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USE OF DEPLETED URANIUM IN THERMAL REACTORS
WITH SLIGHTLY ENRICHED FUEL, TO ACHIEVE
HIGH NEUTRON ECONOMY AND HIGH BURNUP

by

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I. INTRODUCTION

In thermal reactors, the excess reactivity required for a pre-assigned core life is ordinarily controlled by the use of some parasitic burnable poison or by shim control rods. Such practice impairs the neutron economy of the core and, in effect, shortens its life.

Improved economy of neutrons and increase of core life may be achieved by the use of fertile or depleted "D" fuel elements (of 0.2 to 0.6% enrichment) properly distributed in between slightly enriched (of 2 to 4% enrichment) fuel elements. This concept has previously been discussed⁽¹⁾ and it will be shown in more detail in this report.

II. INITIAL CALCULATIONS

In a study for a possible Core-2 design for the Experimental Boiling Water Reactor (EBWR), the use of depleted (0.4% enriched) with slightly enriched (2.7%) fuel elements in a different arrangement than shown here (see Fig. 1), was proposed by B. I. Spinrad⁽²⁾ (see Fig. 2). The volume ratio of depleted to slightly enriched elements was to be approximately 1 to 3. There was to be no shim control by the use of soluble H_3BO_3 or boron-steel strips. A nine-rod cold-shutdown requirement without the use of any other parasitic absorber was mandatory.

A number of core configurations were considered, and PDQ calculations (in XY geometry) were made to determine the reactivities of the cold, clean core with all of the nine rods down by the method described in Ref. 3. The following pertinent information was obtained:

(1) Homogenizing the slightly enriched fuel "E" (2.7%) with depleted "D" (0.4%) fuel elements in such a manner as to give an equivalent uniform enrichment of 2.14% in the core yielded a value for k_{eff} of 1.0709.

(2) Having the control rods face only the "E" elements for a maximum worth of the rods was not adequate for a nine-rod cold shutdown. A calculated value for k_{eff} of 1.0626 was obtained.

(3) Control of reactivity with nine rods alone could be obtained by breaking up local criticalities, achieved by interspersing the "E" elements and "D" elements (see Fig. 1). It was found that $k_{eff} = 0.99$ for the configuration of Fig. 1 in which the control rods face only "E" elements.

Fig. 1

XY Geometry of Distribution of Fuel Elements in Cores that Yield Satisfactory Results. "D" indicates depleted and "E" enriched fuel elements.

112-2284

CYLINDRICAL CORE

112-3694 VERTICAL CROSS SECTION

CYLINDRICAL CORE

CONCENTRIC E AND D SECTIONS

Fig. 2. Initial Configuration Proposed for Possible Use of Depleted "D" and Slightly Enriched "E" Elements

The results obtained in (3) above imply a high importance function associated with the "D" elements. The latter had a high thermal flux and also a high internal conversion ratio (ICR) by virtue of their low initial enrichments:

$$ICR \simeq \frac{\sum_a^{28}}{\sum_a^{25}} + \epsilon \eta^{25} (1 - p) e^{-\tau B^2},$$

where Σ_a denotes the macroscopic cross section for absorption, ϵ the fast-fission factor, η the number of neutrons produced per neutron absorption in fissile nuclei, p the resonance escape probability, τ the age of neutrons at the U^{238} resonance energy, and B^2 the geometric buckling.

Buildup of fissionable plutonium in the "D" elements should, then, have a greater magnitude of reactivity worth, than it would, or burnup of U^{235} would, have in an adjoining "E" element. Thus, it might be possible to build up reactivity in the core with burnup.

The concept of using depleted fuel elements, such as in the well-known centrally located seed and outer ring of blanket, does not give this rising characteristic of reactivity.

An apparent drawback to the use of "D" elements in the core would be a resultant reduction of reactor power. This is not, however, very serious, since the thermal flux would be high initially in the "D" elements, and a buildup of plutonium in the "D" elements with high ICR would increase the power obtainable as the reactor progresses in operation. Another difficulty might arise from a hot spot at the interface of "E" and "D" elements, due to the rise of thermal flux in the "D" elements. These hot spots could be reduced (but not eliminated) by a tapering of the fuel enrichments from "D" to "E" elements.

