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FILAMENTATION INSTABILITY OF
LOWER HYBRID WAVES IN A PLASMA

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Filamentation Instability of Lower Hybrid Waves in a Plasma

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ABSTRACT

It is shown that a strong lower hybrid wave is modulationally unstable to perturbations propagating along its own wave vector. The instability relies critically on the finite thermal corrections to the lower hybrid dispersion relation.

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Recently, there has been considerable interest in the possible use of intense radiofrequency fields at the lower hybrid frequency for heating confined plasmas to thermonuclear temperatures.¹⁻³ It is generally agreed that once the energy reaches the plasma interior, it will be efficiently absorbed by several linear and nonlinear mechanisms. However, the problem of transport of energy from the plasma edge to the interior appears to be more controversial. Whereas linear calculations² indicate that lower hybrid energy readily propagates to the center of plasma column along resonance cones directed parallel to the local group velocity, a recent nonlinear calculation due to Morales and Lee³ shows that ponderomotive force effects can lead to filamentation of lower hybrid fields near the plasma surface. Such a filamentation may then strongly alter the wave energy transport to the center.

In their nonlinear calculation, Morales and Lee have investigated the steepening of lower hybrid waves and the nonlinear evolution of self-trapped filamentary solutions in a homogeneous plasma. Steady state, two-dimensional solutions in which the ponderomotive force effects within the (k_o, B_o) plane balance the dispersive thermal effects are found. In this paper we investigate the filamentation instability of a finite wavelength lower hybrid pump wave in a warm homogeneous plasma. Long wavelength perturbations in the (k_o, B_o) plane are found to be unstable. We show that the growth length of these instabilities is comparable to the corresponding evolution length of the Morales-Lee solutions. However, since these lengths

turn out to be rather large compared to the typical dimensions available for growth in a tokamak plasma, our conclusion is that filamentation cannot play a crucial role in lower-hybrid plasma interactions in a tokamak.

We consider the filamentation instability of a plane lower hybrid wave

$$\Phi_0 = \phi_0 \cos(k_{ox}x + k_{oz}z - \omega_0 t) \quad (1)$$

(where z is the direction of magnetic field) to perturbations propagating in the $x-z$ plane. Since none of the perturbed quantities varies in the y -direction, the dominant $\underline{E} \times \underline{B}$ coupling never comes into the picture. This is in the same spirit as the nonlinear calculation of Morales and Lee³ and the consequences of this assumption will be discussed at the end. Our starting equation is the two-dimensional wave equation describing propagation of lower hybrid waves³, viz.,

$$\begin{aligned} \partial_x (K_{\perp} \partial_x \phi) + \partial_z (K_{\parallel} \partial_z \phi) + \partial_x^2 (a \partial_x^2 \phi) + \partial_z^2 (b \partial_z^2 \phi) + \partial_x^2 \left(\frac{c}{2} \partial_z^2 \phi\right) \\ + \partial_z^2 \left(\frac{c}{2} \partial_x^2 \phi\right) = 0 \end{aligned} \quad (2)$$

where

$$K_{\perp} = 1 + \omega_{pe}^2 / \Omega_e^2 - \omega_{pi}^2 / \omega^2$$

$$K_{\parallel} = 1 - \omega_{pi}^2 / \omega^2 - \omega_{pe}^2 / \omega^2$$

$$a = (\omega_{pi} / \omega)^2 (\bar{v}_i / \omega)^2 + (\omega_{pe} / \Omega_e)^2 (\bar{v}_e / \Omega_e)^2$$

$$b = (\omega_{pi}/\omega)^2 (\bar{v}_i/\omega)^2 + (\omega_{pe}/\omega)^2 (\bar{v}_e/\omega)^2 ,$$

$$\frac{c}{2} = (\omega_{pi}/\omega)^2 (\bar{v}_i/\omega)^2 + (\bar{v}_e/\omega)^2 (\omega_{pe}/\Omega)^2 ,$$

in which $\bar{v}_i, \bar{v}_e$ refer to the two thermal velocities, Ω_i and Ω_e the two cyclotron frequencies, and ω_{pi}, ω_{pe} the two plasma frequencies. It is assumed, as usual, that $\Omega_i \ll \omega \ll \Omega_e$. Equation (2) differs from the corresponding equation of Morales and Lee³ by the inclusion of terms proportional to c ; these terms seem to have been ignored by them through oversight.

