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ON THE SOLUTION OF THE NUCLEAR THREE
BODY PROBLEM IN COORDINATE SPACE†

MASTER

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ABSTRACT

A new method to find the wave functions and binding energy of the three nucleon system is discussed. Results of the calculations with local, central potentials are given. The wave functions are found to be sensitive to the shape of the nucleon-nucleon potential. The corresponding binding energies differ by less than 0.6 MeV.

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In the many attempts to find the binding energy (B.E.) of the three nucleon system it has most often been assumed that it can be found without a detailed knowledge of the radial wave functions (the part of the wave function depending on three scalar coordinates only, referred to later as r.w.f.). This is true in many applications of the variational method¹⁻⁵, where the r.w.f. are explicitly approximated by simple analytical expressions. The more recent methods⁶⁻⁹ use different expansions of the wave function (or potential) and only the first few terms of the expansion are used to estimate the B.E. The methods mentioned here work well in the cases of model nucleon-nucleon interactions as for example a pure central interaction³ or a separable interaction.⁷ In the most interesting case of the complete realistic interactions only the variational method has been tried^{4,5} and so far it has not been fully successful.

Recently, a new method was reported¹⁰ which is aimed at finding the r.w.f. as well as the B.E. In this method the values of the r.w.f. at certain points (mesh points) of configuration space are used as variational parameters. The configuration space is a space of the three distances between three nucleons. To define a continuous r.w.f. the configuration space is divided into tetrahedron-shaped cells. This is done so that the overlapping faces of adjacent tetrahedrons are always identical and their overlap is complete. On each tetrahedron there are 4 mesh points in the corners and 6 mesh points on the edges halfway between the corners. The r.w.f. inside a tetrahedron is defined by the second order polynomial interpolation from the values at the 10 mesh points lying on this tetrahedron. The r.w.f. so defined is continuous everywhere in configura-

tion space and its dependence on variational parameters is linear. There are no further restrictions on the choice of the tetrahedrons. Usually it is profitable to make them larger in the regions of space where less accuracy is needed. It is because of this change in the size of tetrahedrons the mesh points in Fig. 1 are not equally spaced. Because of the linear dependence of the parameters the usually difficult search for variational parameters is reduced to the matrix eigenvalue problem. This allows for a large number of parameters necessary to map the r.w.f. accurately. The total number of mesh points used is 2270. An iterative procedure¹⁰ has been used to solve the eigenvalue problem. This procedure was described independently by Sebe and Nachamkin¹¹.

In the work reported here the described method has been used to find the S-state component of the G.S. wave function of ${}^3\text{H}$ and ${}^3\text{He}$ with the like nucleons in the singlet state. After other components of the wave function are neglected the Schrodinger equation is reduced³ to the following form:

$$\{K + V^S(r_2) + V^S(r_1)/4 + V^S(r_3)/4 + 3V^t(r_1)/4 + 3V^t(r_3)/4 - E\} \psi(r_1, r_2, r_3) = 0,$$

where

$$K = \frac{\hbar^2}{M} \left[\text{cyclic} \left\{ \frac{1}{r_1} \frac{\partial^2}{\partial r_1^2} r_1 + \frac{r_2^2 + r_3^2 - r_1^2}{2r_2 r_3} \frac{\partial^2}{\partial r_2 \partial r_3} \right\} \right],$$

r_2 is the distance between the like nucleons, r_1 and r_3 are the other two distances between nucleons, V^S and V^t are the central parts of the singlet and triplet potentials and $\frac{\hbar^2}{M} = 41.47 \text{ MeV f}^2$.

If an average singlet-triplet potential is used then ψ becomes the totally symmetric S-state¹². Central potentials with a hard core, reproducing the low energy scattering parameters, have been used and are listed in Table I. The computing time to find a totally symmetric ψ for each potential is about 10 min. on the CDC 6500 computer with 34000 locations of the central memory used. This time can be reduced by a factor of 4 or more if a big auxiliary storage is used.