A compensating feature of this flux rise in the "D" elements is that it also flattens radially the power characteristic of the core, increasing the output power of the core. The power peaks will also be reduced with burnup. These points will be shown in this report.

III. WORK DONE

The initial work⁽¹⁾ for EBWR was intended to indicate the feasibility of the approach. To obtain properties for various isotopic concentrations of plutonium, use was made of data extrapolated from a previous study on a similar boiling reactor core.⁽⁴⁾ This extrapolation had limited accuracy when applied to "D" elements with low initial enrichments.

Extended calculations have been made with the ANL CYCLE Code⁽⁵⁾ to determine the burnup characteristics of two lattice designs of core with overall dimensions like those of EBWR. In these calculations, the core was assumed to be hot with a nonboiling water density of 0.792 gm/cm^3 . Single-cycle, one-batch loadings only were considered. No allowance was made for Xe + Sm poisoning. Two cases have been studied:

A. a loosely packed lattice with a volume ratio of H_2O to UO_2 of 2.82: Core A (the same as Ref. 1);

B. a more tightly packed core with a volume ratio of H_2O to UO_2 of 1.29: Core B, such as might be used in a pressurized water reactor or in a boiling water reactor with forced circulation of coolant.

A. Core A ($\text{H}_2\text{O}/\text{UO}_2 = 2.82$)

This loosely packed core is suitable for boiling reactors with natural circulation. It had been designed for possible use as Core 2 in EBWR. The constants of the core are listed in Table I, and the cross sections are shown in Tables II and III.

Table I

VOLUME FRACTIONS (SQUARE LATTICES)

	Core A ($\text{H}_2\text{O}/\text{UO}_2 = 1.82$)	Core B ($\text{H}_2\text{O}/\text{UO}_2 = 1.29$)
$V_F \text{UO}_2$	0.2085	0.3480
$V_F \text{H}_2\text{O}$	0.5888	0.4493
$V_F \text{Zircaloy}$	0.1970	0.1970
$V_F \text{He}$	0.0057	0.0057
Diameter of Pellet, cm	0.927	0.8

$$N^{\text{H}_2\text{O}} = 0.0265 \times 10^{24}; N^{\text{Zircaloy}} = 0.0423 \times 10^{24};$$

$$N^{\text{UO}_2} = 0.0229 \times 10^{24}$$

$$\text{Density of H}_2\text{O (hot)} = 0.792 \text{ gm/cm}^3;$$

$$\text{Density (UO}_2) = 10.2 \text{ gm/cm}^3.$$

Table II

THERMAL CROSS SECTIONS USED FOR CALCULATIONS OF BURNUP IN CORE A ($\text{H}_2\text{O}/\text{UO}_2 = 2.82$)

Isotopes	Fuel Elements (or Rings)	$\hat{\sigma}_c$ (b)	$\hat{\sigma}_f$ (b)	Isotopes	Fuel Elements (or Rings)	$\hat{\sigma}_c$ (b)	$\hat{\sigma}_f$ (b)
U238	E, E', D	2.72		Pu242	D	124	
Pu239	E, E'	625	1196.6	U235	E, E'	111.3	560.5
Pu239	D	562.5	1077	U235	D	108.1	560.5
Pu240	E, E'	760		U236	E, E', D	7	
Pu240	D	510		Fission		50	
Pu241	E, E'	496.2	1368	Product	E, E', D	(per fission)	
Pu241	D	487.2	1343	H ₂ O	E, E', D	0.664	
Pu242	E, E'	180		Zircaloy	E, E', D	0.20	

NOTES:

1. Westcott cross sections⁽⁷⁾ used in accordance with the prescription of Crowther and Weil.⁽⁸⁾
2. $\sigma(E, E')$ refers to element types E, E', with initial enrichments of 2.7% and 1.8%, respectively.
3. Microscopic cross sections for the E' elements (of small physical dimensions) were taken to be the same as for E elements, due to limitations of the CYCLE code.

