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ACCELERATOR DEPARTMENT
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Internal Report

MASTER

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PERTURBATION TECHNIQUES FOR THE DEFLECTING MODE

H. Hahn and H.J. Halama

August 22, 1963

I. Introduction

The descriptive parameters of a waveguide with smooth or periodic structure are most easily measured in a waveguide section of suitable length which is transformed into a resonant cavity by placing short circuits at both ends¹. Measurements of dispersion diagram, phase velocity, group velocity, voltage attenuation coefficient, shunt impedance, field configuration, etc. all involve some form of perturbation technique. The introduction of a perturbing object in a resonant cavity changes the stored electric energy W_e and magnetic energy W_m by ΔW_e and ΔW_m , resulting in a frequency shift Δf of the resonant frequency f , which is given by the perturbation formula of J. Müller²:

$$\frac{\Delta f}{f} = \frac{\Delta W_e - \Delta W_m}{W_e + W_m} \quad (1.1)$$

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Perturbation methods for the iris loaded waveguide of a linear electron accelerator are well established and described in many places (e.g. reference 3). Axial bead pulling as aid for a mode identification of the deflecting mode is mentioned by P. Bramham⁴ and in a previous report⁵. Y. Garault used dielectric spheres to obtain a relative field plot of the deflecting mode⁶. R.R. Larsen and G.A. Loew suggest an axial dielectric rod together with an axial dielectric bead perturbation technique for the shunt impedance measurement of a deflecting structure⁷. Electric field measurements done at Stanford are described in reference⁸. G. Saxon, T.R. Jarvis and I. White recently published their measurements of the deflecting mode in linear accelerator structures¹⁴.

The aim of this report is to give an account of the perturbation methods used to obtain a relative plot of the electric field and the value of R/Q. The method proposed here for the R/Q measurement involves radial pulling of a metallic and dielectric bead. The experimental results are compared with theoretical predictions as presented in a previous report⁹.

The measurements were done on a cavity with the dimensions of the ten iris prototype, now being constructed by ARCO, Walnut Creek, Calif. In a unit system with $c = 1$, $\mu_0 = 1$ and a unit of length = $\lambda_0 / 2\pi$, the waveguide dimensions are

$$b = 3.489414$$

$$w = \pi/2 = (1.033 \text{ inch})$$

$$d = 1.256029$$

The geometrical inner radius is

$$a = 1.440784$$

and the equivalent electric aperture, after correction for the rounded iris edges is

$$\begin{aligned} a &= a_{\text{uncorrected}} + (1-\pi/4) (w-d) / 2 \\ &= 1.474541 \end{aligned} \quad (1.2)$$

II. Field Perturbation By a Sphere

A bead of volume V placed in an electric field with the unperturbed total strength E reduces the stored electric energy by an amount

$$\Delta W_e = - \kappa_e V E^2 / 4 \quad (2.1)$$

The correction factor κ is a function of

1. Material, shape and size of the bead
2. Unperturbed field configuration

The bead has to be small enough so that wall effects are negligible.

The change of the stored magnetic energy is given by

$$\Delta W_m = - \kappa_m V H^2 / 4 \quad (2.2)$$

If the bead is placed at a point where both electric and magnetic fields originally existed, the superposition principle may be used to calculate the total effect¹⁰.

An approximate value for κ_e may be obtained from the electrostatic case of a sphere (dielectric constant ϵ) in a uniform electric field. It was pointed out by A. Septier¹¹ that the energy change equals that of the induced electric dipole with the moment

$$M = 3 \frac{\epsilon - 1}{\epsilon + 2} V E \quad (2.3)$$

in the inducing electric field¹² E :

$$\Delta W_e = - \frac{1}{2} M E = - 3 \frac{\epsilon - 1}{\epsilon + 2} V \frac{E^2}{2} \quad (2.4)$$

from what follows

$$\kappa_e = 3 \frac{\epsilon - 1}{\epsilon + 2} \quad (\text{dielectric}) \quad (2.5)$$

The value for a metallic sphere is obtained from the limiting case¹³ $\epsilon \rightarrow \infty$:

$$\kappa_e = 3 \text{ (metal)} \quad (2.6)$$

The correction factors thus derived are identical to those obtained by a different method in reference 10.

