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QUARTERLY PROGRESS REPORT NO. 14

"Plasma Instabilities and Waves Excited
by an Anisotropic Energy Distribution
Produced by Electron Cyclotron Resonance"

U.S. Atomic Energy Commission
Contract No. AT(11-1) 1695

REPORT NO. COO-1695-31

September 15, 1970

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Madison, Wisconsin 53706

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QUARTERLY PROGRESS REPORT NO. 14

Plasma Instabilities and Waves Excited by an Anisotropic Energy
Distribution Produced by Electron Cyclotron Resonance

Contract No. AT(11-1) 1695

U.S. Atomic Energy Commission

J.L. Shohet
J.C. Sproff
D.G.S. Greene
K.A. Connor
S.K. Nijhawan
K.E. Trnka
R.J. Pedley
J. Schneider

Principal Investigator
Co-Investigator
Graduate Research Assistant
Graduate Research Assistant
Graduate Project Assistant
Student Technician
Student Technician
Student Technician

September 15, 1970

Department of Electrical Engineering
The University of Wisconsin
Madison, Wisconsin 53706

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TABLE OF CONTENTS

- I. INTRODUCTION, - J.C. Sprott
- II. OFF-RESONANCE HEATING STUDIES, K.A. Connor, J.C. Sprott
- III. PLASMA BREMSSTRAHLUNG MEASUREMENTS, D.G.S. Greene,
- IV. A NEW EXPERIMENTAL FACILITY, R.J. Pedley

I. INTRODUCTION

J. C. Sprott

During the period June 15, 1970 to September 14, 1970, the off-resonance heating phenomena discussed in the last Quarterly Progress Report have been studied in considerably more detail, both experimentally and theoretically. Bremsstrahlung measurements have been made and compared with theoretical predictions. A new experimental facility has become operational, and studies of computer simulation of plasmas have continued.

The fluctuation that accompanies lower off-resonance heating has been tentatively identified as a flute instability, and it now appears that plasma is expelled both axially and radially from the mirror field in the presence of lower off-resonance power. Upper off-resonance heating has been observed and measured for plasmas with $kT_e \sim 10$ eV. These measurements required extensive redesign of the magnetron pulse circuitry and diagnostic system to eliminate electrical noise from the diamagnetic loop signal, which is typically 10-100 times smaller than for resonance heating and the construction of a 10 kW, 10 μ sec, 120 pulses/second, 3250 MHz magnetron to produce hotter background resonance plasmas. The upper off-resonance heating rate is proportional to neutral pressure for high pressures and to $\sqrt{kT_e}$ for low pressures. The observations are interpreted in terms of a heating model in which the electron bounce frequency plays the role of a collision rate. Above the cyclotron frequency, the heating is nearly isotropic, but below the cyclotron frequency, the parallel heating dominates, giving rise to enhanced axial diffusion.

Free-free bremsstrahlung measurements have been made and compared with theoretical predictions in an attempt to determine the spatial distribution of energetic electrons in the mirror device. Calculations are for Maxwellian loss cone distributions with various characteristic electron temperatures and various loss cone angles. The experimental results may be matched by the calculated radiation from either a two component model consisting of 31 keV plasma in the heating zone and 15.5 keV plasma filling the remaining volume between the magnetic mirrors, or by many other models with various temperature and density profiles, but not by a single temperature plasma.

A new experimental facility has become operational. It is a 4" diameter glass vacuum system with a set of coils that can be connected to produce a mirror field ($R = 3.8$) or a cusp. Plasmas are produced by a dc discharge, 400 kHz, 1 kW, CW, inductive heating, 10 MHz, 1 kW, CW, resistive heating, or by ECRH using either a 50 W, 3.0 GHz, CW source, or a 10 kW, 10 μ sec, 3.25 GHz pulsed source at 120 pulses/sec. The device is being used to study ion cyclotron resonance heating (using the 400 kHz source), and lower off-resonance heating (using the 10 MHz source).

The computer simulation program is being expanded to utilize more simulation particles, thereby reducing the number of real particles per simulation particle for a given plasma density. This will permit more accurate investigations into thermalization by coulomb collisions and the heating mechanism itself.

Large scale production runs for electron cyclotron simulation will be made in collaboration with S. J. Gitomer.

II. OFF-RESONANCE HEATING STUDIES

K.A. Connor and J.C. Sprott

A. Introduction

Recent experiments at Oak Ridge¹ using microwave heating both above and below the electron cyclotron frequency have produced some spectacular effects. Upper off-resonance heating is found to be very efficient, while lower off-resonance heating results in expelling the plasma from the mirror field. Similar experiments in toroidal octupoles at Wisconsin have also indicated the existence of an upper off-resonance heating mechanism, but lower off-resonance heating does not expel the plasma in an octupole with B_0^2 . As reported in the previous Quarterly Progress Report, similar effects have been observed in a magnetic mirror here at Wisconsin. During the period covered by this report, we have studied the experimental properties of these effects with the goal of unfolding the mechanisms responsible for them. A theoretical model has been developed that explains the results in terms of an effective collision rate. The model predicts nearly isotropic heating above the cyclotron frequency, and strong parallel heating below the cyclotron frequency. Experimental results are presented which confirm the theoretical predictions.

