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COMPRESSIBLE FLUID FLOW
(BNW PROGRAM GAS)

Bernard H. Duane

September 1969

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COMPRESSIBLE FLUID FLOW

(BNW PROGRAM GAS)

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Theoretical and Neutron Physics Section
Physics Research Department
Physics and Engineering Division

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ABSTRACT

Battelle-Northwest Program Gas constructs analytic streamtube integrals for the supersonic or subsonic flow of a real gas (having temperature-variant specific heats) through an open-loop or closed-loop succession of nozzles, oblique-shock diffusers, normal shocks, bypass flows with pressure-matched reentries, heat exchangers, burners, turbines, compressors, shock pumps, and other such elements simply characterizable in terms of mass, energy, momentum, or entropy transfer. Analysis versatility includes coupled variation of design specifications to meet performance requirements.

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SUMMARY

Battelle-Northwest Program Gas constructs one-dimensional analytic streamtube integrals for the supersonic or subsonic flow of a real gas (having temperature-variant specific heats) through an open-loop or closed-loop succession of nozzles, oblique-shock diffusers, normal shocks, bypass flows with pressure-matched reentries, heat exchangers, burners, turbines, compressors, shock pumps, and other such elements simply characterizable in terms of mass, energy, momentum, and entropy transfer. Analysis versatility includes coupled variation of design specifications to meet performance requirements.

Physics Representation

$$\text{Mass Balance: } \nabla \cdot \rho \underline{v} = Q_1 \quad (1)$$

$$\text{Energy Balance: } \nabla \cdot \rho \underline{v} (h + v^2/2) = Q_2 \quad (2)$$

$$\text{Momentum Balance: } \nabla \cdot (\rho \underline{v} \underline{v} + p) = Q_3 \quad (3)$$

$$\text{Entropy Balance: } \nabla \cdot \rho \underline{v} s = Q_4 \quad (4)$$

$$\text{Equations of State: } p = r \rho T \quad (5)$$

$$c_p(T) = c_v(T) + r \quad (6)$$

$$c_v(T) = r \sum_m (N_m/N) [3/2 + R_m/2 + \sum_n \{ (T_{mn}/2T) \operatorname{csch} (T_{mn}/2T) \}^2] \quad (7)$$

Here ρ is fluid density (mass/volume), $\underline{v}$ is fluid velocity, h is fluid enthalpy/mass, p is fluid pressure, s is fluid entropy/mass, T is fluid absolute temperature, $r = p/\rho T$ is the gas constant, $c_p(T) = (\partial h/\partial T)_p$ is specific heat/mass at constant pressure, $c_v(T) = T(\partial s/\partial T)_v$ is specific heat/mass at constant volume, and N_m/N is the fraction of molecules having rotational degrees of excitation freedom R_m and internal vibration characteristic temperatures T_{mn} . The flexible source-sinks Q_1 , Q_2 , Q_3 , and Q_4 provide for piecewise variant transfer of mass, energy, momentum, and entropy respectively.

These equations are integrated analytically over each streamtube segment by conversion to surface integrals, to provide firm numeric reliability for any segmentation mesh size. The transcendental integrated equations are solved exactly, by iterating second-order expansion logic to solid convergence. Traveling or static oblique shocks are generated as Galilean velocity transforms of static normal shocks.

The source deck consists of a 3307-card, 7-link overlay chain, formulated in the machine-independent dialectal intersection of Univac-1108 Fortran-V-CSC and IBM-7090 Fortran-IV-13, and designed for both modular growth versatility and freedom from obsolescence. Input consists, in essence, of gas compositions (r , N_m/N , R_m , T_{mn}), shock intensities and angles, implicit or explicit transverse areas, implicit or explicit source-sinks (Q_1 , Q_2 , Q_3 , Q_4) for any three of the four balance equations, and design-variation specifications, all fed in via teletype, cards, bulk peripheral storage, remote transmitter, or fast memory input linkage. Output consists of thermodynamically complete portrayals of both design and performance, fed out via plotter, teletype, line printer, remote transmitter, or fast memory output linkage.

