Lawrence Radiation Laboratory
UNIVERSITY OF CALIFORNIA
LIVERMORE

UCRL-50973

**RESPONSE OF A RIGID BODY WITH SIX DEGREES OF
FREEDOM ON NONLINEAR SUPPORTS SUBJECTED TO
GROUND SHOCK FROM AN UNDERGROUND
NUCLEAR DETONATION**

H. Joseph Weaver, Jr.

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Abstract

A new shock mounting configuration, called the Universal Guided Column (U.G.C.) System, being used to protect sensitive electronic equipment subjected to ground shock from an underground nuclear detonation, is described. An analytical study of the U.G.C. system is presented.

A new shock mounting configuration, called the Universal Guided Column (U.G.C.) System, is being used to protect sensitive electronic equipment subjected to ground shock from an underground nuclear detonation.¹ The system can basically be described as a rigid body (which houses the electronic equipment) resting on several columns of a crushable material (foam). The base of each column rests on a low friction pad. The rigid body is also constrained by elastic rope tie-downs.

Figure 1 shows an idealized model of the U.G.C. system (shown with only 4 foam columns). The foam columns have a stress-strain curve (Fig. 2), and the approximate stress-strain relation of the rope is shown by the curve in Fig. 3. Each foam column is centered on a low friction pad which has two regions as shown in Fig. 4. The coefficients of friction between the foam column base and regions one and two is μ_1 and μ_2 , respectively, where $\mu_1 < \mu_2$. The assumption that the foam columns have a damping effect proportional to the (negative) rate of strain can also be made.

Shown in Fig. 5 is a F.B.D. (Free Body Diagram) of the U.G.C. system, with forces $\underline{F}_i$ acting on the rigid body. A coordinate system denoted as (x, y, z) is fixed to the center of mass of the rigid body. We fix in space a coordinate system (X, Y, Z) . The vector from (X, Y, Z) to (x, y, z) is $\underline{R}$. (x, y, z) rotates with respect to (X, Y, Z) with angular velocity $\dot{\theta}$. $\underline{L}_i$ denotes the distance from (x, y, z) to the point of application of the force $\underline{F}_i$ (note there are $\bar{m}$ such forces). To the ground we fix the coordinate system (x_g, y_g, z_g) with the vector from (X, Y, Z) to (x_g, y_g, z_g) denoted as $\underline{G}$.

$$\underline{R} = [x, y, z]$$

For the system shown in Fig. 5, the equations of motion are given by²

$$\sum_{i=1}^{\bar{m}} \underline{L}_i \times \underline{F}_i = \frac{d\underline{L}_A}{dt}$$

Fig. 1. Idealized model of the U.G.C. System.

$$\sum_{i=1}^m \underline{F}_i = \frac{M d^2 \underline{R}}{dt^2},$$

where

$$\underline{L}_A \approx I \frac{d\theta}{dt} = I \underline{w},$$

$$I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}.$$

For the coordinate system (x, y, z) fixed in the trailer, we have

$$\frac{d\underline{L}_A}{dt} = \frac{dI\underline{w}}{dt} = \frac{Id\underline{w}}{dt} + \underline{w} \times I\underline{w}.$$

Fig. 2. Stress-strain curve for foam column.

Fig. 3. Stress-strain curve for elastic tie-down rope.

Fig. 4. Low friction pad.

Fig. 5. Coordinate system and free body diagram.

Thus, the equations of motion become

$$\sum_{i=1}^{\bar{m}} \underline{L}_i \times \underline{F}_i = \frac{I d^2 \theta}{dt^2} + \frac{d\theta}{dt} \times \frac{I d\theta}{dt}$$

and

$$\sum_{i=1}^{\bar{m}} \underline{F}_i = \frac{M d^2 \underline{R}}{dt^2} + [0, 1, 0] M \underline{g}. \quad (1)$$