B. Experimental Results

1. Lower off-resonance heating ($\omega < \omega_c$).

One of the effects noted at Oak Ridge during lower off-resonance heating (hereafter LORH) was an increase in radial dif-

fusion by as much as a factor of 2. To see if this effect accompanies LORH in our magnetic mirror device, a Langmuir probe was used to measure the radial density profile. Figure 1 shows a typical case of $\langle n(r) \rangle$ as determined by the ion saturation current to a Langmuir probe for various values of LORH power, 50 μ sec. after the resonant power pulse. The LORH source is a 50 W, C.W. magnetron tuneable from 2.975 - 3.170 GHz. Note that the characteristic annular structure of the plasma is unchanged. The profiles show no distinctive differences other than a decrease with increasing power. The highest LORH power indicated (4 watts) is approximately the maximum that could be applied in this case without extinguishing the plasma. The radial profiles at other times after the resonant power pulse evolved in the same way with increasing LORH power.

When Langmuir probes were used to measure $\langle n(r) \rangle$, fluctuations appear on the probe signal as reported in QPR 12 and QPR 13. Figure 2 shows how these fluctuations are affected by LORH. Notice that they are not normally found in the center of the machine with no LORH, but that they appear there when the LORH is applied. Figures 3 and 4 show the time evolution of the probe signal at various radial positions without and with LORH respectively.

A floating double Langmuir probe was used to measure the fluctuations in electric field found on the edge of the plasma with no LORH and on the axis of the plasma with LORH applied. Figure 5a shows the cases of the double probe electrodes perpendicular and parallel to $\vec{B}$. The absence of the fluctuations in the

Figure 1

Figure 2

Amplitude of Density Fluctuations as Percent
of Density Amplitude vs. Radial Position

50 μ sec after Resonance Pulse

$R = 1.19$

$P = 7 \times 10^{-4}$ Torr

$Z = 15.0$ cm

(a) $r = 0$ cm, $.2$ v/cm
 (b) $r = 1$ cm, $.2$ v/cm
 (c) $r = 2$ cm, $.2$ v/cm

(d) $r = 4$ cm, $.2$ v/cm
 (e) $r = 6$ cm, $.5$ v/cm
 (f) $r = 7$ cm, $.2$ v/cm

Figure 3

Time Evolution of Langmuir Probe Ion Saturation
 Current at Various Radial Positions

For 8.54 GHz Resonant Heating (100 kw-pulsed)

$R = 1.19$

$P = 7 \times 10^{-4}$ Torr

$Z = 15.0$ cm

(a) $r = 0$ cm, $.2$ v/cm
 (b) $r = 1$ cm, $.2$ v/cm
 (c) $r = 2$ cm, $.5$ v/cm

(d) $r = 4$ cm, $.5$ v/cm
 (e) $r = 6$ cm, $.2$ v/cm
 (f) $r = 7$ cm, $.2$ v/cm

Figure 4

Time Evolution of Langmuir Probe Ion Saturation Current
 For 8.54 GHz Resonant Heating (100 kw-pulsed)
 Plus 3.0 GHz LORH (2.5 watts - CW)
 $R = 1.19$; $P = 7 \times 10^{-4}$ Torr; $Z = 15.0$ cm

E

E

(a) No LORH $r = 7$ cm

E

E

← 2.8 msec →
(b) LORH = 2.5 watts $r = 0$ cm

Figure 5
Floating Double Langmuir Probe Measurements
of Fluctuations

parallel direction indicates the existence of flutes. That is to be expected in a mirror confined plasma. The frequency of the observed oscillations is approximately that of the theoretical value for an $m = 1$ flute ($\omega \sqrt{v_B}/r$, where r is distance from mirror axis). Figure 5b shows the results of the same test made when the probe is at the center of the plasma when LORH is applied. The tests in this case show that LORH causes flutes to appear in the center. These fluctuations always appear when LORH is applied but not always with the same amplitude.

A measurement of the axial density profile was made to determine the effect of LORH on it. Figure 6 shows that the axial density peak moves toward the throats of the mirror when LORH is applied. Note that the density gradient to the right of the peak remains approximately the same.

For the purpose of measuring the axial and radial particle fluxes, a new type of electrostatic particle collector³ was used. Figure 7 shows its construction. It consists of fine copper conductors etched 40 to an inch onto a printed circuit board in the pattern shown. The two copper "combs" are completely separate and thus may be biased with different voltages. If they are biased to equal magnitude voltages with opposite polarities both electrons and ions may be collected without producing electric fields more than $\sim 1/40$ " away from the collector.

