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Peter G. O. Freund

The Enrico Fermi Institute

and the Department of Physics

The University of Chicago, Chicago, Illinois, 60637

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THE THREE COMPONENTS OF D-SPIN^{*}

Peter G. O. Freund

The Enrico Fermi Institute

and the Department of Physics

The University of Chicago, Chicago, Illinois, 60637

ABSTRACT

The concept of D-spin, previously introduced to account for the selection rules for exotic hadron couplings, is further investigated. The three components of D-spin are identified. D_3 turns out to be a simple topological characteristic of the way a hadron couples at a vertex. D_3 is conserved at hadronic vertices. In hadronic N-point functions an "effective D_3 " is conserved. It is shown that in the presence of three species of quarks (B, N, λ) there are three D-spins. An application of the D-spin formalism to diffractive exotic hadron production is presented.

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1. INTRODUCTION

No hadrons have ever been observed to decay into two protons, into two positive pions, into two K-mesons, etc.... This was originally interpreted as evidence against the existence of exotic hadrons, i.e. of hadrons with total quark number (number of quarks plus number of antiquarks) $N > 3$. Using the dual¹⁾ two component (background and resonance) picture²⁾ of hadronic reactions, it was shown³⁾ that exotic hadrons of arbitrarily large⁴⁾ N ought to exist. The absence of resonances from channels like pp , $\pi^+\pi^+$, K^+K^+ , etc.... is then accounted for, by a selection rule.⁵⁾ It was suggested that this selection rule can be interpreted as the conservation of a new spin-like quantity: the D-spin.⁶⁾ For this interpretation to hold up, one has to identify the components D_1 , D_2 and D_3 of D-spin and see what one means by, say, a baryon of $D = 1/2$, $D_3 = -1/2$. In this paper we shall identify the components of D-spin and show that D_3 is indeed a new additive quantum number at hadronic vertices. We shall also find that at hadronic vertices D_3 and $\vec{D}^2$ are conserved but not D_1 and D_2 . In hadronic N-point functions D_3 conservation can be violated as propagators do not conserve D_3 . However, even for N-point functions, a conserved additive quantum number the "effective D_3 " or $\bar{D}_3$ can be defined.

We then show that for the important case of three species of quarks $\mathcal{Q}, \mathcal{N}, \lambda$ one can define three types of D-spin.

Finally we apply the D-spin formalism to the problem of exotic hadron production.

2. REVIEW OF D-SPIN^{*}

To start our discussion let us disregard baryon number, hypercharge and isospin and simply label a hadron by its total quark number N . Thus a nonexotic (i.e. $q\bar{q}$ -type) meson becomes H_2 , a baryon (qqq) H_3 , etc... From fig. 1a we see that in H_2H_2 scattering, H_2 Regge-exchange builds up H_2 resonances in the direct channel. On the other hand in H_3H_3 scattering, the same Regge exchange builds up direct channel exotic H_4 resonances (fig. 1b).⁷⁾ From H_3H_4 scattering we then see, that H_2 exchange builds up H_5 resonances in the direct channel (fig. 1c). Continuing this procedure we see that H_2 exchange builds up through duality exotic hadrons H_N with arbitrarily high⁴⁾ total quark number N . Now in a process like H_2H_2 scattering we were able to insure duality without H_4 intermediate states or H_4 exchanges. Nevertheless from H_3H_3 scattering we were led to the existence of H_4 mesons. In order not to upset the carefully achieved duality-exchange degeneracy pattern in H_2H_2 scattering, it is required that the coupling $H_4H_2H_2$ be forbidden³⁾. Similarly the couplings $H_6H_3H_3$, $H_6H_4H_2$ etc..... have to be forbidden. One readily convinces oneself that all these couplings are indeed forbidden and a consistent duality-exchange degeneracy pattern emerges if at any allowed hadronic vertex⁵⁾ :

A) each of the three hadrons meeting at the vertex, exchanges at least one quark line with each of the remaining two hadrons and

^{*}The reader familiar with the work of references 5) and 6) may omit this section.

B) no quark line is connected with an antiquark line from the same hadron.