If we now write Eq. (1) in component form, we arrive at the following 6 equations:

$$M_x = I_{xx} \ddot{\theta}_x - I_{xy} \ddot{\theta}_y - I_{xz} \ddot{\theta}_z - \dot{\theta}_z (-I_{xy} \dot{\theta}_x + I_{yy} \dot{\theta}_y - I_{yz} \dot{\theta}_z) \\ + \dot{\theta}_y (-I_{xz} \dot{\theta}_x - I_{yz} \dot{\theta}_y + I_{zz} \dot{\theta}_z),$$

$$M_y = -I_{xy} \ddot{\theta}_x + I_{yy} \ddot{\theta}_y - I_{yz} \ddot{\theta}_z - \dot{\theta}_x (-I_{xz} \dot{\theta}_x - I_{yz} \dot{\theta}_y + I_{zz} \dot{\theta}_z) \\ + \dot{\theta}_z (I_{xx} \dot{\theta}_x - I_{xy} \dot{\theta}_y - I_{xz} \dot{\theta}_z),$$

$$M_z = -I_{xz} \ddot{\theta}_x - I_{yz} \ddot{\theta}_y + I_{zz} \ddot{\theta}_z - \dot{\theta}_y (I_{xx} \dot{\theta}_x - I_{xy} \dot{\theta}_y - I_{xz} \dot{\theta}_z) \\ + \dot{\theta}_x (-I_{xy} \dot{\theta}_x + I_{yy} \dot{\theta}_y - I_{yz} \dot{\theta}_z),$$

$$\sum_{i=1}^{\bar{m}} F_{xi} = M\ddot{x},$$

$$\sum_{i=1}^{\bar{m}} F_{yi} = M\ddot{y} + M_g,$$

$$\sum_{i=1}^{\bar{m}} F_{zi} = M\ddot{z}, \quad (2)$$

$$\text{where } [M_x, M_y, M_z] = \sum_{i=1}^{\bar{m}} \underline{L}_i \times \underline{F}_i.$$

Equation (2) describes the motion of the mass center of the rigid body. The problem at hand is to determine what forces and what torques (about cm) act on the rigid body.

A dissected view of the rigid body is shown in Fig. 6. The i -th support with its tie-down ropes, is shown in greater detail in Fig. 7 where are also shown the various angles associated with the tie-down ropes. Supports may or may not have one or more tie-downs. For this reason the tie-down rope on the i -th support is designated as the j -th tie-down. Keep in mind that the vector $\hat{\underline{L}}_j$, associated with the location of the j -th tie-down rope, is the same as the $\underline{L}_i$ vector associated with the support to which the rope is connected. $\underline{a}_i$ (Fig. 7) is the position of the rigid body beam which rests upon the i -th foam column. To refer to the j -th tie-down rope, we use $\hat{\underline{a}}_j$ ($\hat{\underline{a}}_i = \underline{R} + \underline{L}_i$, $\hat{\underline{a}}_j = \underline{R} + \underline{L}_j$).

The force acting on the body via the j -th tie-down rope is designated as $\underline{fr}$. With the magnitude of $\underline{fr}$ designated as fr , then

$$\underline{fr}_j = [fr_j \cos \psi_{xj} \cos \psi_{yj}, fr_j \sin \psi_{yj}, fr_j \sin \psi_{xj} \cos \psi_{yj}]$$

Fig. 6. Disected view of rigid body.

where ψ_{xj} and ψ_{yj} (Fig. 7) are given as

$$\psi_{yj} = \tan^{-1} \left[\frac{h + \hat{a}_{yj} - g_y}{((c_j - \hat{a}_{zj} + g_z)^2 + (d_j - \hat{a}_{xj} + g_x)^2)^{1/2}} \right]$$

and

$$\psi_{xj} = \tan^{-1} \left[\frac{c_j - \hat{a}_{zj} + g_z}{d_j - \hat{a}_{xj} + g_x} \right].$$

Fig. 7. i-th support and j-th tie-down.

f_r is a function of the tension in the rope. $\underline{a}$ and $\underline{g}$ are functions of time. The length of the rope (b_j) is also a function of time. The length of the rope is given as

$$b_j = [(c_j - \hat{a}_{zj} + g_z)^2 + (d_j - \hat{a}_{xj} + g_x)^2 + (h + \hat{a}_{yj} + g_y)^2]^{1/2}.$$

In the vertical direction (Y), we assume that the force (f_{yi}) transmitted through the foam supports is a function of the crush of the foam and its rate. In the radial (Z) and tangential (X) direction, we assume that force is a function of 1) the coefficient of friction between the i-th column and the low friction pad upon which it rests, 2) the relative motion between the pad and the i-th column, and 3) the vertical force in the i-th column. These forces are written as f_{zi} and f_{xi} , respectively. We also assume there are N supports and M tie-down ropes.

