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MIXING OF A COLD LIQUID JET WITH
A BOILING LIQUID STREAM

by

S. G. Bankoff

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S. G. Bankoff

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TABLE OF CONTENTS

	<u>Page</u>
INTRODUCTION.	3
TRANSPORT IN A FREE JET.	4
TRANSPORT OF A SCALAR QUANTITY Γ	9
TRANSPORT OF ENTHALPY FROM A BUBBLY MIXTURE.	11
ILLUSTRATIVE EXAMPLE	14
ACKNOWLEDGMENT	16
NOMENCLATURE	16
REFERENCES.	18

MIXING OF A COLD LIQUID JET WITH A BOILING LIQUID STREAM

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INTRODUCTION

A problem of considerable practical importance is the turbulent mixing process involved when a cold liquid jet intrudes into a stream of vapor bubbles and saturated liquid. This process assumes importance, for example, in the downcomer of the natural circulation boiling loop, where it is desired to remove entrained vapor by means of cold water injection at the top of the downcomer. We shall consider first a single axial liquid jet. The results obtained can be extended to any number of liquid jets because of the linearity of the formulation used. The present interest is in jets directed vertically downward, but the results may be applied to any jet orientation providing the turbulent intensity is sufficient to make buoyant effects negligible.

Turbulent mixing in a round free jet, like all free turbulent shear flows, is tractable only where similarity considerations can be applied. It has been shown (Ref. 1) that similarity criteria can be deduced in either of two limiting cases: (1) the issuing jet velocity, U_p , is very much greater than the free stream velocity, U_s , and (2) U_p is nearly equal to U_s . In determining the radial transport properties the classical deductive assumptions of the Prandtl "momentum transport theory" or of the Taylor "vorticity transport theory" concerning a "mixing length" may be made. Equivalently the inductive approach due originally to Reichardt (Refs. 2 and 3) may be employed, which is based upon the observation that the experimental radial distribution curves are closely fitted by Gaussian error functions. The latter approach is preferred here since all methods give good agreement with experimental distributions, while the inductive method, because of the linearity of the resulting equations, allows a superposition principle to be employed. Hence it is theoretically possible to obtain the velocity and mixing characteristics for any assembly of parallel jets.

The central core of the jet (see Fig. 1) will contain no voids, but the outer portion of the turbulent mixing zone will exhibit appreciable density variations. It will be assumed herein that the effect of density variations on the turbulent mixing process can be calculated by the "equivalent orifice diameter" concept of Thring and Newby (Ref. 4). Also it will be assumed that the bubble collapse processes are rapid compared to the turbulent mixing of cold liquid with warmer liquid surrounding the bubbles. In this

event the bubble dynamics do not have to be considered, the average void fraction at any point being proportional to the local average excess enthalpy over saturation enthalpy. The time-average enthalpy at any point therefore is considered to be a transported scalar, and is considered to be analogous to the transport of a scalar, such as heat or mass, in a single-phase turbulent jet mixing process.

Fig. 1

Conceptual Model for the Mixing of a Cold Liquid Jet with a Boiling Liquid Stream

TRANSPORT IN A FREE JET

Consider a liquid jet, of density ρ_L , issuing into a stream of boiling liquid, whose free stream velocity is U_s . In the jet region, the axial velocity component, U_x , is assumed to be composed of the velocity U_s , and the velocity U_1 , which is the deviation from U_s caused by the jet. Outside the jet region the pressure is uniform ($dP/dx = 0$). Rotational symmetry is assumed, and cylindrical coordinates (x, r) with the axial coordinate x along the jet axis are chosen.

