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TRIP-1--A TWO-DIMENSIONAL P-3 PROGRAM IN X-Y GEOMETRY FOR THE IBM-704

July 1960

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PITTSBURGH, PENNSYLVANIA**

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A TWO-DIMENSIONAL P-3 PROGRAM IN X-Y GEOMETRY
FOR THE IBM-704

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ABSTRACT

TRIP-1 solves the one-group P_3 equations in XY geometry. It has been used to investigate two-dimensional transport effects, and to study the feasibility of the method of solution. The program is an extensively modified version of PDQ-3 (Ref. 3).

TRIP-1
A TWO-DIMENSIONAL P-3 PROGRAM IN X-Y GEOMETRY
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E. Gelbard, J. Davis, J. Dorsey, H. Mitchell, J. Mandell

I. INTRODUCTION

The TRIP-1 program, written for the IBM-704^{*}, is designed to solve the P_3 equations in X-Y geometry. Since two-dimensional transport effects are not yet well understood, TRIP-1 has been regarded, primarily, as an exploratory device rather than a production tool. Consequently all non-essential features have been excluded from the program, which treats only one-group cell problems. The cell is assumed to be rectangular, with regionwise constant cross sections. The source is isotropic and regionwise flat. Anisotropic scattering is dealt with rigorously (within the limitations of a P_3 approximation) and P_0 through P_3 scattering components are allowed.

In two dimensions, as in one, the P_3 approximation has important limitations. For this reason, a cell problem should be examined closely before the TRIP-1 solution is requested. It is often possible to assess the accuracy of P_3 in a given X-Y problem through study of related one-dimensional problems. Further, it may be desirable in some cases to check selected features of the TRIP-1 solution against a Monte Carlo calculation.

The derivation of TRIP-1 finite difference equations is discussed in Section III, below. It will be seen that the TRIP-1 and PDQ (Ref. 1) difference equations are quite similar, and that the order of accuracy of the difference equations is the same in both programs. In general, however, the P_3

^{*}The program requires 32,768 words of core storage. See Section X for other requirements.

scalar flux varies more sharply with position than does the P_1 . This is particularly true in the neighborhood of interfaces. If the fine detail of the P_3 solution is not to be lost or distorted, it may be necessary to use more mesh points in a TRIP-1 problem than in the corresponding PDQ. Again, it is often possible to arrive at a satisfactory and economical mesh structure through a one-dimensional P_3 study. Since TRIP-1 allows no more than 2500 interior mesh points, and since the running time increases rapidly with the number of points, the mesh layout should be chosen with great care.

II. DERIVATION OF DIFFERENTIAL EQUATIONS

It is convenient to take as a point of departure the spherical harmonics equations in Davison's (Ref. 2) compact notation:

$$\begin{aligned} (2n - 1) \bar{U} \cdot \nabla_r \psi_{n-1} - U^2 \nabla_u \cdot \nabla_r \psi_{n-1} + \nabla_u \cdot \nabla_r \psi_{n+1} \\ + (2n + 1) (\Sigma_T - \Sigma_{sn}) \psi_n = (2n + 1) S_n \end{aligned} \quad (1)$$

Here Σ_{sn} is the n 'th Legendre component of the scattering cross section.

Equations (1) are identical with Davison's except for the addition of anisotropic source and scattering terms. In a P_3 approximation

$$\nabla_u \cdot \nabla_r \psi_1 + \Sigma_0 \psi_0 = s_0 \quad , \quad (2)$$

$$\nabla_u \cdot \nabla_r \psi_2 + \bar{U} \cdot \nabla_r \psi_0 + 3 \Sigma_1 \psi_1 = 0 \quad , \quad (3)$$

$$\nabla_u \cdot \nabla_r \psi_3 + 3 \bar{U} \cdot \nabla_r \psi_1 - U^2 \nabla_u \cdot \nabla_r \psi_1 + 5 \Sigma_2 \psi_2 = 0, \text{ and} \quad (4)$$

$$5 \bar{U} \cdot \nabla_r \psi_2 - U^2 \nabla_u \cdot \nabla_r \psi_2 + 7 \Sigma_3 \psi_3 = 0 \quad (5)$$

if the source is isotropic.