When the $\vec{E} \times \vec{B}$ coupling is ignorable, the dominant nonlinear effect is the modification of density [and hence the coefficients $K_{\perp}, K_{\parallel}$, a, b, c in (2)] by ponderomotive force effects. One can readily convince oneself that the most important ponderomotive force component is that along the field lines, viz., $(\mathbf{v} \cdot \nabla) \mathbf{v}_{\parallel}$ and leads to the following expression for density modifications for magnetized electrons [Eq. (3a)] and unmagnetized ions [Eq. (3b)] respectively.

$$\frac{n_e}{n_0} = \exp \left\{ -\frac{1}{T_e} \left[-e\phi_s + \left(\frac{\omega_{pe}}{\omega} \right)^2 \frac{|\partial_z \phi|^2}{8\pi n_0} - \left(\frac{\omega_{pe}}{\Omega_e} \right)^2 \frac{|\partial_x \phi|^2}{8\pi n_0} \right] \right\}, \quad (3a)$$

$$\frac{n_i}{n_0} = \exp \left\{ \frac{1}{T_i} \left[e\phi_x + \frac{\omega_{pi}^2}{\omega} \frac{|\nabla \phi|^2}{8\pi n_0} \right] \right\}. \quad (3b)$$

Here ϕ_s is the self-consistent ambipolar potential which may be eliminated by the quasi-neutrality condition yielding (for small density changes)

$$\frac{\delta n}{n_o} \simeq - \frac{|\nabla \phi|^2}{8\pi n_o (T_e + T_i)} + C, \quad (4)$$

where C is a constant to take care of the fact that if ϕ has a uniform amplitude, then there are no density deviations. In deriving Eq. (4), the cold plasma dispersion relation for lower hybrid waves has been used. Substitution in (2) gives the desired nonlinear equation for ϕ :

$$\begin{aligned} K_{\perp o} \partial_x^2 \phi - |K_{\parallel o}| \partial_z^2 \phi + a_o \partial_x^4 \phi + b_o \partial_z^4 \phi + c_o \partial_x^2 \partial_z^2 \phi \\ + \alpha_o \partial_x \left[\left(C - \frac{|\nabla \phi|^2}{8\pi n_o T} \right) \partial_x \phi \right] + \beta_o \partial_z \left[\left(\frac{|\nabla \phi|^2}{8\pi n_o T} - C \right) \partial_z \phi \right] = 0, \end{aligned} \quad (5)$$

where the subscript o refers to unperturbed quantities and our notation is identical to Morales and Lee³:

$$\alpha = (\omega_{pe}/\Omega_e)^2 - (\omega_{pi}/\omega)^2; \quad \beta = (\omega_{pe}/\omega)^2 + (\omega_{pi}/\omega)^2.$$

Substitution of (1) into (5) gives the dispersion relation for pump waves

$$K_{\perp o} k_{ox}^2 + |K_{\parallel o}| k_{oz}^2 + a_o k_{ox}^4 + b_o k_{oz}^4 + c_o k_{ox}^2 k_{oz}^2 = 0. \quad (6)$$

We now introduce a perturbation

$$\delta \phi = \delta \phi_1(x, z) \cos(k_{ox} x + k_{oz} z - \omega t) + \delta \phi_2(x, z) \sin(k_{ox} x + k_{oz} z - \omega t), \quad (7)$$

where $\delta\phi_1$ and $\delta\phi_2$ are assumed to be slow functions of x and z (in the usual spirit of modulational instability calculations). Substituting (7) into a linearized (in $\delta\phi$) version of Eq. (5), using (6), equating the coefficients of sine and cosine terms, retaining only derivatives upto second order in $\delta\phi$, and assuming a perturbation of the form $\exp[i(Kx + \chi z)]$, one obtains the two equations