Some of the results obtained with the calculated wave functions are given in the rows 2 to 7 of the Table 2. In the row 1 a variational wave function³ was used. The only results for ${}^3\text{He}$ are in row 4. Numbers in column 2 refer to a potential in the Table 1. It is indicated if an averaged singlet-triplet potential was used. The error with which energies in the columns 3,4,5 were calculated depends on the size of the mesh only and is difficult to estimate. A very conservative estimate gives the error value 0.2 MeV which is probably much too large. In the columns 6 and 7 r.m.s. radii are given. They include nucleon $R_{\text{r.m.s.}} = 0.77\text{f}$. Therefore a proton radius is equal to a nuclear charge radius. Coulomb energy difference is found from 3 and 4 to be 0.73 MeV. The same value is obtained when the expectation value of the Coulomb potential is calculated with the functions 3 or 4. This value is too small by about 0.02 MeV because the proton-neutron mass difference has been neglected in the calculation.

The comparison of the variational³ and the exact calculations (1 and 2) shows that the correct B.E. is obtained with the variational function although it fails to reproduce the radius and potential and kinetic energies.

Only a part of so called S' state¹² is included in this calculation. In the case of the exponential potential (2 and 3) its probability is 1.0% and the B.E. gain is 0.40 MeV. For the Gaussian potential (5 and 6) the B.E. gain is 0.55 MeV. A rough estimate gives 1% probability for the remaining part of the S'-state. The total probability 2% is in agreement with other calculations¹³.

The r.w.f. calculated (3,6 and 7) with the three central potentials are different mainly at small distances as shown in Fig. 1. The corresponding differences in radii are less than 2% and the differences in the kinetic and potential energies are as big as 7 MeV (12%). The resulting B.E. are less than 1 MeV from the experimental value and lie within a half MeV range from each other. This supports a view that the B.E. of three nucleons is independent of the shape of nucleon-nucleon potential--at least for central potentials with the same hard core radius. So far it has been shown only the separable potentials⁷ that the B.E. is not changed much when a tensor force is included. The extension of the present method to the case of a realistic interaction is straightforward and the computing time should not be prohibitive with existing computers.

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FIGURE CAPTION

Figure 1. R.w.f. $\psi(r_1, r_2, r_3)$ at the mesh points lying on the line $r_1=r_2=r_3$. The cut-off distance not shown here is at $7f$.

TABLE CAPTIONS

Table 1. The potentials and their parameters.

Table 2. The energies and radii calculated with different potentials.

TABLE 1.--The potentials and their parameters.

No.	Potential	$V(r)$	Singlet		Triplet	
			V_0 (MeV)	μ (f ⁻¹)	V_0 (MeV)	μ (f ⁻¹)
1 ^a	exponential	$-V_0 e^{-\mu(r-d)}$	330.794	2.4021	475.044	2.5214
2	Gaussian	$-V_0 e^{-[\mu(r-d)]^2}$	98.799	.9652	164.253	1.0832
3	modified Yukawa	$\frac{-V_0 (r-d)^{1/2} f}{0.2f^{3/2} + (r-d)^{3/2}} e^{-\mu(r-d)}$	202.006	1.7902	274.425	1.8453
	hard core radius $d=0.4f$		$a_s = -23.06f$ $r_{os} = 2.4012f$		$a_t = 5.353f$ $r_{ot} = 1.722f$	

∞

^aThe same as in Ref. 3.

TABLE 2.--The energies and radii calculated with different potentials.

Case	Potential	Energy (MeV)			Nuclear Density radius(f)	
		Binding	Potential	Kinetic	Unlike	Like
1 ^a ${}^3\text{H}$	1 averaged ^b	8.85	-76.99	68.14	1.62	1.62
2 ${}^3\text{H}$	1 averaged ^b	8.78	-70.55	61.77	1.72	1.72
3 ${}^3\text{H}$	1	9.21	-71.95	62.74	1.65	1.75
4 ${}^3\text{He}$	1+Coulomb	8.48	-70.27	61.79	1.655	1.77
5 ${}^3\text{H}$	2 averaged ^b	8.33	-62.87	54.54		
6 ${}^3\text{H}$	2	8.84	-64.41	55.57	1.68	1.78
7 ${}^3\text{H}$	3	9.30	-73.74	64.44		
Experimental		8.49(${}^3\text{H}$) 7.73(${}^3\text{He}$)			1.70 ^c (${}^3\text{H}$)	1.87 ^c (${}^3\text{He}$)

^aVariational calculation Ref. 3. ^bR.w.f. totally symmetric in space. ^cCharge radius.

Figure 1

$$\psi(r_1, r_2, r_3)$$