Upon elimination of the odd moments, one is led to two second-order equations:

$$\begin{aligned} -\nabla_r \cdot \left\{ \frac{1}{3\Sigma_1} \left[\nabla_u (\nabla_u \cdot \nabla_r) \psi_2 + \nabla_u (\bar{U} \cdot \nabla_r) \psi_0 \right] \right\} + \Sigma_0 \psi_0 = S_0, \text{ and} \quad (6) \\ -\nabla_r \cdot \left\{ \frac{1}{7\Sigma_3} \left[5 \nabla_u (\bar{U} \cdot \nabla_r) \psi_2 - \nabla_u (U^2 \nabla_u \cdot \nabla_r) \psi_2 \right] - \frac{1}{\Sigma_1} \left[\bar{U} (\nabla_u \cdot \nabla_r) \psi_2 \right. \right. \\ \left. \left. + \bar{U} (\bar{U} \cdot \nabla_r) \psi_0 \right] \right\} \\ + 5 \Sigma_2 \psi_2 = U^2 (S_0 - \Sigma_0 \psi_0). \quad (7) \end{aligned}$$

In the above equations ψ_0 is simply the scalar flux, henceforth denoted by ϕ , while

$$\begin{aligned} \psi_2 = \frac{1}{5} U^2 \left[\psi_{20} P_2^0(\mu) + \psi_{21} \cos \lambda P_2^1(\mu) + \psi'_{21} \sin \lambda P_2^1(\mu) \right. \\ \left. + \psi_{22} \cos (2\lambda) P_2^2(\mu) + \psi'_{22} \sin (2\lambda) P_2^2(\mu) \right]. \quad (8) \end{aligned}$$

The angles λ and $\theta = \cos^{-1} \mu$ are, respectively, azimuthal and polar angles with respect to the z axis, while the P_ℓ^m are associated Legendre polynomials.

Introducing the components of $\bar{U}$, one may write:

$$\psi_2 \equiv \frac{1}{5} \left[\frac{1}{2} (3 U_z^2 - U^2) \psi_{20} + 3 U_x U_z \psi_{21} + 3 U_y U_z \psi_{21} + 6 U_x U_y \psi_{22} + 3 (U_x^2 - U_y^2) \psi_{22} \right] \quad (9)$$

If the vector flux is symmetric about the xy plane, then $\psi_{21} = \psi_{21}' = 0$. In terms of the new variables

$$M \equiv -\frac{1}{10} \psi_{20} + \frac{3}{5} \psi_{22} \quad (10)$$

$$P \equiv -\frac{1}{10} \psi_{20} - \frac{3}{5} \psi_{22} \quad , \quad \text{and} \quad (11)$$

$$B \equiv \frac{6}{5} \psi_{22}' \quad (12)$$

ψ_2 then takes the form

$$\psi_2 \equiv (U_x^2 - U_z^2) M + (U_y^2 - U_z^2) P + U_x U_y B. \quad (13)$$

Substituting this explicit expression for ψ_2 into Eq. (6), one finds that

$$-\nabla_r \cdot \left\{ \frac{1}{3\epsilon_1} \left[\left(\frac{\partial T}{\partial x} + 2 \frac{\partial M}{\partial x} + \frac{\partial B}{\partial y} \right) \hat{i} + \left(\frac{\partial T}{\partial y} + 2 \frac{\partial P}{\partial y} + \frac{\partial B}{\partial x} \right) \hat{j} \right] \right\} + \epsilon_0 \varphi = S_0 \quad (14)$$

Similarly, definition (13) may be introduced into Eq. (7). It is easy to show that the resulting coefficients of U_x^2 , U_y^2 , U_z^2 and $U_x U_y$ must vanish. In consequence, four conditions are imposed on the moments, but only three are linearly independent:

$$\begin{aligned}
& - \nabla_{\mathbf{r}} \cdot \left\{ \left[\frac{1}{7\Sigma_3} \left(9 \frac{\partial M}{\partial x} - 3 \frac{\partial B}{\partial y} \right) + \frac{1}{\Sigma_1} \left(2 \frac{\partial M}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial \Phi}{\partial x} \right) \right] \hat{i} \right. \\
& \quad \left. + \left[\frac{1}{7\Sigma_3} \left(4 \frac{\partial B}{\partial x} + 5 \frac{\partial M}{\partial y} - 2 \frac{\partial P}{\partial y} \right) \right] \hat{j} \right\} \\
& + 5 \Sigma_2 M = S_0 - \Sigma_0 \Phi, \quad (15)
\end{aligned}$$

$$\begin{aligned}
& - \nabla_{\mathbf{r}} \cdot \left\{ \left[\frac{1}{7\Sigma_3} \left(9 \frac{\partial P}{\partial y} - 3 \frac{\partial B}{\partial x} \right) + \frac{1}{\Sigma_1} \left(2 \frac{\partial P}{\partial y} + \frac{\partial B}{\partial x} + \frac{\partial \Phi}{\partial y} \right) \right] \hat{j} \right. \\
& \quad \left. + \left[\frac{1}{7\Sigma_3} \left(4 \frac{\partial B}{\partial y} + 5 \frac{\partial P}{\partial x} - 2 \frac{\partial M}{\partial x} \right) \right] \hat{i} \right\} \\
& + 5 \Sigma_2 P = S_0 - \Sigma_0 \Phi, \quad \text{and} \quad (16)
\end{aligned}$$

