320
8/28/64

UCRL-7864

MASTER

University of California
Ernest O. Lawrence
Radiation Laboratory

LOOP CURRENTS IN A SYSTEM OF
INDUCTIVELY COUPLED, SEQUENTIALLY
ENERGIZED AND CROWBARRED COILS

Livermore, California

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

UCRL-7864

Controlled Thermonuclear
Processes, UC-20
TID-4500 (30th Ed.)

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Livermore, California

Contract No. W-7405-eng-48

LOOP CURRENTS IN A SYSTEM OF INDUCTIVELY COUPLED,
SEQUENTIALLY ENERGIZED AND CROWBARRED COILS

R. E. Ellis

May 19, 1964

LOOP CURRENTS IN A SYSTEM OF INDUCTIVELY COUPLED,
SEQUENTIALLY ENERGIZED AND CROWBARRED COILS

R. E. Ellis

Lawrence Radiation Laboratory, University of California
Livermore, California

May 19, 1964

ABSTRACT

A computer program designated M-PULSE, designed for use on the IBM 7090, has been developed for the purpose of computing the loop currents, as a function of time, in a circuit consisting of up to 14 inductively coupled loops. A loop may typically consist of a pulsed coil and its associated capacitor bank (RLC loop), or the shielding for a dc coil (RL loop). Loops may be energized or crowbarred in sequence (when crowbarred an RLC loop becomes an RL loop). The program is specifically designed for use in calculating the currents in the pulsed coils of a plasma fusion device.

An IBM 7090 code, called M-PULSE, has been developed for determining the current, as a function of time, in each of the conductors of a system of pulsed coils and shielded dc coils which may be inductively coupled to a high degree. A sequence of sets of coupled linear differential equations, representing the pulsed coils and passive shielding loops which are involved at any particular time in the time-sequenced operation of a multistage magnetic mirror plasma fusion device,¹ are solved for the current as a function of time in each loop of the circuit. The values of current thus obtained are to be used in calculating the time-varying magnetic field produced by the system of coils.

¹The particular device for which the code was developed is known as Toy Top, a plasma fusion device operated under the direction of Dr. Frédéric H. Coensgen at LRL, Livermore.

A rather precise knowledge of the magnetic field as a function of time may be required for an understanding of the behavior of the plasma which one may attempt to confine and heat by means of such fields. In the multistage high compression magnetic mirror machine developed at this laboratory,² a coaxial system of pulsed cylindrical coils are arranged within the volume defined by the inner diameters of a number of large, cylindrical dc coils which are shielded from the pulsed fields by copper or aluminum shields. We assume here that such shields are entirely effective in protecting the dc circuits from induced currents. The shield for each dc coil is then included as a passive loop of the system. Since the magnitude of the magnetic field due to a current-carrying conductor is proportional to the magnitude of the current through it, the solution for the current in each coil or loop may then be used in the input for any code which calculates the dc magnetic field of a coil, resulting in a determination of the magnetic field as a function of both time and space, which may then be suitably added to the fields of all other pulsed coils in the system, as well as any dc fields present, to obtain the total resultant field.

We consider inductive coupling only. Any pulsed coils or separate conducting loop (such as a dc shielding can) along with its associated resistance and the capacitance of the associated capacitor bank (if any) is denoted as a loop of the system. Figure 1 shows two typical pulsed coil loops. Whether such a loop is an RLC loop or simply an RL loop depends on whether crowbar switches (S_{1c} , S_{2c}) are open or closed. An RL loop representing a dc shielding can would be similar to either of the two loops shown when the crowbar switch was closed in that loop. It is evident that the resistance or self-inductance of a loop may be different after crowbarring occurs, but the mutual inductance is assumed to remain unchanged.

For the j th loop in the system, the loop elements are thus as follows:

- L_{jj} = total self-inductance in the loop,
- $L_{jk} = L_{kj}$ = mutual inductance with the k th loop,
- $R_{jj} = R_j$ (no mutual resistance) = total loop resistance,
- $C_{jj} = C_j$ (no mutual capacitance) = total loop capacitance
= capacity of capacitor bank energizing the j th loop,

²F. H. Coensgen, W. F. Cummins, W. E. Nexsen, Jr., and A. E. Sherman, Nuclear Fusion, 1962 Supplement, Part 1, p. 125.

Fig. 1. Two typical coupled loops.

V_j = capacitor bank voltage.

The current in the j th loop is $I_j(t)$ and the charge on the capacitor bank is $Q_j(t)$. When a loop is crowbarred (or is a passive loop) $Q_j(t)$ no longer appears as a variable.

The first set of equations is generated when the first energizing switch (say S_{1E} of Fig. 1) is closed, and will represent all passive loops for which there is inductive coupling with the energized loop, plus the energized loop itself. (The code is actually set up to include all passive RL loops in the first set, whether there is any mutual with the first energized loop or not. Thus, each loop in the system is numbered, so that as the RLC loops are energized and crowbarred in any given sequence, the solutions for $I_j(t)$ are arranged ordinally in accord with the original numbering.) The solutions for this set of equations will then be generated for the time interval beginning with $t_1 = 0$ when S_{1E} is closed until the next switch is closed at $t_2 = 0$. The values of I and Q for each loop represented in the first set of equations, evaluated at the time the second switch is closed, are then used as initial values for the solution of the second set of equations which is generated when the second switch is closed at $t_2 = 0$. Succeeding sets of equations are generated as further switches are closed and the initial conditions are obtained from the previous set. Several loops may be energized or crowbarred simultaneously.

Thus if switches have been closed at $t_1 = 0, t_2 = 0, \dots, t_\mu = 0$, we will have generated μ sets of coupled equations, the number of equations in each set being given by $n = p + r$, where p is the number of passive loops in the system and r is the number of loops added by the closing of switches. Apply Kirchhoff's law for the voltage in each loop, and for the μ th set of equations we have

$$\sum_{j=1}^n L_{kj} \dot{I}_j(t) + R_k I_k(t) + C_k^{-1} \int_0^t I_k(t) dt - V_k(0) = 0, \quad (1)$$

$k = 1, 2, \dots, n$. Here $V_k(0) = C_k^{-1} Q_k(0)$ is the voltage on C_k at the time $t_\mu = 0$, which is the zero time for this set. If the i th loop were a passive or crowbarred loop, the last two terms in equation (1) would be missing for $k = i$.

The solution to this set of equations could now be obtained by the method of Laplace transforms in which the set of ordinary differential equations is transformed into a set of simultaneous algebraic equations. The transforms

$I_k(s)$ of the loop currents could then be solved for in terms of the normal modes of the system, and the loop currents $I_k(t)$ obtained by inversely transforming the $I_k(s)$. This method is outlined in Appendix I, from which it may be seen that it involves the determination of the roots of an algebraic equation of degree $2n - q$, where q is the total number of passive and crowbarred loops in the μ th set, there being $n - q$ complex conjugate pairs of roots and q real roots. The precision required in the values of circuit parameters in order to successfully evaluate these roots suggests that another approach, proposed by R. E. von Holdt, is more desirable for computer calculations. In this method a matrix formulation of the problem is used in which it is unnecessary to determine explicitly the eigenfrequencies of the system, and in which the loop currents $I_k(t)$ are obtained from the initial values $I_k(0)$ and $Q_k(0)$ by iterative operation of a suitable matrix on the initial-value current-charge vector, as will be shown below. The problem was coded by W. G. Cunningham for the IBM 7090 on the basis of the following mathematical development.

We can rewrite equation (1) as follows:

$$\sum_{j=1}^n L_{kj} \dot{I}_j(t) = -R_k I_k(t) - C_k^{-1} Q_k(t), \quad (2)$$

$$\dot{Q}_k(t) = I_k(t), \quad k = 1, 2, \dots, n$$

where we have introduced the auxiliary variables $\dot{Q}_k = I_k$ for convenience, making a set of $2n$ equations in $2n$ variables. Using matrix notation, we may now write (2) in the form

$$\left[\begin{array}{c|c} L & 0 \\ \hline 0 & E_n \end{array} \right] \left[\begin{array}{c} \dot{I} \\ \dot{Q} \end{array} \right] = \left[\begin{array}{c|c} R & D \\ \hline E_n & 0 \end{array} \right] \left[\begin{array}{c} I \\ Q \end{array} \right] \quad (3a)$$

(see Appendix II for explicit representation of this equation) or

$$Y\dot{J} = XJ. \quad (3b)$$

In the last equation, the vector $J = \{I, Q\}$ has the components $I_1, I_2, \dots, I_n, Q_1, Q_2, \dots, Q_n$. The matrix Y is a $(2n \times 2n)$ matrix with the $(n \times n)$ inductance submatrix L , the $(n \times n)$ unit submatrix E_n , and the $(n \times n)$ null submatrices 0 . The $(2n \times 2n)$ matrix X consists of the $(n \times n)$ submatrix R , the $(n \times n)$ elastance submatrix D ($D_k = C_k^{-1}$), and the $(n \times n)$ unit and null submatrices. R and D are both diagonal. Actually, if as mentioned above, a number

q of passive and crowbarred loops are present, then q of the charge components $Q_k, \dot{Q}_k$ in the vectors $J, \dot{J}$ will be missing, and the corresponding $(n+k)$ th rows and columns will be missing in the matrices Y and X .

Premultiplying (3b) by Y^{-1} (the reciprocal of Y) we have

$$\dot{J}(t) = Y^{-1} X J(t) = A J(t), \quad (4)$$

since the determinant $|Y| \neq 0$. The matrix A is a $(2n \times 2n)$ matrix whose elements are constant, real functions of the loop parameters. The solution of (4) is (see Appendix III)

$$J(t) = e^{tA} J(0), \quad (5)$$

where $J(0)$ is the vector whose elements are the initial values of loop current and charge at $t = t_\mu = 0$ for the μ th set. The exponential matrix is defined as

$$e^{tA} = E + tA + \frac{t^2 A^2}{2!} + \dots + \frac{t^r A^r}{r!} + \dots, \quad (6)$$

where E is the unit matrix of same order as A . Let $t = P \delta t$, where $\delta t \ll t$ is a small time interval and P is an integer. Then

$$J(t) = \left[e^{\delta t \cdot A} \right]^P J(0). \quad (7)$$

Thus the solution $J(t)$ is obtained by operating iteratively P times on the initial-value vector $J(0)$ with the matrix $e^{\delta t \cdot A}$. It is thus necessary to select δt small enough that the terms of the matrix series (7) converge sufficiently rapidly that a reasonable number of terms is required in the series (6) to provide the accuracy needed. Writing the matrix series in (7) in the form

$$e^{\delta t \cdot A} = \sum_{j=0}^{\nu} \frac{[\delta t \cdot A]^j}{j!} + R_\nu, \quad (8)$$

we determine that the number of terms of the matrix series $e^{\delta t \cdot A}$ required is given by:

$$\left| e^{\delta t \cdot A} - \sum_{j=0}^{\nu} \frac{[\delta t \cdot A]^j}{j!} J(0) \right| = \left| R_\nu J(0) \right| \leq \sum_{j=\nu+1}^{\infty} \left| \frac{[\delta t \cdot A]^j}{j!} J(0) \right| \leq \epsilon |J(0)| \quad (9)$$

where ϵ is a small number, chosen to be $\epsilon = 10^{-8}$ (the limit of computer single-precision).

To establish an upper bound on ν which meets the precision criterion (9) we use the method of iterating on an arbitrary vector V to find an upper bound on the largest eigenvalue of A . Since the eigenvalues of A may be complex, we instead iterate on V with $A'A = B$, where A' is the transpose of A . Since A is real, B is real and symmetric. For our purposes we have chosen the elements of V to be equal to the sum of elements in the corresponding row of A . (See Appendix IV for an outline of this procedure.)