$$\delta\phi_1(A + B_1) + i\delta\phi_2(D + B_2) = 0 \quad , \quad (8a)$$

$$i\delta\phi_1(-D + B_3) + \delta\phi_2(A + B_4) = 0 \quad , \quad (8b)$$

where

$$A = -K^2 k_{\perp}^2 + \chi^2 |K_{\parallel}|^2 + 6a_o K^2 k_{oz}^2 + 6b_o \chi^2 k_{oz}^2 \\ + c_o (K^2 k_{oz}^2 + \chi^2 k_{ox}^2) + 4c_o k_{oz} k_{ox} K \chi \quad ,$$

$$D = 2k_{ox} K K_{\perp} - 2|K_{\parallel}| k_{oz} \chi - 4a_o K k_{ox}^3 - 4b_o \chi k_{oz}^3 \\ - 2c_o k_{ox} k_{oz} (k_{ox} \chi + k_{oz} K) \quad ,$$

$$B_1 = \Lambda (\alpha_o k_{ox}^2 - \beta_o k_{oz}^2) \quad , \quad B_2 = -\frac{K}{k_{ox}} B_1 \quad ,$$

$$B_3 = \Lambda (\alpha_o k_{ox} K - \beta_o k_{oz} \chi) \quad , \quad B_4 = \frac{K}{k_{ox}} B_3 \quad ,$$

and $\Lambda = (k_{ox}^2 \phi_o^2) / 8\pi n_o T$ is the strength parameter of the pump wave.

The dispersion relation, correct to quadratic order in K , χ may now be readily written

$$-D^2 + A(B_1 + B_4) + D(B_3 - B_2) = 0 \quad (9)$$

To analyze this dispersion relation for convective amplification we look for solutions with real χ and complex K . This corresponds, for example, to a situation where the initial amplitude of the perturbation is given at $x=0$ for all z (such as at one edge of the plasma, defined by the direction of magnetic field, viz., $x = \text{constant}$), and we look for solutions completely periodic in z and with a growing modulation envelope along x . Equation (9) may now be rewritten in the form

$$\Delta K^2 + \Theta K + \Psi = 0 \quad (10)$$

which can be solved for complex K .

We first look at the special case in which the thermal dispersive terms proportional to a_0 , b_0 , c_0 are negligible. One can now readily show that Δ , Θ , Ψ satisfy the relation

$$\Theta^2 = 4 \Delta \Psi \quad ,$$

so that K is purely real and approximately given by

$$K \approx k_{ox} \chi / k_{oz} \quad (11)$$

where we have used Eq. (6) (with a_0 , b_0 , c_0 negligible). Thus, there is no instability in the absence of thermal dispersive effects. Since the thermal

dispersive terms are small, we now use the approximate Eq. (11) to simplify them. In this case, we obtain

$$\begin{aligned}\Delta &= \frac{1}{k_{ox}^2} \left[-4K_{\perp}^2 + 16K_{\perp} \frac{\Theta}{k_{ox}^2} + 3\Lambda\alpha_o K_{\perp} - 2\Lambda\alpha_o \frac{\Theta}{k_{ox}^2} - \Lambda\beta_o \frac{k_{oz}^2}{k_{ox}^2} K_{\perp} \right. \\ &\quad \left. - 2\Lambda\beta_o \Theta \frac{k_{oz}^2}{k_{ox}^2} \right] \\ \Theta &= \frac{\chi k_{oz}}{k_{ox}^3} \left[8|K_{\parallel}| K_{\perp} - 16|K_{\parallel}| \frac{\Theta}{k_{ox}^2} + 4\Lambda\beta_o \frac{\Theta}{k_{ox}^2} - 2\Lambda\beta_o K_{\perp} - 4\Lambda\alpha_o |K_{\parallel}| \right. \\ &\quad \left. + 2\Lambda\beta_o |K_{\parallel}| \frac{k_{oz}^2}{k_{ox}^2} \right] \\ \Psi &= \frac{\chi^2}{k_{ox}^2} \left[-4K_{\parallel}^2 \frac{k_{oz}^2}{k_{ox}^2} + \Lambda\alpha_o |K_{\parallel}| + \Lambda\beta_o |K_{\parallel}| k_{oz}^2 \right],\end{aligned}$$