$$\begin{aligned}
& - \nabla_{\mathbf{r}} \cdot \left\{ \left[\frac{1}{7\Sigma_3} \left(8 \frac{\partial B}{\partial x} + 10 \frac{\partial M}{\partial y} - 4 \frac{\partial P}{\partial y} \right) + \frac{1}{\Sigma_1} \left(\frac{\partial B}{\partial x} + 2 \frac{\partial P}{\partial y} + \frac{\partial \Phi}{\partial y} \right) \right] \hat{i} \right. \\
& \quad \left. + \left[\frac{1}{7\Sigma_3} \left(8 \frac{\partial B}{\partial y} + 10 \frac{\partial P}{\partial x} - 4 \frac{\partial M}{\partial x} \right) + \frac{1}{\Sigma_1} \left(\frac{\partial B}{\partial y} + 2 \frac{\partial M}{\partial x} + \frac{\partial \Phi}{\partial x} \right) \right] \hat{j} \right\} \\
& + 5 \Sigma_2 B = 0. \quad (17)
\end{aligned}$$

The functions Φ , M , P and B are, then, determined by Eqs. (14) through (17), supplemented by approximate boundary conditions. From the definitions of these functions it follows that Φ , M and P are symmetric about the cell boundaries, while B is antisymmetric.* Continuity conditions at

*It is assumed here that the boundaries are parallel to the coordinate axes.

interfaces will be stated in Section III, below.

It has been verified that Eqs. (14) through (17) are invariant under 45° and 90° rotation, and that they pass to the correct one-dimensional limit.

From Eq. (13), it follows that

$$F_{2x} \equiv \int \frac{(3\Omega_x^2 - 1)}{2} F(\hat{\Omega}, \hat{r}) d\hat{\Omega} = M \quad \text{and}$$

$$F_{2y} \equiv \int \frac{(3\Omega_y^2 - 1)}{2} F(\hat{\Omega}, \hat{r}) d\hat{\Omega} = P \quad .$$

Therefore, in one-dimensional x and y geometries, M and P, respectively, should play the role of F_2 . Examination of Eqs. (14) through (17) will show that this is, indeed, the case.

III. DERIVATION OF DIFFERENCE EQUATIONS

The two-dimensional P_3 equations may be cast into finite difference form, and then solved by standard iteration procedures. In the following discussion it will be assumed that all interfaces lie on mesh lines. Nine neighboring mesh points are shown in Fig. 1. Difference equations at point 0 will be derived by integration, as in PDQ. In order to illustrate this procedure, all the necessary integrals will be evaluated over region A, the region bounded by lines (0,9), (9,10), (10,11) and (11,0). First it is convenient to abbreviate the P_3 equations which may be written in the simple form:

$$\nabla_r \cdot \bar{V}_I + F_I = 0, \quad 1 \leq I \leq 4. \quad (18)$$

MESH RECTANGLE SURROUNDING ZEROth MESH POINT

Figure 1

As in PDQ,

$$\int_A \nabla_r \cdot \bar{V}_I dA + \int_A F_I dA \approx \int_C V_{In} dS + \frac{1}{4} h_1 h_2 F_{I0} , \quad (19)$$

where V_{In} is the normal component of $\bar{V}_I$. Ultimately, contributions from the four quadrants surrounding point 0 will be added. Now, it can be shown, through a "pillbox" argument, that V_{In} is continuous across interfaces. Consequently, the line integrals along segments (0,9) and (0,11) will be cancelled by contributions from other quadrants. Only integrals along (9,10) and (10,11) need be computed. These integrals consist of sums of terms, of the following types:

$$I_1 = \int_{(9)}^{(10)} \frac{\partial}{\partial x} G(x,y) ds , \quad (20)$$

$$I_2 = \int_{(9)}^{(10)} \frac{\partial}{\partial y} G(x,y) ds , \quad (21)$$

$$I_3 = \int_{(10)}^{(11)} \frac{\partial}{\partial y} G(x,y) ds, \quad \text{and} \quad (22)$$

$$I_4 = \int_{(10)}^{(11)} \frac{\partial}{\partial x} G(x,y) ds , \quad (23)$$

where the symbol $G(x,y)$ may designate any one of the functions ϕ , M , P or B .

Integrals of type 20 and 22 occur in the derivation of PDQ difference equations.

They will be approximated as in PDQ:

$$I_1 \approx \frac{h_2}{2} \left[\frac{G_1 - G_0}{h_1} \right] , \quad (24)$$

$$I_3 \approx \frac{h_1}{2} \left[\frac{G_2 - G_0}{h_2} \right] \quad (25)$$

To the same order

$$I_2 \approx \frac{1}{2} \left[\frac{G_2 + G_5 - (G_0 + G_1)}{2} \right], \quad \text{and} \quad (26)$$

$$I_4 = \frac{1}{2} \left[\frac{G_2 + G_5 - (G_0 + G_2)}{2} \right].$$

The above approximations define the TRIP-1 difference equations. Boundary conditions may be imposed without difficulty. The cell boundaries are located halfway between mesh rows or mesh columns. Suppose, for example, that a cell boundary passes between points 0 and 1 (Fig. 1). Then, for symmetric G's, $G_5 = G_2$, $G_1 = G_0$, $G_8 = G_4$. If G is to be antisymmetric, $G_5 = -G_2$, $G_1 = -G_0$, $G_8 = -G_4$.