The theoretical correction factor is not reliable enough to be used in absolute R/Q measurements. The exact value of κ_e for a metal sphere was therefore obtained by a calibration in a cavity with known field configuration, similar to that of the deflecting mode. The use of metal beads for absolute measurements makes a comparison of results from various laboratories easier and should always be preferred. Dielectric beads (teflon with $\epsilon = 2.1$) were used only for relative field measurements.

III. Bead Calibration

Two beads are used for the perturbation measurements.

1. Metal sphere with diameter $\phi = 0.360$

$$\text{and volume } V = 2.45 \times 10^{-2}$$

2. Teflon sphere with diameter $\phi = 0.450$

$$\text{and volume } V = 4.79 \times 10^{-2}$$

Anticipating the specific application, the beads have to be calibrated in a cylindrical cavity resonator (radius b , length $\ell = 7.021$) which is excited in the TM_{110} mode. Rods as azimuthal mode stabilizers are necessary; their effect on the calibration is however negligible.

The expressions for the field components of the TM_{110} mode are

$$E_r = 0$$

$$E_\theta = 0$$

$$E_z = J_1(kr) \cos \theta$$

(3.1)

$$H_r = -j \frac{J_1(kr)}{kr} \sin \theta$$

$$H_\theta = -j J_1'(kr) \cos \theta$$

$$H_z = 0$$

The resonant frequency $\omega = k$ is given by

$$k = \frac{\chi}{b} \quad (3.2)$$

with $J_1(\chi) = 0$

whence $\chi = 3.8317, J_0(\chi) = -0.4028$

and $k = 1.10$

The stored electric energy is obtained after integration of

$$W_e = \frac{1}{4} \int E_z^2 r dr dz \quad (3.3)$$

The total stored energy $W = 2W_e$ is then

$$W = \frac{\pi}{4} \ell b^2 J_0^2(\chi) \quad (3.4)$$

1. Calibration of Metal Bead

The metal bead will be used to determine the configuration of the electric field in the vicinity of the axis. For the TM_{110} mode, is valid ($\theta = 0, kr < 1$):

$$\begin{aligned} E^2 &= \left\{ \frac{kr}{2} \right\}^2 + \dots \\ H^2 &= \left\{ \frac{1}{2} \right\}^2 + \dots \end{aligned} \quad (3.5)$$

The observed frequency shift vs radial bead position has, therefore, in the vicinity of the axis, the shape of a parabola with the osculation circle ρ and may be written as:

$$\frac{\Delta f}{f} = \frac{\Delta W_e}{W} = - \frac{V}{2\rho} (kr)^2 \quad (3.6)$$

The value of ρ is found from the data in Fig. 1 as

$$\rho \approx 20 \pm 2$$

ΔW_e is defined by (2.1). Together with (3.5):

$$\Delta W_e = - \frac{\kappa_e}{4} V \left(\frac{kr}{2}\right)^2 \quad (3.7)$$

Combination of (3.6), (3.7) and (3.4) leads to

$$\kappa_e = 8 \frac{W}{\rho} = 2\pi \ell b^2 J_0^2(x) / \rho \quad (3.8)$$

with the numerical result

$$\kappa_e = 4.4 \pm 0.4$$

The experimental value for κ_e is considerably larger than the theoretical value of (2.6). It is assumed that this discrepancy is due mainly to the fact that the comparatively large bead is used in a strongly inhomogeneous field.

2. Calibration of the Dielectric Bead

The teflon bead will be used to plot the electric field throughout the cavity. In Fig. 2 the measured frequency shift vs. radial bead position ($\theta = 0$) is compared with the theoretical curve as given by

$$\begin{aligned} \frac{\Delta W_e}{VW} &= - \frac{\kappa_e}{4} \frac{J_1^2(kr)}{W} \\ &= - \frac{3}{\pi} \frac{\epsilon-1}{\epsilon+2} \frac{1}{\ell b^2} \left\{ \frac{J_1(kr)}{J_0(x)} \right\}^2 \end{aligned} \quad (3.9)$$

And with the numerical values:

$$\frac{\Delta f}{Vf} = \frac{\Delta W_e}{VW} = - 1.80 \times 10^{-2} J_1^2(kr)$$

It is found that electric field measurements can be done with an error of $< \pm 5\%$. These limits, however, are no more valid in the vicinity of the axis ($r < 1$).