We are now able to rewrite Eq. (2) as

$$\sum_{i=1}^N f_{xi} + \sum_{j=1}^M fr_j \cos \psi_{xj} \cos \psi_{yj} = M\bar{x},$$

$$\sum_{i=1}^N f_{yi} + \sum_{j=1}^M fr_j \sin \psi_{yj} = M\bar{y} + M_g,$$

$$\sum_{i=1}^N f_{zi} + \sum_{j=1}^M fr_j \sin \psi_{xj} \cos \psi_{yj} = M\bar{z},$$

$$\sum_{i=1}^N f_{zi} \bar{L}_{yi} - \sum_{i=1}^N f_{yi} L_{zi} + \sum_{j=1}^M (fr_j \sin \psi_{yj}) \hat{L}_{zj} - \sum_{j=1}^M (fr_j \sin \psi_{xj} \cos \psi_{yj}) \hat{L}_{yj}$$

$$= I_{xx} \bar{\theta}_x - I_{xy} \bar{\theta}_y - I_{xz} \bar{\theta}_z - \dot{\theta}_z (-I_{xy} \theta_x + I_{yz} \dot{\theta}_y - I_{yz} \dot{\theta}_z) \\ + \dot{\theta}_y (-I_{xz} \dot{\theta}_x - I_{yz} \dot{\theta}_y + I_{zz} \dot{\theta}_z),$$

$$\sum_{i=1}^N f_{xi} L_{zi} + \sum_{i=1}^N f_{zi} L_{xi} + \sum_{j=1}^M (fr_j \cos \psi_{xj} \cos \psi_{yj}) \hat{L}_{zj} - \sum_{j=1}^M (fr_j \sin \psi_{xj} \cos \psi_{yj}) \hat{L}_{xj}$$

$$= -I_{xy} \bar{\theta}_x + I_{yy} \bar{\theta}_y - I_{yz} \bar{\theta}_z - \dot{\theta}_x (-I_{xz} \dot{\theta}_x - I_{yz} \dot{\theta}_y + I_{zz} \dot{\theta}_z) \\ + \dot{\theta}_z (I_{xx} \dot{\theta}_x - I_{xy} \dot{\theta}_y - I_{xz} \dot{\theta}_z), \text{ and}$$

$$\sum_{i=1}^N f_{yi} L_{xi} - \sum_{i=1}^N f_{xi} \bar{L}_{yi} - \sum_{j=1}^M (fr_j \cos \psi_{xj} \cos \psi_{yj}) \hat{L}_{yj} + \sum_{j=1}^M (fr_j \sin \psi_{yj}) \hat{L}_{xj}$$

$$= -I_{zx} \bar{\theta}_x - I_{zy} \bar{\theta}_y + I_{zz} \bar{\theta}_z - \bar{\theta}_y (I_{xx} \dot{\theta}_x - I_{xy} \dot{\theta}_y - I_{xz} \dot{\theta}_z) \\ + \dot{\theta}_x (-I_{xy} \dot{\theta}_x + I_{yy} \dot{\theta}_y - I_{yz} \dot{\theta}_z)$$

(3)

where $\bar{L}_{yi} = L_{yi} - h - (\alpha_i - g_y) - y_{\text{static deflection}}$.