The exact form of the difference equations is exhibited in full detail in Section VI.

IV. METHOD OF SOLUTION

Equations (14) through (17), in difference form, are solved by a combination of inner and outer iterative processes. Equation (14) is used to compute ϕ , with terms in M, P and B treated as constituents of the source. Similarly, Eqs. (15), (16) and (17) are solved for M, P and B, respectively.

A regionwise flat guess for ϕ is entered as input; initial values of M, P and B are taken to be zero. Equation (14) is then solved by simultaneous line overrelaxation, as in PDQ-3 (Ref. 3) using PDQ-3 routines. It should be

noted that Eq. (14), with M, P and B set equal to zero, is simply the P_1 diffusion equation.

Having solved Eq. (14), the newly computed ϕ is used to calculate the source in Eq. (15), (with P and B still zero); Eq. (15) is solved next. Subsequently, Eq. (16) and (17) are solved (in that order), whereupon the outer cycle is repeated.

Throughout the outer cycles, all source terms are computed from the latest iterates of the moments. All equations are solved by simultaneous line overrelaxation, with optimum ω 's computed for each equation. These ω 's are determined during the course of the first outer iteration. Convergence of the outer cycle is not accelerated, but the unaccelerated convergence rate is generally fast.

V. CONVERGENCE CRITERIA

Separate convergence criteria are used to terminate the inner and outer iterative processes. Inner iteration is continued, in each equation, until

$$\left| \frac{G_I^{(n+1)} - G_I^{(n)}}{G_I^{(n+1)}} \right| < \epsilon (2 - \omega_I)$$

at all mesh points. Here $G_I^{(n)}$ and $G_I^{(n+1)}$ denote successive inner iterates of the independent variable in the I 'th equation, while ω_I is the computed overrelaxation factor. It will be seen that a single ϵ is specified for all four equations. However, the factor $2 - \omega_I$ automatically refines the convergence criterion when the convergence rate is low. As in PDQ-3, this device is used to prevent false convergence.

The outer cycle is terminated when

$$\frac{\varphi^{(N+1)} - \varphi^{(N)}}{\varphi^{(N+1)}} \leq \eta \quad (28)$$

In Eq. (28) $\varphi^{(N+1)}$ and $\varphi^{(N)}$ denote successive outer iterates of the scalar flux. No attempt is made, here, to adjust the convergence criterion to the convergence rate. This precaution does not seem to be necessary in view of the rapid cycle.

Most TRIP-1 problems have been run, at Bettis, with $\eta = .001$, $\epsilon = .001$. It should be noted that the TRIP-1 ϵ is, generally, coarser than the inner cycle criterion required in a one-energy P_1 problem. This is permissible since each TRIP-1 equation is entered repeatedly during successive outer cycles.

VI. DIFFERENCE EQUATIONS IN DETAIL

The TRIP-1 difference equations are exhibited below^{*}:

$$\begin{aligned} - \left[\sum_{n=1}^4 \alpha_n + \frac{3}{2} X \right] \frac{\varphi_0}{2} + \frac{1}{2} \sum_{n=1}^4 \alpha_n \varphi_n - (\alpha_1 + \alpha_3) M_0 + \alpha_1 M_1 + \alpha_3 M_3 - (\alpha_2 + \alpha_4) P_0 \\ + \alpha_2 P_2 + \alpha_4 P_4 + \frac{\delta}{2} B_0 + \frac{1}{2} \sum_{n=5}^8 (-1)^{n+1} \frac{B_n}{\delta_{n-4}} = - \frac{3}{4} \xi; \quad (29) \\ - \left[\alpha_1 + \alpha_3 + \frac{9}{14} (\omega_1 + \omega_3) + \frac{5}{14} (\omega_2 + \omega_4) + \frac{5}{4} Y \right] M_0 + \left(\alpha_1 + \frac{9}{14} \omega_1 \right) M_1 \\ + \left(\frac{5}{14} \omega_2 \right) M_2 + \left(\alpha_3 + \frac{9}{14} \omega_3 \right) M_3 + \left(\frac{5}{14} \omega_4 \right) M_4 - \left(\alpha_1 + \alpha_3 + \frac{X}{2} \right) \frac{\varphi_0}{2} + \left(\frac{\alpha_1}{2} \right) \varphi_1 \end{aligned}$$

^{*}Subscripts on φ , M , P and B denote mesh points, as in Fig. 1.