The eigenvalues of B are thus real, and the largest eigenvalue Λ_M of B is related to the eigenvalues λ_i of A by³

$$\Lambda_M \geq \lambda_i^2. \quad (10)$$

The quantity $\alpha \Lambda_M^{1/2} > \lambda_i$, $\alpha \geq 1$, is then used to replace the matrix A in evaluating ν . Letting $H = \alpha \Lambda_M^{1/2}$ (see Appendix V for additional test the code makes to ensure that H is an upper bound for λ_i) and using equation (9)

$$\begin{aligned} \sum_{j=1}^{\infty} \left| \left[\frac{(\delta t)^j A^j}{j!} \right] J(0) \right| &\leq \frac{[H \delta t]^{\nu+1}}{(\nu+1)!} \left\{ 1 + \frac{H \delta t}{\nu+2} + \frac{H^2 (\delta t)^2}{(\nu+2)(\nu+3)} + \dots \right\} |J(0)| \\ &\leq \frac{[H \delta t]^{\nu+1}}{(\nu+1)!} \left[\frac{1}{1 - \frac{H \delta t}{\nu+2}} \right] |J(0)| < \epsilon |J(0)|, \end{aligned} \quad (11)$$

since the infinite series in $\{ \}$ is less than the infinite geometric series

$$\sum_{j=0}^{\infty} \left(\frac{H \delta t}{\nu+2} \right)^j. \quad \text{By requiring that the value of } H \delta t \text{ lie in the range } 0.1 \leq H \delta t \leq 10,^4$$

³E. Bodewig, Matrix Calculus (Interscience Publishers, Inc., New York, 1959), p. 75.

⁴With these limits on $H \delta t$, the extreme limits of ν (the number of terms retained in the exponential series (8)) are determined to be $\nu = 5$ for $H \delta t = 0.1$ and $\nu = 40$ for $H \delta t = 10$. The range of $H \delta t$ was empirically determined for the type of input data corresponding to the experimental setup in order to effect the best compromise between computer running time and the degree of accuracy desired. An initial value of δt is required to be specified in the input data. The computer then adjusts ν until $H \delta t$ lies within the range mentioned. If the value of δt specified in the input data results in $H \delta t > 10$, the

a value of δt is determined which gives a value of $H \delta t$ to be used in (11). The computer, by substitution of values of ν in (11), determines that least value of ν which satisfies the inequality (11). The series (8) is then truncated for that value of ν and the computer iterates P times as in (7) to give the solution for $J(t) = J(P \delta t)$ at the time t (where t is measured from the closing of the last switch).

For the type of circuit for which these calculations were undertaken (for coils with inductance of a few microhenries, energized by capacitor banks of several hundreds of microfarads capacity giving rise times of typically ten to hundreds of microseconds) values of δt ranging from 10^{-7} to 10^{-9} second are most frequently encountered. For time intervals of several hundred microseconds, values of P ranging up to $\sim 10^6$ are obtained with results accurate to within 1% or less in most of the results checked. For values of $\delta t \leq 10^{-10}$, running time may become a problem and for larger P the accuracy is diminished.

To test the accuracy of the code, two general procedures have been followed. Solutions for cases in which no mutual inductance is involved may be readily hand-calculated (see Appendix VI) since if the matrix L is diagonal the solutions are uncoupled and the current and charge in each loop develop independently of the other loops. Oscillatory solutions (no crowbarring) have been found in such a case for 10 RLC loops energized simultaneously and followed out to 1 millisecond ($\delta t = 10^{-8}$, $P = 10^5$). Deviations of the order of 1% or less were obtained. Another test utilized a third-order differential equation with suitable auxiliary variables and initial conditions, to which solutions are known, and which involves coupling between the three variables which would be related to resistive coupling for an actual electrical system. The results and procedures of these tests are outlined in Appendix VI.

value of δt to be used by the computer in determining ν is decreased until $H \delta t \leq 10$. If $H \delta t < 0.1$ initially, δt is increased until $H \delta t \geq 0.1$. If the initial value of $H \delta t$ lies within the range $0.1 \leq H \delta t \leq 10$, the input value of δt is retained. The final value of δt used, in any case, is printed out so that it is always known. In general, it has appeared desirable to select initially as large a value of δt as is compatible with the above in order to minimize the number of matrix iterations.

Following the appendices of this report is a write-up of the procedure to be followed in using the code, and a reproduction of the FORTRAN listing.

I wish to acknowledge the essential contribution of Dr. Richard E. von Holdt in the mathematical approach used and that of Warren G. Cunningham who did the actual coding of the problem. Dr. Frederic H. Coensgen provided encouragement and support for my efforts.

APPENDIX I

Using equation (1) let

$$F(t) = \left[\sum_{j=1}^n L_{kj} \dot{I}_j(t) \right] + R_k I_k(t) + C_k^{-1} \int_{-\infty}^t I_k(t) dt = 0, \quad k = 1, 2, \dots, n. \quad (A-1)$$

Then the Laplace transform is

$$L[F(t)] = \sum_{j=1}^n s L_{kj} I_j(s) - L_{kj} I_j(0) + R_k I_k(s) + (C_k s)^{-1} I_k(s) - s^{-1} V_k(0) = 0. \quad (A-2)$$

Multiply through by s and rearrange (A-2) so that

$$\left(L_{kk} s^2 + R_k s + C_k^{-1} \right) I_k(s) + \sum_{\substack{j=1 \\ j \neq k}}^n \left(L_{kj} s^2 \right) I_j(s) = \sum_{j=1}^n \left(L_{kj} s \right) I_j(0) + V_k. \quad (A-3)$$

This set of n equations in the $I_k(s)$ may now be solved by the method of determinants of the coefficients of $I_j(s)$, using Cramer's rule. The right-hand side of (A-3) involves only initial values of current and voltage (charge). In matrix form (A-3) becomes

$$ZI = G \quad (A-4)$$

where $I = I_1, I_2, \dots, I_n$, $G = G_1, G_2, \dots, G_n$ are vectors and

$$Z = \begin{bmatrix} Z_{11}, Z_{12}, \dots, Z_{1n} \\ Z_{21}, Z_{22}, \dots, Z_{2n} \\ Z_{n1}, Z_{n2}, \dots, Z_{nn} \end{bmatrix}$$

where $Z_{kk} = \left(L_{kk} s^2 + R_k s + C_k^{-1} \right)$,

$$Z_{kj} = L_{kj} s^2, \quad j \neq k.$$

Thus

$$I_k(s) = \frac{\Delta_k}{|Z|} \quad (A-5)$$

where $|Z|$ is the determinant of Z and Δ_k is the determinant obtained by replacing the k th column of $|Z|$ by the corresponding elements of the vector G .

We note that if a number q of the circuit loops have no capacitive element (as in the case of crowbarred or shielding loops), by expanding the numerator and denominator of (A-5),

$$I_k(s) = \sum_{j=0}^{2n-q-1} b_{kj} s^j \left\{ a_{2n-q} \left[s^{2n-q} + \sum_{j=0}^{2n-q-1} \frac{a_j s^j}{a_{2n-q}} \right] \right\}^{-1} \quad (A-6)$$

where now the b_{kj} in the numerator are functions of the loop parameters and initial conditions, and the a_j are functions of the loop parameters, being the same for all k . In order to reduce this expression to one suitable for the inverse transformation, it is expanded by the method of partial fractions into the form

$$I_k(s) = \frac{1}{a_{2n-q}} \left[\sum_{j=1}^n \frac{K_{kj}}{(s - s_{kj})} + \sum_{j=1}^{n-q} \frac{K_{kj}^*}{(s - s_{kj}^*)} \right] \quad (A-7)$$

where the constants K_{kj} , K_{kj}^* are to be evaluated by the method of residues and the s_{kj} , s_{kj}^* are the roots of the expression in square brackets in the denominator of (A-6). There will thus be $n - q$ complex conjugate pairs of roots and q real roots. In principle these roots may be found by methods such as Graeffe's squaring method. The K_{kj} may then be evaluated by

$$K_{kj} = \left[\sum_{\ell=0}^{2n-q-1} b_{\ell j} (s_{kj})^{\ell} \right] \left[\prod_{\substack{\ell=1 \\ \ell \neq j}}^n (s_{kj} - s_{\ell j}) \cdot \prod_{\ell=1}^{n-q} (s_{kj} - s_{\ell j}^*) \right]^{-1} \quad (A-8)$$

and similarly for K_{kj}^* except that in the denominator, the factor for $\ell = j$ is excluded from the second product instead of the first. The solutions $I_k(t)$ are finally found by the inverse transformation

$$\begin{aligned} L^{-1} \left[I_k(s) \right] &= I_k(t) \\ &= \frac{1}{a_{2n-q}} \left[\sum_{j=1}^n K_{kj} e^{s_{kj} t} + \sum_{j=1}^{n-q} K_{kj}^* e^{s_{kj}^* t} \right] \end{aligned} \quad (A-9)$$

The initial conditions for the closing of the $(\mu + 1)$ th switch could then be obtained by evaluating $I_k(t_{\mu+1} - t_\mu) = I_k(0)$ for the μ th set. The initial voltage $V_k(0)$ may be found from the integral of $I_k(t)$ over the interval $t = 0$ to $t = t_{\mu+1} - t_\mu$, since

$$\begin{aligned} Q_k(0) &= \int_0^{t_{\mu+1} - t_\mu} I_k(t) dt \\ &= \frac{1}{a_{2n-q}} \left\{ \sum_{j=1}^n \frac{1}{s_{kj}} \left[K_{kj} \left(\exp \left[s_{kj} (t_{\mu+1} - t_\mu) \right] - 1 \right) \right] \right. \\ &\quad \left. + \sum_{j=1}^{n-q} \frac{1}{s_{kj}^*} \left[K_{kj}^* \left(\exp \left[s_{kj}^* (t_{\mu+1} - t_\mu) \right] - 1 \right) \right] \right\} \quad (A-10) \end{aligned}$$

so that $V_k(0) = C_k^{-1} Q_k(0)$ for the $(\mu + 1)$ th set of equations.

APPENDIX II

Equation (3a) is given by

$$\begin{bmatrix} L_{11} & L_{12} & \dots & L_{1n} \\ L_{21} & L_{22} & \dots & L_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ L_{n1} & L_{n2} & \dots & L_{nn} \end{bmatrix} \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_n \end{bmatrix} = \begin{bmatrix} -R_1 & 0 & \dots & 0 \\ 0 & -R_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & -R_n & 0 \end{bmatrix} \begin{bmatrix} -D_1 & 0 & \dots & 0 \\ 0 & -D_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & -D_n & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \\
 \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \vdots \\ \dot{Q}_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 1 & \dots & 0 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix}$$

where $L_{kj} = L_{jk}$ are the mutual inductance between loops k and j , and $D_j = C_j^{-1}$, where C_j is the capacitance of the j th loop.

APPENDIX III

To show that equation (5) is correct, we differentiate the matrix series on the right-hand side term by term, so that

$$\begin{aligned} \frac{d}{dt}(e^{tA}) &= \frac{d}{dt}\left(E + tA + \frac{t^2 A^2}{2!} + \dots\right) \\ &= A + tA^2 + \frac{t^2 A^3}{2!} + \dots \\ &= A e^{tA}. \end{aligned} \tag{A-12}$$

Thus,

$$\dot{J}(t) = A e^{tA} J(0) = AJ(t)$$

as was to be shown.