The collector was first placed on one of the end plugs to monitor axial flux. V-I plots were made with the collector mounted using plasmas produced without LORH. Figure 8 shows a

Figure 6
 Axial Density Profile
 $R = 1.19$, $P = 5 \times 10^{-4}$ Torr,
 50 μ sec after Resonance Pulse

Figure 7
Particle Collector
40 lines/inch

Figure 8

V-I Characteristics of Particle Collector

1 msec after Resonance Pulse

No LORH

$R = 1.02$

$P = 7 \times 10^{-7}$ Torr

$Z = 15.0$ cm

typical V-I characteristic. When the LORH power is increased, the axial flux monotonically decreases. The collector was then placed in the machine as a radial collector. Similar V-I plots were obtained. In this case the particle flux increases by about a factor of two with increasing LORH. This increase is observed when fluctuations appear on the Langmuir probe ion saturation current signal early in time (< 50 μ sec). If fluctuations are present but not early in time the collector signals will decrease monotonically just as in the axial case. The decrease in axial flux is just a result of the decrease in density and does not indicate a decrease in the axial diffusion coefficient. In fact, most of the plasma is lost axially from the device, and so the integrated axial flux is a measure of the total number of trapped particles, whether off-resonance heating is applied or not.

The x-rays produced by wall and free-free bremsstrahlung were measured using a geiger counter with and without the end plugs in place. As reported previously, the time integrated x-ray flux increases monotonically with LORH power both in the radial and axial directions. The radial increase is usually about a factor of 2 while the axial increase has been as high as a factor of 60 with the end plugs in place. When the end plugs are removed, thus decreasing the target area for particles moving parallel to $\vec{E}$, the radial increase in x-ray flux is about the same as with end plugs while the axial increase has been no larger than 6. The radial collimation does not allow x-rays from the end plugs to reach a detector, so one would expect approximately the same flux increase

with and without the end plugs.

The increase in x-ray flux can be attributed to the fact that the confinement time is reduced by the LORH. Thus particles are lost on the average with higher energies. The larger increase with the end plugs in indicates that the particles being lost due to LORH have a more efficiently placed target. That is, particles are less effectively confined and are lost out the mirror throats and strike the end plugs.

The CW magnetron used for LORH was tuneable from 2.925 - 3.170 GHz. When it was tuned through its range, the LORH effect was found to depend on frequency. Depending on the microwave frequency and the position and orientation of the coupling loop, cases were observed where <4 watts of lower off-resonance power would expel the plasma, while other cases showed no effect on the plasma with powers of >50 watts.

The important features of LORH are thus a decrease in density with increasing LORH, the appearance of fluctuations on axis that look like flutes, enhanced radial diffusion, no change in the shape of the radial density profile, enhanced x-ray flux, and change in axial profile.

In an experiment performed on a different mirror machine (see section IV) using plasmas produced by a 10 kW, 10 μ sec, 3250 MHz resonance source, the plasma could be extinguished by applying a 10 MHz, CW, electric field parallel to $\vec{B}$, but not if the same electric field was perpendicular to $\vec{B}$.

2. Upper off-resonance heating ($\omega > \omega_c$).

As reported in QPR #13 an attempt was made to observe upper off-resonance heating (hereafter UORH) using plasmas produced by ECRH with the 50 W, 3.0 GHz C.W. source mentioned above. Any affect present was obscured by electrical noise picked up by the diamagnetic loop. The experimental setup was subsequently modified to minimize electrical noise, and a 10 kW pulsed 3.25 GHz source was built to produce hotter plasmas and thus to improve the signal-to-noise ratio. This source produces 10 μ sec pulses at 120 sec^{-1} while the 8.54 GHz source used for UORH gives 100 kW, 2 μ sec pulses at 360 sec^{-1} .

The high power, short pulse source is ideally suited for UORH because the heating time is short compared with the ionization time and the scattering time into the loss cone, and so the diamagnetic loop signal should give a reliable measure of the power absorbed by the plasma. The loop circuit was modified to extend its bandwidth to 1 MHz, and this has proved adequate for the measurements made. The loop was also calibrated using resonant heating at 8.54 GHz, and the observed signals were in good agreement with the predictions of the resonant heating theory in QPR #12.

If the 3.25 GHz source is operated so as to be at cyclotron resonance with the $\vec{B}$ -field and the 8.54 GHz source being triggered by it to provide the UORH, heating will take place under three distinct conditions of temperature and density. These latter quantities decay in time in the afterglow, which should allow us to see at a glance how the UORH qualitatively depends on $\langle n \rangle$ and

and T_e . We can also vary the pulse spacing continuously. Figure 9 shows the density and temperature as measured by a Langmuir probe in the afterglow of the 3.25 GHz heating. It also shows the average heating rate as derived from the average density ($\langle n \rangle$) and diamagnetic loop voltage (V) as follows:

$$\frac{d\bar{W}}{dt} = \frac{3BV}{2\mu_0 NA \langle n \rangle} ,$$

where N is the number of turns on the loop (16) and A is the loop area. The density used in this equation was derived from Langmuir probe data so it is a local density rather than the average density that the formula calls for. However, comparisons with cavity perturbation density measuring techniques shows this to be a reasonable choice for $\langle n \rangle$. Figure 10 indicates that $d\bar{W}/dt$ is linearly dependent on $\sqrt{kT_e}$ or velocity. The experiments were carried out at high pressures ($\sim 50\mu$) so it was suspected that the heating indicated by the loop signals was a result of electron-neutral collisions. When the 3.25 GHz source was turned off leaving just the 8.54 GHz source on a faint glow could be seen at times indicating that resistive heating was taking place.