$$\begin{aligned}
& + \left(\frac{\alpha_3}{2} \right) \varphi_3 + \frac{(\omega_2 + \omega_4)}{7} P_0 - \left(\frac{\omega_2}{7} \right) P_2 - \left(\frac{\omega_4}{7} \right) P_4 + \left(\bar{\delta} + \frac{\bar{\mu}}{7} \right) \frac{B_0}{4} \\
& + \frac{1}{4} \sum_{n=1}^4 B_n C_{n,n-1} + \frac{1}{4} \sum_{n=5}^8 (-1)^{n+1} B_n a_n = -\frac{\xi}{4} ; \quad (30) \\
& 1 \leq (n-1) \leq 4
\end{aligned}$$

$$\begin{aligned}
& - \left[\alpha_2 + \alpha_4 + \frac{5}{14} (\omega_1 + \omega_3) + \frac{9}{14} (\omega_2 + \omega_4) + \frac{5}{4} Y \right] P_0 + \left(\frac{5}{14} \omega_1 \right) P_1 \\
& + (\alpha_2 + \frac{9}{14} \omega_2) P_2 + \left(\frac{5}{14} \omega_3 \right) P_3 + (\alpha_4 + \frac{9}{14} \omega_4) P_4 - (\alpha_2 + \alpha_4 + \frac{Y}{2}) \frac{\varphi_0}{2} \\
& + \left(\frac{\alpha_2}{2} \right) \varphi_2 + \left(\frac{\alpha_4}{2} \right) \varphi_4 + \frac{(\omega_1 + \omega_3)}{7} M_0 - \left(\frac{\omega_1}{7} \right) M_1 - \left(\frac{\omega_3}{7} \right) M_3 + (\bar{\delta} + \frac{\bar{\mu}}{7}) \frac{B_0}{4} \\
& - \frac{1}{4} \sum_{n=1}^4 B_n C_{n,n-1} + \frac{1}{4} \sum_{n=5}^8 (-1)^{n+1} B_n A_n = -\frac{\xi}{4} ; \quad (31) \\
& 1 \leq n-1 \leq 4
\end{aligned}$$

$$\begin{aligned}
& - \left[\sum_{n=1}^4 (\alpha_n + \frac{8}{7} \omega_n) + \frac{5}{2} Y \right] B_0 + \sum_{n=1}^4 (\alpha_n + \frac{8}{7} \omega_n) B_n + (\bar{\delta}) \varphi_0 + \sum_{n=5}^8 (-1)^{n+1} \frac{\varphi_n}{\delta_{n-4}} \\
& + (\bar{\delta} + \frac{3}{7} \bar{\mu}) P_0 + \sum_{n=1}^4 C_{n,n-1} P_n + \sum_{n=5}^8 (-1)^{n+1} b_n P_n + (\bar{\delta} + \frac{3}{7} \bar{\mu}) M_0 \\
& - \sum_{n=1}^4 C_{n,n-1} M_n + \sum_{n=5}^8 (-1)^{n+1} b_n M_n = 0 ; \quad (32) \\
& 1 \leq n-1 \leq 4
\end{aligned}$$

where φ , M , P and B are the variables, $X \equiv \Sigma_0$, $\delta \equiv \Sigma_1$, $\rho \equiv \Sigma_2$, $\mu \equiv \Sigma_3$
and $S \equiv S_0$.

Equations 29 through 32 are derived from 14 through 17, respectively.

Coefficients in the difference equations are defined through the following relations:

$$\bar{\delta} = -\frac{1}{\delta_1} + \frac{1}{\delta_2} - \frac{1}{\delta_3} + \frac{1}{\delta_4} \quad \bar{\mu} = -\frac{1}{\mu_1} + \frac{1}{\mu_2} - \frac{1}{\mu_3} + \frac{1}{\mu_4}$$

$$\alpha_1 = \frac{1}{h_1} \left(\frac{h_2}{\delta_1} + \frac{h_4}{\delta_4} \right) \quad \omega_1 = \frac{1}{h_1} \left(\frac{h_2}{\mu_1} + \frac{h_4}{\mu_4} \right)$$

$$\alpha_2 = \frac{1}{h_2} \left(\frac{h_3}{\delta_2} + \frac{h_1}{\delta_1} \right) \quad \omega_2 = \frac{1}{h_2} \left(\frac{h_3}{\mu_2} + \frac{h_1}{\mu_1} \right)$$

$$\alpha_3 = \frac{1}{h_3} \left(\frac{h_4}{\delta_3} + \frac{h_2}{\delta_2} \right) \quad \omega_3 = \frac{1}{h_3} \left(\frac{h_4}{\mu_3} + \frac{h_2}{\mu_2} \right)$$