IV. Field Expansions

A valid interpretation of the perturbation measurements is only possible if the character of the investigated mode is qualitatively known. The mode is completely described by the electric field $\vec{E}$, since the magnetic field is given by $\nabla \times \vec{E} = -jk\vec{H}$. Using the expressions for the field configuration of the deflecting mode in a waveguide⁹, the electric fields in a cavity may be represented by the following infinite series:

$$\begin{aligned} E_r = & 2a_o \beta_o a \frac{J_2(\gamma_o r)}{\gamma_o a J_1(\gamma_o a)} \sin \beta_o z \cos \theta \\ & - \sum_n' 2a_n \frac{\beta_n a}{(\gamma_n a)^2} \frac{\gamma_n a J_1'(\gamma_n r)}{J_1(\gamma_n a)} \sin \beta_n z \cos \theta \\ & - \sum_n 2b_n \frac{J_1(\gamma_n r)}{\gamma_n r J_1'(\gamma_n a)} \sin \beta_n z \cos \theta \end{aligned} \quad (4.1)$$

$$\begin{aligned} E_\theta = & 2a_o \beta_o a \frac{J_2(\gamma_o r)}{\gamma_o a J_1(\gamma_o a)} \sin \beta_o z \sin \theta \\ & + \sum_n' 2a_n \frac{\beta_n a}{(\gamma_n a)^2} \frac{a J_1(\gamma_n r)}{r J_1(\gamma_n a)} \sin \beta_n z \sin \theta \\ & + \sum_n 2b_n \frac{J_1'(\gamma_n r)}{J_1'(\gamma_n a)} \sin \beta_n z \sin \theta \end{aligned} \quad (4.2)$$

$$E_z = \sum_n 2a_n \frac{J_1(\gamma_n r)}{J_1(\gamma_n a)} \cos \beta_n z \cos \theta \quad (4.3)$$

The slope of E_z in radial direction is given by

$$k^{-1} \partial_r E_z = \sum_n 2a_n \frac{\gamma_n J_1'(\gamma_n r)}{k J_1(\gamma_n a)} \cos \beta_n z \cos \theta \quad (4.4)$$

with the shorthand notation $\partial_r = \frac{\partial}{\partial r}$

The field expansion used here is not the only possible one. It has, however, several outstanding properties which strongly recommend its use: The coefficients are all and always real; the expressions remain finite if $\gamma_0 \rightarrow 0$ and the deflecting properties are completely defined by the coefficient a_0 . The most general field representation requires two sets of coefficients a_n and b_n . A minimum of two field measurements is therefore mandatory for their determination. The set a_n alone may be obtained by one measurement of E_z in radial direction. The set b_n then follows from an axial field measurement. The equivalent deflecting field strength E_0 is closely linked to the slope of E_z in radial direction. It is thus evident, that a R/Q measurement is most easily performed by radial bead pulling.

The investigations are limited to the case where the phase velocity equals the velocity of light, i.e. $\gamma_0 = 0$ and $\beta_0 = k$. The field expressions now take the form:

$$E_r = 2a_0 ka \left(\frac{r}{2a} \right)^2 \sin kz \cos \theta - \sum' - 2b_0 \sin kz \cos \theta - \sum' \quad (4.5)$$

$$E_\theta = 2a_0 ka \left(\frac{r}{2a} \right)^2 \sin kz \sin \theta + \sum' + 2b_0 \sin kz \sin \theta + \sum' \quad (4.6)$$

$$E_z = 2a_0 \frac{r}{a} \cos kz \cos \theta + \sum' \quad (4.7)$$

$$k^{-1} \partial_r E_z = 2 \frac{a_0}{ka} \cos kz \cos \theta + \sum' \quad (4.8)$$

The equivalent deflecting field strength E_0 on a synchronous ultra-relativistic particle is therefore

$$E_0 = a_0/ka \quad (4.9)$$

The deflection is aberration-free over the total aperture and the shunt impedance R is a constant: $R \neq R(r, \theta)$.