EQUATIONS OF STATE

Evaluative comparisons both with measurements and with more refined theory (as summarized, for example, by Shapiro¹, by Mayer and Mayer², by Richards³, by Wilson⁴, and by Landau and Lifshitz⁵) seem to indicate that, with comparatively few exceptions over the range of intermediate pressures and temperatures of interest in gas dynamics, adequate equations of state are obtainable by neglecting both interaction between molecules and coupling between molecular rotation and vibration. Quantum thresholds tend to dominate the statistical physics of a gas molecule by creating sharply temperature-variant constraints upon its internal excitation freedom.

The relevant quantum trace partition function $Z = \langle \exp(-H/kT) \rangle$, involving the system Hamiltonian H , the Boltzmann constant k , and the temperature T , takes the form

$$Z = \langle \exp(- \sum_m \sum_{N_m} [p^2/2mkT + \underline{J} \cdot \underline{I}_m^{-1} \cdot \underline{J}/2kT + \sum_n \{P_n^2 + \omega_{mn}^2 Q_n^2\}/2kT]) \rangle,$$

$$= \prod_m (N_m!)^{-1} [\langle \exp(- p^2/2mkT - \underline{J} \cdot \underline{I}_m^{-1} \cdot \underline{J}/2kT - \sum_n \{P_n^2 + \omega_{mn}^2 Q_n^2\}/2kT) \rangle]^{N_m}, \quad (8)$$

¹ A. H. Shapiro, "The Dynamics and Thermodynamics of Compressible Fluid Flow". Vol. I-II, Ronald Press, New York, 1953-1954.

² J. E. Mayer, M. G. Mayer, "Statistical Mechanics", John Wiley & Sons, New York, 1959.

³ P. I. Richards, "Manual of Mathematical Physics", Pergamon Press, New York, 1959.

⁴ A. H. Wilson, "Thermodynamics and Statistical Mechanics", Cambridge University Press, London, 1957.

⁵ L. D. Landau, E. M. Lifshitz, "Statistical Physics", Pergamon Press, London, 1958.

where N_m is the number of molecules having mass m , moment of inertia tensor I_m , and characteristic vibrational frequencies $\omega_{mn}/2\pi$. The factor $(N_m!)^{-1}$ stems from exchange symmetrization of the wave functions for indistinguishable molecules. In the single-molecule Hamiltonian which remains, $\underline{p}$ is translational momentum, $\underline{J}$ is angular momentum about the center of mass, and P_n and Q_n are canonical momentum and displacement for vibratory normal mode n .

Translation

Momentum eigenfunctions spanning a volume V quantize as the discrete set

$$\psi = \bar{V}^{-1/2} \exp(i2\pi \underline{p} \cdot \underline{r}/h), \quad \underline{p} = (h/V^{1/3}) \underline{n}, \quad (9)$$

where $\underline{r}$ is the position vector, h is Planck's constant, and $\underline{n} = (n_x, n_y, n_z)$ is an integer lattice vector. The translation partition function thus becomes

$$\begin{aligned} \langle \exp(-\underline{p}^2/2mkT) \rangle &= \sum_{\underline{n}} \exp[-(h^2/2mV^{2/3}kT) \underline{n}^2], \\ &= \left[\int_{-\infty}^{\infty} \exp\{-(h^2/2mV^{2/3}kT) n^2\} dn \right]^3, \\ &= V(2\pi mkT)^{3/2}/h^3. \end{aligned} \quad (10)$$

Use of the continuum limit here is quite rigorous, since any macroscopic volume spans many thermal De Broglie wave lengths.