We now transform the equations of motion of the rigid body (Eq. [3]) to the state space notation (i.e. we make the following substitution).

$$\begin{aligned}
 x_1 &= x, & x_2 &= \dot{x}_1 = \dot{x}, \\
 y_1 &= y, & y_2 &= \dot{y}_1 = \dot{y}, \\
 z_1 &= z, & z_2 &= \dot{z}_1 = \dot{z}, \\
 \theta_{x1} &= \theta_x, & \theta_{x2} &= \dot{\theta}_{x1} = \dot{\theta}_x, \\
 \theta_{y1} &= \theta_y, & \theta_{y2} &= \dot{\theta}_{y1} = \dot{\theta}_y, \\
 \theta_{z1} &= \theta_z, & \text{and } \theta_{z2} &= \dot{\theta}_{z1} = \dot{\theta}_z.
 \end{aligned} \tag{4}$$

Equation (3) now becomes

$$\begin{aligned}
 \dot{x}_1 &= \dot{x}_2, \\
 \ddot{x}_2 &= \frac{1}{M} \left(\sum_{i=1}^N f_{xi} + \sum_{j=1}^M fr_j \cos \psi_{xj} \cos \psi_{yj} \right), \\
 \dot{y}_1 &= y_2, \\
 \ddot{y}_2 &= \frac{1}{M} \left(\sum_{i=1}^N f_{yi} + \sum_{j=1}^M fr_j \sin \psi_{yj} \right) - g, \\
 \dot{z}_1 &= z_2, \\
 \ddot{z}_2 &= \frac{1}{M} \left(\sum_{i=1}^N f_{zi} + \sum_{j=1}^M fr_j \sin \psi_{xj} \cos \psi_{yj} \right), \\
 \dot{\theta}_{x1} &= \theta_{x2}, \\
 \dot{\theta}_{y1} &= \theta_{y2}, \\
 \dot{\theta}_{z1} &= \theta_{z2}, \\
 I_{xx} \dot{\theta}_{x2} - I_{xy} \dot{\theta}_{y2} - I_{xz} \dot{\theta}_{z2} &= \hat{b}_1, \\
 -I_{xy} \dot{\theta}_{x2} + I_{yy} \dot{\theta}_{y2} - I_{yz} \dot{\theta}_{z2} &= \hat{b}_2, \\
 -I_{xz} \dot{\theta}_{x2} - I_{yz} \dot{\theta}_{y2} + I_{zz} \dot{\theta}_{z2} &= \hat{b}_3
 \end{aligned} \tag{5}$$

where

$$\begin{aligned}
 \hat{b}_1 &= \sum_{i=1}^N f_{zi} L_{yi} - \sum_{i=1}^N f_{yi} L_{zi} + \sum_{j=1}^M (fr_j \sin \psi_{yj}) \hat{L}_{zj} - \sum_{j=1}^M (fr_j \sin \psi_{xj} \cos \psi_{yj}) \hat{L}_{yj} \\
 &+ \theta_{z2} (-I_{xy} \theta_{x2} + I_{yy} \theta_{y2} - I_{yz} \theta_{z2}) - \theta_{y2} (-I_{xz} \theta_{x2} - I_{yz} \theta_{y2} + I_{zz} \theta_{z2}),
 \end{aligned}$$

$$b_2 = \sum_{i=1}^N f_{xi} L_{zi} - \sum_{i=1}^N f_{zi} L_{xi} + \sum_{j=1}^M (fr_j \cos \psi_{xj} \cos \psi_{yj}) L_{zj} - \sum_{j=1}^M (fr_j \sin \psi_{xj} \cos \psi_{yj}) L_{xj} \\ + \theta_{x2} (-I_{xz} \theta_{x2} - I_{yz} \theta_{y2} + I_{zz} \theta_{z2}) - \theta_{z2} (I_{xx} \theta_{x2} - I_{xy} \theta_{y2} - I_{xz} \theta_{z2}),$$

and

$$b_3 = \sum_{i=1}^N f_{yi} L_{xi} - \sum_{i=1}^N f_{xi} L_{yi} - \sum_{j=1}^M (fr_j \cos \psi_{xj} \cos \psi_{yj}) \hat{L}_{yj} + \sum_{j=1}^M (fr_j \sin \psi_{yj}) \hat{L}_{xj} \\ + \theta_{y2} (I_{xx} \theta_{x2} - I_{xy} \theta_{y2} - I_{xz} \theta_{z2}) - \theta_{x2} (-I_{xy} \theta_{x2} + I_{yy} \theta_{y2} - I_{yz} \theta_{z2}). \quad (6)$$