V. Computed Coefficients

The coefficients in the field expansions a_n and b_n have been computed⁹ on an IBM 7094. The coefficients, normalized to give a unity equivalent electric field strength $E_0 = 1$, are tabulated in Table 1. The correctness of the coefficients obtained may be checked by verifying that the boundary conditions for E_z and E_θ at the (square) irises are satisfied. To do so, the following two field components are computed and plotted in Figs.3 and 4.

1. E_z at $r = a$ and $\theta = 0^\circ$

$$E_z = \sum_{-15}^{15} 2 a_n \cos \beta_n z \quad (5.1)$$

2. E_θ at $r = a$ and $\theta = 90^\circ$

$$E_\theta = 2(a_0 \frac{ka}{4} + b_0) \sin \beta_0 z + \sum_{-15}^{15} 2 \left\{ a_n \frac{\beta_n a}{(\gamma_n a)^2} + b_n \right\} \sin \beta_n z \quad (5.2)$$

It can be seen that the boundary conditions are satisfied: The E_z curve has three zero crossings and E_θ has two zero crossings at the boundary. It is interesting to note, that the maximum of E_z does not occur in the center

of two irises but directly at the irises, thus increasing the danger of breakdown:

$$\frac{\text{maximum } E_z}{\text{center } E_z} \approx \frac{5.08}{3.78} = 1.35$$

The correction factor $2E_o/k^{-1}_0 E_z$ will be required for the R/Q measurement and may be found from

$$\frac{2 E_o}{k^{-1}_0 E_z} = a_o / \left\{ a_o + \sum_n \frac{a_n \gamma_n a}{2 J_1(\gamma_n a)} \right\} \quad (5.3)$$

$$\approx 0.8973$$

VI. The Electric Field Configuration

It was indicated under IV, that the complete determination of the coefficients requires two appropriate measurements.

1. Radial teflon bead measurement

At $z = 0$, $\lambda/2$ etc. and $\theta = 0^\circ$, the total field E equals E_z and is given by:

$$E = 2 a_o \frac{r}{a} + \sum_n 2 a_n \frac{J_1(\gamma_n r)}{J_1(\gamma_n a)} \quad (6.1)$$

In Fig. 5, the computed field strength in the center region is compared to the measured frequency shift caused by the bead pulling. The two curves agree rather well in the vicinity of axis (arbitrary units) and not as well in the neighborhood of the irises, where the influence of the higher harmonics is more pronounced. The exploitation of the experimental curve for the determination of the a_n will be treated under VII.

The curves of Fig. 5 permit to determine the ratio of maximum field to deflecting field in the center of the slot.

$$E_{\max}(\text{center})/E_0 \approx 3.8$$

The maximum occurs inside the slot but close to the slot mouth.

In the case of rounded iris edges this value equals the ratio of absolute maximum to deflecting field. In the case of square irises, the absolute maximum is larger (cf.V) and we find

$$E_{\max}/E_0 \approx 1.35 \times 3.8 = 5.1$$

The value found for the ratio of maximum field to useful deflecting field is measured on a standing wave and has to be halved for a travelling wave.

2. Axial Bead Measurement

The total field on the axis ($r = 0$) is given by

$$E = - 2 b_0 \sin \beta_0 z$$

$$- \sum' \left\{ a_n \frac{\beta_n^2 a^2}{(\gamma_n a)^2} \frac{\gamma_n a}{J_1(\gamma_n a)} + b_n \frac{1}{J_1'(\gamma_n a)} \right\} \sin \beta_n z \quad (6.2)$$

In Fig. 6 the computed field strength is compared to the measured frequency shift. The units are the same as in Fig. 5. A good agreement of the curves is found.