$$\alpha_4 = \frac{1}{h_4} \left(\frac{h_1}{\delta_4} + \frac{h_3}{\delta_3} \right) \quad \omega_4 = \frac{1}{h_4} \left(\frac{h_1}{\mu_4} + \frac{h_3}{\mu_3} \right)$$

$$a_n = \frac{1}{\delta_{n-4}} + \frac{1}{7\mu_{n-4}} \quad b_n = \frac{1}{\delta_{n-4}} + \frac{3}{7\mu_{n-4}} \quad 5 \leq n \leq 8$$

$$c_{14} = -\frac{1}{\delta_1} + \frac{1}{\mu_1} + \frac{1}{\delta_4} - \frac{1}{\mu_4} \quad c_{21} = -\frac{1}{\delta_2} + \frac{1}{\mu_2} + \frac{1}{\delta_1} - \frac{1}{\mu_1}$$

$$c_{32} = -\frac{1}{\delta_3} + \frac{1}{\mu_3} + \frac{1}{\delta_2} - \frac{1}{\mu_2} \quad c_{43} = -\frac{1}{\delta_4} + \frac{1}{\mu_4} + \frac{1}{\delta_3} - \frac{1}{\mu_3}$$

$$Y = \rho_1 h_1 h_2 + \rho_2 h_2 h_3 + \rho_3 h_3 h_4 + \rho_4 h_4 h_1$$

$$X = \chi_1 h_1 h_2 + \chi_2 h_2 h_3 + \chi_3 h_3 h_4 + \chi_4 h_4 h_1$$

$$\xi = s_1 h_1 h_2 + s_2 h_2 h_3 + s_3 h_3 h_4 + s_4 h_4 h_1$$

Boundary conditions have already been discussed in Section III.

VII. METHOD OF SOLUTION FORMULATED IN MATRIX NOTATION

To simplify notation, it is convenient to introduce the vectors $\Phi_1 \approx \varphi$, $\Phi_2 \approx M$, $\Phi_3 \approx P$, $\Phi_4 \approx B$, vectors whose components are ordered as in PDQ-3. The TRIP-1 difference equations can then be cast into matrix form, as follows:

$$A_{11}\Phi_1 = A_{12}\Phi_2 + A_{13}\Phi_3 + A_{14}\Phi_4 + \psi_1, \quad (33)$$

$$A_{22}\Phi_2 = A_{21}\Phi_1 + A_{23}\Phi_3 + A_{24}\Phi_4 + \psi_2, \quad (34)$$

$$A_{33}\Phi_3 = A_{31}\Phi_1 + A_{32}\Phi_2 + A_{34}\Phi_4 + \psi_3, \quad (35)$$

$$A_{44}\Phi_4 = A_{41}\Phi_1 + A_{42}\Phi_2 + A_{43}\Phi_3. \quad (36)$$

These matrix equations are solved by a Gauss-Seidel iterative process, defined below:

$$A_{11}\Phi_1^{(N+1)} = A_{12}\Phi_2^{(N)} + A_{13}\Phi_3^{(N)} + A_{14}\Phi_4^{(N)} + \psi_1, \quad (37)$$

$$A_{22}\Phi_2^{(N+1)} = A_{21}\Phi_1^{(N+1)} + A_{23}\Phi_3^{(N)} + A_{24}\Phi_4^{(N)} + \psi_2, \quad (38)$$

$$A_{33}\Phi_3^{(N+1)} = A_{31}\Phi_1^{(N+1)} + A_{32}\Phi_2^{(N+1)} + A_{34}\Phi_4^{(N)} + \psi_3, \quad (39)$$

$$A_{44}\Phi_4^{(N+1)} = A_{41}\Phi_1^{(N+1)} + A_{42}\Phi_2^{(N+1)} + A_{43}\Phi_3^{(N+1)}. \quad (40)$$

Equations 37 through 40 may be condensed further:

$$A_{II}\Phi_I^{(N+1)} = \psi_I^{(N+1)} \quad 1 \leq I \leq 4, \quad (41)$$

where $\Psi_I^{(N+1)}$ is known. The function $\Phi_I^{(N+1)}$ is computed by a simultaneous line overrelaxation process, as in PDQ-3.

VIII. INPUT PREPARATION

The routine which processes the input is a slightly modified version of that used in PDQ-3.

Card Input

All of the input to this program is from punched cards.

Title Card. A title card must precede the input deck of each problem. Columns 1-8 must contain an eight-digit numeric identification. Columns 65-67 must be blank. Columns 68-72 must contain TRIPL.

Input Deck. All input numbers are expressed in the fixed-point form. The card format is such that columns 1-7 and 11 are blank, columns 8-10 contain "DEC", and columns 12-15 contain the card number, followed by a comma in column 16. The input parameters begin in column 17 and may extend through column 72. Successive numbers are separated by commas, but no comma is allowed following the last number on a card. The first blank column indicates the end of the information on that card. The numbers may be preceded by signs, but only minus signs are necessary.