To facilitate the analysis of Eq. (5), we make the following substitutions:

$$A = \begin{bmatrix} I_{xx} - I_{xy} - I_{xz} \\ -I_{xy} \quad I_{yy} - I_{yz} \\ -I_{xy} - I_{yz} \quad I_{zz} \end{bmatrix},$$

$$\underline{\theta}_1 = [\theta_{x1}, \theta_{y1}, \theta_{z1}],$$

$$\underline{\theta}_2 = [\theta_{x2}, \theta_{y2}, \theta_{z2}],$$

$$\underline{X}_1 = (x, y, z),$$

$$\underline{X}_2 = (\dot{x}, \dot{y}, \dot{z}),$$

and

$$\underline{b} = \{\dot{b}_1, \dot{b}_2, \dot{b}_3\}.$$

With the above substitutions the last 3 parts of Eq. (5) may be written as

$$A \dot{\underline{\theta}}_2 = \underline{\dot{b}}.$$

We shall now assume that determinant $A = |A| \neq 0$. Thus A has an inverse, denoted as

$$A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

such that

$$\dot{\underline{\theta}}_2 = A^{-1} \underline{\dot{b}}$$

or

$$\begin{aligned}\dot{\theta}_{x2} &= a_{11}b_1 + a_{12}b_2 + a_{13}b_3, \\ \dot{\theta}_{y2} &= a_{21}b_1 + a_{22}b_2 + a_{23}b_3, \\ \dot{\theta}_{z2} &= a_{31}b_1 + a_{32}b_2 + a_{33}b_3.\end{aligned}\quad (7)$$

where

$$\begin{aligned}|A| &= I_{xx}I_{yy}I_{zz} - I_{xx}I_{yz}^2 - I_{yy}I_{xz}^2 - I_{zz}I_{xy}^2 - 2I_{xy}I_{xz}I_{yz}, \\ a_{11} &= (I_{yy}I_{zz} - I_{yz}^2)/|A|, \\ a_{12} &= (I_{zz}I_{xy} + I_{xz}I_{yz})/|A|, \\ a_{13} &= (I_{yy}I_{xz} + I_{yz}I_{xy})/|A|, \\ a_{21} &= a_{12}, \\ a_{22} &= (I_{xx}I_{zz} - I_{xz}^2)/|A|, \\ a_{23} &= (I_{xx}I_{yz} - I_{xz}I_{xy})/|A|, \\ a_{31} &= a_{13}, \\ a_{32} &= a_{23}, \\ a_{33} &= (I_{xx}I_{yy} - I_{xy}^2)/|A|.\end{aligned}\quad (8)$$

The equations of motion (Eq. [5]) now become

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{1}{M} \left(\sum_{i=1}^N f_{xi} + \sum_{j=1}^M fr_j \cos \psi_{xj} \cos \psi_{y_j} \right), \\ \dot{y}_1 &= y_2, \\ \dot{y}_2 &= \frac{1}{M} \left(\sum_{i=1}^N f_{yi} + \sum_{j=1}^M fr_j \sin \psi_{y_j} \right) - g, \\ \dot{z}_1 &= z_2, \\ \dot{z}_2 &= \frac{1}{M} \left(\sum_{i=1}^N f_{zi} + \sum_{j=1}^M fr_j \sin \psi_{xj} \cos \psi_{y_j} \right),\end{aligned}$$

$$\dot{\theta}_{x1} = \theta_{x2} ,$$

$$\dot{\theta}_{x2} = a_{11}\dot{b}_1 + a_{12}\dot{b}_2 + a_{13}\dot{b}_3 ,$$

$$\dot{\theta}_{y1} = \theta_{y2} ,$$

$$\dot{\theta}_{y2} = a_{21}\dot{b}_1 + a_{22}\dot{b}_2 + a_{23}\dot{b}_3 ,$$

$$\dot{\theta}_{z1} = \theta_{z2} ,$$

and

$$\dot{\theta}_{z2} = a_{31}\dot{b}_1 + a_{32}\dot{b}_2 + a_{33}\dot{b}_3 . \quad (9)$$