The 3.0 GHz C.W. source was used to determine the heating rate of the UORH. Figures 11 and 12 show two sets of experimental data for $d\bar{W}/dt$ vs. pressure as determined from the above equation. Note that there is a linear relationship between $d\bar{W}/dt$ and pressure over much of the range of the log-log plot. At lower pressures the heating rate is linearly related to $\sqrt{kT_e}$ or velocity. This suggests that at high pressures, electron-neutral collisions dominate the

FIGURE 9

HEATING RATE vs $\sqrt{\frac{kT_e}{e}}$

(SAME CASE AS FIGURE 9)

FIGURE 10

Figure 11
Perpendicular Heating Rate vs. Pressure

Heating Rate $\frac{12}{\text{Figure}}$ vs. Pressure

heating, while at low pressures, some other velocity dependent mechanism becomes dominant.

C. Theoretical Model

1. Upper off-resonance heating ($\omega > \omega_c$)

The heating rate for a cold, tenuous, uniform plasma in a uniform magnetic field can be expressed in terms of a real conductivity tensor $\vec{\sigma}$ as

$$\frac{dW}{dt} = \frac{(\vec{\sigma} \cdot \vec{E}) \cdot \vec{E}}{n} = \frac{e^2 E_{\perp}^2 \nu (\omega^2 + \omega_c^2)}{m[\omega^2 - \omega_c^2]^2 + 4\omega^2 \nu^2} + \frac{e^2 E_{\parallel}^2 \nu}{m\omega^2}, \quad (1)$$

where $E_{\perp}$ ($E_{\parallel}$) is the perpendicular (parallel) component of the rf electric field, ω is the applied rf frequency, ω_c is the electron cyclotron frequency, and ν is the collision frequency ($\nu \ll \omega$).

Equation (1) was used in QPR #12 to calculate the stochastic heating of a collection of electrons that traverse a resonance in a non-uniform magnetic field. This was done by integrating equation (1) over the volume of the plasma, assuming $\nu \ll \omega$ so that the dominant contribution to the integral comes from the immediate vicinity of the resonance. The magnetic field was then expanded in a Taylor series about the resonance value, and the resulting heating rate is independent of the collision frequency ν . The result was shown to be consistent with the single particle models of Lichtenberg,⁴ Kuckes,⁵ and Guest,⁶ and with the non-relativistic limit of the results of Grawe.⁷

For $\omega^2 \gg \omega_c^2$ (extreme upper off-resonance heating), equation (1) becomes

$$\frac{d\bar{W}}{dt} = \frac{e^2 E^2 v}{m\omega^2}, \quad (2)$$

which is the usual result for collisional heating in the absence of a magnetic field. The more general case of ω only slightly greater than ω_c is more complicated because the heating rate depends on the geometry of the field and on the particle trajectories (or density distribution). The heating rate (averaged over a bounce period) is calculated from the expression

$$\langle \frac{d\bar{W}}{dt} \rangle = \int n(l) \frac{d\bar{W}}{dt}(l) \frac{dl}{B(l)} / \int n(l) \frac{dl}{B(l)}. \quad (3)$$

Calculations of the heating rate of a hot, relativistic plasma should take into account the following effects:

- 1) Variation of ω_c with energy: $\omega_c = eB/8m$
- 2) Doppler shift of ω due to v : $\omega_D = \omega \pm k v$
- 3) Variation of E over the gyrodiameter of the particle
- 4) Effect of E on the bounce motion
- 5) Non-conservation of the particle's magnetic moment
- 6) Shift in gyrofrequency by perpendicular field gradients (∇B)
- 7) The $\vec{v} \times \vec{B}$ force of the rf magnetic field.

Existing heating models which claim to be relativistic generally consider only a couple of the above effects. By restricting the discussion to the case $\omega^2 \gg \omega_c^2$, the present model can be made relativistic by choosing an appropriate, phenomenological, energy dependent, collision frequency v , which, when substituted into equation (2), gives a heating rate in agreement with experiment.

Physically, ν can result from real particle collisions or from some other mechanism that interrupts the simple periodic variation of the particle's energy in a sinusoidal rf electric field. In the weakly ionized plasmas studied here, electron-ion collisions are negligible compared with electron-neutral collisions. For electron energies greater than a few eV, the electron-neutral collision frequency in helium is

$$\nu_{e-n} = 2.3 \times 10^9 p \text{ (torr)},$$

and the resulting heating rate should vary linearly with neutral pressure in agreement with observations as shown in Figs. 11 and 12. The rf electric field calculated from the input microwave power ($p_0 = 100 \text{ kW}$) and from the unperturbed cavity Q ($=1000$) is given by

$$E_{\text{RMS}} = \frac{p_0 Q}{\epsilon_0 \omega V} = 1.8 \text{ kV/cm}.$$

The unperturbed Q is used because the off-resonance absorption of power by these cold, tenuous plasmas is quite small. Substituting these values into equation (2) gives a heating rate of

$$\frac{d\bar{W}}{dt} \text{ (eV/sec)} = 4.5 \times 10^9 p \text{ (torr)}$$

which agrees within experimental error with the heating rates measured at high pressure.