Table III

CROSS SECTIONS (cm^{-1}) USED IN CORE A
(INITIAL HOT CLEAN CORE)

<u>Group 1 (j=1)</u>				
<u>Fuel Element</u>	<u>Σ_{REM}</u>	<u>$3\Sigma_{\text{TR}}$</u>	<u>$\epsilon\nu\Sigma_{\text{F}}$</u>	<u>$\Sigma_{j \rightarrow j+1}$</u>
E ₁	0.03843	0.64250	0	0.03843
D ₁	↓	↓	↓	↓
E ₂				
D ₂				
E ₃	0.03843	0.64250	↓	0.03843
Reflector	0.052346	0.47938	0	0.052346
<u>Group 2 (j=2)</u>				
<u>Fuel Element</u>	<u>Σ_{REM}</u>	<u>$3\Sigma_{\text{TR}}$</u>	<u>$\epsilon\nu\Sigma_{\text{F}}$</u>	<u>$\Sigma_{j \rightarrow j+1}$</u>
E ₁	0.056816	1.25220	0	0.048547
D ₁	0.057028	↓	↓	↓
E ₂	0.056816			
D ₂	0.057028	↓	↓	↓
E ₃	0.056816	1.25220	↓	0.048547
Reflector	0.082214	1.30700	0	0.082214
<u>Group 3 (j=3)</u>				
<u>Fuel Element</u>	<u>Σ_{REM}</u>	<u>$3\Sigma_{\text{TR}}$</u>	<u>$\epsilon\nu\Sigma_{\text{F}}$</u>	<u>$\Sigma_{j \rightarrow j+1}$</u>
E ₁	0.096280	2.2613	0.156476	0
D ₁	0.028972	↓	0.0136109	↓
E ₂	0.096280		0.156476	
D ₂	0.028972	↓	0.0136109	↓
E ₃	0.096280	2.2613	0.156476	↓
Reflector	0.017596	3.7500	0	0

The values of k_{eff} and breeding ratio versus burnup characteristics of the core are shown in Fig. 3. These data were obtained for a cylindrical equivalent geometry, shown in Fig. 4, from the fuel element distribution shown in Fig. 1. The value of k_{eff} for the hot clean core is 1.047, and there is a maximum rise in k_{eff} of $\delta k_{\text{eff}} = +0.0104$. It was found that $\delta k_{\text{eff}} \geq 0$ for a maximum burnup of 6000 MWd/tonne of "E" element. This value of maximum burnup would improve if calculations were made for a core with boiling water (lower p and hence higher rate of plutonium formation). For the present nonboiling case, allowing for Xe + Sm poisoning (approximately 3% ρ), the values of k_{eff} will be ≥ 1 for a burnup of 8500 MWd/tonne of the

central "E₁" element (see Fig. 1). There will be, during this period of time, average burnups of 5700 MWd/tonne and 2060 MWd/tonne in the "E" and "D" elements, respectively.

Fig. 3
k_{eff} and Breeding Ratio
Versus Burnup in Core A

112-3692

Fig. 4
Cylindrical Equivalent of Fuel
Distribution with XY Geometry
of Fig. 1

112-3681

The breeding ratio of the core, defined as

$$\frac{\Sigma_a^{28} + \Sigma_a^{40}}{\Sigma_a^{25} + \Sigma_a^{49} + \Sigma_a^{41}},$$

is quite low (approximately 0.5) for this nonboiling case of a loosely packed core. The radial distributions, at midcore, of thermal neutron flux and of power are shown in Figs. 5 and 6. The burnup rates and buildup of plutonium in the various fuel elements are shown in Figs. 7 and 8, respectively.

112-3684

Fig. 5. Radial Distribution of Thermal-neutron Flux at Midcore Plane for Initial Core and for Core with Burnup of 10,000 MWd/tonne of Central Fuel Element E_1

The above data serve to indicate the operational characteristics of a loosely packed core, for which the advantages of the proposed scheme are not great.

Fig. 6
Radial Distribution of Power at
Midcore Plane for Initial Core
and after 10,000 MWd/tonne
Burnup of Central Element E_1

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112-3690

Fig. 7. Relative Burnup Rates of
Different Fuel Elements
in Core A

112-3683

Fig. 8. Buildup of Pu^{239} in
Elements of Core A

B. Core B ($\text{H}_2\text{O}/\text{UO}_2 = 1.29$)

This tightly packed core with a high conversion ratio is of greater interest than Core A. Dimensionally, this core is the same as Core A except that small sections of E_1 and E_2 , facing D_1 , have been converted into subrings E'_1 and E'_2 with the reduced enrichment of 1.8%.