If we neglect the pressure and viscosity terms (free turbulence), the equation of motion for the instantaneous velocity U_x becomes

$$\frac{dU_x}{dt} = \frac{\partial U_x}{\partial t} + U_r \frac{\partial U_x}{\partial r} + U_x \frac{\partial U_x}{\partial x} = 0 \quad (1)$$

The continuity equation states ($\rho = \text{const}$)

$$\frac{\partial U_r}{\partial t} + \frac{U_r}{r} + \frac{\partial U_x}{\partial x} = 0 \quad (2)$$

On multiplying Equation (2) by U_x , adding the results to Equation (1), taking time-mean values (denoted by the bar superscript), and assuming steady-state conditions, one obtains

$$\frac{\partial \overline{U_x^2}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r \overline{U_x U_r}) = 0 \quad (3)$$

According to Reichardt's hypothesis (Ref. 2)

$$\overline{U_x U_r} = -L \frac{\partial \overline{U_x^2}}{\partial r} \quad (4)$$

where $L = L(x)$. Define

$$\overline{U_x^2} = \overline{U_s^2} + \overline{\eta^2} \quad (5)$$

where

$$\overline{\eta^2} = 2\overline{U_s U_1} + \overline{U_1^2} + \overline{u_1^2} \quad (6)$$

and

$$U_1 = \overline{U_1} + u_1 \quad (7)$$

u_1 being the turbulent fluctuation in the velocity defect, U_1 . From Equations (3), (4), and (5)

$$\frac{\partial \overline{\eta^2}}{\partial x} = \frac{L}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \overline{\eta^2}}{\partial r} \right) \quad (8)$$

This is of the same form as the heat-conduction equation. In order to deduce the form of $L(x)$, similarity concepts must be employed. With the usual boundary-layer approximations, the steady-state equation of motion in the axial direction reads (Ref. 1)

$$\rho_L \left(\overline{U_x} \frac{\partial \overline{U_x}}{\partial x} + \overline{U_r} \frac{\partial \overline{U_x}}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} (r \sigma_{xr}) \quad (9)$$

where σ_{xr} is the shear stress in the x direction on an element of surface normal to the radius. The time-mean axial velocity is by definition

$$\overline{U_x} = U_s + \overline{U_1} \quad (10)$$

which when inserted in Equation (9) yields

$$\left(\frac{U_s}{U_p} + \frac{\overline{U_1}}{U_p} \right) \frac{\partial}{\partial x} \left(\frac{\overline{U_1}}{U_p} \right) + \frac{\overline{U_r}}{U_p} \frac{\partial}{\partial r} \left(\frac{\overline{U_1}}{U_p} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{r \sigma_{xr}}{\rho_L U_p^2} \right) \quad (11)$$

where U_p is the issuing velocity from the jet of diameter d . Assuming that the flow patterns in subsequent sections are similar beyond a certain

distance from the orifice, one puts

$$\frac{\overline{U_1}}{\overline{U_{1,\max}}} = f(\xi_2); \quad \frac{\overline{U_r}}{\overline{U_{1,\max}}} = g(\xi_2); \quad \frac{\overline{U_{1,\max}}}{U_p} = \psi(\xi_1); \quad \frac{\sigma_{xr}}{\rho_L \overline{U_{1,\max}}^2} = h(\xi_2) \quad (12)$$

where

$$\xi_2 = \frac{r}{d} \phi(\xi_1); \quad \xi_1 = \frac{x+a}{d} \quad (13)$$

Here a is the distance between the geometrical origin of similarity and the origin of the coordinate system, taken as the plane of the orifice.

In terms of these new variables, Equation (11) becomes

$$(\mu + f\psi) \left(\frac{\psi}{\phi} \frac{d\phi}{d\xi_1} \xi_2 \frac{df}{d\xi_2} + \frac{d\psi}{d\xi_1} f \right) + \psi^2 \phi g \frac{df}{d\xi_2} = \frac{\psi^2 \phi}{\xi_2} \frac{d}{d\xi_2} (\xi_2 h) \quad (14)$$

where

$$\mu = \frac{U_s}{U_p} \quad (15)$$

the velocity ratio between the secondary and the primary flow.