The principles A) and B) then lead without much further ado to the selection rule⁵⁾:

S) for the coupling ABC to be allowed it is necessary that the following three inequalities hold (N_A = total quark number of A, etc...)

$$\begin{aligned} N_A &\leq N_B + N_C - 2 \\ N_B &\leq N_C + N_A - 2 \\ N_C &\leq N_A + N_B - 2 \end{aligned} \quad (2.1)$$

Defining the D-spin of a hadron⁶⁾ by

$$D = \frac{N}{2} - 1, \quad (2.2)$$

the relations (1) are equivalent to

$$|D_B - D_C| \leq D_N \leq D_B + D_C \quad (2.3)$$

Baryon conservation and the definition (2) then require the differences $D_B + D_C - D_A$ and $D_B + D_C - |D_B - D_C|$ to be integers. Thus D_A can take only the values $|D_B - D_C|, |D_B - D_C| + 1, \dots, D_B + D_C$. At this point the identification of D with a spin-like quantity becomes highly suggestive. The selection rule S then states that at hadronic vertices D-spin is conserved. To complete the identification of D with a spin-like quantity we have to find the individual components of D-spin. This will be done in the next section.

Here we still make some useful observations.

i) D-spin conservation holds not only at hadronic vertices but for N-point functions in the tree-approximation. Diagrams with loops break D-spin conservation. This is illustrated in fig. 2 where a $D=1$ hadron H_4 decays into two nonexotic ($D=0$) mesons via a loop diagram. As in a dual theory resonances are narrow, diagrams with loops (i.e. final state interactions with the exception of Pomanchuk type diagrams) are expected to be small and consequently D-spin conservation is expected to hold rather accurately. As we shall see later, the small branching ratio $\phi \rightarrow \rho\pi / \phi \rightarrow K\bar{K}$ is a measure of D-spin breaking.

ii) Inclusion of baryon number and of other additive quantum numbers would not change our discussion of this section. There is only one point to keep in mind, the proliferation of exotica discussed above, leads to arbitrarily high values of all these quantum numbers. Thus, for instance there must exist H_6 with baryon number $B = 0$ ($q^3 \bar{q}^3$) but also H_6 with $B = 2$ (q^6). Similarly H_9 with $B = 1, 3$ and so on. That this is so can be seen from fig. 3.

We now give a few applications of D-spin conservation. First, a meson H_4 has $D = 1$ so that it cannot decay into two nonexotic ($D = 0$) mesons but can decay in a baryon-antibaryon ($H_3 H_3$) pair as for H_3 . $D = 1/2$. This explains why the channels $\pi^+ \pi^+$, $K^+ K^+$ are empty of resonances. As a further example, the D-spin of the proton being $1/2$, a pp state can only couple to a $D = 0$ or a $D = 1$ hadron. Baryon conservation, on the other hand requires that the baryon number of this hadron equal two, whence its $D \geq 2$. So D-spin and baryon conservation

together forbid the appearance of resonances in the pp-system. Of course, through final state interactions, small resonant effects may be induced even in the pp system (see remark i above).

3. IDENTIFICATION OF THE THREE COMPONENTS OF D-SPIN

If the interpretation of D as a spin is to make sense, then we have to understand what we mean by D_3 (the third component of D-spin). Consider three hadrons A, B and C such that the ABC vertex be allowed. To this vertex we attach a sense of rotation say clockwise $A \rightarrow B \rightarrow C \rightarrow A$ (fig. 4). The total quark number of A is N_A . Of these N_A quark lines N_{AQ} go clockwise (i.e. to the hadron B) and $N_{A\bar{Q}}$ counter-clockwise (i.e. to the hadron C). Obviously (because of rule B section 2)

$$N_{AQ} + N_{A\bar{Q}} = N_A = 2D_A + 1 \quad (3.1)$$

Define

$$(D_3)_A = \frac{1}{2}(N_{AQ} - N_{A\bar{Q}}) \quad (3.2)$$

and similar definitions for $(D_3)_B$ and $(D_3)_C$. It is then clear that

$$(D_3)_A + (D_3)_B + (D_3)_C = 0 \quad (3.3)$$

so that D_3 is conserved at the ABC vertex. Because of rule A section 2, for any hadron

$$N_{\overline{Q}} \geq 1, N_Q \geq 1 \quad (3.4)$$

so that from (3.1) and (3.2) we see that the possible values of D_3 are

$$-D, -D+1, \dots, D \quad (3.5)$$

These are precisely the possible values of the third component of a spin whose value is D . The additive quantity

$$D_3 = \frac{1}{2} (N_{\overline{Q}} - N_Q) \quad (3.6)$$

is thus conserved and has the obvious interpretation of third component of D-spin. As simple examples, we present in fig. 5 the various D_3 - states of H_2, H_3, H_4 . From the above, it is obvious that the D_3 state of a hadron A in the ABC vertex depends on the cyclic order of the hadrons A, B, C at the vertex.