where

$$\Theta = a_o k_{ox}^4 + b_o k_{oz}^4 + c_o k_{ox}^2 k_{oz}^2, \quad (12)$$

is the thermal correction term. The quadratic for K can now be solved to yield

$$K_R \approx k_{ox} \chi / k_{oz} \quad (13a)$$

$$K_i \approx K_R \left[\frac{3}{4} \frac{\Lambda\Theta}{K_{\perp}^2 k_{ox}^2} \left(\frac{k_{oz}^2}{k_{ox}^2} \beta_o - \alpha_o \right) \right]^{1/2} \quad (13b)$$

where use has been made of the pump dispersion relation [Eq. (6)] and the various symbols are defined after Eqs. (2), (5), (8), and (12).

Let us now interpret the above results. Equation (13a) shows that the wave-vector for the modulation instability is parallel to that of the pump wave. This is readily understandable if one regards this instability as the linear stage of the trapping of the lower hybrid wave in a modulation envelope. The linear dispersion relation for a cold plasma

$$\omega^2 = \frac{\omega_{pi}^2}{1 + \omega_{pe}^2 / \Omega_e^2} \left[1 + \frac{M}{m} \frac{k_z^2}{k^2} \right],$$

shows us that no trapping of lower hybrid waves is possible in the underdense region ($\omega > \omega_{pi}$) unless k_z/k is held constant. More specifically, if k_z/k is free to change an increase in background density due to a ponderomotive bunching, can always be counterbalanced by a change in k_z/k so that the wave continues to propagate. Thus, the only possibility of trapping in the underdense region exists if k_z/k is constrained to remain constant. Hence the modulational instability also arises only for $\vec{K} \parallel \vec{k}_0$.

The above discussion also reveals why there is no modulational instability for zero temperature. Note that when k_z/k is held constant, the only dependence of ω on k arises through thermal effects. For modulational instability we should have,

$$(\partial \omega / \partial A_0^2) (\partial^2 \omega / \partial k^2) < 0,$$

where A_0 is the amplitude, (following Lighthill⁴ and Karpman and Kruskal⁵) and this is indeed satisfied if thermal dispersive terms are taken into account.

Order of magnitude wise, Eq. (13b) indicates that the inverse growth for modulational instability discussed above is given by

$$K_i \sim K_R (k_0 \lambda_D) \Lambda^{1/2} . \quad (14)$$

This may be compared with the evolution length of steepening and filamentation obtained by comparing the first two terms of Eq. (7) of Morales and Lee³ (and converting evolution length along z into one along x by the scaling factor):

$$K_i \sim K_R \Lambda . \quad (15)$$

Thus, we note that the filamentation instability discussed above is comparable to the Morales-Lee nonlinear evolution as long as $k_0 \lambda_D \sim \Lambda^{1/2}$.

To assess the importance of these instabilities for a tokamak heating experiment we make an upper estimate of k_i using Eq. (14) and taking $K_R \sim k_0$. Note that the temperature dependence drops out in (14). We thus obtain

$$K_i \sim k_0^2 (E/4\pi n_e) . \quad (16)$$

Taking typical values for $k_0 \sim 1 \text{ cm}^{-1}$, $E \sim 10^3 \text{ v/cm}$, $n \sim 10^{12} \text{ cm}^{-3}$ at the surface of a tokamak plasma, one obtains

$$K_i \sim 5 \times 10^{-4} ,$$

which means that one requires tens of meters for instability e-folding.

In the interior we may take $k_o \sim 10 \text{ cm}^{-1}$, $E \sim 10^4 \text{ v/cm}$, $n \sim 10^{13} \times 5$ so that

$$K_i \sim 10^{-2} .$$

so that one still needs a length of order a meter. Thus our conclusion is that filamentation cannot play a crucial role in the lower hybrid heating of tokamaks.

Finally, it should be mentioned that the $\vec{E} \times \vec{B}$ coupling, which has been ignored in this 2-dimensional treatment, leads to the more dominant and well-known parametric effects of a lower hybrid wave.⁶ It is, therefore, quite likely that the effects considered here and also discussed by Morales and Lee are strongly modified by inclusion of such a coupling. A model calculation showing this effect is in progress.

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