APPENDIX IV

The determination of the largest eigenvalue of a real, symmetric matrix³ may be accomplished by iteration on an arbitrary vector V , where $V = x_1 + x_2 + x_3 + \dots + x_{2n}$, and x_j is the eigenvector corresponding in our case to Λ_j , an eigenvalue of the matrix $B = A'A$. We choose V to have the components

$$x_i = \left[\sum_{j=1}^n B_{ij} \right] / |V| \quad (A-13)$$

where B_{ij} is the element in the i th row and j th column of B , and the starting vector V is normalized to its length $|V| = \sqrt{V'V}$. Then, letting $W^{(0)} = V/|V|$,

$$BW^{(0)} = \frac{1}{|V|} \left(\Lambda_1 x_1 + \Lambda_2 x_2 + \dots + \Lambda_{2n} x_{2n} \right) \quad (A-14)$$

where we have chosen the notation so that $\Lambda_1 \geq \Lambda_2 \geq \dots \geq \Lambda_{2n}$. Now let

$$|BW^{(0)}| = h_0, \quad \frac{BW^{(0)}}{|BW^{(0)}|} = \frac{BW^{(0)}}{h_0} = W^{(1)}. \quad (A-15)$$

Then, for the m th iteration of B , using the normalizing process in each iteration, we have

$$W^{(m)} = \frac{BW^{(m-1)}}{|BW^{(m-1)}|} = \frac{B^m W^{(0)}}{h_0 h_1 \dots h_{m-1}} = \frac{B^m W^{(0)}}{|B^m W^{(0)}|}$$

where

$$h_k = |BW^{(k)}| = \frac{|B^{k+1} W^{(0)}|}{h_0 h_1 \dots h_{k-1}} = \frac{|B^{k+1} W^{(0)}|}{|B^k W^{(0)}|}. \quad (A-16)$$

Also, we have

$$\begin{aligned} B^m W^{(0)} &= \frac{1}{|V|} \left(\Lambda_1^m x_1 + \Lambda_2^m x_2 + \dots + \Lambda_{2n}^m x_{2n} \right) \\ &= \frac{\Lambda_1^m}{|V|} \left[x_1 + \left(\frac{\Lambda_2}{\Lambda_1} \right)^m x_2 + \left(\frac{\Lambda_3}{\Lambda_1} \right)^m x_3 + \dots + \left(\frac{\Lambda_{2n}}{\Lambda_1} \right)^m x_{2n} \right]. \end{aligned} \quad (A-17)$$

From this last equation we see that for m sufficiently large,

$$B^m W^{(0)} \approx \frac{\Lambda_1^m}{|V|} x_1. \quad (A-18)$$

The ratio

$$\frac{h_m}{h_{m-1}} = \frac{|B^{m+1} W^{(0)}|}{|B^m W^{(0)}|} \cdot \frac{|B^{m-1} W^{(0)}|}{|B^m W^{(0)}|} \quad (A-19)$$

evidently approaches as close to unity as desired for m sufficiently large.

Then, for a value of the ratio h_m/h_{m-1} sufficiently near 1,

$$h_m = \frac{|B^{m+1} W^{(0)}|}{|B^m W^{(0)}|} \approx \frac{\Lambda_1^{m+1}}{\Lambda_1^m} = \Lambda_1 = \Lambda_M, \quad (A-20)$$

and taking $h_m^{\frac{1}{2}} = \Lambda_M^{\frac{1}{2}} \geq |\lambda_1|$, as in equation (10), we see that we have thus evaluated the quantity H which is to be used in equation (11).

APPENDIX V

To ensure that $H = \alpha \Lambda_M^{\frac{1}{2}} > |\lambda_1|_{\max}$ where $|\lambda_1|_{\max}$ is the largest eigenvalue of the matrix A , an additional requirement is made that the matrix

$$(A'A - HE) = F \quad (A-21)$$

be negative definite, which is true if and only if $(-1)^i B_i > 0$, where B_i is the $(i \times i)$ minor in the upper left-hand corner of the determinant $|F|$. The proof of this statement is not shown here, but has been carried out by R. E. von Holdt. E is again the unit matrix of the same order as A .

APPENDIX VI

Several ways of testing the code have been considered and used. As mentioned before, if the inductance matrix L is diagonal so that the matrix Y is also diagonal, the solutions are uncoupled and the current and charge may be determined as a function of time by the relations

$$Q(t) = Q(0) \left[\cos \omega t + \frac{R}{2\omega L} \sin \omega t \right] e^{-Rt/2L}, \quad (A-22.1)$$

$$I(t) = -Q(0) \left[\frac{R^2}{4L^2\omega} + \omega \right] e^{-Rt/2L} \sin \omega t, \quad (A-22.2)$$

where these two equations are applied to each loop. We assume here that experimentally we are interested in the oscillatory case where

$$\omega = \left[\frac{R^2}{4L^2} - \frac{1}{LC} \right]^{\frac{1}{2}}.$$

Solutions for such a case have been obtained, as mentioned before, for a system of 10 RLC loops and the results compared with hand-calculated values from equations (A-22.1, 2). The parameters used were those corresponding to two of the coils and associated capacitor banks used in one of the Toy Top experiments, the parameters for each loop being entered 5 times for a total of 10 loops. With a δt of 10^{-8} sec, at $t = 900 \mu\text{sec}$ (9×10^4 iterations), the values of current in the two loops deviated at the most about 0.3% from the hand-calculated value. The loop parameters used were:

$$\begin{aligned} L_{11} &= 5 \times 10^{-6} \text{ henry}, & R_1 &= 2 \times 10^{-4} \text{ ohm}, & D_1 &= 2 \times 10^3 \text{ farad}^{-1}, \\ L_{22} &= 1 \times 10^{-5} \text{ "}, & R_2 &= 1 \times 10^{-3} \text{ "}, & D_2 &= 3.3 \times 10^3 \text{ "}. \end{aligned}$$

At $t = 900 \mu\text{sec}$, the computed values of loop currents were:

$$I_1 = -4.182 \times 10^5 \text{ A}, \quad I_2 = 1.251 \times 10^5 \text{ A}.$$

The hand-calculated values were:

$$I_1 = -4.184 \times 10^5 \text{ A}, \quad I_2 = 1.247 \times 10^5 \text{ A}.$$

Another manner of testing is by using the equation

$$\ddot{y}(t) - 5\dot{y}(t) + 9y(t) - 5y(t) = 0 \quad (A-23)$$

which has a solution of the type

$$y = C_1 e^t + C_2 e^{2t} \sin(t + k). \quad (A-24)$$

Assuming that $y(0) = 1$, $\dot{y}(0) = 2$, $\ddot{y}(0) = 3$, and choosing $k = \pi/2$ for convenience, we find that $C_1 = 4$, $C_2 = -1$, and

$$\begin{aligned} y &= 4e^t - e^{2t} \cos t, \\ \dot{y} &= 4e^t + (\sin t - 2 \cos t) e^{2t}, \\ \ddot{y} &= 4e^t + (4 \sin t - 3 \cos t) e^{2t}. \end{aligned} \tag{A-24.1}$$

Letting $\dot{y} = u$, $\ddot{y} = \dot{u} = v$, we may rewrite (A-23) as

$$\begin{aligned} \dot{v} &= 5v - 9u + 5\dot{y}, \\ \dot{u} &= v, \\ \dot{y} &= u. \end{aligned} \tag{A-25}$$

In the matrix form, (A-25) becomes

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{u} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 5 & -9 & 5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ u \\ y \end{bmatrix}. \tag{A-26}$$

Comparing this equation with (A-11) we see that it corresponds to a system with $L_{11} = L_{22} = L_{33} = 1$; $R_{11} = 5$; $R_{12} = -9$; $R_{13} = 5$; $R_{21} = 1$; $R_{22} = R_{23} = R_{31} = 0$; $R_{23} = 1$; $R_{33} = 0$; $v = I_1$; $u = I_2$; $y = I_3$; and with $\dot{Q}_1, \dot{Q}_2, \dot{Q}_3, Q_1, Q_2, Q_3, D_1, D_2, D_3$ missing. Thus it would correspond to three RL circuits with initial values of currents which are coupled by mutual resistive elements so that there are off-diagonal terms in the R submatrix. The code has been modified to include off-diagonal elements in the R submatrix in order to handle this problem. (Off-diagonal elements may also be used in the D submatrix, for completeness, although they are not required here nor in the problems anticipated experimentally.)

This problem has been run with code input of $\delta t = 0.01$ to $\delta t = 10^{-5}$. Since a value of $H \approx 11.4$ is obtained, this gives values of $H \delta t$ ranging from 0.114 to 1.14×10^{-4} . These values of $H \delta t$ are small compared to the range of $H \delta t$ presently built into the code (i.e., $0.1 \leq H \delta t \leq 10$), so the limits were removed for the purpose of testing. For $\delta t = 10^{-3}$, the computed solutions at a value of $t = 10$ deviated from the hand-calculated solutions by $\sim 1.5\%$, 0.004% , and 0.1% for the three variables. A total of four terms of the exponential series were used by the code ($v = 4$). The value of v is printed out in the output, as are the values of H as determined by iteration on an arbitrary vector (see Appendices IV and V). Ten iterations are carried out and the results printed out so that one may see whether the iteration process has

proceeded far enough to produce the desired result. Several of the values of H which are printed out should be identical.

WRITE-UP FOR M-PULSE CODE

The M-PULSE code provides for the sequential solutions of sets of coupled differential equations, such solutions representing the current flowing in each of as many as 14 loops of a circuit at a given time. Each loop of the circuit is taken to be one of the pulsed coils (RLC loop) or one of the passive conducting elements (RL loop), such as a conducting shield for a dc coil.

Each loop of the complete circuit is to be assigned a number, using the ordinal numbers 1, 2, 3, depending on the order in which the loop becomes effective in the circuit. To simplify matters all passive (RL loops) are assigned the numbers 1, 2, 3, . . . , m , where m is the total number of passive loops in the complete circuit, and the data cards covering all the passive loops are entered for the first time interval along with the first RLC loop, or loops, to be initially energized. Thus the first RLC loop to be energized will be numbered $m + 1$, or if there are more than one loop energized initially, they will be numbered $m + 1, m + 2, \dots, m + n$, where n is the number of RLC loops to be energized initially at the same time. At subsequent times, as more RLC loops are energized, they will be numbered in the order in which they are to be introduced into the circuit. When an RLC loop is energized and becomes effective in the circuit, it will be seen that all the values of mutual inductance, L_{ij} , between the loop and all passive loops and all RLC loops previously or simultaneously energized are entered in the inductance matrix. When a loop is crowbarred it retains its original number, even though new data are entered for the elements in that loop. The maximum number of loops permitted in any one problem is 14.

Since the subroutine which accomplishes the matrix inversion of the Y matrix (or inductance grand matrix), as represented by equation (4), is not suitable for inverting a (1×1) matrix, it is necessary to provide a dummy circuit loop whenever a problem is initiated with a single RLC loop. Thus when there are no passive RL loops in the circuit (recall that all passive RL loops which are present initially are to be entered as loops number 1 through m , regardless of whether they are mutually coupled to the RLC loop to be energized initially), a dummy RL loop must be entered as loop number 1 to

accompany the first RLC loop to be energized. There will be no mutual inductance $L_{ij} = L_{j1}$ between this dummy loop and the other loops of the system. Thus L_{11} and R_1 will be the (arbitrary) self-inductance and resistance of the dummy, conveniently taken, say, to be identical to the inductance L_{22} and R_2 of the first RLC loop to be energized. This artifice is necessary to avoid including separately the solutions for a single RLC loop and a single RL loop in the code.

The input data cards for any M-PULSE problem are to be made out according to the attached data card instruction sheets, where the identifying name or symbol for each instruction is listed, with the format for the entry and the number of the card in the data deck on which it is to be entered. The cards for each type of entry are numbered for reference purposes. There may be more than one card of a type, however, and some may be eliminated, so that the total number of cards in the data deck will generally be other than 22.

1st Card - (IDNAME), format (3A6)

The first 18 columns on this first card may be used to cause to be printed out any identifying letters or numbers for a given problem.