VII. Space Harmonic Analysis

The coefficients a_n in the field expansion may be determined from the experimental curve of Fig. 5. The knowledge of the a_n is required for the correction factor to be used in the R/Q measurement. The experimental data represents $E_z^2(r, 0, 0)$ and may be developed in the region $0 \leq r \leq a$ into the series.

$$E_z(r) = 2 a_0 \frac{r}{a} + \sum_{-K}^K 2 a_n \frac{J_1(\gamma_n r)}{J_1(\gamma_n a)} \quad (7.1)$$

$$= \sum 2 a_n R_n(r)$$

$E_z(r_i)$ are the M ($M > 2K + 1$) experimental points in the region $0 < r < a$. The condition for the best fitting of the experimental data is given by the matrix equation¹⁵

$$M_{mn} a_n = D_m \quad (7.2)$$

with $M_{mn} = \sum_i R_n(r_i) R_m(r_i)$ (7.3)

$$D_m = \sum_i E_z(r_i) R_m(r_i) \quad (7.4)$$

A FORTRAN program was written for the IBM 7094 at Brookhaven to do the numerical computations. The coefficients obtained from the measurement for $K = 1$, $M = 6$ are listed in Table I.

The experimentally determined correction factor is therefore

$$\frac{a_0}{a_0 + \sum_n \frac{a_n \gamma_n a}{2 J_1(\gamma_n a)}} = 0.892 (1 \pm 0.05)$$

which has to be compared with the theoretical value of 0.8973 (cf chapter V).

VIII. Definition of Waveguide Parameters

The maximum transverse impulse p_T on a synchronous ultrarelativistic particle ($v = c = 1$) with the charge q in a lossless waveguide structure of length ℓ is given by

$$p_T = q E_0 \ell = q \sqrt{2P} \ell \quad (8.1)$$

E_o = maximum equivalent electric field strength

P = power flow

and the series impedance Z is defined by

$$Z = \frac{E_o^2}{P} \quad (8.2)$$

In a structure with losses, equation (8.1) has to be substituted by

$$P_T = q \sqrt{Z P_o} \ell \frac{1 - \exp(-\alpha \ell)}{\alpha \ell} \quad (8.3)$$

where the voltage attenuation coefficient α is defined by

$$\alpha = - \frac{\partial_z P}{2 P} \quad (8.4)$$

$\partial_z P$ = powerloss per unit length in waveguide.

If one prefers to use the concept of shunt-impedance R , one has to write

$$P_T = q \sqrt{2RP\ell} \frac{1 - \exp(-\alpha \ell)}{\sqrt{\alpha \ell}} \quad (8.5)$$

where the shunt impedance is defined by

$$R = - \frac{E_o^2}{\partial_z P} \quad (8.6)$$

Shunt impedance and series impedance are related by (8.2, 8.4 and 8.6)

$$Z = 2 \alpha R \quad (8.7)$$

The performances of a deflecting waveguide are completely described by three parameters

1. Z, α, ℓ , or
2. R, α, ℓ

The deflection in an iris loaded waveguide is aberration free and R and Z are therefore constant over the aperture.

Other parameters are occasionally used:

1. Quality factor Q_{guide} which is defined by

$$Q_{\text{guide}} = - \frac{kW}{\partial_z P} \quad (8.8)$$

The stored energy per unit length W is related to the power flow (because of $v_{\text{group}} = v_{\text{energy}}$ in vacuum)

$$P = v_g W \quad (8.9)$$

and therefore (8.4, 8.8, and 8.6):

$$\alpha = \frac{k}{2 v_g Q_{\text{guide}}} \quad (8.10)$$

It is easy to show, that $Q_{\text{guide}} = Q_{\text{cavity}} = Q$ and α may be obtained from a cavity measurement of Q and the group velocity v_g .

2. The parameter R/Q which is given by (8.6 and 8.8)

$$\frac{R}{Q} = \frac{E_o^2}{kW} \quad (8.11)$$

We find the simple relation (8.7 and 8.10)

$$Z = \frac{k}{v_g} \left\{ \frac{R}{Q} \right\} \quad (8.12)$$

and obviously

$$R = \left\{ \frac{R}{Q} \right\} Q \quad (8.13)$$

The importance of R/Q lies in the fact that it is directly measurable by perturbation methods (cf chapter IX).

3. The average power flux density for unit equivalent field strength σ , which is defined by⁹

$$\sigma = \frac{P}{\pi a^2 E_o^2} \quad (8.14)$$

σ is related to Z and R by

$$Z = \frac{1}{\pi a^2 \sigma} \quad (8.15)$$

$$R = \frac{1}{2\pi a^2 \sigma \alpha} \quad (8.16)$$

The use of σ is recommended for a theoretical determination of the waveguide parameters.