The number zero need not have a decimal point. The nonzero numbers on card 1002, those contained in the 2000 and 3000 series, and the mesh intervals in the 4000 and 5000 series must contain decimal points. The nonzero numbers on cards 1001 and 1003, the row and column numbers in the

4000 and 5000 series, and the nonzero numbers in the 6000, 7000, and 8000 series must not contain decimal points. A detailed description of each card follows.

Card Number	Description
1001	<p>K, n, ss, tt, a, b, c, d</p> <p>K must be one and $a = b = c = d = \text{one}$. n is the largest composition number for which data is provided in the 2000 series ($1 \leq n \leq 35$). Data must be provided for all compositions $c = 1, 2, \dots, n$.</p> <p>ss is the last column in the mesh ($3 \leq ss \leq 87$). Column ss is half an interval beyond the symmetry boundary.</p> <p>tt is the last row in the mesh [$3 \leq tt \leq 174, ss \leq tt, (ss - 1)(tt - 1) \leq 2500$]. Row tt is half an interval beyond a symmetry boundary.</p>
1002	<p>$f, \eta, \epsilon, g, h, \ell, m, n$</p> <p>$f, g, h, \ell, m$, and n must always be zero.</p> <p>η is the parameter used for stopping the problem with the criterion being</p> $\frac{ \varphi_i^{(N+1)} - \varphi_i^{(N)} }{ \varphi_i^{(N+1)} } \leq \eta$ <p>where $\varphi_i^{(N)}$ is the i-th component of the flux, φ, after iteration N.</p> <p>ϵ is the parameter used for stopping the iteration in each of the four matrix equations. The criterion is</p> $\left \frac{\Phi_{Ii}^{(n+1)} - \Phi_{Ii}^{(n)}}{\Phi_{Ii}^{(n+1)}} \right \leq \epsilon(2 - \omega_I)$ <p>where ω_I is the overrelaxation factor computed for matrix equation I, n is the iteration index, and Φ_{Ii} is the i-th component of Φ_I.</p>
2101 2102 : 21n	<p>$\Sigma_1, \Sigma_a, \Sigma_3, \Sigma_2, S_0$</p> <p>These are the composition-variable parameters for compositions 1 through n. Σ_1 and Σ_3 must always be greater than zero and all the rest must be nonnegative. In the above notation, $\Sigma_i = \Sigma_a + \Sigma_{S0} - \Sigma_{Si}$, for $i = 1, 2, 3$.</p>

Card Number

Description

3000

ϕ

ϕ is a constant input flux approximation. This card is optional. If this card is present, there may be no 3000 series. If this card is absent, there must be a 3000 series.

3001

ϕ

3002

$\vdots$

$\vdots$

30n

ϕ is a flux approximation for compositions 1 through n. This series is optional. See card 3000.

4001

$H, s_1, H, s_2, \dots$

4002

$\vdots$

$\vdots$

This is the sequence of mesh intervals in the x coordinate direction. The first value is the mesh interval between columns 0 and s_1 , the second is the interval between columns s_1 and s_2 , and so forth. No set H, s_i may overlap two cards, the sequence s_i must be strictly increasing, and the last value must equal ss on card 1001.

5001

$H, t_1, H, t_2, \dots$

5002

$\vdots$

$\vdots$

This is the sequence of mesh intervals in the y coordinate direction. The first value is the mesh interval between rows 0 and t_1 , the second is the interval between columns t_1 and t_2 , and so forth. No set H, t_i may overlap two cards, the sequence t_i must be strictly increasing, and the last value must equal tt on card 1001.

6001

$c, s_1, s_2, t_1, t_2, \dots$

6002

$\vdots$

$\vdots$

The material composition described by successively laying rectangular blocks of specified composition over the mesh. Any block of composition may be laid over all or part of other blocks specified previously. For each mesh rectangle, the last specification which includes this rectangle determines its composition. It is not necessary that every composition $c, 1 \leq c \leq n$, appear in the mesh.

s_1, s_2, t_1 , and t_2 are the left-hand, right-hand, upper, and lower boundaries, respectively, of composition number c , where $0 \leq s_1 < s_2 \leq ss$ and $0 \leq t_1 < t_2 \leq tt$.

No specification of a composition block may overlap two cards.

Card NumberDescription

7001

7002

⋮

 $g, s_1, s_2, t_1, t_2, \dots$

The integrals of the flux and the absorption are automatically calculated for each composition. If there are any additional rectangular regions over which the flux and absorption are to be integrated, they are specified in this series by five-word sets. The first word of a set, g , is one when designating the flux and is two when designating the absorption. The next four words describe the outer boundaries of the rectangle over which the integration is desired. If a rectangle includes a symmetry boundary, the region outside this boundary is not considered.

s_1, s_2, t_1, t_2 are as defined in the description of the 6000 series.