2nd Card - (iL, iR, id, iQ, iC) format (5i12)

With respect to the second card, we may note that here an attempt has been made to generalize the code so that the solution of the coupled differential equations is not necessarily restricted to the purpose for which it was originally intended. Thus, although in the circuit problem for which the code was designed, the inductance matrix is symmetric so that $L_{ij} = L_{ji}$, the code provides for the case where $L_{ij} \neq L_{ji}$. Similarly provision is made for nonzero off-diagonal elements in the R and D submatrices, corresponding to resistive or capacitive coupling between loops of the circuit. Usually, the correct entry on the 2nd card, format (5i12), is zero for iL, iR, and id. This signifies that $L_{ij} = L_{ji}$ and that all off-diagonal elements in the resistance (R) and the elastance (D) submatrices are zero. The entry $iQ = 0$ provides that the value of the charge on the capacitor bank in any loop will not be printed out for such time t as the desired solutions for the loop currents, I_i , are printed out. These quantities Q_i are calculated by the code, of course, and may be printed out by the entry $iQ = 1$.

The fifth entry on the 2nd card provides a choice between allowing the code to calculate the time at which all crowbars are to be accomplished ($IC = \emptyset$) or to permit the designation of the time at which each loop is to be crowbarred ($IC = 1$).

The entry $iC = \emptyset$ causes the code to determine the time at which the peak current is attained in any circuit loop and to then crowbar (i.e., remove the loop capacitance (elastance), so that the loop inductance is effectively shorted through the loop resistance). In this case, the crowbar time will be printed out and the code continues to calculate the loop currents on the basis of the data entered under the other instructions, which are discussed below. Under this automatic crowbar mode of calculation it is not possible to specify the time at which any loop is crowbarred — all RLC loops will be automatically crowbarred when the current reaches its peak value in the loop. Thus no entries under the instruction $NT = 3$ (5th card) are permitted when the instruction $IC = 0$ is entered on the second card. The instruction $IC = 1$ eliminates the automatic crowbar feature and the code then relies on the entry $NT = 3$ (coil is crowbarred) on the 6th card to specify that a loop is to be crowbarred, and the time at which it is to be crowbarred will be specified by the entry for $T\emptyset$ on the 5th card. For a circuit which has already been designed and is in operation, the crowbar time will have been determined experimentally and the proper entry is then $IC = 1$.

3rd Card — (NCROW), format (5i12)

The instruction NCROW provides that when $IC = \emptyset$ on the 2nd card, and the automatic crowbar feature is being used, the resistance or inductance elements of any of the circuit loops may be changed when a given loop is crowbarred. This might occur, for example, if a significant portion of the loop inductance or resistance were associated with the loop capacitance. Since the loop capacitance (elastance) is eliminated from the circuit by the crowbar, the remaining loop inductance or resistance would be different from the values previously entered (see 6th and subsequent cards). On the other hand, if any significant value of loop capacitance were to remain in a loop after crowbar (this is very unlikely), the automatic crowbar, $IC = \emptyset$, cannot be used. The proper instruction in this case would be $IC = 1$ on the 2nd card and $NT = 2$ (coil is energized) on the 6th card rather than $NT = 3$ (coil is crowbarred). This would permit the entry of a new value of capacitance (elastance) as well as different values of resistance and inductance in any loop if required. It

would be necessary to enter also the initial charge, Q_i , which would be present on the capacitance remaining in the loop. If this could be ascertained from the value of charge on the original capacitance at the time of crowbar, the entry $NT = 2$ would permit this type of solution; if Q_i is not known, it is not possible. This type of solution is not possible when the automatic crowbar feature is being utilized (i.e., when $IC = \emptyset$ on the 2nd card) since in that case no capacitance (elastance) is retained in the circuit and the solution for the particular loop involved will always be that of an exponentially decaying current plus the induced current from inductive coupling with other active loops, if any. Thus, when a loop is crowbarred, if no changes are to be made in the inductance and resistance elements, and no capacitance remains in the loop, use $NCROW = \emptyset$. This is usually the case. If any changes in inductance or resistance are to be made when $IC = \emptyset$ (automatic crowbar) on the 2nd card, then $NCROW = N$ is the proper entry on the 3rd card, where N is the total number of resistance and inductance elements to be changed for all the automatic crowbars which would occur in the total time interval for the complete problem. If no resistance or inductance element is to be changed for any automatic crowbar, or if automatic crowbar is not being used, then $NCROW = \emptyset$ is the correct entry on the 3rd card. A 3rd card with the $NCROW$ entry must be used whether IC is zero or 1. If $IC = 1$, the proper entry is $NCROW = \emptyset$, and any changes in the values of inductance or resistance will be made under the entry $NT = 3$ on the 6th card.

4th Card - (NLOOP, NTYPE, N_i , N_k , VALUE) format (4i12, E12.0)
(up to $(3 + N)$ th card if $IC = \emptyset$, $NCROW = N$)

If $IC = \emptyset$ on the 2nd card, and $NCROW = N$ on 3rd card, then one card is used for each of the N elements of inductance or resistance which are to be changed in any loop, when the automatic crowbars occur. (NOTE: This 4th card is eliminated if $IC = 1$ on the 2nd card, since then the automatic crowbar feature is not being used, and all data for loop parameters are determined by the NT entries on the 6th card and the data entries on the 7th to 19th cards.) NLOOP specifies the loop number of the loop which is being automatically crowbarred. The particular type of element to be changed is specified by NTYPE, and N_i , N_k are the matrix element subscripts for the row and column of the inductance or resistance element to be changed.

5th Card - (T_0 , T_1 , $dELT$), format (3E12.0)

If $IC = 1$ on the 2nd card (no automatic crowbar) T_0 designates the time at which a loop (or more than one loop) is energized or crowbarred, and the entry T_1 designates the time at which the next loop (or set of loops) is to be energized or crowbarred, or the problem is to be terminated. The entry T_0 for any time interval will be the same as T_1 for the previous interval, except, of course, for the initial T_0 , which will be $T_0 = 0$. During a time interval T_0 to T_1 , the code will produce a new solution for the loop currents I_i (and charges Q_i) at the end of each iteration interval $dELT$, and these solutions may be printed out for any desired time by the proper instructions on the 22nd card. The iteration interval $dELT$ is of course usually very small compared to any interval from T_0 to T_1 (see below). The time interval dT (see entry for dT on 22nd card) which specifies the time lapse between the printouts for the solutions I_i will be some multiple of $dELT$ but generally smaller than T_0 to T_1 .

If $IC = 0$ on the 2nd card (automatic crowbar) then T_0 is used only to designate the time at which an RLC loop is energized. The solutions then proceed to T_1 , for which time another card is prepared with T_0 coinciding with the previous T_1 , the new T_0 being the time at which the next RLC loop is energized. The entry $dELT$ is an estimate of the largest iteration time interval which will yield a reasonable number of terms in the series of equation (11). As pointed out in the mathematical text, a largest value of 10^{-7} sec has been found generally for the type of circuit investigated. Using the largest estimated value, the code then calculates a value of δt (code name $dELT$) which is the largest value of δt which, for a given value of H (see eq. (11)), meets the criterion $0.1 \leq H \delta t \leq 10$. If a smaller value of δt is entered under $dELT$, an unnecessarily large number of iterations may be required for a given time interval from T_0 to T_1 , increasing the running time. If the value of $dELT$ chosen is such as to cause $H \delta t$ to satisfy the above criterion, the code will use that value of δt .

6th Card - (NT), format (i12)

(For subsequent cards, the card number is noted in parentheses as it is introduced.)

The entry $NT = 1, 2$, or 3 is prepared on one card for each shield (at initial T_0) or coil which is to be added to the circuit at T_0 for any given time interval T_0 to T_1 . The entry $NT = 1$ shows that a shield (or passive RL loop)

is being placed in the circuit. $NT = 2$ means that an RLC loop is being energized in the circuit. $NT = 3$ means that an RLC loop is being crowbarred. This last entry is not used if $IC = \emptyset$ on the 2nd card (automatic crowbar). For each passive loop placed in the circuit initially ($NT = 1$), a card (No. 7) is made out for each inductance element L_{ij} , where i has the values 1 to j ($L_{ij} = L_{ji}$ always when L represents inductance).

This means that when $j = 1$, for example, only one value of inductance, L_{11} , which is the self-inductance of the first passive loop, is to be entered. For the second passive loop, $j = 2$, and L_{12} , L_{22} are the two values of inductance to be entered, each on one card, the cards arranged in order of increasing i as shown. Since the entry for the loop resistance R_{ii} (No. 8) has the same format (E12.0), it is essential that the correct order of the inductance and resistance cards following each NT card be maintained. Normally only one value of resistance is used for each loop, since we are not concerned with the solution involving off-diagonal elements R_{ij} , except perhaps for test purposes, so that one card with one entry for the loop resistance will follow the inductance cards. If the code is being used in such a way that the off-diagonal R_{ij} terms, $i \neq j$, are not all zero, the $R_{ij} \neq R_{ji}$, for example, then if $j = 3$, say, the cards would be made for R_{13} , R_{23} , R_{33} , R_{31} , R_{32} , in that order. Thus first the entries are made for the column containing R_{33} , and this is followed by the row containing R_{33} , but the entry R_{33} is not repeated, of course. (The same would apply to entries for L_{ij} above if $L_{ij} \neq L_{ji}$.)

The third item to be entered for the i th passive loop, following the inductance and resistance entries, is the initial value of the loop current, I_i (No. 9). Normally, for any loop $I_i = 0$, so that a blank card is inserted for this item.

For any RLC loop which is energized at time interval T_0 to T_1 , an NT card, format (i12), with the entry 2 is made, followed by a card (or cards) for each of the following items: loop inductance L_{ij} (No. 10), elastance d_{ij} (No. 11), resistance R_{ij} (No. 12), charge Q_i (No. 13), and current I_i (No. 14), in that order. Normally, only one card for elastance, d_{ii} , and resistance, R_{ii} , will be required. (The elastance, d_{ii} is just the reciprocal of the loop capacitance.) Q_i is the product of the voltage (in volts) on the loop capacitance and the loop capacitance (in farads) and is thus in coulombs. Again I_i (amperes) is normally zero, and a blank card is used.

An NT card with the entry 3 is prepared for any loop which has previously been energized and is to be crowbarred at a time T_0 for an interval T_0 to T_1 , if IC = 1 on the 2nd card. Following this card is a card for NC (No. 15), format (i12), the coil number of the coil being crowbarred. The coil number is the number originally assigned to the RLC loop containing this coil. The next card is for NL (No. 16), the number of inductances changed by the crowbarring. Here, inductances refer to the loop self-inductance and any mutual inductances with other coils which might be altered. Normally, $NL = 0$, so a blank card would be used for this entry. If $NL = 0$, the next card for i, j, AEE (No. 17) is omitted. If $NL \neq 0$, the subscript for row and column, i and j, of the inductance matrix element in which the new value of inductance is to be entered, and the value itself (AEE) are entered on the next card with format (2i12, E12.0). For each inductance element to be changed, a card is made. Similarly, for any resistances changed by the crowbarring, an NR card (No. 18) is prepared giving the number of resistances to be changed, and a card for each resistance element changed, giving its matrix position and new value. Normally $NR = 0$, so that a blank card is inserted for NR and the i, j, XXX (No. 19) card is eliminated.