A steady-state momentum-flux integral balance gives

$$2\pi \int_0^\infty r U_x (U_x - U_s) dr = U_p (U_p - U_s) (\pi d^2/4) \quad (16)$$

where the right hand side of the equation is the excess momentum flux of the fluid issuing from the orifice, and has a constant value. Taking time-averages and substituting the variables in Equations (12) and (13), Equation (16) becomes

$$\frac{\mu\psi}{\phi^2} \int_0^\infty \xi_2 f d\xi_2 + \frac{\psi^2}{\phi^2} \int_0^\infty \xi_2 f^2 d\xi_2 = \text{const.} \quad (17)$$

Similarity conditions cannot be satisfied for both the equation of motion (14) and the momentum-flux integral balance (17) except for the two limiting cases given by

$$\psi f \gg \mu; \quad \frac{\overline{U_1}}{U_s} \gg 1 \quad (18)$$

or

$$\psi f \ll \mu ; \quad \frac{\overline{U_1}}{U_s} \ll 1 \quad . \quad (19)$$

If the stream velocity is much less than the jet velocity (18) Equation (17) reduces to

$$\frac{\psi^2}{\phi^2} \int_0^\infty f^2 \xi_2 d\xi_2 = \text{const.} \quad (20)$$

from which it follows that

$$\psi = A_1 \phi \quad (21)$$

where A_1 is a constant. Employing Equations (18) and (21) in Equation (14), it follows that

$$\frac{1}{\phi^2} \frac{d\phi}{d\xi_1} = \text{const.} \quad (22)$$

whence

$$\phi = \frac{1}{\xi_1} = \frac{d}{x + a} \quad . \quad (23)$$

For the case where the jet and stream velocities are nearly equal (Equation 19), it follows from similar arguments that

$$\psi = A_2 \phi^2 \quad (24)$$

and

$$\phi = \xi_1^{-1/3} \quad . \quad (25)$$

This is to say that in the region where the jet velocity is large compared to the stream velocity the spread of the jet is linear with $(x + a)$ and the jet velocity decreases in the axial direction hyperbolically with increasing distance, $(x + a)$. When the jet has spread sufficiently so that its velocity is nearly that of the stream velocity, the spread of the jet is proportional to $(x + a)^{1/3}$ and the velocity decreases as $(x + a)^{-2/3}$.

Consider now the case given by Equation (18). This will be the case if U_s is very nearly equal to zero. If we consider that $U_s = 0$ Equation (6) reduces to

$$\overline{\eta^2} = \overline{U_1^2} \quad (26)$$

where the similarity assumption has been made and where it is also assumed that the turbulent intensity is small compared to the mean velocity defect, $\bar{U}_1$. From Equations (12), (21), and (23) it follows

$$\psi_1(\xi_1) = \frac{B_1}{\xi_1^2} \quad (27)$$

where B_1 is a constant. On substituting Equation (26) into Equation (8) and transforming to the variables given by Equation (13), similarity is achieved if and only if

$$\frac{L}{x+a} = \text{const.} \quad (28)$$

Then f_1 satisfies the differential equation

$$\frac{d}{d\xi_2} \left(\xi_2^2 f_1 \right) = \frac{L}{x+a} \frac{d}{d\xi_2} \left(\xi_2 \frac{df_1}{d\xi_2} \right) \quad (29)$$

The solution of this differential equation which satisfies the condition $f_1 = 1$ for $\xi_2 = 0$ is the Gaussian error law,

$$f_1 = \exp \left(- \frac{x+a}{2L} \xi_2^2 \right) \quad (30)$$

Hence,

$$\frac{\eta^2}{U_p^2} = B_1 \left(\frac{d}{x+a} \right)^2 \exp \left(- \frac{x+a}{2L} \xi_2^2 \right) \quad (31)$$

The quantities B_1 , a , and L are determined from experiments.

From Equations (12), (21), and (23) one obtains

$$\frac{\bar{U}_{1,\max}}{U_p} = A_1 \frac{d}{x+a} \quad (32)$$

On combining with Equation (31) and employing the condition $\bar{U}_1 = \bar{U}_{1,\max}$ when $\xi_2 = 0$, this leads to

$$\frac{\bar{\eta}^2}{U_{1,\max}^2} = \exp \left(- \frac{x+a}{2L} \xi_2^2 \right) \quad (33)$$

whence, if $\overline{u^2} \ll \overline{U_1^2}$,

$$\frac{\overline{U_1}}{\overline{U_{1,\max}}} = e^{C \xi_2^2} \quad (34)$$

where

$$C = \frac{x + a}{4L} , \quad a \text{ constant} . \quad (35)$$

Experimental measurements have been made to determine the coefficients in Equations (31) and (32) (Refs. 6, 7, and 8). For the axial decrease in momentum Hinze and van der Hegge Zijnen found that

$$\frac{U_{1,\max}}{U_p} = 6.4 \frac{d}{x + a} ; \quad a = 0.6 d ; \quad x > x_c \quad (36)$$

where x_c represents the length of the initial potential-flow core region and is given by the following empirical relation (Ref. 9).