Figure 12 shows, however, that at low pressure the linear dependence on pressure no longer holds, and the heating rate varies with $\sqrt{kT_e}$. This behavior suggests that the bounce frequency or

the inverse transit time through a cavity mode ($v/\lambda = \omega v/c$), or some other mechanism with a constant mean free path plays the role of an effective collision rate at low pressures. Smullen⁸ and Grawe⁷ assumed (somewhat arbitrarily) that phase randomization occurs after a bounce time, although Grawe strangely calculates a heating rate less than would be obtained by substituting the bounce frequency (ω_β) for v in equation (2). Equation (2) with $v = \omega_\beta$ gives a heating rate in agreement with Grawe's prediction at relativistic energies, but Grawe's result decreases sharply with decreasing energy in contrast to the slow energy variation of the present theory. The heating rate in the present experiment has not been measured with sufficient accuracy to establish that $v = \omega_\beta$, but the heating rate is of the right order, apparently in contradiction to the much smaller result predicted by Grawe.

2. Lower off-resonance heating ($\omega < \omega_c$)

For $\omega^2 \ll \omega_c^2$, equation (1) predicts a heating rate of

$$\frac{d\bar{W}}{dt} = \frac{e^2 E^2 v}{m\omega_c^2} + \frac{e^2 E^2 v}{m\omega^2} . \quad (4)$$

Lower off-resonance heating is anisotropic, in contrast to the upper off-resonance case of equation (2). The first (second) term on the right of equation (4) represents the rate of change of perpendicular (parallel) energy. This result is reasonable since the magnetic field reduces the mobility of the electrons in the perpendicular direction and limits the time during which the

electron stays in phase with the rf electric field. In a multi-mode cavity, on the average, $E_{\perp}^2 = 2E_{\parallel}^2$, so that for $\omega^2 \ll \omega_c^2$, the parallel heating dominates the perpendicular heating by a factor of ω_c^2/ω^2 . The anisotropic heating should cause enhanced axial diffusion in agreement with observations. In the present experiments, the cavity mode number is rather small (~ 65), and so we expect small changes in frequency to change the ratio of $E_{\perp}$ to $E_{\parallel}$ (and thus the diffusion rate). This effect has also been observed. The most convincing experiment, however, is the one in which a 10 MHz electric field was applied perpendicular or parallel to $\vec{B}$ in a mirror system with a plasma produced by ECRH at 3250 MHz. The plasma was expelled with $\vec{E}$ parallel to $\vec{B}$ and unaffected with $\vec{E}$ perpendicular to $\vec{B}$.

The theoretical model can be generalized to arbitrary ω by defining a heating anisotropy,

$$A = \frac{E_{\parallel}^2}{E_{\perp}^2} \frac{d\bar{W}_{\perp}}{dt} / \frac{d\bar{W}_{\parallel}}{dt} = \frac{(\omega^2 + \omega_c^2)\omega^2}{[(\omega^2 - \omega_c^2)^2 + 4\omega^2\nu^2]} \quad , \quad (5)$$

such that $A = 1$ represents isotropic heating. Equation (5) is plotted in Fig. 13 as a function of ω/ω_c for $\nu = 0$ and for $\nu = 0.1$. For $\omega \sim \omega_c$ the heating is highly anisotropic with most of the energy in the perpendicular component as expected for resonance heating. For $\omega > \omega_c$, the anisotropy is always greater than one, and approaches unity at large ω . For $\omega < \omega_c/\sqrt{3}$, the anisotropy is less than one, enhancing axial diffusion. For ω only slightly

FIGURE 13

below the cyclotron frequency ($\omega_c/\sqrt{3} < \omega < \omega_c$), we expect heating to occur without enhanced diffusion.

These results are changed if the rf electric field is strongly polarized, and we must also allow for the possibility of different effective collision frequencies in the perpendicular and parallel directions. The expulsion of the plasma in the Oak Ridge experiments probably results from the strong off-resonance absorption expected and observed for relativistic plasmas. In the present experiment, the plasma is probably easily expelled because of the small mirror ratio (1.02 or 1.19), and because the resonance heating occurs in short pulses after which the large anisotropy can disappear. For relativistic plasmas, the hot electron cyclotron frequency should be used in Fig. 13, so that particles of different energy are heated differently. In a non-uniform magnetic field, such as a mirror, an exact calculation of the heating anisotropy would require an average along the particle trajectories, as indicated in equation (3).