When all the NT and associated cards discussed above have been prepared for a particular T_0 , T_1 , dELT entry, the next card will be the end sentinel (No. 20), which is blank (this is the $NT = 0$ card shown as the 20th card in the data card instructions). Following this end sentinel for the period T_0 to T_1 under consideration is a MAX card (No. 21). On the MAX card is entered the number of print/plot time interval cards which will provide the printed values of the loop currents I_i for various times during the interval T_0 to T_1 , as well as plots of I_i , if so desired, over the corresponding interval. It should be noted here that it is necessary to use one MAX for each interval T_0 to T_1 in which it is desired to produce print/plot output. The print/plot time interval cards (No. 22) follow the MAX card, one card for each interval within T_0 to T_1 for which it is desired to specify a different subinterval dT for which the loop currents I_i (and loop charge Q_i if iQ = 1 on 2nd card of the data deck) are to be printed or printed and plotted. On each time interval card the items TMIN, TMAX, dT, iPLOT, format (3E12.0, i12), are to be entered. All times are in seconds. The first printout will occur at TMIN, a second at $TMIN + dT$, the third at $TMIN + 2dT$, etc., and the last at TMAX. The plot sentinel entry, iPLOT, determines whether the solutions for the

current I_i (and charge Q_i , if $iQ = 1$) are plotted as well as printed. For a given TMIN to TMAX time interval, the entry $iPLOT = 0$ provides that no points are stored for future plotting and no points are plotted at TMAX for this interval. The entry $iPLOT = 1$ provides that the values of I_i and Q_i obtained as a function of time during the interval TMIN to TMAX will be stored for plotting. Thus, if several intervals TMIN to TMAX are connected to form a complete larger interval, it would be desirable to have the solutions plotted as a function of time for the complete interval, rather than for each interval TMIN to TMAX separately. On the other hand, if there is more than one interval TMIN to TMAX, separated by some interval for which no print or plot is desired, then the entry $iPLOT = 2$ should be used for each interval TMIN to TMAX for which a plot is desired. Also, the entry $iPLOT = 2$ should be entered on the card for the last interval of several connected intervals for which a plot is desired covering the complete interval and where the entry $iPLOT = 1$ has been made on the cards for the preceding intervals. It should be kept in mind that each time a loop (or set of loops) is energized and/or crowbarred, a new $T0$ to $T1$ interval is thus created, and following the end sentinel card for this interval (card No. 20), one MAX card, followed by the number of print/plot time interval cards specified thereon, must be prepared if any printed (or printed and plotted) output is desired for any points in the interval $T0$ to $T1$.

Following a set of print/plot time interval cards there may be a new time interval $T0$ to $T1$, and the cards for this interval (5th card through 22nd card) will follow. When the cards for the final $T0$ to $T1$ interval have been entered, a blank card (No. 23) is entered, indicating that the next problem is to be read in, or that control is returned to monitor.

The following input information is printed out for a given time interval $T0$ to $T1$, with iteration interval $dELT$: $T0$, $T1$, $dELT$; N (number of loops in the circuit for this time interval); the inductance submatrix L ; the resistance submatrix R ; the elastance submatrix D ; the current vector I , showing the initial current in each loop; the initial charge vector Q , showing the initial charge on each capacitor bank.

The output information printed out is as follows: the reciprocal of the inductance matrix L ; the A matrix (see equation (4)); 10 values of H (see equation (11) and Appendices IV and V); a new value of $dELT$ if such is required; the number of terms v required in the exponential series $d^{A \delta t}$ (see

equations (7) and (9)); TMIN, TMAX, and DT, specifying the interval in which solutions are to be printed out; and finally, the time (in microseconds) at which a particular solution of the current in each loop is desired, followed by the value of the current in each loop, under the heading "J vector," in the order in which the loops have been numbered in the input. Since 7 columns are available for the values of the loop currents, the values for any particular time are continued on a second line if more than 7 loops are involved, up to the maximum of 14 entries, or 2 lines. It will be noted that the notation used to denote the values of loop currents and other quantities in the printout is a decimal number followed by the letter E, in turn followed by two more digits. The two latter digits are the power of 10 by which the decimal number is to be multiplied to obtain the correct value of loop current.

Finally, if the automatic crowbar feature is being used, the time at which a particular loop is crowbarred will be printed out following the solutions for the loop currents.

The units in which input and output quantities are expressed are as follows: inductance (L) in henries; resistance (R) in ohms; elastance (D) in reciprocal farads (i.e., capacitances are in farads); current (I) in amperes; charge (Q) in coulombs; time (T) in seconds (except that the time at which solutions are printed out is in microseconds in the output).

DATA CARD INSTRUCTIONS (M-PULSE)

* Usual option

<u>Card No.</u>	<u>Code Symbol</u>	<u>Format</u>
1	<u>IDNAME</u>	(3A6)
	Identification for this run (1st 18 columns of card)	
2	<u>iL, iR, id, iQ, iC</u>	(5i12)
	* iL = $\emptyset$ means	$L_{ij} = L_{ji}$ (symmetric inductance submatrix)
	iL = 1 "	$L_{ij} \neq L_{ji}$
	* iR = $\emptyset$ "	$R_{ij} = \emptyset, i \neq j$ (off-diagonal resistance elements zero)
	iR = 1 "	$R_{ij} \neq \emptyset, i \neq j$
	* id = $\emptyset$ "	$d_{ij} = \emptyset, i \neq j$ (off-diagonal elastance elements zero)
	id = 1 "	$d_{ij} \neq \emptyset, i \neq j$

- * $iQ = \emptyset$ means Q_i not printed or plotted
 - $iQ = 1$ " Q_i printed and plotted if I_i printed and plotted
 - $iC = \emptyset$ " Automatic crowbar, crowbar time printed out
 - * $iC = 1$ " No automatic crowbar
- 3 NCROW (i12)
- * $NCROW = \emptyset$ if $iC = 1$ on card No. 2, or if no inductance or resistance elements are to be changed when $iC = \emptyset$ on card No. 2
 - $NCROW = N$ if $iC = \emptyset$ on card No. 2, where N is the number of inductances and resistance elements to be changed for all automatic crowbars which may occur
- 4 NLOOP, NTYPE, N_i , N_k , VALUE (4i12, E12.0)
- (This card is omitted if $NCROW = \emptyset$ on card No. 3)
- (N of these cards if $NCROW = N$ on card No. 3)
- NLOOP = the loop number of the loop being automatically crowbarred
- NTYPE = 1 specifies that an inductance element is to be changed
- NTYPE = 0 specifies that a resistance element is to be changed
- N_i , N_k are the row, column subscripts of the element in the inductance submatrix or resistance submatrix which is to be changed
- VALUE is the new value of inductance (in henries) or the value of resistance (in ohms) to be inserted when the loop designated by NLOOP is automatically crowbarred
- 5 $T\emptyset$, $T1$, dELT (3E12.0)
- $T\emptyset$ is the initial time or when the nth switch (or set of switches) is closed
- $T1$ is the terminal time or when the (n+1)st switch (or set of switches) is closed
- dELT is the iteration interval of time t
- 6 NT (i12)
- NT = 1 means that a passive (RL) loop is placed in the circuit
 - NT = 2 " that an RLC loop is energized
 - NT = 3 " that an RLC loop is crowbarred (not used when $iC = \emptyset$ on card No. 2)
 - NT = $\emptyset$ (see card No. 20, end sentinel)

If NT = 1,

7

L_{ij}

(E12.0)

If $iL = \emptyset$, enter L_{ij} , $i = 1$ to j

If $iL = 1$, enter L_{ij} , $i = 1$ to j , and L_{ji} , $i = 1$ to $j - 1$

8

R_{ij}

(E12.0)

If $iR = \emptyset$, enter R_{ii} , $i = i$

If $iR = 1$, enter R_{ij} , $i = 1$ to j , and R_{ji} , $i = 1$ to $j - 1$

9

I_i

(E12.0)

I_i = initial current when RL loop is placed in circuit (normally $I_i = 0$, use blank card)

If NT = 2,

10

L_{ij}

(E12.0)

If $iL = \emptyset$, enter L_{ij} , $i = 1$ to j

If $iL = 1$, enter L_{ij} , $i = 1$ to j , and L_{ji} , $i = 1$ to $j - 1$

11

d_{ij}

(E12.0)

If $id = \emptyset$, enter d_{ii} , $i = i$

If $id = 1$, enter d_{ij} , $i = 1$ to j , and d_{ji} , $i = 1$ to $j - 1$

12

R_{ij}

(E12.0)

If $iR = \emptyset$, enter R_{ii} , $i = i$

If $iR = 1$, enter R_{ij} , $i = 1$ to j , and R_{ji} , $i = 1$ to $j - 1$

13

Q_i

(E12.0)

Q_i = initial charge on loop capacitance when RLC loop is energized

14

I_i

(E12.0)

I_i = initial current in RLC loop when loop is energized
(Normally, $I_i = 0$, so use blank card here)

If NT = 3,

15

NC

(i12)

NC is the loop number of the loop being crowbarred

16

NL

(i12)

NL is the number of inductance elements to be changed
when coil number NC is crowbarred

If NL = $\emptyset$, this card is blank

- 17 i, j, AEE (2i12, E12.0)
 If $NL = 0$, omit this card
 If $NL \neq 0$, i, j are row, column subscripts of inductance element to be changed;
 AEE is new value of inductance
 NL of these cards are required if $NL \neq 0$
- 18 NR (i12)
 NR is the number of resistance elements to be changed when coil number NC is crowbarred.
 If $NR = 0$, this card is blank
- 19 i, j, XXX (2i12, E12.0)
 If $NR = 0$, omit this card
 If $NR \neq 0$, i, j are row, column subscripts of resistance elements to be changed;
 XXX is new value of resistance
 NR of these cards are required if $NR \neq 0$
- 20 NT (end sentinel) (i12)
 A blank card is inserted here to denote that all data have been entered for this interval T_0 to T_1
- Card after each end sentinel:
- 21 MAX (i12)
 MAX is the number of print/plot cards, equal to the number of TMIN, TMAX, dT, iPLOT cards (see below)
- 22 TMIN, TMAX, dT, iPLOT (3E12.0, i12)
 TMIN is the initial time for print/plot interval
 TMAX is the terminal time for print/plot interval
 dT is the time sub-interval for print/plot
 iPLOT is the plot sentinel
 If $iPLOT = 0$, no points are stored or plotted
 If $iPLOT = 1$, points are stored for plotting
 If $iPLOT = 2$, points are stored and plotted over complete interval ending at time TMAX
- 23 Blank card (end of problem)

```

*ID 410MMJ          MPLSE-14-AUTO 6/4/64      WARREN CUNNINGHAM BOX 67
*   XEQ
*   MAP
*   PANIC          DEC,*
*   CARDS COLUMN
*   LIST
*   FORTRAN        MAIN
C   MAIN
   CALL AL200
   END
*   CARDS COLUMN
*   LIST
*   FORTRAN        MPLS
C   SUBROUTINE AL200
M-PLUSE AL200 WARREN CUNNINGHAM BOX 662 1/17/63 (14 COILS)
COMMON AE, B, D, DI, F, FA, FI, HH, IDNAME, NCOIL, PLOTX, PLOTY,
1 RR, RI, V, VJ, VJD, W, X, XR, XRR, XS, XV, Y, YMIN1, YMAX1, Z,
2 AA, AEE, AJ, BB, BWT, DELT, DEM, DT, FMAX, FMIN, H, I, ID, II,
3 III, IL, INP, IPLOT, IQ, IR, IX, J, K, L, LP, LPP, L11, L111,
4 L112, MAX, MIN, N, NAA, NAAA, NC, NCRT, NDATE, NL, NN, NP,
5 NPLOT, NPOS, NR, NT, NTIME, N1, N11, N111, N2, N22, N221, P, PF,
6 PP, TEST, TMAX, TMAX1, TMIN, TMIN1, TO, TOO, T1, XMAX, XMIN,
7 XVSC, XXX, YMAX, YMIN
COMMON NLOOP, NTYPE, NI, NK, VALUE
COMMON NTSW
COMMON TMIN2, TMAX2, DT2
COMMON IC, KC, NCG, NCROW, NSH, TOM1, TOM
COMMON TO1
COMMON NJ
DIMENSION AE(28,28), B(784), D(28,28), F(28,28), FA(28,28),
1 FI(28,28), RR(784), V(784), W(784), X(28,28), XRR(28,28),
2 Z(28,28), VJ(28), VJD(28), XR(28), XS(28), XV(28), Y(28),
3 YMIN1(28), YMAX1(28), PLOTY(28,501), PLOTX(501), DI(14,14),
4 RI(14,14), NCOIL(14), HH(100), IDNAME(3)
DIMENSION NLOOP(100), NTYPE(100), NI(100), NK(100), VALUE(100)
DIMENSION NJ(14)
EQUIVALENCE (F,W), (FA,V)
CALL ERRORS
55 WRITE OUTPUT TAPE 3, 156
156 FORMAT (1H1)
   NIIPRE = 0
   CALL CLOCK (NTIME, NDATE)
   READ INPUT TAPE 2, 154, (IDNAME(I), I = 1, 3)
154 FORMAT (3A6)
   READ INPUT TAPE 2, 135, IL, IR, ID, IQ, IC
135 FORMAT (5I12)
   DO 149 NCR = 1, 100
149 NLOOP(NCR) = 0
   READ INPUT TAPE 2, 135, NCROW
   IF (NCROW) 147, 148, 147
147 READ INPUT TAPE 2, 140, (NLOOP(NCR), NTYPE(NCR), NI(NCR), NK(NCR),
1 VALUE(NCR), NCR = 1, NCROW)
140 FORMAT (4I12,E12.0)
148 NI1 = 0
   N22 = 0
   L11 = 0
   TMIN1 = 0
   NAA = 0
   NSH = 0
   KC = 0
   TO = 0.
   DO 441 I = 1, 14
441 NCOIL(I) = 1
   DO 403 I = 1, 28
   DO 403 J = 1, 28
   AE(I,J) = 0.
403 X(I,J) = 0.
   DO 499 I = 1, 28
499 AE(I,I) = 1.
   DO 35 I = 1, 14
   DO 35 J = 1, 14
   RI(I,J) = 0.
35 DI(I,J) = 0.
400 IF (NI1) 455, 456, 457
455 STOP
457 DO 453 I = 1, NI1
   DO 453 J = 1, NI1
453 X(I,J) = RI(I,J)