Equation (9) is the set of motion equations of the rigid body in the state space notation, where

$$\underline{X}_1 = [x, y, z] = \text{translational displacement,}$$

$$\underline{X}_2 = [\dot{x}, \dot{y}, \dot{z}] = \text{translational velocity,}$$

$$\underline{\dot{X}}_2 = [\ddot{x}, \ddot{y}, \ddot{z}] = \text{translational acceleration,}$$

$$\underline{\theta}_1 = [\theta_x, \theta_y, \theta_z] = \text{angular displacement,}$$

$$\underline{\theta}_2 = [\dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z] = \text{angular velocity,}$$

and

$$\underline{\dot{\theta}}_2 = [\ddot{\theta}_x, \ddot{\theta}_y, \ddot{\theta}_z] = \text{angular acceleration.}$$

The initial conditions on Eq. (9) are as follows:

$$\theta_{x1}(0) = \theta_{x2}(0) = \theta_{y1}(0) = \theta_{y2}(0) = \theta_{z1}(0) = \theta_{z2}(0) = 0 ,$$

$$x_1(0) = x_2(0) = z_1(0) = z_2(0) = y_2(0) = 0 ,$$

$$y_1(0) = \text{static deflection} = - \frac{(\text{wt of trailer})(\text{height of foam columns})^*}{K_1 (\text{total area of foam columns})} .$$

The ground motion is assumed to be known as a function of time. It is given as the displacement, velocity, and acceleration in the vertical, radial, and tangential directions.

* We assume no initial crush.

FORCE DUE TO TIE-DOWN ROPES (f_{r_j})

As mentioned earlier the magnitude of f_{r_j} is a function of the tension in the j -th rope. This is shown graphically in Fig. 3. We assume that the force is described by two linear curves of slopes S_1 and S_2 . Initially

$$f_{r_j} = (\text{cross sectional area of rope}) \times S_1 \times \text{strain}.$$

Once the strain reaches the value 0.15^*

$$f_{r_j} = (\text{cross sectional area of rope}) \times S_2 \times (\text{strain} - \gamma).$$

The tie-down ropes are unable to support compression

$$\text{Note: Strain} = (\text{length} - \text{original length}) / (\text{original length})$$

VERTICAL FORCE IN FOAM COLUMN (f_{y_i})

The vertical force in the foam column is a function of both the "crush" and the rate of "crush" of the foam. The "crush" of the foam is given as

$$\alpha_i(t) - g_y(t).$$

$$f_{y_i} = f_{y_i}(\text{due to crush}) + f_{y_i}(\text{due to rate of crush}).$$

Obviously

$$f_{y_i}(\text{crush}) = (\text{cross sectional area of } i\text{-th foam column}) \times \text{strain}_i$$

where

$$\text{strain}_i = \frac{\text{crush}_i}{\text{height of column}_i}$$

We are therefore able to obtain f_{y_i} from the stress strain curve of Fig. 2.

We assume that the force due to the rate of crush is a linear function of the rate of crush, i.e.,

$$f_{y_i}(\text{rate of crush}) = -C_1 (\text{rate of crush}).^\dagger$$

* This value is experimentally determined for.

† The stress strain curve in Fig. 2 and value of C_1 are conjecture based on experimental data.

RADIAL AND TANGENTIAL FORCE IN FOAM COLUMN (f_{xi}, f_{zi})

The radial and tangential force in the foam column is due to the friction between the base of the column and the pad upon which it rests. It is given as 1) if the foam column is within region 1,^{*}

$$f_{xi} = \mu_1 f_{yi}$$

(the direction of f_{xi} is opposite to the relative motion), and 2) if the foam column is within region 2 and outside region 1,

$$f_{xi} = \mu_2 f_{yi}$$

(again the direction of f_{xi} is opposite to the relative motion). The same remarks are true for f_{zi} .