IX. R/Q Measurement

In vicinity of the axis, the fields of the deflecting mode may be expanded into a power series:

$$E(r, 0, 0) = k^{-1} \partial_r E_z \times kr + \dots \quad (9.1)$$

$$H(r, 0, 0) = \text{const} + \dots \quad (9.2)$$

The measured frequency shift in a radial metal bead measurement is therefore written as the parabola

$$\frac{\Delta f}{f} = \frac{\Delta W_e}{2W\ell} = -\frac{1}{2\rho} \times V(kr)^2 \quad (9.3)$$

with

$2W\ell$ = total stored energy in cavity.

The measured data of Fig. 7 yields the value

$$\rho = 55 \pm 5$$

The predicted energy change ΔW_e for the calibrated metal bead is given by (2.1 and 9.1)

$$\Delta W_e = -\frac{\pi}{4} \left\{ k^{-1} \partial_r E_z \right\}^2 \times V(kr)^2 \quad (9.4)$$

Comparing (9.3) and (9.4) leads to

$$\frac{W\ell}{\rho} = \kappa \left\{ \frac{k^{-1} \partial_r E_z}{2} \right\}^2 \quad (9.5)$$

and obviously

$$\frac{W\ell}{\rho} = \kappa E_o^2 \left\{ \frac{k^{-1} \partial_r E_z}{2E_o} \right\}^2 \quad (9.6)$$

It follows directly after rearrangement (5.3, 8.11)

$$\frac{R}{Q} = \frac{E_o^2}{\kappa W} = \frac{\ell}{\pi \rho k} \left\{ \frac{a_o}{a_o + \sum \frac{a_n}{2} \frac{Y_n a}{J_1(Y_n a)}} \right\}^2 \quad (9.7)$$

From the numerical values

$$\ell = 5\pi$$

$$k = 1.00$$

$$\rho = 55 (1 \pm 0.1)$$

$$\kappa = 4.4 (1 \pm 0.1)$$

$$a_o / (a_o + \sum ') = 0.892 (1 \pm 0.05)$$

follows the experimental value

$$\frac{R}{Q} = 5.2 \times 10^{-2} (1 \pm 0.17)$$

The computed value⁹ may be found from

$$\frac{R}{Q} = \frac{v_g}{\pi a^2 k} \frac{1}{\sigma} \quad (9.8)$$

$$\left. \begin{array}{l} \sigma = -0.0637 \\ v_g = -0.0198 \end{array} \right\} \rightarrow \frac{R}{Q} = 4.59 \times 10^{-2}$$

An agreement of computed and measured value within the experimental errors is found.

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n	Theory	Experiment
-15	-0.035	
-14	-0.022	
-13	0.083	
-12	-0.124	
-11	0.128	
-10	-0.088	
-9	0.013	
-8	0.125	
-7	-0.209	
-6	0.220	
-5	-0.137	
-4	-0.034	
-3	0.270	
-2	-0.545	
-1	0.962	1.059
0	1.475	1.487
+1	-0.335	-0.717
+2	0.048	
+3	0.133	
+4	-0.222	
+5	0.221	
+6	-0.145	
+7	0.028	
+8	0.095	
+9	-0.126	
+10	0.126	
+11	-0.086	
+12	0.023	
+13	0.040	
+14	-0.082	
+15	0.092	

TABLE I: Coefficients in field expansions

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Diotr,: AD B1, B2

Fig. 1: Calibration of metal head: TM₁₁₀

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Fig. 2: Calibration of teflon bead: TM_{110}

Fig. 3: Computed field component E_z : HEM₁₁

Fig. 4: Computed field component E_θ : HEM_{11}

Fig. 5: Radial bead measurement: HEM₁₁
(Teflon bead)

at $r = 0$

— computed
 x experimental points

Fig. 6: Axial bead measurement: HEM₁₁
 (Teflon bead)

x experimental points

$$— \frac{\Delta f}{Vf} = \frac{-1}{110} (kr)^2$$

Fig. 7: Radial bead measurement: HEM₁₁
 (metal bead)