```

```

IF (N22) 458, 456, 459
458 STOP
459 DO 454 I = 1, N11
DO 454 J = 1, N22
K = N11 + J
454 X(I,K) = D1(I,J)
456 READ INPUT TAPE 2, 363, T01, T1, DELT
SENSE LIGHT 1
T00 = T0
IF (T1) 487, 487, 523
481 IF (NCRT) 482, 483, 484
482 STOP
483 NCRT = 1
CALL CRTID (2HAL, 1)
484 IF (NPLOT) 19, 19, 20
20 CALL CARTMM (NPLOT, XMIN, XMAX, PLOTX, 1)
NAAA = 14 + NAA
IF (IQ) 74, 76, 74
76 DO 77 I = 1, 14
77 CALL CARTMM (NPLOT, YMIN1(I), YMAX1(I), PLOTY(I), 14)
CALL CARTMM (14, YMIN, YMAX, YMIN1, 1, YMAX1, 1)
GO TO 78
74 DO 485 I = 1, 28
485 CALL CARTMM (NPLOT, YMIN1(I), YMAX1(I), PLOTY(I), 28)
CALL CARTMM (28, YMIN, YMAX, YMIN1, 1, YMAX1, 1)
78 CALL GRID (14, XMIN, XMAX, 0, 24H TIME IN MICRO-SECONDS, 3,
1 YMIN, YMAX, 0, 48H CURRENT IN AMPERES X 10E4 / CHARGE IN COULOMBS
2, 1)
DO 486 I = 1, N11
CALL XPLOT (PLOTX, 1, NPLOT)
486 CALL YPLOT (PLOTY(I,1), 28, 1)
IF (IQ) 79, 81, 79
79 DO 541 I = 1, NAAA
CALL XPLOT (PLOTX, 1, NPLOT)
541 CALL YPLOT (PLOTY(I,1), 28, 1)
81 DO 540 I = 1, NAAA
DO 540 J = 1, NPLOT
PLOTX(J) = 0
540 PLOTY(I,J) = 0
19 IF (T1) 1, 487, 2
2 NPLOT = 0
GO TO 3
1 NPLOT = 0
GO TO 456
523 READ INPUT TAPE 2, 135, NT
N111 = N11 + 1
N221 = N22 + 1
L111 = L11 + 1
L112 = L11 + 2
IF (NT) 512, 512, 502
502 IF (NT - 2) 139, 501, 506
139 NCOIE(N111) = 0
501 READ INPUT TAPE 2, 56, (AE(I,N111), I = 1, N111)
56 FORMAT (E12.0)
IF (N111) 445, 446, 447
445 STOP
447 DO 525 I = 1, N11
IF (IL) 31, 32, 31
32 AE(N11+1,I) = AE(I,N11+1)
GO TO 525
31 READ INPUT TAPE 2, 56, AE(N11+1,I)
525 CONTINUE
446 IF (NT - 2) 504, 505, 506
505 IF (ID) 26, 27, 26
26 READ INPUT TAPE 2, 56, (X(I,L112), I = 1, N111)
IF (N22) 28, 29, 28
28 DO 30 I = 1, N22
J = N111 + I
30 READ INPUT TAPE 2, 56, X(N111,J)
GO TO 29
27 READ INPUT TAPE 2, 56, X(N111,L112)
29 X(L112,N111) = 1
504 IF (N22) 507, 507, 508
508 DO 509 K = 1, N22
J = L11 + 1 - K

```

```

DO 509 I = 1, N11
X(I,J+1) = X(I,J)
509 X(I,J) = 0.
DO 510 K = 1, N22
I = L11 + I - K
DO 510 J = 1, N11
X(I+1,J) = X(I,J)
507 IF (IR) 33, 34, 33
34 READ INPUT TAPE 2, 56, X(N111,N111)
GO TO 511
33 READ INPUT TAPE 2, 56, (X(I,N111), I = 1, N111)
IF (N11) 13, 511, 17
13 STOP
17 DO 25 I = 1, N11
25 READ INPUT TAPE 2, 56, X(N11+1,I)
GO TO 511
506 READ INPUT TAPE 2, 135, NC
132 NPOS = 0
DO 442 I = 1, NC
IF (NCOIL(I)) 460, 442, 443
460 STOP
443 NPOS = NPOS + 1
442 CONTINUE
NCOIL(NC) = 0
IF (N22 - 1) 463, 61, 462
463 STOP
61 NC = N11 + NPOS
DO 62 J = NC, NC
DO 63 I = 1, N11
63 X(I,J) = 0.
62 VJ(J) = 0.
DO 64 I = NC, NC
DO 64 J = 1, N11
64 X(I,J) = 0.
GO TO 461
462 NC = N11 + NPOS + 1
IF (NC - 28) 124, 124, 125
125 DO 126 I = 1, N11
126 X(I,28) = 0.
VJ(28) = 0.
DO 127 J = 1, N11
127 X(28,J) = 0.
GO TO 461
124 DO 517 J = NC, L11
DO 513 I = 1, N11
X(I,J-1) = X(I,J)
513 X(I,J) = 0.
517 VJ(J-1) = VJ(J)
DO 514 I = NC, L11
DO 514 J = 1, N11
X(I+1,J) = X(I,J)
514 X(I,J) = 0.
461 IF (KC) 141, 133, 141
141 IF (NCROW) 171, 171, 142
142 DO 143 NCR = 1, 100
IF (NCC - NLOOP(NCR)) 143, 144, 143
144 I = NI(NCR)
J = NK(NCR)
IF (INTYPE(NCR) - 1) 145, 146, 145
146 AE(I,J) = VALUE(NCR)
GO TO 143
145 X(I,J) = VALUE(NCR)
143 CONTINUE
171 N22 = N22 - 1
N11 = N11 + N22
IF (NJ(1)) 512, 512, 134
134 DO 136 K = 2, 14
IF (NJ(K)) 172, 172, 136
136 CONTINUE
K = 15
172 NC = NJ(K-1)
NCC = NC
NJ(K-1) = 0
GO TO 132
133 READ INPUT TAPE 2, 135, NL

```



```

      IF (NL) 526, 526, 464
464 DO 527 KAT = 1, NL
      READ INPUT TAPE 2, 528, I, J, AEE
528 FORMAT (2I12,E12.0)
527 AE(I,J) = AEE
526 READ INPUT TAPE 2, 135, NR
      IF (NR) 522, 522, 476
476 DO 529 KAT = 1, NR
      READ INPUT TAPE 2, 528, I, J, XXX
529 X(I,J) = XXX
510 X(I,J) = 0.
      GO TO 522
511 IF (NT - 2) 515, 516, 516
516 READ INPUT TAPE 2, 56, VJ(L112)
515 IF (N22) 448, 449, 450
448 STOP
450 DO 518 K = 1, N22
      J = L11 + 1 - K
518 VJ(J+1) = VJ(J)
449 READ INPUT TAPE 2, 56, VJ(N111)
      IF (NT - 2) 520, 521, 521
521 N22 = N22 + 1
      NAA = NAA + 1
520 N11 = N11 + 1
524 L11 = N11 + N22
      GO TO 523
522 N22 = N22 - 1
      GO TO 524
512 N = N11
      KC = 0
      WRITE OUTPUT TAPE 3, 416, (IDNAME(I), I = 1, 3), INTIME, INDATE
416 FORMAT (1H1,40X,10HINPUT FOR ,5A6)
      WRITE OUTPUT TAPE 3, 417, T01, T1, DELT
417 FORMAT ( // 6H T0 = 1PE12.4,2X,6H T1 = 1PE12.4,2X,8H DELT =
1PE12.4)
      WRITE OUTPUT TAPE 3, 418, N
418 FORMAT ( // 5H N = 12)
      WRITE OUTPUT TAPE 3, 419
419 FORMAT ( // 12H L SUBMATRIX)
      DO 420 I = 1, N
420 WRITE OUTPUT TAPE 3, 421, (AE(I,J), J = 1, N)
421 FORMAT (1P10E12.4)
      WRITE OUTPUT TAPE 3, 422
422 FORMAT ( // 12H R SUBMATRIX)
      IF (N11PRE) 27000, 27000, 27001
27001 DO 27002 I = 1, N11PRE
      DO 27002 J = 1, N11PRE
27002 X(I,J) = - X(I,J)
27000 DO 423 I = 1, N
      423 WRITE OUTPUT TAPE 3, 421, (X(I,J), J = 1, N)
      DO 27003 I = 1, N
      DO 27003 J = 1, N
27003 X(I,J) = - X(I,J)
      N11PRE = N
      IF (N22) 467, 468, 469
467 STOP
469 WRITE OUTPUT TAPE 3, 424
424 FORMAT ( // 12H D SUBMATRIX)
      MIN = N + 1
      DO 425 I = 1, N
425 WRITE OUTPUT TAPE 3, 421, (X(I,J), J = MIN, L11)
468 WRITE OUTPUT TAPE 3, 432, (VJ(I), I = 1, N)
432 FORMAT ( // 9H Q VECTOR / (1P10E12.4))
      IF (N22) 470, 471, 472
470 STOP
472 MIN = N + 1
      WRITE OUTPUT TAPE 3, 433, (VJ(I), I = MIN, L11)
433 FORMAT ( // 9H Q VECTOR / (1P10E12.4))
471 DO 451 I = 1, N11
      DO 451 J = 1, N11
451 RI(I,J) = X(I,J)
      IF (N22) 473, 474, 475
473 STOP
475 DO 452 I = 1, N11
      DO 452 J = 1, N22