The lower off-resonance power required to expel the plasma in the Oak Ridge experiment can be estimated by assuming a multi-mode cavity with total absorption of both the resonance power (P_R) and off-resonance power (P_{OR}). The plasma is expelled if the midplane anisotropy satisfies

$$\frac{W_{\perp}(0)}{W_{\parallel}(0)} < \frac{1}{R-1},$$

where R is the mirror ratio. Neglecting scattering or other

processes that cause exchange of perpendicular and parallel energy, the respective energies are determined by the ratio

$$\frac{W(0)}{W(0)} = \frac{P_R + P_{OR}(I)}{P_{OR}(II)},$$

where $P_{OR}(I) + P_{OR}(II) = P_{OR}$. Combining these equations with $A = P_{OR}(I)/P_{OR}(II)$ from equation (5), gives

$$P_{OR} = \frac{(R-1)(A+1)}{1-A(R-1)} P_R. \quad (6)$$

For the Oak Ridge case, $A \ll 1$, and the required power is simply

$$P_{OR} = (R-1)P_R \quad (7)$$

In the Oak Ridge experiments, $R \approx 2$, and so we expect the required off-resonance power to be about equal to the resonance power in agreement with observations.

The off-resonance power required to expel the plasma in the present experiment can be estimated in a slightly different way;

$$\frac{d\bar{W}_n}{dt} = \frac{W_n(0)}{t} = \frac{W_{\perp}(0)(R-1)}{t} = \frac{e^2 E_{\perp}^2 v}{m\omega^2},$$

where t is the time between resonance pulses (2.8 msec), and $E_{\perp}^2$ is given by

$$E_{\perp}^2 = \frac{P_{OR} Q_0}{3\epsilon_0 \omega V}.$$

Solving for P_{OR} gives

$$P_{OR} = \frac{3\epsilon_0 m\omega^3 V W_{\perp}(0)(R-1)}{e^2 v t Q_0}$$

Taking $Q_0 = 600$, $R = 1.02$, and $W(0) = 10$ eV, and assuming that the collision frequency measured for upper off-resonance heating holds also for the lower off-resonance case, the calculated power is 2.8 watts, in reasonable agreement with experimentally observed values (Fig. 1).

It may not be necessary to add enough parallel energy to all the particles to put them into the loss cone in order to expel the plasma. A slight axial diffusion, as indicated for example in Fig. 6, could cause flute instabilities resulting in enhanced radial loss as observed experimentally. Furthermore, in a mirror field, there would be a resonance in the heating rate when the parallel rf electric field frequency is equal to the bounce frequency, and the coupling would then become stronger than indicated by equation (4). This effect is probably not very important in the Oak Ridge experiment because the coupling is already very strong, or in the present experiment because the bounce frequency is too small with the relatively cold plasmas produced.

III. BREMSSTRAHLUNG MEASUREMENTS AND COMPARISON TO CALCULATIONS

Figure 14 shows a top view of the experimental apparatus. The plasma at H is confined in the long evacuated cylinder I by the mirror magnetic field from coils J. The mirror ratio is 1 to 1.19 which corresponds to a loss cone angle of 66.5° . Axial x-rays are measured by the counter F through port E. Radial x-rays are measured through port B. The counters are beryllium faced NaI scintillator crystals backed by a 6810A photomultiplier tube. The pulses from the phototube are shaped and then counted by a pulse height analyzer to yield an intensity versus energy spectrum for the x-rays.

A computer program was used to calculate the magnetic field due to the field coils J for a current of 400 amperes, as reported in the 1968 annual progress report.⁹ Since the magnetic field is proportional to the current in the field coils, a contour of constant magnetic field could be relabeled with the value of the magnet current which would produce resonance heating on that contour. This has been done for a resonant field of 3050 gauss in Fig. 15. The magnet current was measured by the voltage drop across a series resistance. The voltage in millivolts multiplied by $20/3$ gives the magnet current in amperes. It was found that on increasing the magnetic field the plasma would go out at 56.30 mv; on decreasing the field the plasma would ignite at 55.20 mv and go out at 46.00 mv or sometimes as high as 47.00 mv.

Bremsstrahlung Experimental Apparatus

(Top View Cut-Away)

- H plasma
- I confining evacuated stainless steel cylinder
- M end plates of confining cylinder
- G tungsten limiter
- J magnetic field coils
- K movable copper end plug
- A movable brass plug
- B,L,E,N,D ports
- C,F photomultiplier tubes; shaded parts are NaI scintillators

Figure 14

A. The scintillator-photomultiplier tube-pulse height analyzer combination was calibrated using various radioactive sources. At different analyzed gain settings, the graphs of the x-ray energy versus analyzer channel number were found to be curves. On log-log paper these graphs were straight lines. These straight lines were interrelated in that the gain versus the inverse slope of the log-log graph was a straight line. See Fig. 16.

When the x-ray source was too intense, pulse pile up resulted. Pile up means that two or more smaller pulses arrive simultaneously and are recorded as a larger pulse. This tends to increase the channel numbers of the x-rays and so move the characteristic peaks from the calibrating radioactive sources to larger channel numbers. The percent of dead time for the analyzer is a measure of the frequency of arrival of x-rays. For dead times less than 10% no displacements of calibration peaks nor changes of slope were observed. For later measurements dead times were kept to less than 2% and usually much less by limiting x-ray intensities with pinholes.