The constants of the core are listed in Table I and the cross sections in Tables IV and V. The enrichments of the elements E_1' and E_2' (see Fig. 9) were taken to be 1.8%, in order to reduce the magnitudes of hot spots at the interfaces between the D_1 and E_1 , E_2 elements (prior to this subdivision).

Table IV

THERMAL CROSS SECTIONS USED FOR BURNUP CALCULATIONS IN CORE B ($H_2O/UO_2 = 1.29$)

Isotopes	Fuel Elements (or Rings)	$\hat{\sigma}_c$ (b)	$\hat{\sigma}_f$ (b)	Isotopes	Fuel Elements (or Rings)	$\hat{\sigma}_c$ (b)	$\hat{\sigma}_f$ (b)
U ²³⁸	E, E', D	272		Pu ²⁴²	D	125	
Pu ²³⁹	E, E'	686	1300	U ²³⁵	E, E'	113	561.5
Pu ²³⁹	D	549	1040	U ²³⁵	D	106	561.5
Pu ²⁴⁰	E, E'	1000		U ²³⁶	E, E', D	7	
Pu ²⁴⁰	D	600		Fission	E, E', D	50	
Pu ²⁴¹	E, E'	512	1412	Product		(per fission)	
Pu ²⁴¹	D	491	1355	H ₂ O	E, E', D	0.664	
Pu ²⁴²	E, E'	250		Zircaloy	E, E', D	0.20	

NOTES

- Westcott cross sections⁽⁷⁾ used in accordance with the prescription of Crowther and Weil (8)
- $\sigma(E, E')$ refers to element types E and E' with initial enrichments of 2.7% and 1.8%, respectively.
- Microscopic cross sections for the E' elements (of small physical dimensions) were taken to be the same as for E elements, due to limitations of the CYCLE code.

Table V

CROSS SECTIONS (cm^{-1}) USED IN CORE B
(INITIAL HOT CLEAN CORE)

Group 1				
Fuel Element	Σ_{REM}	$3\Sigma_{TR}$	$\epsilon\nu\Sigma_F$	$\Sigma_{j \rightarrow j+1}$
E ₁	0.03524	0.71110	0	0.03524
E ₁ '	↓	↓	↓	↓
D ₁				
E ₂				
E ₂ '				
D ₂				
E ₃	0.03524	0.71110	↓	0.03524
Reflector	0.052346	0.47938	0	0.052346
Group 2				
Fuel Element	Σ_{REM}	$3\Sigma_{TR}$	$\epsilon\nu\Sigma_F$	$\Sigma_{j \rightarrow j+1}$
E ₁	0.051041	1.27330	0	0.037240
E ₁ '	0.051169	↓	↓	↓
D ₁	0.051396			
E ₂	0.051169			
E ₂ '	0.051041			
D ₂	0.051396	↓	↓	↓
E ₃	0.051041	1.27330	↓	0.037240
Reflector	0.082214	1.30700	0	0.082214
Group 3				
Fuel Element	Σ_{REM}	$3\Sigma_{TR}$	$\epsilon\nu\Sigma_F$	$\Sigma_{j \rightarrow j+1}$
E ₁	0.175738	2.8500	0.306685	0
E ₁ '	0.127575	↓	0.204457	↓
D ₁	0.0418438		0.0227174	
E ₂	0.127575		0.204457	
E ₂ '	0.175738		0.306685	
D ₂	0.0418438	↓	0.0227174	↓
E ₃	0.175738	2.8500	0.306685	↓
Reflector	0.017596	3.7500	0	0

Fig. 9

Cylindrical Equivalent of Fuel
Distribution of Fuel Elements
in Core B

112-3689

The variation of k_{eff} versus burnup characteristic of the core is shown in Fig. 10. There is a maximum reactivity rise of 4.7%, and $\delta k_{eff} \geq 0$ for a total burnup in the control E_1 element of 20,000 MWd/tonne. During this period of time, the E and E' elements will have had an average burnup of ~14,000 MWd/tonne and the D elements an average of 8500 MWd/tonne. The core average will be 12,600 MWd/tonne.

112-3664 Rev.