Now, D_1 and D_2 or more conveniently, $D_{\pm} = D_1 \pm iD_2$, are easily identified. As D_+ (D_-) has to be a raising (lowering) operator, it must lead from a state $|D, D_3\rangle$ to a state $|D, D_3 + 1\rangle$ ($|D, D_3 - 1\rangle$), so that it has to flip a counterclockwise (clockwise) quark line into a clockwise (counterclockwise) line. So, now we have identified $D_{\pm}$, D_3 and $\vec{D}^2$. We also know that D_3 and $\vec{D}^2$ are conserved. How about

D_+ and D_- ? To answer this question, all one has to consider is the $H_4 H_4 H_2$ vertex. If all three components of D-spin were conserved then the H_2 (with $D = 0$) ought to couple with equal strengths to an H_4 ($D = 1$) with D_3 value of $+1$, 0 or -1 . From fig. 6 however, we see that for the $H_4 H_4 H_2$ coupling to be possible, the D_3 value of H_4 must be $+1$ or -1 . It is impossible to couple H_2 to an H_4 in the $D_3 = 0$ state. This proves that D_1 and D_2 cannot be conserved. Thus D_3 and $\vec{D}^2$ are conserved at a hadronic vertex but not D_1 and D_2 . This is not hard to picture. For instance, if the interaction hamiltonian depended on D_3 , then it would commute with D_3 and $\vec{D}^2$ but not with D_1 or D_2 .

We now come to the problem of D_3 - conservation in hadronic N-point functions. This will be discussed in the next section.

4. EFFECTIVE D_3 VALUES AND N-POINT FUNCTIONS

D_3 being conserved at hadronic vertices, one would naively think that it must be conserved in hadronic N-point functions as well. After all, the latter are built out of vertices. Unfortunately this is not quite so. The point is that the vertices, making up an N-point function, are connected by "propagators" and these do not necessarily conserve D_3 . As a simple example consider the quark diagram of fig. 7. Here the s-channel H_4 intermediate state couples with $D_3 = 0$ to the baryons on the left but with $D_3 = +1$ to the $H_4 H_2$ on the right. Thus while propagating, its D_3 has changed from 0 to $+1$. Does this mean that we cannot speak of D_3 for hadronic N-point functions? No. To see how we can define D_3 for N-point functions we shall consider the special case

of 4-point functions. We divide the quark diagrams for 4-point functions in three categories:

I Planar diagrams. By a planar diagram we mean a quark diagram for which the four external hadrons can be labeled 1, 2, 3, 4 in such a way that every quark line that "enters" the diagram at the hadron i "exits" either at the hadron $i + 1$ or $i - 1 \pmod{4}$. An example of a planar diagram is given in fig. 8.

II Quasi planar diagrams. These are all quark diagrams which are not planar in the sense of I, but can be drawn in a plane without any lines crossing. For an example see Fig. 9a.

III Nonplanar diagrams. These are all quark diagrams which cannot be drawn in a plane without lines crossing. An example is given in figures 10a - 10c which show really the same diagram once as an st -once as a tu - and once as an su -diagram. As a rule nonplanar diagrams have poles in all three (s , t and u) channels. Such diagrams were previously considered in reference 6 (under the name of s - t - u diagrams). It was observed there that in a dual resonance model, such diagrams could be described by Virasoro⁸⁾, rather than Veneziano⁹⁾ type terms. The conformal theory of such Virasoro terms has been recently developed¹⁰⁾.

This solves the ambiguity, as to the treatment of such " s - t - u " diagrams, mentioned in ref. 6. Nonplanar 4-point diagrams can be drawn without crossing lines on a sphere.

With this classification of diagrams we are now in measure to discuss D_3 -conservation for the 4-point function.

It is obvious that D_3 is exactly conserved in planar diagrams.

