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SUBJECT: Steady State Heat Losses from Radio Waste Storage
Tanks in Oak Ridge National Laboratory Tank Farm

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ABSTRACT

Six cylindrical radio waste storage tanks in an Oak Ridge National Laboratory tank farm are arranged in a rectangular array two tanks wide and three tanks long with a 60 ft distance between tank centers. The tanks are 50 ft in diameter and 13 ft to 15 ft tall. They are buried in the earth with five foot of cover soil. An electrical analog of the farm was constructed in order to determine the steady state heat loss rates from tanks at the boiling point, 230°F, to the winds over the earth with an average year round temperature of 60°F.

The steady state heat loss rate per tank ranged from 4,000 Btu/hr to 44,000 Btu/hr depending upon the location of the tank and whether the soil was dry or moist.

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1.0 INTRODUCTION

At ORNL radio waste solutions are stored in a tank farm consisting of six cylindrical tanks placed underground and arranged as specified in Clinton Engineering Works Engineering Department Drawing W-68334. A schematic of this arrangement is shown in Fig. 1. An elevation of a typical tank is shown in Fig. 2.

The tank farm poses a problem in heat conduction because the tank contents generate heat. If the tanks actually boiled, the radio vapors would pollute the surroundings since the tanks are not equipped with condensers. It is obvious then that there is a limit to the amount of activity that can be stored in each tank and this is determined by the steady state rate of heat conduction from the radio waste at its boiling point to the winds passing over the surface of the earth.

The unsteady state rate of heat conduction at the boiling point is not the governing factor in long term radio waste storage because this rate is always greater than that for steady state. One must therefore always consider the severest limitation which is that brought on by steady state heat conduction.

2.0 THE STEADY STATE HEAT CONDUCTION PROBLEM AND ITS ELECTRICAL ANALOG

The basic problem of heat conduction through the earth may be stated in terms of the conduction equation

$$\frac{\partial \tau}{\partial t} = \alpha \left[\frac{\partial^2 \tau}{\partial x^2} + \frac{\partial^2 \tau}{\partial y^2} + \frac{\partial^2 \tau}{\partial z^2} \right] \quad (1)$$

where

τ = temperature at point x, y, z and time, t .

α = the thermal diffusivity of the earth.

Since the problem at hand involves the steady state, equation 1 reduces to

$$\frac{\partial^2 \tau}{\partial x^2} + \frac{\partial^2 \tau}{\partial y^2} + \frac{\partial^2 \tau}{\partial z^2} = 0 \quad (2)$$

for which one must specify six boundary conditions. If x is measured normal to the surface of the earth, then when $x = 0$, i.e., at the surface of the earth, one may write that

$$k \left(\frac{\partial \tau}{\partial x} \right)_{x=0} = h(\tau_{x=0} - \tau_g) \quad (3)$$

which is in accord with Fourier's third law of heat conduction and Newton's law of cooling. Here

Fig. 1. Schematic layout of tank farm.

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Fig. 2. Elevation of a typical tank.

$T_a = 60^\circ\text{F}$, the year-round average bulk temperature of the winds over the surface of the earth.

h = film coefficient.

k = thermal conductivity of the earth.

The remaining five boundary conditions involve complicated functions of x , y , and z . These functions derive from the fact that the surface temperature of the tanks must be specified and the surfaces themselves are complicated functions of x , y , and z . For all practical purposes then, the formal solution to the problem meets with insuperable mathematical difficulty and one must resort to some other method of solution such as the electrical analog.

If the solution to equation 2 could be subjected to the boundary conditions, one would find that

$$T = T(x, y, z) \quad (4)$$

This expression would be substituted in the relation that

$$q'' = k \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(\frac{\partial T}{\partial x} \right)_{x=0} dy dz \quad (5)$$

to determine the total steady state heat loss rate q'' from the entire tank farm. The individual tank heat loss rates q' may be obtained by integrating the relation

$$q' = k \int_0^A \left(\frac{\partial T}{\partial n} \right)_{\text{surface}} dA \quad (6)$$

where

n = the direction normal to the elemental surface area, dA , of the tank.