The phototubes were always shielded with both lead foil and netic, conetic, and soft iron shields so that calibration curves in the axial and radial positions were indistinguishable, even with the magnetic field on.

B. Tests to determine whether the bremsstrahlung occurs from wall encounters or free-free interplasma encounters were conducted. Referring to Fig. 14, counter C was aligned with port B and the

Typical Calibration Curves for X-ray Spectrum Measurements

Fig 16

diametrically opposite port D using a laser beam. Pinholes limited the field of observation to the brass plug A. As A was inserted into the cylinder the x-ray intensity sharply increased. See Fig. 17. The slope of the x-ray spectra stays nearly constant as the plug is inserted, perhaps becoming only slightly flatter at the discontinuity of Fig. 17. With the brass plug removed and as counter C was moved in the z direction or in the up and down direction, the x-ray intensity again sharply increased as the field of observation began to include the confining cylinder wall at the edge of port D. See Fig. 18. The x-ray intensity dropped to background intensity at sufficiently large displacements such that the phototube could no longer see through the pinhole at port B into the plasma. The asymmetry appeared because the pinhole at B was not carefully set perpendicular to the confining cylinder. The asymmetry was later shown to be reversible by purposefully misaligning the pinhole.

Similar tests were made on the axial x-rays. Up and down displacement and radial displacement of the counter F produced the same characteristic graphs as in Fig. 18. In this case the x-ray intensity sharply increased as the limiter G entered the field of observation. Corresponding to the radial insertion of the brass plug A, a perforated copper end plug K was inserted. However, as the plug entered the mirror throat, the plasma was extinguished. On withdrawing the plug some distance the plasma reignited. Fig. 19 is a graph of this effect as a function of plasma resonance heating location. Axial x-rays increased sharply on insertion of

Insertion of Brass Plug

Fig 17

Variation in Position of Photomultiplier
Tube for Viewing Plasma

Fig 18

Condition of Plasma as a Function of End Plug Position for Various Resonant Heating Locations

Fig 19

a target into port L. Large deflecting magnets placed outside the confining magnetic fields were meant to deflect the electrons escaping axially from striking either of the end plates M. No changes in x-ray intensity or slope of spectra were observed on employing these magnets.

In conclusion it is assumed that only free-free bremsstrahlung will be observed when the fields for counters C and F are blank spaces and not metallic targets.

C. Radial spectra through both ports B and N and axial spectra were observed for various conditions of the plasma. A coincidence gating circuit allowed x-ray counts only during the power pulse and for about 20 microseconds thereafter. Background radiation during that time was negligible. Photographs of the pulse height analyzer display were taken. See Fig. 20. The photographs were measured with a movable stage microscope. This measured data was supplied to a computer programmed to make calculations and corrections to yield final data for a graph of x-ray intensity (from the plasma source) versus x-ray energy. The following corrections were made:

1. The decade markings on the photograph did not form an exactly rectangular grid and so corrections for distortion were employed such that the measured positions of the decade markings would be adjusted to lie on evenly spaced horizontal lines. This also would correct for any tilt of the photograph on the microscope stage.

Drawing of Photograph of Pulse Height Analyzer Display

Fig 20

2. Channel number was translated into x-ray energy by considering the calibration curves referred to in part A.
3. Absorption corrections due to the mylar window covering the port, the air path, and the beryllium window of the NaI crystal were made. The correction due to the air path was the largest of all the corrections considered in either the theoretical calculations or the experimental measurements.
4. The efficiency of the NaI crystal was calculated.
5. The pinhole geometry and volume of plasma observed were considered.

Graphs of the final data for the resonance heating zone located at the center of the plasma and the lowest pressure for a sustained plasma are given in Fig. 21 for axial x-rays and radial x-rays viewed at the plasma's center. Figure 22 shows a graph of the central radial x-ray spectra for various locations of the resonance zone. The axial x-rays remain essentially unchanged for these different locations. The off-center radial x-ray spectra viewed from port N (which is located 15.2 cm from the center of the plasma and 7.6 cm from the mirror throat) have not yet been analyzed.

D. Calculations of x-ray spectra have been reported in previous progress reports.¹⁰ The experimental axial and radial spectra of Fig. 21 have a greater vertical separation and also diverge much more rapidly than any of the calculated spectra reported so far. Electron energy distributions with characteristic

X-ray Spectra

Fig 21

Parallel X-ray Spectra for Various Magnet Currents

Fig 22

temperatures of above 160 keV are required to produce theoretical spectra with either sufficient vertical separation or sufficient divergence to match the experimental results, but at such high electron temperatures the x-ray spectra are much too flat.

Although the perpendicular experimental spectrum is matched quite well by the perpendicular theoretical spectrum for a loss cone electron energy distribution with characteristic temperature of 31 keV and loss cone angle of 67.5° , the parallel spectrum is not at all matched. See Fig. 21. Thus with the given theoretical calculations, the experimental results cannot be matched by a single electron temperature for the extended plasma volume.