In the case of quasiplanar and nonplanar diagrams, it is not in general possible, to attach a given D_3 value to each of the external particles. Take diagram 9a for instance. The baryon C has $D_3 = -1/2$ in the CDH_{3s} but $D_3 = +1/2$ in the BCH_{4t} vertex. To be able to attach a unique D_3 value to each external or internal hadron one has to first isolate a "planar core" of the diagram by removing the "diagonal" lines. Fig. 9b shows the planar core of fig. 9a. For this planar core we can attach uniquely a new "effective" D_3 value $\bar{D}_3$ to each hadron. For instance the baryon C now carries the "effective D_3 " value $\bar{D}_3 = 0$. Obviously since lines have been removed from the original diagram, it can happen, as indeed it does in the example under consideration, that $\bar{D}_3$ should be integer (half-integer) even though D_3 and D are half integer (integer). In any case $\bar{D}_3 \leq D$. It is readily seen that the effective D_3 of a hadron, as defined above, is really the average value of D_3 over all the vertices in which this hadron participates. As an example, for the baryon C in fig. 9, $\bar{D}_3 = 0, D_{3av} = (+\frac{1}{2} - \frac{1}{2})/2 = 0$. So for quasiplanar diagrams $\bar{D}_3$ is conserved. Similarly, for the hadrons in intermediate states in such diagrams, $\bar{D}_3$ is just the average value of D_3 of that hadron over the two vertices (with proper relative orientation!) in which it participates. Thus for instance the baryon H_{3s} as incoming to the vertex $H_{3s}DC$ has $D_3 = +1/2$ while as outgoing from the vertex ABH_{3s} has $D_3 = -1/2$ so that its $\bar{D}_3 = 0 = \bar{D}_{3A} + \bar{D}_{3B} = -\bar{D}_{3C} - \bar{D}_{3D}$.

Planar and quasiplanar diagrams for 4-point functions, can have poles in two channels only. Now, if we look at nonplanar diagrams, these have poles in all three channels so there are three different ways of drawing any nonplanar diagram (s-t, t-u, u-s) so that it has poles in

both "cyclic" channels appearing in the plane of the figure. We call these the three versions of the diagram. An example of the three versions of one diagram is presented in fig. 10. For each of these versions one can define a $\bar{D}_3$ for each particle as the average of the D_3 's with which the particle participates in the two vertices corresponding to the cyclic channels of that version. The so defined $\bar{D}_3$ is conserved just as in the quasiplanar case. However one and the same particle will have different values of $\bar{D}_3$ in the three versions of the diagram. This can be seen for example in figures 10 a-c, where the values of $\bar{D}_3$ are written in. So, for a nonplanar diagram one has to indicate the $\bar{D}_3$ values for each of its three versions.

To sum up:

- A) for planar diagrams D_3 is conserved;
- B) for quasiplanar diagrams D_3 is neither uniquely defined for each hadron, nor conserved, but for each hadron there exists a unique value $\bar{D}_3$ (the average or effective D_3) which is conserved ;
- C) for nonplanar diagrams neither D_3 nor $\bar{D}_3$ are uniquely defined for each hadron, but for each version of that diagram, there exists a uniquely defined $\bar{D}_3$ for each hadron appearing in the diagram, and moreover this $\bar{D}_3$ is conserved.

It is obvious that this procedure can be applied also to N-point functions with $N \geq 5$.

5. THREE KINDS OF D-SPIN

The quark model used in hadrodynamics, employs three species of quarks, $\mathcal{P}$, $\mathcal{N}$ and λ . For each of these three species individually, a principle of the type B of section 2 holds, however a principle of the type A does not hold. For example, two of the hadrons meeting at one vertex could exchange no λ -like quarks, but only $\mathcal{P}$ and/or $\mathcal{N}$ quarks. (see fig. 7). It is therefore necessary to define

$$D^{(i)} = \frac{N^{(i)}}{2} \quad (i = \mathcal{P}, \mathcal{N}, \lambda) \quad (4.1)$$

where $N^{(i)}$ is the number of i type quarks and antiquarks contained in the hadron. From here on, it will be convenient to use the notation $i = 1, 2, 3$ for $\mathcal{P}, \mathcal{N}, \lambda$ respectively. Then the total D-spin of section 2 is

$$D = \sum_{i=1}^3 D^{(i)} - 1 \quad (4.2)$$

Similarly

$$D_3^{(i)} = \frac{1}{2} (N_{\mathcal{Q}}^{(i)} - N_{\overline{\mathcal{Q}}}^{(i)}) \quad (4.3)$$

Here $N_{\mathcal{Q}}^{(i)}$ ($N_{\overline{\mathcal{Q}}}^{(i)}$) is the number of quark and antiquark lines of type i that leave the hadron clockwise (counterclockwise) at the vertex.