$$\frac{x_c}{d} = 4 + 12\mu = 4 + 12 \frac{U_s}{U_p} . \quad (37)$$

The measured half-value point (Ref. 6) is 0.08 which fixes the dispersion of the radial distribution:

$$\left(\xi_2 \right)_{1/2} = \left(\frac{\ln 2}{C} \right)^{1/2} = 0.08 \quad (38)$$

TRANSPORT OF A SCALAR QUANTITY Γ

The transport of any scalar quantity, Γ , is handled in a similar fashion to the transport of axial momentum. Neglecting molecular effects, the transport equation reads, on making the boundary-layer assumptions,

$$U_x \frac{\partial \Gamma}{\partial x} + U_r \frac{\partial \Gamma}{\partial r} = 0 . \quad (39)$$

Combining with the continuity equation in the form

$$\Gamma \left(\frac{\partial U_x}{\partial x} + \frac{\partial U_r}{\partial r} + \frac{U_r}{r} \right) = 0 \quad (40)$$

we obtain

$$\frac{\partial}{\partial x} \left(\overline{U_x \Gamma} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r \overline{U_r \Gamma} \right) = 0 \quad . \quad (41)$$

On employing Reichardt's inductive hypothesis,

$$\overline{U_r \Gamma} = -L\gamma \frac{\partial}{\partial r} \left(\overline{U_x \Gamma} \right) \quad (42)$$

Equation (41) transforms into the heat-conduction equation

$$\frac{\partial}{\partial x} \overline{U_x \Gamma} = \frac{L\gamma}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \overline{U_x \Gamma} \right) \quad . \quad (43)$$

Letting

$$\left. \begin{aligned} U_x &= U_s + \overline{U}_1 + u_1 \\ \Gamma &= \Gamma_s + \overline{\Gamma}_1 + \gamma \end{aligned} \right\} \quad (44)$$

where γ is the instantaneous fluctuation of the scalar quantity Γ from its time mean value; $\overline{\Gamma}_1$ is the defect in the scalar quantity Γ (in this case enthalpy) from the value in the free stream. Taking time-mean averages one obtains

$$\overline{U_x \Gamma} = U_s \Gamma_s + U_s \overline{\Gamma}_1 + \overline{U}_1 \Gamma_s + \overline{U}_1 \overline{\Gamma}_1 + \overline{u_1 \gamma} \quad . \quad (45)$$

If now the reference enthalpy state is taken to be that of the free stream, such that $\Gamma_s = 0$, and defining

$$\overline{\eta_\gamma^2} = \overline{U}_1 \overline{\Gamma}_1 + \overline{u_1 \gamma} \quad (46)$$

and putting

$$\frac{\overline{\eta_\gamma^2}}{U_p \Gamma_p} = \psi_\gamma(\xi_1) f_\gamma(\xi_2) \quad (47)$$

where Γ_p is the value of the scalar quantity in the issuing jet, we obtain, by similar reasoning to that employed in obtaining Equations (26) to (31)

$$\psi_\gamma = \frac{B\gamma}{\xi_1^2} \quad (48)$$

where B_γ is again a constant. Also, the similarity condition may be written

$$\frac{L_\gamma}{x+a} = \text{const.} \quad (49)$$

from which an analogous Gaussian law is eventually derived

$$\frac{\overline{\eta_\gamma^2}}{U_p \Gamma_p} = B_\gamma \left(\frac{d}{x+a} \right)^2 \exp \left(- \frac{x+a}{2L_\gamma} \xi_2^2 \right) \quad (50)$$

which is the counterpart of Equation (31) for the transportation of a scalar quantity Γ .