Fig. 10. k_{eff} and Breeding Ratio Versus Burnup in Core B

The radial flux and power distribution characteristic of the initial core at its midplane are shown in Figs. 11 and 12. Reduction of fuel enrichments from 2.7% to 1.8% in the subdivisions E'_1 and E'_2 , referred to above, resulted in a peak power reduction of from 3.66 to 2.85 (see Fig. 12) at the interface of E'_1 and D_1 , and a new peak of 3.16 at the interface of E_1 and E'_1 . The ratio of this peak to power at core center, is 1.45.

Fig. 11

Radial Distribution of Thermal-neutron Flux at Midcore Plane for Initial Core and for Burnups of 10,000 and 20,000 MWd/tonne of Central Element E_1

112-3682

112-3663
Rev.

Fig. 12. Radial Distribution of Power at Midcore Plane for Initial Core and after 10,000 and 20,000 MWd/tonne Burnups of Central Element E_1

The peaks between E_2 , D_2 and D_2 , E_3 interfaces may be reduced in a similar manner by the use of E' elements between E_2 , D_2 and D_2 , E_3 interfaces.

The above power peaks may appear to be quite objectionable; however, it is important to note that the flux peaking caused by "D" elements flattens also the overall radial characteristic of the core. This is shown by a comparison with the radial power distribution characteristic of a similar core having a uniform enrichment of 2.07%. The following were the results of calculations for the hot clean cores:

1) Radial maximum/average power ratio for Core B, with maximum peak at interface of E' , D_1 , is equal to 1.94.

2) Radial maximum/average power ratio for the core with a uniform enrichment of 2.07%, is equal to 2.01%.

It is seen that the values of maximum/average power ratios are nearly the same for both cores.

112-3688

Fig. 13. Buildup of Pu^{239} in Various Elements of Core B Versus Burnup of Fuel in E_1 Element

of 0.365×10^{20} and 0.34×10^{20} atoms/cc, respectively (see Figs. 13 and 14). This amounts to a little over 0.4% enrichment of Pu^{239} . Concentrations of Pu^{240} and Pu^{241} are shown in Fig. 15.

The power developed in the "E" and "D" elements of the "D, E" core was initially greater than 91% and 7% of the total power, respectively. After 20,000 MWd/tonne burnup in " E_1 " element, the powers developed in the "E" and "D" elements became 77% and 21.5%, respectively (see Fig. 12). With burnup, the power peaks are reduced (see Fig. 12).

Thus, by virtue of the high neutron economy of the proposed arrangement shown in Fig. 9, the "E" and "E'" elements with an average enrichment of 2.66% will have had an average burnup of approximately 14,000 MWd/tonne and the "D" elements an average of 8500 MWd/tonne. These " D_1 " and " D_2 " elements will, when $\delta k_{eff} \rightarrow 0$, have Pu^{239} concentrations

Fig. 15. Buildup of Pu²⁴⁰ and Pu²⁴¹ in Elements of Core B Versus $\phi_3 t$

A comparison of burnup rates in different fuel elements is shown in Figs. 16 and 17.

112-3686

Fig. 16. Comparative Burnup Rates of Fuel Elements in Core B

112-3687

Fig. 17

Variations in Burnup Rates of E_3 and D Elements with Core Burnup

IV. CONCLUSIONS

In this report are given in some detail the average burnup capabilities of two cores with Zircaloy as structural material, and which use slightly enriched (2.3 to 2.7% enrichment) and depleted (0.2% enrichment) fuel elements in the ratio of 1 to 3 in numbers, or their volume equivalents.

The possibilities for high burnup in Core B were contrasted with the hot spots occurring at the interfaces between the "D" and "E" elements. It was, however, shown that for Core B there is a flattening in the power characteristics because of the rise of thermal flux in the "D" elements, which compensates for the deleterious effects of the hot spots. The latter were somewhat reduced by the use of fuel sections with enrichments intermediary to those of the "D" and "E" elements.

In our calculations, no allowance has been made for Xe + Sm poisoning. This will amount to about 3% ρ , which requires, according to some of our calculations, an increase in the enrichment of "E" elements of Core B from the present 2.7% to 3.0%. The average enrichment of the core will then be 2.3%.