However a variety of models with regions of different electron temperatures can be used to match the experimental results of Fig. 8. For example: a two component model consisting of 31 keV plasma in the heating zone and 15.5 keV plasma filling the remaining volume between the magnetic mirrors produces the spectra marked by (x) on Fig. 8.

E. Proposed research in an effort to find a unique model of the plasma to match Fig. 21 will include:

1. Analysis of the off-center spectra taken through port N.
2. Analysis of the central radial spectra for heating not in the center of the confining cylinder.
3. Axial insertion of a scintillator probe to determine electron energies as a function of position.

IV. A NEW EXPERIMENTAL FACILITY

R. J. Pedley

Since the last progress report, a new piece of apparatus has become available. It is a pyrex system, similar in structure to the other available apparatus which is described in a previous progress report.¹¹ The pyrex system, shown basically in Fig. 23, although lacking the sophistication of the other apparatus, has certain unique advantages due to its nonconducting properties and its transparency. It has a roughing pump, a diffusion pump, and necessary pressure monitoring equipment with a standard helium leak. The brass plates have ports for probes and also serve as electrodes. When a D.C. discharge is used, electrodes can be inserted within the system to increase stability. The magnets are water cooled and are rated at 18 volts, 3.5 Gauss per amp with a maximum of 550 amps. The current source for the magnets is a 3 phase 220 volt arc welder. After modification, the welder can produce 0 to 550 amps at 22 volts D.C. with less than 5% ripple. It was determined that, due to the heating and subsequent change in magnet resistance, a period of about 15 minutes is required for the magnet current to reach equilibrium. At a steady state current of 370 amps, the magnetic field along the center of the system was determined using a small Gaussmeter and a transverse probe. The results are shown in Fig. 24. From this, the mirror ratio was found to be 3.8:1.

PYREX SYSTEM

FIGURE 23

FIGURE 24

Several methods are available for producing and heating plasmas in this system. Some methods produced problems which are unique to the pyrex system. Since the system is new, many methods were investigated so as to determine which were acceptable. The different arrangements are listed.

D.C. discharge - a high voltage supply produces 5 kilovolts at 23 milliamps. When this is applied between the end plates, instabilities in the plasma occur due to varying discharge characteristics caused by pressure gradients within the system. Applying the D.C. voltage between two closer spaced electrodes solves the problem.

A.C. discharge (60 Hz) - supplied by a 10 kilovolt current limited transformer. When this is applied between the end plates, the plasma is again somewhat unstable.

10 MHz discharge - supplied to the end plates by a 1 kilowatt R.F. generator. This produces a stable plasma.

400 kHz inductive heating - several turns of wire around the cylinder near the center are excited by a 1 kilowatt generator.

C.W. microwave heating - radiated into the cylinder at one of the end plates. The supply is a 2.975 - 3.170 GHz, 50 watt magnetron unit. This unit can be pulsed when coupled to a pulse generator. At times, a rectangular screen waveguide was inserted in the cylinder in order to better locate the resonance heating zone where desired.

Pulsed microwave heating - supplied by a 10 kilowatt magnetron unit. The pulses are 10 microseconds in duration at a rate of

120 per second with a frequency of 3.294 - 3.263 GHz.

Using the various heating and discharge schemes, the various plasmas were thoroughly investigated. Considerable data was taken using basic Langmuir probe techniques.¹² Having overcome some of the inherent problems associated with the pyrex system, such as insufficient ground paths and R.F. interference appearing in the probe circuits from the heating source, acceptable data were obtained. Single and double probes of various tip area and shape were used both axially and radially. A typical V.I. characteristic is shown in Fig. 25. Assuming thermal equilibrium, the current in the transition region from V_f to V_p is given by

$$i(V) = i(V_p) \exp \left[\frac{e(V-V_p)}{kT_e} \right]$$

Solving for the electron temperature gives

$$T_e = \frac{k}{e} \left[\frac{d(\ln i(V))}{dV} \right]^{-1}$$

From Fig. 25, the electron temperature is calculated to be 570 eV.

Once the electron temperature has been determined, the density (n) can be derived from the electron saturation current:

$$i_{oe} = \frac{1}{4} n e \bar{V}_e$$

where $\bar{V}_e$ is the average electron speed. For a Maxwellian distribution

$$\bar{V}_e = \sqrt{\frac{8kT_e}{\pi m_e}}$$

FIGURE 25

therefore

$$n = \frac{4 I_{oe}}{e \sqrt{\frac{8kT_e}{\pi m_e}}}$$

From Fig. 25 , the density can be calculated to be $5 \times 10^9 / \text{cm}^3$.

At the time of this publication, off-resonance heating is just being started on this system and the results are only qualitative. A resonance heated plasma is produced by the pulsed microwave unit. The 10 MHz is applied between the end plates, parallel to the magnetic field. When the 10 MHz is turned on the plasma is extinguished. When the power of the 10 MHz is increased the plasma is again present, indicating only that the 10 MHz discharge is producing the plasma. When the 10 MHz is applied to plates perpendicular to the magnetic field, the plasma could not be extinguished. This is evidence in support of the off-resonance heating theory by J.C. Sprott to be found elsewhere in this progress report.

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