From steps similar to those leading from Equation (31) to Equation (36), it was found (Ref. 6) that for the spread of heat and matter

$$\frac{\Gamma_{1,\max}}{\Gamma_p} = 5.27 \frac{d}{x+a} ; \quad a = 0.8 d \quad x > x_c \quad (51)$$

whereas the ratio of the eddy diffusivities or mixing lengths for the spread of mass or heat to those for the spread of momentum was found to be approximately

$$\frac{L_\gamma}{L} = \frac{C_\gamma}{C} \simeq 1.4 \quad (52)$$

TRANSPORT OF ENTHALPY FROM A BUBBLY MIXTURE

Consider now a single jet of cold water at high velocity, U_p , issuing into a relatively quiescent stream of bubbly liquid. Assume the entering volumetric flow concentration of vapor in the mixture to be β_s . The excess enthalpy of the entering mixture over saturated liquid is

$$H_s = X \lambda \quad (53)$$

where

X = quality

λ = latent heat of vaporization .

The enthalpy defect of the entering cold liquid compared to saturation is H_p . Since

$$\frac{1}{X} = 1 + \frac{\rho_L}{\rho_G} \left(\frac{1}{\beta_s} - 1 \right) \quad (54)$$

from Equations (53) and (54) one obtains

$$H_s = \frac{\lambda}{1 + \frac{\rho_L}{\rho_G} \left(\frac{1}{\beta_s} - 1 \right)} \quad (55)$$

If the assumption is made that enthalpy is the transported scalar, by definition

$$\Gamma_p = H_s - H_p \quad ; \quad \Gamma_s = 0 \quad (56)$$

$$H_s - H_1 = \Gamma_1 = \bar{\Gamma}_1 + \gamma = H_s - \bar{H}_1 - h \quad (57)$$

where $\bar{H}_1$ is time-mean difference of enthalpy in the jet from saturation enthalpy; h the enthalpy fluctuation due to turbulence; and

$$\bar{H}_1 = \frac{\lambda}{1 + \frac{\rho_L}{\rho_G} \left(\frac{1}{\bar{\beta}} - 1 \right)} \quad (58)$$

where $\bar{\beta}$ = time-mean local value of β . Since

$$\overline{\eta^2 \gamma} = \bar{U}_1 (H_s - \bar{H}_1) + \overline{u_1 h} \quad (59)$$

from Equation (50)

$$\frac{\bar{U}_1 (H_s - \bar{H}_1) + \overline{u_1 h}}{U_p (H_s - H_p)} = B_\gamma \left(\frac{d}{x+a} \right)^2 \exp \left[- \frac{x+a}{2L\gamma} \xi_2^2 \right] \quad (60)$$

Following the steps which lead to Equation (33), one obtains for the radial distribution of enthalpy

$$\frac{\bar{\Gamma}_1}{\bar{\Gamma}_{1,\max}} = \frac{H_s - \bar{H}_1}{H_s - \bar{H}_{1,\min}} = e^{-C_\gamma \xi_2^2} \quad (61)$$

We assume that no voids exist for $\bar{\Gamma}_1 \leq \Gamma_{\text{sat}} = H_s$. Hence

$$-C\gamma \left(\frac{r_{\text{sat}}}{x+a} \right)^2 = \ln \left(\frac{H_s}{H_s - \bar{H}_{1,\min}} \right) = \ln \frac{\Gamma_{\text{sat}}}{\bar{\Gamma}_{1,\max}} \quad (62)$$

The void fraction at $r > r_{\text{sat}}$ is given by

$$\alpha = K\beta \quad (63)$$

where K is a flow parameter which is a function only of pressure (Ref. 5). From Equation (55) one obtains

$$H_s = \frac{\lambda}{1 + \frac{\rho_L}{\rho_G} \left(\frac{K}{\alpha_s} - 1 \right)} ; \quad H_{1,\min} = \frac{\lambda}{1 + \frac{\rho_L}{\rho_G} \left(\frac{K}{\alpha_{\min}} - 1 \right)} \quad (64)$$