There will be a rise in core reactivity of about 4.6% ρ with burnup. In general, however, it may not be feasible to absorb the Xe + Sm poisoning effect with this rise. The increased enrichment and reactivity to be controlled will reduce the conversion ratio of the core, and the total average calculated burnups of 12,600 MWd/tonne will be reduced by about (estimated by extrapolation) 600 MWd/tonne.

The economic advantages in the use of the proposed core are:

- (1) decreasing feed enrichment and, therefore, inventory;
- (2) burning more plutonium in place (also a result of lower enrichment);
- (3) building up more plutonium from the higher conversion ratio (initially, average breeding rate is 0.74 for the core).

V. METHOD OF CALCULATIONS

The CYCLE-II code⁽⁵⁾ written at Argonne National Laboratory recently consists of a step-by-step evaluation of criticality by the diffusion theory subroutine (RE-122) and of burnup calculations from an exact solution of the isotopic differential equations, making use of the neutron flux distribution obtained from the preceding diffusion theory subroutine.

In the study of Core A, a cylindrical equivalent of Fig. 1 was used in which the geometry of similar elements was approximated by concentric rings, as shown in Fig. 4. In the study of Core B, rings of the "E" element were added. They would correspond approximately to a row of fuel rods in an element "E₁" of Fig. 1, with a lower enrichment of 1.8%, and facing a "D₁" element.

VI. CROSS SECTIONS

Three-group diffusion theory was used for determining the core reactivity and the neutron fluxes ϕ :

$$-D_1 \nabla^2 \phi_1 + \Sigma_{s1_1} \phi_1 = \Sigma_2 \Sigma_{f3} \phi_3;$$

$$-D_2 \nabla^2 \phi_2 + (\Sigma_{s1_2}/p) \phi_2 = \Sigma_{s1_1} \phi_1;$$

$$-D_3 \nabla^2 \phi_3 + \Sigma_{a3} \phi_3 = \Sigma_{s1_2} \phi_2,$$

where the cross sections are as described below.

The three energy groups used were as follows:

Group 1: $\infty \rightarrow 0.18$ MeV (includes all fission neutrons);

Group 2: 0.18 MeV $\rightarrow 0.625$ eV;

Group 3: 0.625 eV $\rightarrow 0$.

A. Fast Cross Sections

The cross sections in Groups 1 and 2 were computed as described by Deutsch⁽⁶⁾ by means of the equivalence factors method. For U, y_1 , and y_2 previously calculated⁽³⁾ values of $y_1 = 0.34$ and $y_2 = 0.213$ were used. Epithermal absorption was accounted for by the use of the Westcott cross sections,⁽⁷⁾ except for U²³⁸. The latter was assumed to be a $1/v$ absorber, thermally, plus a resonant absorber. Capture by resonance was allowed for in the epithermal energy group.

B. Thermal Cross Sections

Westcott cross sections⁽⁷⁾ were used in accordance with the Crowther-Weil⁽⁸⁾ prescription, except that U²³⁸ was assumed to be a $1/v$ absorber.

VII. RESONANCE INTEGRAL OF U^{238} (Ref. 9)

A. Core A

$$\sigma_0^{28} = 4.45 + 26.6 \sqrt{S/M},$$

where

$$S/M = 2/\rho a = 0.423 \left(\begin{array}{l} \rho = 10.2 \\ \text{pellet radius } a = 0.4635 \text{ cm} \end{array} \right).$$

The Dancoff self-shielding factor is found⁽¹⁰⁾ to be $C = 0.035$ for two rods. For eight rods surrounding a rod, $C_{\text{total}} = 0.18$. Doppler broadening coefficient⁽¹¹⁾ for $a = 0.463$, $\beta = 0.0084/^{\circ}\text{C}$. For the hot reactor (525°K),

$$\sigma_0^{28} = 21.4.$$

$$\sigma_{\text{eff resonance}}^{28} = 22.4 / \ln \left(\frac{0.18 \times 10^6}{0.625} \right) = 1.68b.$$

B. Core B

Pellet radius $a = 0.4 \text{ cm}$.

Similar to Core A, C and β are found to be

$C = 0.052$ for two rods, and $C_{\text{total}} = 0.28$

$\beta = 0.0083$

and

$$\sigma_0^{28} = 1.68b, \text{ same as above for Core A.}$$

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