```

```

K = N11 + J
452 D1(I,J) = X(I,K)
474 WRITE OUTPUT TAPE 3, 426, (IDNAME(I), I = 1, 3), NTIME, NDATE
426 FORMAT (1H1, 40X, 11HOUTPUT FOR, 5A6)
DO 40 I=1, N11
DO 40 J=1, L11
D(I,J)=X(I,J)
40 X(I,J) = 0.
Y(I)=1.0
CALL MIR (28, N, N, AE, Y, XR, B, RR, W, V)
DO 60 I=1, N
Y(I)=1.0
CALL MIRA
DO 41 L=1, N
XRR(L, I) = XR(L)
DO 11 NNN = 1, N
11 X(L, NNN) = -D(I, NNN) * XR(L) + X(L, NNN)
IF (N22) 38, 41, 39
38 STOP
39 DO 59 NN = MIN, L11
X(L, NN) = -D(I, NN) * XR(L) + X(L, NN)
59 CONTINUE
41 CONTINUE
60 Y(I) = 0.0
WRITE OUTPUT TAPE 3, 427
427 FORMAT ( // 20H L INVERSE SUBMATRIX)
DO 428 I = 1, N
428 WRITE OUTPUT TAPE 3, 421, (XRR(I, J), J = 1, N)
429 WRITE OUTPUT TAPE 3, 429
429 FORMAT ( // 9H A MATRIX)
DO 430 I = 1, L11
430 WRITE OUTPUT TAPE 3, 421, (X(I, J), J = 1, L11)
L = L11
DO 80 I=1, L
DO 80 J=1, L
AA=0.0
DO 75 K=1, L
75 AA=AA+X(K, I)*X(K, J)
80 F(I, J)=AA
DO 97 I=1, L
XV(I) = 0.
DO 97 J = 1, L
97 XV(I) = XV(I) + F(I, J)
XVSC = 1.
83 DEM = 0.
DO 352 I = 1, L
IF (ABSF(XV(I)/XVSC) - 1.E18) 352, 352, 82
82 XVSC = XVSC + 100.
GO TO 83
352 DEM = DEM + (XV(I) / XVSC) ** 2
DEM = SQRTF (DEM) * XVSC
DO 353 I = 1, L
353 XV(I) = XV(I) / DEM
N1=1
N2=10
84 DO 85 I=1, N
DO 85 J=1, N
85 Z(I, J)=F(I, J)
98 DO 120 I1=1, N2
IF (N1 - 100) 354, 354, 355
355 STOP
354 I1 = I1
BWT = 1.
58 BB = 0.
DO 100 M=1, L
B=0.0
DO 99 K=1, L
99 B=B+F(M, K)*XV(K)
XS(M)=B
B = B / BWT
IF (ABSF(B) - 1.E18) 100, 100, 57
57 BWT = BWT * 100.
GO TO 58
100 BB=BB+B**2
DO 104 IZ=1, L

```

```

104 XV(IZ)=XS(IZ)
H = SQRTF(BB) * BWT
HH(II)=H
DO 105 M=1,L
105 XV(M)=XV(M)/H
IF (II - N2) 120, 107, 107
107 IF (HH(II)/HH(II-1)-0.98) 118,118,109
109 IF (HH(II) / HH(II-1) - 1.02) 114, 114, 118
118 WRITE OUTPUT TAPE 3, 119, HH(II), HH(II-1), II
119 FORMAT (//,29H HH(II)/HH(II-1) RATIO FAILED ,2E17.7,17)
N1=II+1
N2=N1
GO TO 98
114 DO 113 I = 1, L
Z(I,I) = F(I,I) - 1.02 * HH(II)
IF (Z(I,I)) 113, 110, 110
110 WRITE OUTPUT TAPE 3,111,I,F(I,I),HH(N2),II
111 FORMAT (//, 28H 1.02H LESS THAN SOME F(I,I) ,I4,2E18.8,17)
112 N1=II+1
N2=N1
GO TO 98
113 CONTINUE
120 CONTINUE
H=1.02*H
NN=N-1
DO 180 I=1,NN
IX = I + 1
DO 180 J = IX, N
AA=Z(J,I)/Z(I,I)
DO 180 K = IX, N
180 Z(J,K)=AA*Z(I,K)+Z(J,K)
DO 195 I=1,N
IF (Z(I,I)) 195,189,189
189 WRITE OUTPUT TAPE 3,190,I,Z(I,I)
190 FORMAT (/,36H NON-NEGATIVE ELEMENT IN GAUSS ELIM. ,I4,E18.7,///)
N1=II+1
N2=N1
GO TO 84
195 CONTINUE
DO 36 JACK = 1, II
36 HH(JACK) = SQRTF(HH(JACK))
WRITE OUTPUT TAPE 3, 37, (HH(JACK), JACK = 1, II)
37 FORMAT (//20H THE VALUES OF H ARE / (1P8E15.7))
H = SQRTF(H)
116 P = H * DELT - 1.
NP=P
IF (NP) 210,210,215
210 NP=1
P=1
215 Z(1,1) = H * DELT
IF (Z(1,1) - .1) 101, 102, 103
101 DELT = DELT * 10.
GO TO 106
103 IF (Z(1,1) - 10.) 102, 108, 108
108 DELT = DELT / 10.
106 Z(1,1) = H * DELT
IF (Z(1,1) - .1) 115, 122, 117
122 WRITE OUTPUT TAPE 3, 123, DELT
123 FORMAT (//26H DELT HAS BEEN CHANGED TO 1PE12.4)
GO TO 116
115 DELT = DELT * 10.
GO TO 106
117 IF (Z(1,1) - 10.) 122, 121, 121
121 DELT = DELT / 10.
GO TO 106
102 DO 230 I = NP, 1000
PP=I+1
INP=I+1
TEST = 1.
DO 439 J = 1, INP
AJ = INP + 1 - J
439 TEST = TEST * (Z(1,1) / AJ)
TEST = TEST / (1.0 - Z(1,1) / (PP + 1.))
IF (ABSF(TEST) - 1.E-8) 235, 230, 230
230 CONTINUE

```

```

STOP
235 LP = PP
LPP = LP - 1
WRITE OUTPUT TAPE 3, 236, LP
236 FORMAT ( // 28H NUMBER OF TERMS REQUIRED = 13)
L = L11
DO 250 I = 1, L
DO 250 J = 1, L
250 FA(I, J) = X(I, J) * DELT
DO 270 I = 1, L
DO 270 J = 1, L
270 FI(I, J) = FA(I, J) / PP
IF (LPP) 47, 48, 49
47 STOP
49 DO 310 II = 1, LPP
PF = 11
DO 280 I = 1, L
280 FI(I, I) = F(I, I) + 1.0
DO 300 J = 1, L
DO 300 K = 1, L
AA = 0.0
DO 290 M = 1, L
290 AA = AA + (FA(M, K) / (PP - PF)) * F(J, M)
300 FI(J, K) = AA
DO 309 IG = 1, L
DO 309 IIG = 1, L
309 F(IG, IIG) = FI(IG, IIG)
310 CONTINUE
48 DO 330 I = 1, L
330 FI(I, I) = F(I, I) + 1.0
434 DO 340 I = 1, L
AA = 0.0
DO 335 J = 1, L
335 AA = AA + F(I, J) * VJ(J)
340 VJD(I) = AA
TOM1 = TO * 1.E6
TOM = (TO + DELT) * 1.E6
IF (IC) 131, 155, 131
155 DO 128 I = 1, 14
128 NJ(I) = 0
I = 0
DO 129 J = 1, N11
IF (NCOIL(J)) 129, 129, 169
169 IF (ABSF(VJD(J)) - ABSF(VJ(J))) 170, 129, 129
170 I = I + 1
WRITE OUTPUT TAPE 3, 138, J, TOM
138 FORMAT ( // 10H LOOP NO. 12, 17H WAS CROWBARED AT F7.1, 15H MICRO-
1 SECONDS.)
NJ(I) = J
129 CONTINUE
IF (NJ(1)) 131, 131, 130
130 NC = NJ(1)
NCC = NC
NJ(1) = 0
NTSW = 1
KC = 1
TO = TO + DELT
TOL = TO
DO 150 I = 1, N11
VJ(I) = VJD(I)
DO 150 J = 1, N11
150 X(I, J) = R1(I, J)
IF (N22) 151, 151, 152
151 STOP
152 DO 153 I = 1, N11
DO 153 J = 1, N22
K = N11 + J
153 X(I, K) = D1(I, J)
GO TO 132
131 IF (SENSE LIGHT 1) 360, 361
360 READ INPUT TAPE 2, 135, MAX
IF (MAX) 86, 86, 96
86 MAX = MAX - 1
GO TO 361
96 READ INPUT TAPE 2, 363, TMIN, TMAX, DT, IPLOT

```

```

363 FORMAT (3E12.0,112)
    MAX = MAX - 1
    NTSW = 1
    TMIN2 = TMIN
    TMAX2 = TMAX
    DT2 = DT
    IF (TMIN - TMIN1) 533, 361, 361
533 STOP
361 TO = TO + DELT
    IF (TO - TMIN) 364, 87, 87
    87 IF (MAX) 364, 365, 365
365 TMIN1 = TMIN
    TMIN = TMIN + DT
    IF (TMIN - TMAX) 366, 366, 21
    21 IF (IPLOT) 367, 367, 22
    22 SENSE LIGHT 3
    IF (IPLOT - 1) 50, 50, 51
    51 SENSE LIGHT 4
    TMAX1 = TMAX
    50 IPLOT = 0
367 IF (MAX) 366, 366, 369
369 READ INPUT TAPE 2, 363, TMIN, TMAX, DT, IPLOT
    SENSE LIGHT 2
    MAX = MAX - 1
    IF (NTSW) 69, 66, 69
    69 WRITE OUTPUT TAPE 3, 431, TMIN2, TMAX2, DT2
431 FORMAT (19H1 TMIN = 1PE12.4, 2X, 8H TMAX = 1PE12.4, 2X, 6H DT =
    1PE12.4 // 19H TIME J VECTOR)
    NTSW = 0
    66 IF (TO - DELT - T00) 91, 91, 159
    91 WRITE OUTPUT TAPE 3, 440, TOM1, (VJ(I), I = 1, N11)
    IF (IQ) 160, 67, 160
160 WRITE OUTPUT TAPE 3, 137, (VJ(I), I = N11, L)
    GO TO 67
159 WRITE OUTPUT TAPE 3, 440, TOM, (VJD(I), I = 1, N11)
440 FORMAT (F7.1, 1X, 1P7E16.7 // (8X, 1P7E16.7))
    IF (IQ) 65, 67, 65
    65 WRITE OUTPUT TAPE 3, 137, (VJD(I), I = N11, L)
137 FORMAT (8X, 1P7E16.7)
    67 NPLOT = NPLOT + 1
    PLOTX(NPLOT) = TO * 1.E+6
    IF (PLOTX(NPLOT) - PLOTX(NPLOT - 1)) 14, 14, 15
    14 PLOTX(NPLOT) = 0
    NPLOT = NPLOT - 1
    GO TO 489
    15 DO 477 I = 1, N
    IF (TO - DELT - T00) 165, 165, 166
165 PLOTY(I, NPLOT) = VJ(I) * 1.E-4
    GO TO 477
166 PLOTY(I, NPLOT) = VJD(I) * 1.E-4
477 CONTINUE
    IF (N22) 488, 489, 490
488 STOP
490 J = N11
    DO 491 K = 1, NAA
    IF (NCOIL(K)) 492, 491, 493
492 STOP
493 J = J + 1
    IF (TO - DELT - T00) 167, 167, 168
167 PLOTY(K+14, NPLOT) = VJ(J)
    GO TO 491
168 PLOTY(K+14, NPLOT) = VJD(J)
491 CONTINUE
489 IF (TMIN - TMIN1) 530, 531, 531
530 STOP
531 NTSW = 1
    TMIN2 = TMIN
    TMAX2 = TMAX
    DT2 = DT
    GO TO 364
366 IF (TO - DELT - T00) 437, 437, 438
437 IF (NTSW) 72, 88, 72
    72 WRITE OUTPUT TAPE 3, 431, TMIN2, TMAX2, DT2
    NTSW = 0
    88 WRITE OUTPUT TAPE 3, 440, TOM1, (VJ(I), I = 1, N11)