If the earth and air film over it were replaced by an electrical conducting medium and voltages were substituted for temperatures, the steady state voltage distribution in the medium would be represented by the differential equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = 0. \quad (7)$$

The current from each tank would be given by a variation of Ohms law, namely

$$I = 30.48 \frac{\Delta n}{R\Delta A} \int_0^A \left(\frac{\partial E}{\partial n} \right)_{\text{surface}} dA \quad (8)$$

Here $\frac{R\Delta A}{\Delta n}$ is the electrical resistivity normally given in ohm-cm. Since n and A are in ft and ft² the conversion factor 30.48 is necessary. Equations 7 and 8 are the dynamic analogs of equations 2 and 6. In fact if we define

$$N = \frac{E - E_s}{E_t - E_s} = \frac{\tau - \tau_s}{\tau_t - \tau_s} = \frac{E - E_s}{\Delta E} = \frac{\tau - \tau_s}{\Delta \tau} \quad (9)$$

where

τ_t = the tank surface temperature

τ_s = the average bulk temperature of the winds

E_t and E_s are the counterpart of τ_t and τ_s

$\Delta E = E_t - E_s$

$\Delta \tau = \tau_t - \tau_s$

then equations 2 and 7 reduce to

$$\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} + \frac{\partial^2 N}{\partial z^2} = 0 \quad (10)$$

Equation 6 reduces to

$$q' = k\Delta\tau \int_0^A \left(\frac{\partial N}{\partial n} \right)_{\text{surface}} dA \quad (11)$$

and 8 reduces to

$$I = 30.48 \frac{\Delta n}{R\Delta A} \Delta E \int_0^A \left(\frac{\partial N}{\partial n} \right)_{\text{surface}} dA \quad (12)$$

The fact that τ_t and consequently E_t is specified as constant over the surface of a given tank can be justified. In a warm tank or one at the boiling point heat is transferred to the top of the tank by convection and condensation of vapors. Thus, even though the tank may not be completely filled with solution, there exists a mechanism of heat transfer to the surface of the unfilled portion which is much more rapid than steady state conduction through appreciable amounts of earth which is a relatively poor heat conductor. In the bottom of a tank that has been operated for some time, there is a layer

of wet sludge. This layer conducts heat much better than does soil. Also it is located in the bottom of the tank where the heat transfer path to the winds is much longer. Consequently the temperature drop in the sludge is relatively small. Thus, the entire surface of a given tank should tend toward a uniform temperature τ_t .

If one were to build an electrical analog of a tank farm, it would not be practical to build it the actual size of the farm because it would be too expensive and unwieldy. Thus, one is confronted with a problem of scaling.

Scaling can be accomplished by assuming that equations 1-12 apply to the model, i.e., the scaled down version of the tank farm. Thus, for the farm itself one writes equation 11 as

$$q = k\Delta\tau \int_0^{A'} \left(\frac{\partial N}{\partial n} \right)_{\text{surface}} dA' \quad (13)$$

and equation 2 as

$$\frac{\partial^2 \tau}{\partial (x')^2} + \frac{\partial^2 \tau}{\partial (y')^2} + \frac{\partial^2 \tau}{\partial (z')^2} = 0 \quad (14)$$

From equation 14 we see that only one scale factor say "a" is required because if we divide equation 14 by a^2 we obtain

$$\frac{\partial^2 \tau}{\partial (ax')^2} + \frac{\partial^2 \tau}{\partial (ay')^2} + \frac{\partial^2 \tau}{\partial (az')^2} = 0 \quad (15)$$

If we set

$$x = ax'$$

$$y = ay'$$

$$z = az'$$

equation 15 reduces to that for the model namely equation 2.

