0}^{\xi_2}$  / 0.005      { Constraint on  $R_{34}$  }
                                -0.000

      LABEL = FIT2

*LENGTH*      26.0000 FT

```

Interpretation:

So long as B_1 and B_2 fall inside the shaded area (this is the tolerance requirement), then the ellipsoid representing the corresponding deviations of the matrix elements R_{12} and R_{34} is using Eq. (5):

$$\left(\frac{0 - R_{12}}{0.005}\right)^2 + \left(\frac{0 - R_{34}}{0.005}\right)^2 \leq x_{\min}^2 + 1.0 = 2.6$$

or

$$R_{12}^2 + R_{34}^2 \leq 2.6 (0.005)^2$$

Note that it is not enough to prescribe tolerances $|\Delta B_1| < .079$ and $|\Delta B_2| < .038$, since there is unshaded area inside the rectangle defined by these values. The strongly tilted covariance ellipse (i.e., $|r_{12}| \sim 1$) suggests that the triplet power supplies should be designed so that any drift in magnetic field B_1 causes a compensating drift in the magnetic field B_2 so as to stay inside the shaded area shown in Figure 1.