```

```

68 IF (IQ) 68, 70, 68
68 WRITE OUTPUT TAPE 3, 137, (VJ(I), I = N11, L)
70 NPLOT = NPLOT + 1
PLOTX(NPLOT) = (TO - DELT) * 1.E+6
IF (NPLOT - 1) 8, 9, 10
8 STOP
10 IF (PLOTX(NPLOT) - PLOTX(NPLOT - 1)) 12, 12, 9
12 PLOTX(NPLOT) = 0.
NPLOT = NPLOT - 1
GO TO 364
9 DO 478 I = 1, N
478 PLOTY(I, NPLOT) = VJ(I) * 1.E-4
IF (N22) 494, 364, 495
494 STOP
495 J = N11
DO 496 K = 1, NAA
IF (NCOIL(K)) 497, 496, 498
497 STOP
498 J = J + 1
PLOTY(K+14, NPLOT) = VJ(J)
496 CONTINUE
GO TO 364
438 IF (SENSE LIGHT 2) 437, 444
444 IF (NTSW) 89, 90, 89
89 WRITE OUTPUT TAPE 3, 431, TMIN2, TMAX2, DT2
NTSW = 0
90 WRITE OUTPUT TAPE 3, 440, TOM, (VJD(I), I = 1, N11)
IF (IQ) 71, 73, 71
71 WRITE OUTPUT TAPE 3, 137, (VJD(I), I = N11, L)
73 NPLOT = NPLOT + 1
IF (TO - T1) 5, 5, 6
6 PLOTX(NPLOT) = T1 * 1.E+6
GO TO 16
5 PLOTX(NPLOT) = TO * 1.E+6
16 IF (PLOTX(NPLOT) - PLOTX(NPLOT - 1)) 18, 18, 7
18 PLOTX(NPLOT) = 0.
NPLOT = NPLOT - 1
GO TO 364
7 DO 479 I = 1, N
479 PLOTY(I, NPLOT) = VJD(I) * 1.E-4
IF (N22) 535, 364, 536
535 STOP
536 J = N11
DO 537 K = 1, NAA
IF (NCOIL(K)) 538, 537, 539
538 STOP
539 J = J + 1
PLOTY(K+14, NPLOT) = VJD(J)
537 CONTINUE
364 IF (SENSE SWITCH 1) 358, 359
358 WRITE OUTPUT TAPE 3, 156
CALL OFFMON
CALL CRTID (2HAL, 1)
359 DO 351 I = 1, L
351 VJ(I) = VJD(I)
IF (SENSE LIGHT 3) 52, 23
52 IF (SENSE LIGHT 4) 24, 53
53 SENSE LIGHT 3
GO TO 23
24 PLOTX(NPLOT) = TMAX1 * 1.E+6
GO TO 481
23 IF (NPLOT - 50) 23, 43, 4
43 IF (SENSE LIGHT 3) 44, 45
44 SENSE LIGHT 3
GO TO 481
45 DO 46 I = 1, NAAA
DO 46 J = 1, NPLOT
PLOTX(J) = 0.
46 PLOTY(I, J) = 0.
NPLOT = 0
GO TO 3
4 STOP
3 IF (TO - T1) 434, 400, 400
487 DO 54 I = 1, NAAA
DO 54 J = 1, NPLOT

```

```

54  PLOTX(J) = 0.
    PLOTY(I,J) = 0.
    NPLOT = 0
    GO TO 55
END
DATA
*
*  CARDS COLUMN
*  FAP MLR
*  R. H. VON HOLT MINIMIZE LINEAR RESIDUALS - LEAST SQUARE ROUTINE
COUNT 280
ENTRY MLRA
ENTRY MLR
L  PZE 0,0,**
MLRA BCI 1,MLR
    SXA XRA,1
    SXA XRB,2
    SXD L,4
    TXI **1,4,10
    SXA SAVEC,4
    TRA VF
MLR  SXA XRA,1
    SXA XRB,2
    SXD L,4
    SXA SAVEC,4
    GLA* 1,4
    STD RD
    PDX 0,2
    SXA CG,2
    SUB* 2,4
    TMI VHE
    CLA* 2,4
C AT 17
C AT 35
M AT 17
M AT 17

```

	PDX	0,2	
	SXA	M,2	
	SUB*	3,4	N AT 17
	TMI	VHE	
	CLA*	3,4	
	PDX	0,2	
	SXA	N,2	
	TNX	VHE,2,1	
	STZ	WS	
	SXA	WS,2	N-1 AT35
	CLA	4,4	
	ADD	ONE	
	STA	AAA	
	STA	RB	
	CLA	5,4	L(Y)
	ADD	ONE	
	STA	YAA	
	STA	YAB	
	CLA	6,4	L(X)
	ADD	ONE	
	STA	XAA	
	STA	XAB	
	STA	XAC	
	STA	XAD	
	CLA	7,4	L(B)
	ADD	ONE	
	STA	BAA	
	CLA	8,4	L(Z)
	ADD	ONE	
	STA	ZAA	
	STA	ZAB	
	STA	ZAC	
	STA	ZAD	
	STA	ZAE	
	STA	ZAF	
	STA	ZAG	
	CLA	9,4	L(W)
	ADD	ONE	
	STA	WAA	
	STA	WAB	
	STA	WAC	
	STA	WAD	
	CLA	10,4	L(L)
	ADD	ONE	
	STA	LAA	
	STA	DONE	
	LDQ	WS	N-1
	MPY	CC	
	XCA	WS	CN-C
	STO	BAA	
	SUB	BAB	
	STA	WS	
	CLA	M	
	ADM	DIM	
	STA	0,2	
	AXT	0,2	
	CLA	0,2	
	STO	0,2	
	TIX	**2,2,1	
	AXT	0,1	
	AXT	0,2	
	CLA	DTWO	
	STO	LE	
	CLA	BAB	
	SXA	DA,2	
	SXA	I,1	
	STA	DB	
	STA	LA	
	TSX	DP+2,4	
	TZE	VHE-2	
	ACL	**15	
	LDQ	**10	
	LGR	28	
	ALS	27	
	STO	WS+2	

SQUARE ROOT OF NORMALIZED FLOATING
POINT NUMBERS. ARGUMENT IN ACCUMULATOR
ON ENTRY. RESULT IN ACCUMULATOR ON EXIT.


```

TOP    **11
STQ    WS+1
LDQ    **4
VLM    WS+1,0,4
ADD    **4
TRA    **11
OCT    771777777731
OCT    400000000007
DEC    784
OCT    1100000000020
OCT    200777777777
LGR    1
STQ    WS+1
LDQ    *-6
VLM    WS+1,0,3
SUB    *-6
STO    WS
CLA    WS+1
VDH    WS,0,15
RGL    20
XCA
SUB    WS
LRS    1
STO    WS
CLA    WS+1
DVH    WS
XCA
ADM    WS
ARS    9
ADM    WS+2
XCA
LXA    DA,2
CLA    DB
LRS    0
STQ    0,1
WAB    FAD    0,1
STO    DB
XCA
FMP    0,1
WAC    ACL    DBL
CHS
STO    0,1
LAA    STO    WS+1
TNX    I,1,1
CLA    CC
ADM    DC
TSX    DP,4
TSX    LC,4
I    TIX    *-4,1,1
AXT    0,1
LXA    DA,2
CLA    CC
ADM    DB
TIX    **1,2,1
VF    TIX    F,1,1
LXA    N,1
LXA    M,2
CLA    DONE
STO    LE
SXA    DA,2
LXA    M,2
YAA    CLA    0,2
ZAA    STO    0,2
TIX    *-2,2,1
LXA    N,2
XAA    STZ    0,2
TIX    *-1,2,1
CLA    ZAA
STA    DB
TSX    DP,4
STO    Y
STO    S
VB    CLA    BAB
LXA    M,2
VA    STA    LA

```

RESTRICTION NON NEGATIVE ARGUMENT

CALLING SEQUENCE

A TSX LCSQRT,4
A+1 ERROR RETURN (-ARGUMENT)
A+2 NORMAL RETURN

ACCURACY 27 BITS

TIMING 1.092 MS. NON ZERO ARGUMENT
0.048 MS. ZERO ARG.

NOTE 1. IF SQ. RT OF 2 TO THE POWER-129
(SMALLEST NORMALIZED NON-ZERO
ARG.) IS ACCEPTABLE FOR SQ. RT.
OF ZERO, THEN 0.024 MS MAY BE
SAVED BY THE ENTRY
A TSX LCSQRT+1,4

NOTE 2. IF NOTE 1 APPLIES AND SQ. RT.
OF ABS. ARG. IS DESIRED THEN
ANOTHER 0.024 MS. MAY BE SAVED
BY THE ENTRY
A TSX LC SQRT+2,4

NOTE 3. IF NOTES 1 AND 2 APPLY, A TOTAL
OF 0.096 MS MAY BE SAVED BY
USING THE OPEN SUBROUTINE
(02-36)

COL TO COL

COL TO COL

N TO XA
M TO XB
FDH L,1

1+L(Z,1)

1+L(B 1,N)

	SXA	DA, 2	
	TSX	DB	
	TSX	DP+3, 4	
	LXA	LC, 4	
	CLA	DA, 2	
	TNX	LA	
	ADM	++3, 1, 1	
	TIX	CC	
	LXA	VA, 2, 1	
BA	SXA	N, 4	N TO XC
	SXA	BD, 2	
ZAB	STA	BE, 4	
WAD	CLA	BC	
ZAC	FDH	0, 2	Z
	STQ	0, 1	W
	STQ	0, 2	Z
	TNX	WS	
BB	TXI	IN, 4, 1	
BC	LDQ	++1, 2, 1	
	FMP	0, 2	
ZAD	FAD	WS	
ZAE	STO	0, 2	Z
	TIX	0, 2	Z
BD	AXI	BB, 4, 1	COUNT
BE	AXI	0, 2	
	CLA	0, 4	
	SUB	BC	
	TXI	CC	COL TO COL
	TXI	++1, 1, 1	
	TIX	++1, 2, 1	
IN	LXA	BA, 4, 1	
	LXA	M, 2	M TO XB
ZAF	CLS	N, 4	N TO XC
XAB	FAD	0, 2	Z
	FRN	0, 4	X
XAC	STO	0, 4	X
	TIX	++1, 2, 1	
	TIX	ZAF, 4, 1	
	LXA	DIM, 4	
RA	LXA	M, 2	M TO XB
YAB	CLS	N, 1	N TO XA
	STO	0, 2	Y
	STZ	WS	
RB	SXA	WS+1	
XAD	LDQ	RC, 4	
	FMP	0, 4	
	STQ	0, 1	X
	FAD	WS+2	
	STO	WS	
	XCA	WS	
	FAD	WS+2	
	FAD	WS+1	
	FAD	WS	
	STO	WS	
	STQ	WS+1	
RD	TIX	++1, 4, 0	COL TO COL DEC
	TIX	RB, 1, 1	
	CHS		
ZAG	STO	0, 2	Z
RC	AXI	0, 4	
	TIX	++1, 4, 1	
	TIX	RA, 2, 1	
	CLA	ZAA	1+L(Z 1)
	STA	DB	
	TSX	DP+2, 4	
	SUB	S	
	TPL	++7	
	ADD	S	
	STO	S	
	LXA	N, 1	
	LXA	M, 2	
	SXA	DA, 2	
	TRA	VB	

	ADD	S
	SUB	Y
	TMI	XRA
	PXA	0,0
	COM	
VHE	LDI	**2
	TSX	\$(EXE),4
	MZE	12,0,L
XRA	AXT	0,1
XRB	AXT	0,2
SAVEC	AXT	**4
	TRA	11,4
CC	PZE	
DONE	FDH	0,1
DTWO	FDH	WS+1
BAB	HTR	0
DP	STA	LB
	STA	LD
	STZ	DC
DA	AXT	WS
DB	LDQ	0,2
DC	FMP	0,2
	FAD	0,2
	STO	WS
	TIX	DB,2,1
	TRA	1,4
LC	ACL	DBL
LE	FDH	0
	STQ	WS
LA	LXA	DA,2
	LDQ	0,2
LB	FMP	WS
LD	FAD	0,2
	STO	0,2
	TIX	LA,2,1
	TRA	1,4
DBL	DEC	188
ONE	HTR	1
Y	PZE	
S	BSS	
WS	END	

4

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission " includes any employee or contractor of the commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.