When substituted into Equation (61) this gives an expression for the radial void fraction distribution (negative void fractions indicate subcooled liquid):

$$\frac{\bar{\Gamma}_1}{\bar{\Gamma}_{1,\max}} = \frac{H_s - \bar{H}_1}{H_s - \bar{H}_{1,\min}} = \frac{\alpha_s - \alpha}{\alpha_s - \alpha_{\min}} \cdot \frac{b_1 K + \alpha_{\min}(1 - b_1)}{b_1 K + \alpha(1 - b_1)} \quad (65)$$

where b_1 is equal to ρ_L/ρ_G ; $\alpha_{\min}$ is the equivalent void fraction at the jet axis; and α_s is the free-stream void fraction. The mean void fraction at the distance x from the orifice is given by

$$\alpha_{\text{av}} = \frac{2}{R^2} \int_{r_{\text{sat}}}^R \alpha r \, dr \quad (66)$$

where R is the pipe radius, and r_{sat} is the radius of the single-phase core of the jet. From Equations (61), (65), and (52) one obtains

$$\alpha_{\text{av}} = n_1 \left(1 - \frac{w_{\text{sat}}}{w_R} \right) \left(1 + \frac{n_2}{n_3} \right) + \frac{n_1 n_2}{n_3 C\gamma w_R} \ln \left[\frac{n_3 + \exp[-C\gamma w_R]}{n_3 + \exp[-C\gamma w_{\text{sat}}]} \right] \quad (67)$$

where

$$w_{\text{sat}} = \left(\frac{r_{\text{sat}}}{x+a} \right)^2 ; \quad w_R = \left(\frac{R}{x+a} \right)^2 ; \quad n_1 = \frac{b_1 K}{b_1 - 1} ;$$

$$n_2 = b_2 \left(\frac{1}{\alpha_s(b_1 - 1)} - \frac{1}{b_1 K} \right) ; \quad n_3 = \frac{b_2}{\alpha_s(1 - b_1)} ; \quad b_2 = \frac{1 + b_1 \left(\frac{K}{\alpha_{\min}} - 1 \right)}{\frac{1}{\alpha_{\min}} - \frac{1}{\alpha_s}} \quad (68)$$

To take into account the difference in density between the free stream, ρ_s , and the cold liquid, ρ_L , the equivalent orifice diameter concept (Ref. 4) is employed. This assumes that the development of the jet is determined by the total jet momentum, and leads to

$$\frac{d_e}{d} = \sqrt{\frac{\rho_L}{\rho_s}} \quad (69)$$

where the equivalent orifice diameter, d_e , is to be used in the above formulae.

To determine the average void fraction at any distance down stream of the orifice, we employ Equation (67). We assume that the following quantities are specified: R , P , α_s , d , U_1 , U_s , H_p . The flow parameter for downwards two-phase flow is obtained, (Ref. 10)* from

$$\frac{\alpha}{\beta} = 1.5 \quad , \quad \beta < 0.4 \quad . \quad (70)$$

We first determine the enthalpy at the jet axis at some distance x downstream from the plane of the orifice from Equation (51), where a is determined from Equation (51), and using for d the equivalent orifice diameter given by Equation (69). The radial distribution of enthalpy is given by Equation (61). The constant C_γ is now determined from Equations (38) and (52) and this is used in Equation (62) to determine r_{sat} , the radius of the single-phase region. α_s and α_{min} are determined from Equation (64); and all the quantities in Equation (68) can now be calculated. These lead directly to $\bar{\alpha}$, the average void fraction in Equation (67).

ILLUSTRATIVE EXAMPLE

As an example, consider the mixing of a liquid jet with a bubbly stream in a vertical, round downcomer at 600 psia. The inside diameter of the pipe is taken to be one inch and the diameter of the jet is 0.1 inch. The enthalpy of the jet liquid is -200 Btu/lb as compared to saturation enthalpy. The void fraction of the bubbly stream is 0.5. We assume that the velocity of the jet is very much greater than the velocity of the bubbly stream. The constant $\underline{a}$, which locates the geometrical origin of similarity, is taken to be 0.8 $\underline{d}$. C_γ , calculated from Equations (38) and (52) is equal to 151. With these assumptions, enthalpy defect profiles, as a function of dimensionless radial distance, were calculated for various dimensionless axial positions. These results are shown in Fig. 2.