Due to the high degree of nonlinearity of Eq. (9), an analytical solution is just about impossible to obtain. Therefore to "solve" Eq. (9), we must discretize it and iterate over time. There are a number of ways to do this but as a compromise among simplicity, speed, and accuracy, we choose the following

$$\frac{dx}{dt} = \frac{\Delta x}{T} = \frac{x(kT + T) - x(kT)}{T}$$

Note,

$$\lim_{T \rightarrow 0} \frac{\Delta x}{T} = \frac{dx}{dt}.$$

This method of discretizing the equations results in the following set of finite difference equations:

$$\theta_{x1}(k+1) = \theta_{x1}(k) + T\theta_{x2}(k),$$

$$\theta_{x2}(k+1) = T(a_{11}\dot{b}_1(k) + a_{12}\dot{b}_2(k) + a_{13}\dot{b}_3(k)) + \theta_{x2}(k),$$

$$\theta_{y1}(k+1) = \theta_{y1}(k) + T\theta_{x2}(k),$$

$$\theta_{y2}(k+1) = T(a_{21}\dot{b}_1(k) + a_{22}\dot{b}_2(k) + a_{23}\dot{b}_3(k)) + \theta_{y2}(k) - Tg(k),$$

$$\theta_{z1}(k+1) = \theta_{z1}(k) + T\theta_{z2}(k),$$

$$\theta_{z2}(k+1) = T(a_{31}\dot{b}_1(k) + a_{32}\dot{b}_2(k) + a_{33}\dot{b}_3(k)),$$

$$x_1(k+1) = x_1(k) + Tx_2(k),$$

$$x_2(k+1) = x_2(k) + \frac{T}{M} \left[\sum_{i=1}^N f_{xi}(k) + \sum_{j=1}^M fr_j(k) \cos \psi_{xj}(k) \cos \psi_{yj} \right],$$

$$y_1(k+1) = y_1(k) + Ty_2(k),$$

* Any portion of the foam column.

$$y_2(k+1) = y_2(k) + \frac{T}{M} \left[\sum_{i=1}^N f_{yi}(k) + \sum_{j=1}^M fr_j(k) \sin \psi_{yj}(k) \right],$$

$$z_1(k+1) = z_1(k) + T z_2(k),$$

$$z_2(k+1) = z_2(k) + \frac{T}{M} \left[\sum_{i=1}^N f_{zi}(k) + \sum_{j=1}^M fr_j(k) \sin \psi_{xj}(k) \cos \psi_{yj}(k) \right],$$

where it is understood that

$\underline{\theta}(k)$ means $\underline{\theta}(kT)$

and

$\underline{X}(k)$ means $\underline{X}(kT)$.

(10)

Equation (10) represents a mathematical model for the rigid body subjected to ground shock. This model will be used for two main purposes: (1) to perform parametric studies to optimize such quantities as the location, height, and area of the foam columns and the location and size of the tie-down ropes, and (2) to serve as a shock mounting design tool for future events, particularly multiple event ground motion interaction.

A computer code called TRUND has been written to integrate Eq. (10). Two assumptions have been made: the shape of the stress strain curves for the foam and ropes, and that we are dealing with a rigid body. Thus some disagreement with the real physical situation is to be expected. However, to date this is the most complete analysis available and although TRUND is still in its initial development stages, early results indicate our assumptions are not unreasonable. By comparing the output of TRUND with movies of a trailer responding to measured ground motion, we hope to refine our data on the foam, ropes, low friction plates, etc. When available, these results and TRUND will be published.

Acknowledgments

The author is pleased to acknowledge the helpful advice of Mr. Don Bernreuter and Mr. Aubrey Miller on the characteristics of the U.G.C. system. The valuable comments of Dr. John Ruminer on the modeling system used are also greatly appreciated.

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**Printed in USA. Available from the National Technical
Information Center, National Bureau of Standards,
U. S. Department of Commerce, Springfield, Virginia 22151
Price: Printed Copy \$3.00; Microfiche \$0.65.**