The total number of rectangles specified in this series must not exceed 100 and the sequence of values chosen as g must be nondecreasing. No specification of a rectangle may overlap two cards. If there are no additional integrations to be requested in this series, a card numbered 7001 must be provided with no comma following the card number.

8001

8002

⋮

 $\pm g, s_1, s_2, t_1, t_2, \dots$

Pointwise editing is done only in those equations and over those rectangles specified by five-word sets. The first word of a set is the equation number, g , and the next four words, s_1, s_2, t_1, t_2 , are as defined in the 6000 series except that here $1^2 \leq s_1 < s_2 \leq ss - 1$, $1 \leq t_1 < t_2 \leq tt - 1$, and $1 \leq |g| \leq 4$. If g is positive, the editing is done in fixed-point with three digits to the right of the decimal point and up to four digits to the left. If g is negative, the editing is done in floating-point with a two-digit exponent and a four-digit mantissa.

The total number of rectangle specifications may not exceed 100. The sequence of values of $|g|$ must be nondecreasing. No rectangle specification may overlap two cards. If there is no edit control information, a card 8001 must be provided with no comma following the card number.

9000

This card signifies the end of the input.

IX. EDITS

All edits are accomplished by use of the PDQ-3 edit routines. At the end of the problem, the program provides the following edits:

(1) Input Edit

(2) Composition-Integrated Area: $\sum_R h_s h_t$

(3) Composition-Integrated Scalar Flux, ϕ :

$$\sum_R h_s h_t \frac{\{\phi_{s-1,t-1} + \phi_{s-1,t} + \phi_{s,t-1} + \phi_{s,t}\}}{4}$$

(4) Composition Averaged Flux:

$$\bar{\phi}_R \equiv (3) \div (2)$$

(5) Composition-Integrated Absorption: $\sum_O \bar{\phi}_R$

(6) Composition-Averaged Absorption: $(5) \div (2)$,

where in the above expressions R is the region of interest, h_s is the interval between mesh columns s-1 and s, and h_t is the interval between mesh-rows t-1 and t.

(7) Picture indicating mesh lines and compositions.

(8) Pointwise Edit consisting of Φ_1 , Φ_2 , Φ_3 , and Φ_4 .

X. OPERATING INSTRUCTIONS

Card Reader: Use the 72-72 card reader board.

On-Line Printer: Use the SHARE 2 or the GLOUT 2 printer board and paper which has space enough for at least 72 columns.

Card Punch: Standard SHARE board.

Off-Line Printer: Use 120 column paper and set the carriage control switch to PROGRAM.

Tapes: Use 9 tape units designated as 1-9 with the instruction tape designated as tape 1.

Sense Switches: All switches are normally set to UP. If No. 2 is down, this setting signifies a restart.

If 3 is down, this setting signifies a forced edit.

Starting a Problem: See that the necessary tapes are mounted, that tape 1 is rewound, and that sense switches 2 and 3 are properly set. Then depress the CLEAR and LOAD TAPE keys.

Restart Procedure: To restart a problem at the last restart point, see that the original tape reels are remounted and set at the same logical settings as set initially. Then rewind tape 1, depress sense switch 2, CLEAR and LOAD TAPE.

Removing a Problem: To remove a problem from the computer, rewind, remove, and label tape 2-9.

Forced Edit: To force an edit at the end of the current iteration, depress sense switch 3. This will cause an edit to be written on tape 9. When continued, put on a new tape 9. (DO NOT depress sense switch 3 until the results of Eq. (1) are printed.)

Tape Output: The only output tape for off-line printing is tape 9.

Card Output: Each tape check sum or redundancy tape test error causes a card to be punched. Rows 9R and 8R contain the check sum from tape and the computed check sum, respectively. If both 9R and 8R are zero, the error is due to the redundancy tape test. Row 2R contains the record number and file number in the address and decrement, respectively. Row OR contains the RTB or WTB instruction.

Instruction Tape Preparation: To write an instruction tape, ready a blank tape 1, ready the TRIP-1 deck in the card reader, CLEAR, and LOAD CARDS. A stop at 343 indicates that the proper instruction tape consisting of 30 records has been written.

Program Stops:

00006: Error loading binary tape loader from tape 1. Push LOAD TAPE to try again.

00077: Error loading a record of instructions from tape 1. Push START to try again.

77777: The reason for this stop is printed on-line.

References

1. G. G. Bilodeau, et al, "PDQ -- An IBM-704 Code to Solve the Two-Dimensional Few-Group Neutron-Diffusion Equations," WAPD-TM-70, August 1957.
2. B. Davison, Neutron Transport Theory, Oxford (1957), p. 160.
3. W. R. Cadwell, "PDQ-3 -- A Program for the Solution of the Neutron-Diffusion Equations in Two Dimensions on the IBM-704," WAPD-TM-179, March 1960.