*These data were called to the attention of the author by Dr. Novak Zuber.

It will be noted that the dividing surface goes through a maximum and then begins to decrease, corresponding to the mixing of the finite jet with an infinite bubbly stream. From Fig. 2 it would seem that the zone of influence is fairly sharply defined, so that superposition principles can be employed for an assembly of jets.

The assumptions made concerning the equivalence of the transport of enthalpy in the mixing of a jet with a two-phase mixture with the transport of heat in single-phase jet mixing should be emphasized. No experimental data are available for this case, but some measurements have been made where the jet and ambient fluid are of different densities (Ref. 7). In general, the width of the jet becomes smaller if the density of the jet fluid is greater than that of the ambient fluid. This corresponds to a decrease in the half-value radius (Equation 38). The length of the potential core region also increases, and the velocity at the axis decreases at a lower rate with increasing distance from the orifice than it does in the case of a constant-density jet.

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NOMENCLATURE

a	Distance between the geometrical origin of similarity and the origin of the coordinate system, taken as the plane of the orifice.
A_1	Proportionality constant, Eq. (21).
A_2	Proportionality constant, Eq. (24).
B_1	Proportionality constant, Eq. (27).
B_γ	Proportionality constant, Eq. (48).
C	Constant, Eq. (35).
C_γ	Constant, Eq. (52).
d	Diameter of the orifice.
d_e	Equivalent orifice diameter, Eq. (69).
f	Radial distribution function of axial velocity defect, Eq. (12).
g	Radial distribution function of radial velocity, Eq. (12).
h	Radial shear stress distribution function, Eq. (12).
H_p	Enthalpy per unit mass of the liquid leaving the orifice with reference to saturated liquid.

H_s	Enthalpy of the free stream of bubble liquid with reference to saturated liquid.
H_1	Instantaneous local value of the enthalpy at any point in the mixing zone with reference to saturation enthalpy.
h	Instantaneous fluctuation of enthalpy at any point due to turbulence.
K	Two-phase flow parameter, Eq. (63).
L	Mixing length defined by Eq. (4).
L_γ	Mixing length defined by Eq. (42).
b_1	Ratio of liquid to vapor densities.
b_2	Parameter defined by Eq. (68).
n_1, n_2, n_3	Constants defined in Eq. (68).
r	Radial distance from the axis of the jet.
r_{sat}	Radius of the single phase region.
R	Radius of the pipe.
u	Turbulent velocity fluctuation.
U_p	Velocity of liquid leaving the orifice.
U_r	Radial velocity.
U_s	Axial velocity of the free stream.
U_1	Axial velocity defect.
$\bar{U}_{1, \max}$	Time-average velocity defect at axis of the jet.
w_R	Dimensionless pipe area, Eq. (68).
w_{sat}	Dimensionless area of the single-phase core of the jet.
x	Axial distance from the orifice.
X	Quality.
α_{av}	Time-average void fraction over pipe cross section.
α	Time-average local void fraction.
α_{min}	Equivalent negative void fraction at jet axis.
α_s	Void fraction of the free stream.
β	Volumetric flow concentration of vapor.
γ	Turbulent fluctuation in value of scalar quantity.
Γ	Instantaneous value of scalar quantity.
Γ_s	Value of scalar quantity in the free stream.

$\bar{\Gamma}_1$	Defect in the scalar quantity from the value in the free stream.
$\bar{\Gamma}_{1,\max}$	Defect in the scalar quantity at the jet axis from the value in the free stream.
$\bar{\eta}^2$	Momentum transport term defined by Eq. (5).
$\bar{\eta}_\gamma^2$	Scalar transport term given by Eq. (46).
μ	Velocity ratio between the secondary and primary flow, Eq. (15).
ξ_1	Dimensionless axial distance, Eq. (13).
ξ_2	Dimensionless radial distance, Eq. (13).
ρ_g	Density of vapor.
ρ_L	Density of liquid.
ρ_s	Density of the free stream bubbly mixture.
σ_{xr}	Shear stress in the axial direction on an element of surface normal to the radius.
ϕ	Dimensionless function, Eq. (13).
ψ	Dimensionless axial velocity defect, Eq. (12).

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