The scale factor "a" may be injected into equation 13 as follows

$$aq = k\Delta\tau \int_0^{a^2 A'} \left(\frac{\partial N}{\partial (an')} \right) d(a^2 A') \quad (16)$$

Now

$$n = an'$$

$$A = a^2 A'$$

$$q' = aq$$

so that equation 16 becomes

$$q' = aq = k\Delta\tau \int_0^A \left(\frac{\partial N}{\partial n} \right)_{\text{surface}} dA \quad (17)$$

Returning now to equation 12 and comparing it with 17 we see that if numerically we wish

$$\begin{aligned} I &= Kaq \\ \Delta E &= K\Delta\tau \end{aligned} \quad (18)$$

where K is a proportionality constant then we must accept that numerically

$$30.48 \frac{\Delta n}{RAA} = k \quad (19)$$

We can avoid having to determine K by taking the ratio of equations 12 and 17 to show that

$$q = \frac{\Delta\tau}{a} \frac{I}{\Delta E} \quad (20)$$

Thus we have established that the actual tank farm may be scaled down by a factor "a" to a model of the farm which may be simulated by its electrical dynamic analog provided that numerically equation 19 is satisfied. With this being so we can establish ΔE in the simulator and measure I. If we choose $\Delta\tau$, then "a", $\Delta\tau$, I, ΔE may be substituted into equation 20 to compute q for a given tank in the actual farm.

3.0 THE ELECTRICAL ANALOG OR SIMULATOR

With reference to Fig. 1 it is obvious that with six tanks there are numerous combinations of $\Delta\tau$ for the various tanks. This situation may be simplified if one considers one $\Delta\tau$ for the corner tanks and the same or another $\Delta\tau$ for the center tanks, i.e., #112 and #115. In this case because of symmetry there is no heat conduction between the first quadrant (indicated by the dashed lines) and other quadrants. Thus, we need simulate only the first quadrant.

For all practical purposes the earth represents a body of infinite extent. However, in Fig. 1 if one uses the corner of the quadrant as center and draws the smallest circle that will contain within its bound the two tanks #114 and #115, he finds the radius of the circle to be about 93'. From previous experience with similar problems on the Oak Ridge ORACLE, a digital computer, it is known that earth beyond five times the 93' radius or 465' does not contribute noticeably to the heat conduction problem. One thousand feet was chosen just to be safe and then the scale factor "a" was selected as

$$a = 0.001.$$

The simulator is shown in front of the rule in Fig. 3. The left side of the simulator is a 1/8-in. copper plate which represents the surface winds. The rest of the simulator is Lucite. The copper tanks are shown in Fig. 4. They were cemented in place with tiny Lucite pegs. The white patches on the tanks are small amounts of KCl left after drying the simulator for photographic purposes. During a run the simulator is filled with 28 liters of aqueous KCl solution at room temperature. The electrical lines were plastic covered stranded copper wire soldered to the tank bottoms. These wires were fed to the instruments through a glass tube cemented to the simulator wall as shown in Fig. 4.

In Fig. 3 the box with dials contained a voltage regulated line isolation 115 v 60 cycle ac transformer, a fuse, a switch, a panel light, and three variacs connected to the voltmeters, ammeters, and the simulator as shown in the wiring diagram of Fig. 5. Note that the voltmeter for a given tank is connected from ground to the tank and not to the tap on the variac. This eliminates the voltage drop across the ammeter from the voltage read on the voltmeter. Before any runs were made the tanks were shorted to the copper plate and voltages and amperages read. From these it was found that for the corner tank there was a 0.15 v/amp wire loss and for the center tank 0.2 v/amp wire loss. These factors were used to correct the voltage readings in each run.

A plot of the aqueous KCl resistivity data taken from the International Critical tables is given in Fig. 6.

4.0 RESULTS

Two series of runs were made and these are reported in Tables 1-4. The average calculated results from Tables 2 and 4 are plotted in Figs. 7 and 8.

In general three types of runs were made. The first type was made using 28 liters of solution and the hookup of Fig. 5. The variacs were adjusted so that the two tank voltmeters read the same in a given run. The second type was made using 28 liters of solution and the hookup of Fig. 5 except that the green line to the corner tank was disconnected. The third type was the same as the second except the yellow line was disconnected instead of the green line. In all runs the KCl solution concentration was determined from the specified value of k , equation 19, and Fig. 6.

The calculated results are based on equation 20 where

$$\Delta T = 170^{\circ}\text{F}$$

For the center tank the right hand side of equation 20 was multiplied by two because in the simulator only half of a center tank was simulated and the measured I was half that for a whole tank. The value of ΔT was based on a solution boiling point of 230°F and an average year round wind temperature of 60°F .