Again

$$D_3 = \sum D_3^{(i)}. \quad (4.4)$$

We can now easily generalize the arguments of section 3 to find that at a hadronic vertex $\vec{D}, \vec{D}^{(1)}, \vec{D}^{(2)}, \vec{D}^{(3)}, D_3, D_3^{(1)}, D_3^{(2)}, D_3^{(3)}$ are all conserved. Observed that D_3 conservation follows from $D_3^{(i)}$ conservation and eq. (4.4), but the conservation of $\vec{D}^2$ does not follow from those of $\vec{D}^{(i)2}$ (because of the extra -1 in eq. (4.2)). Thus $\vec{D}^2$ conservation is a separate requirement that leads to additional results.

A simple example is now in order. The decay $\phi \rightarrow \rho\pi$ is forbidden by $D^{(3)}$ conservation. Indeed $D^{(3)} = 1$ for ϕ but $D^{(3)} = 0$ for ρ and π . On the other hand this same decay is also forbidden by $D_3^{(1)}, D_3^{(2)}$ and D conservation. Indeed for the decay $\phi \rightarrow \rho^+\pi^-$, as can be seen from fig. 8, for the ρ^+ , $D_3^{(1)} = 1/2$, $D_3^{(2)} = 1/2$, so that $D_3 = 1/2 + 1/2 = 1$. This is impossible since for the ρ , $D = 0$ and therefore, the only possible value of D_3 is zero. This example substantiates the remark made earlier that the small $\phi \rightarrow \rho\pi/\phi \rightarrow K\bar{K}$ branching ratio is a measure of D -(more precisely $D^{(3)}$ -) spin nonconservation.

Effective $D_3^{(i)}$ ($i = 1, 2, 3$) for N -point functions can be constructed along the lines of section 4.

6. D-SPIN AND EXOTIC HADRON PRODUCTION

In diagrams with loops, D-spin is not conserved. Diffractive processes being unitarity effects, are presumably described by diagrams with loops.¹¹⁾ As such, for diffractive processes one does not expect D-spin to be conserved. Thus diffractive processes say of the type $\pi p \rightarrow Mp$ or $\pi p \rightarrow \pi X$ can lead to exotic ($D_M \neq 0$, $D_X \neq 1/2$) hadron production.^{*)} One thus predicts exotic hadron production processes that have an essentially energy independent cross-section.

I wish to thank F. Mansouri and J. L. Rosner for stimulating discussions.

^{*)} The possibility of diffractive exotic meson production has been pointed out in ref. 12.

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Figure Captions:

- Fig. 1 Quark diagrams and duality.
- Fig. 2 D-spin nonconservation in one loop diagram.
- Fig. 3 Duality requires both $B=2$ (a) and $B=0$ (b) hadrons H_6 of D-spin 2.
- Fig. 4 Definition of N_{AQ} and $N_{AQ'}$.
- Fig. 5 D_3 states of $D=0, 1$ and 2 hadrons.
- Fig. 6 D_1 and D_2 are not conserved.
- Fig. 7 D_3 -nonconservation through hadron propagation.
- Fig. 8 A planar quark diagram.
- Fig. 9 a) A quasi planar diagram and
b) its planar core.
- Fig. 10 The three versions of a nonplanar diagram. The $\bar{D}_3$ assignments are obtained either by averaging D_3 over the vertices or more simply by eliminating all diagonal lines and looking at the planar cores of the three versions. It is important to observe that the planar cores are not the same for the three versions.
- Fig. 11 Vertex without a λ -like quark line between two of the hadrons (K^+ and K^-) meeting at the vertex.
- Fig. 12 $D_3^{(1)}$, $D_3^{(2)}$ and $\bar{D}^2$ conservation forbid $\phi \rightarrow \rho\pi$.

(a)

(b)

(c)

Fig.1

Fig.2

(a)

(b)

Fig.3

Fig.4

$$D_3 = 0$$

$$D = 0$$

$$D_3 = -\frac{1}{2}$$

$$D_3 = +\frac{1}{2}$$

$$D = \frac{1}{2}$$

$$D_3 = -1$$

$$D_3 = 0$$

$$D_3 = +1$$

$$D = 1$$

Fig.5

Fig.6

Fig.7

Fig.8

(a)

(b)

Fig. 9

Fig.10

Fig.II

Fig.12