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Fig. 3. Photograph of simulator and auxiliary equipment.

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Fig. 4. Photograph of tanks in simulator.

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Fig. 5. Wiring diagram.

Fig. 6. The resistivity of aqueous KCl solutions at 18°C. Data taken from International critical tables.

Table 1. Series 1, Original Data

Run No.	Solution Volume	Corner Tank (Green)			Center Tank (Yellow)			Remarks
		emf, volts	Corrected emf, volts	ma	emf, volts	Corrected emf, volts	ma	
1	28 liters	4	3.86	900	4	3.96	221	371.8 g KCl k = 0.5559 Btu/hr ft °F KCl mol. wt = 74.6
2		3	2.90	650	3	2.97	159	
3		0	0	0	1	0.99	71	
4		0	0	0	3	2.96	221	
5		0	0	0	6	5.9	482	
6		2	1.93	440	0	0	0	
7		4	3.86	940	0	0	0	
8		3	2.9	700	0	0	0	
9	28 liters	5	4.96	237	0	0	0	62.66 g KCl in 28 liters of solution. k = 0.07976. Btu/hr ft °F
10		2	1.99	94	0	0	0	
11		10	9.92	515	0	0	0	
12		5	4.97	226	5	4.99	53	
13		10	9.93	475	10	9.98	120	
14		2	1.99	89.5	2	1.96	22.5	
15		0	0	0	2	1.99	50.5	
16		0	0	0	4	3.99	60.5	
17		0	0	0	6	5.98	93.5	
18		5	4.97	225	5	4.99	55	
19		5	4.97	225	5	4.99	54.5	
20		5	4.97	224	5	4.99	54	
21		5	4.97	224	5	4.99	54	
22		5	4.97	223	5	4.99	53	
23		5	4.97	223	5	4.99	52.5	
24		5	4.97	222	5	4.99	52.5	
25		5	4.97	219	5	4.99	51.5	

Table 2. Series 1, Calculated Results

Runs 1-8 ($a = 0.001$; $\Delta\tau = 230-60 = 170$; $k = 0.5559$ Btu/hr ft $^{\circ}\text{F}$; $\frac{\Delta\tau}{a} = 170,000$)

Run No.	Solution Volume	Corner Tank, $q = 170,000 \frac{I}{\Delta E}$	Center Tank, $q = 340,000 \frac{I}{\Delta E}$
1	28 liters	39,637	18,975
2		$\frac{38,102}{\text{average} = 38,870}$	$\frac{18,202}{\text{average} = 18,589}$
3		0	24,384
4		0	25,385
5		0	$\frac{27,776}{\text{average} = 25,848}$
6		38,755	0
7		41,399	0
8		$\frac{41,034}{\text{average} = 40,396}$	0

Runs 9-25 ($a = 0.001$; $\Delta\tau = 230-60 = 170$; $k = 0.07976$ Btu/hr ft $^{\circ}\text{F}$; $\frac{\Delta\tau}{a} = 170,000$)

9	28 liters	8,123	0
10		8,030	0
11		$\frac{8,826}{\text{average} = 8,325}$	0
12		7,730	3,611
13		8,132	4,088
14		$\frac{7,646}{\text{average} = 7,834}$	$\frac{3,903}{\text{average} = 3,868}$
15		0	5,984
16		0	5,155
17		0	$\frac{5,316}{\text{average} = 5,485}$
18		7,696	3,747
19	24	7,696	3,713
20	20	7,662	3,679
21	16	7,662	3,679
22	12	7,628	3,611
23	8	7,628	3,577
24	4	7,594	3,577
25	1	7,491	3,509

Table 3. Series 2, Original Data

Run No.	Solution Volume	Corner Tank (Green)			Center Tank (Yellow)			Remarks
		emf, volts	Corrected emf, volts	ma	emf, volts	Corrected emf, volts	ma	
1	28 liters	5	4.94	410	5	4.98	100	139 g KCl k = 0.22086 Btu/hr ft °F
2		3	2.96	240	3	2.99	59	
3		3	2.96	250	0		0	
4		5	4.93	442	0		0	
5		0		0	3	2.98	80	
6		0		0	5	4.97	143	
7	28 liters	5	4.9	642	5	4.97	164	216 g KCl k = 0.35441 Btu/hr ft °F
8		3	2.95	362	3	2.98	89	
9		0		0	3	2.97	126	
10		0		0	5	4.95	226	
11		3	2.94	390	0		0	
12		5	4.9	693	0		0	
13	28 liters	3	2.93	475	3	2.98	119	293 g KCl k = 0.45492 Btu/hr ft °F
14		5	4.87	880	5	4.96	218	
15		0		0	3	2.97	172	
16		0		0	5	4.94	290	
17		5	4.86	942	0		0	
18		3	2.92	520	0		0	

Table 4. Series 2, Calculated Data

Runs 1-6 ($a = 0.001$; $\Delta\tau = 230-60 = 170$; $k = 0.22086$ Btu/hr ft $^{\circ}\text{F}$; $\frac{\Delta\tau}{a} = 170,000$)

Run No.	Solution Volume	Corner Tank, $q = 170,000 \frac{I}{\Delta E}$	Center Tank, $q = 340,000 \frac{I}{\Delta E}$
1	28 liters	14,108	6,827
2		$\frac{13,784}{\text{average} = 13,946}$	$\frac{6,708}{\text{average} = 6,767}$
3		14,356	0
4		$\frac{15,240}{\text{average} = 14,798}$	0
5		0	9,126
6		0	$\frac{9,782}{\text{average} = 9,454}$

Runs 7-12 ($a = 0.001$; $\Delta\tau = 230-60 = 170$; $k = 0.35441$ Btu/hr ft $^{\circ}\text{F}$; $\frac{\Delta\tau}{a} = 170,000$)

7	28 liters	22,273	11,217
8		$\frac{20,861}{\text{average} = 21,567}$	$\frac{10,152}{\text{average} = 10,684}$
9		0	14,423
10		0	$\frac{15,521}{\text{average} = 14,972}$
11		22,550	0
12		$\frac{24,041}{\text{average} = 23,295}$	0

Runs 13-18 ($a = 0.001$; $\Delta\tau = 230-60 = 170$; $k = 0.45492$; $\frac{\Delta\tau}{a} = 170,000$)

13	28 liters	27,559	13,576
14		$\frac{30,717}{\text{average} = 29,138}$	$\frac{14,943}{\text{average} = 14,259}$
15		0	19,689
16		0	$\frac{19,958}{\text{average} = 19,823}$
17		32,949	0
18		$\frac{30,274}{\text{average} = 31,611}$	0

Fig. 7. Steady state heat losses with all tanks at boiling point.

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Fig. 8. Steady state heat losses with some tanks at boiling point and the others not generating heat.

The two extremes for k are 0.5559 Btu/hr ft °F for average moist soil and 0.07976 Btu/hr ft °F for dry soil. The variation of q with k for the tanks is shown in Figs. 7 and 8. The value of q is quite sensitive to changes in k as is evident from the figures. The heat loss rate for average moist soil is roughly four times that for dry soil. Also for a given tank at the boiling point the value of q is higher when some of the other tanks are not generating heat than when all tanks are at the boiling point. The increase is greater for a center tank than for a corner tank because the corner tank is exposed to more earth and less neighboring tanks than is a center tank.

Run 18 in series 1 was made with 28 liters of solution. In each succeeding run a measured quantity of solution was dipped out of the simulator until in run 25, one liter of solution remained. The calculated results in Table 2 show a 2.6% decrease in q for a corner tank and 5.4% decrease in q for a center tank. In other words the original choice of the overall size of the simulator is more than adequate because most of the calculated heat flow occurs in the first liter of solution.

A final point which is based on equation 20 is that the reader may be located in a warmer or colder climate or be dealing with a waste which has a different boiling point. He might like to correct the heat loss rates given in Figs. 7 and 8. He may do this by substituting in the formula

$$q_{\text{reader}} = q_{\text{author}} \frac{(\Delta T)_{\text{reader}}}{170} \quad (21)$$

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