Rotation

The Euler angle representation of the rotational Hamiltonian takes the form

$$\begin{aligned} \underline{J} \cdot \underline{I}^{-1} \cdot \underline{J}/2 &= [(p_\phi - p_\psi \cos\theta) \csc\theta \sin\psi + p_\theta \cos\psi]^2/2I_1 \\ &+ [(p_\phi - p_\psi \cos\theta) \csc\theta \cos\psi - p_\theta \sin\psi]^2/2I_2 + p_\psi^2/2I_3, \end{aligned} \quad (11)$$

where ϕ and ψ are the azimuthal Euler angles about laboratory and body polar axes respectively, and θ is the polar Euler angle between laboratory and body polar axes. The canonical angular momenta are $(p_\phi, p_\theta, p_\psi)$, and the principal moments of inertia are (I_1, I_2, I_3) . The square of the angular momentum (as read off the Hamiltonian by the artifice of replacing the moment of inertia tensor I by the unit matrix) is

$$\underline{J} \cdot \underline{J} = (p_\phi - p_\psi \cos\theta)^2 \csc^2\theta + p_\theta^2 + p_\psi^2. \quad (12)$$

Both p_ϕ and $\underline{J} \cdot \underline{J}$ commute with the Hamiltonian, so the angular momentum quantization is given by

$$p_\phi = \hbar M; M=0, \pm 1, \dots, \pm J; \hbar = h/2\pi; \quad (13)$$

$$\underline{J} \cdot \underline{J} = \hbar^2 J(J+1); J=0, 1, \dots, \infty. \quad (14)$$

The energy quantization logic, as summarized by Landau⁶, involves solving secular determinants of degree $2J+1$. The first few energy levels, which set the quantum excitation thresholds, are:

$$J=0; M=0; E_0=0.$$

$$J=1; M=0, \pm 1; E_1 = \hbar^2 (I_2^{-1} + I_3^{-1})/2, E_2 = \hbar^2 (I_3^{-1} + I_1^{-1})/2, E_3 = \hbar^2 (I_1^{-1} + I_2^{-1})/2. \quad (15)$$

Polyatomic Nonlinear Molecules

For polyatomic molecules not of linear chain structure, with the exception of a few hydrogen derivatives near liquification, both Mayer⁷ and

⁶ L. D. Landau, E. M. Lifshitz, "Quantum Mechanics, Non-Relativistic Theory", Vol. 3 "Course of Theoretical Physics", pp. 373-378, Addison-Wesley, Reading, Mass., 1958.

⁷ Mayer, Op. Cit., p. 454.

Wilson⁸ find that molecular moments of inertia typically are so large that these quantum excitation thresholds are far exceeded for all gaseous-state temperatures. Consequently it becomes quite adequate to use the semi-classical rotation partition function,

$$\langle \exp(-\underline{J} \cdot \underline{I}^{-1} \cdot \underline{J}/2kT) \rangle = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(-\underline{J} \cdot \underline{I}^{-1} \cdot \underline{J}/2kT) dp_{\theta} dp_{\phi} dp_{\psi} d\theta d\phi d\psi / h^3. \quad (16)$$

Glancing back at the sum-of-squares structure of $\underline{J} \cdot \underline{I}^{-1} \cdot \underline{J}$ in Equation (11) suggests doing the momentum integration first, with the help of the linear transformation

$$\begin{aligned} J_1 &= (p_{\phi} - p_{\psi} \cos\theta) \csc\theta \sin\psi + p_{\theta} \cos\psi, \\ J_2 &= (p_{\phi} - p_{\psi} \cos\theta) \csc\theta \cos\psi - p_{\theta} \sin\psi, \\ J_3 &= p_{\psi}. \end{aligned} \quad (17)$$

The transformation determinant can be read off as

$$|\partial(J_1, J_2, J_3)/\partial(p_{\theta}, p_{\phi}, p_{\psi})| = \begin{vmatrix} \cos\psi & \csc\theta \sin\psi & -\cot\theta \sin\psi \\ -\sin\psi & \csc\theta \cos\psi & -\cot\theta \cos\psi \\ 0 & 0 & 1 \end{vmatrix} = \csc\theta, \quad (18)$$

and the rotational partition function thus integrates to

$$\begin{aligned} \langle \exp(-\underline{J} \cdot \underline{I}^{-1} \cdot \underline{J}/2kT) \rangle &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[-(J_1^2/I_1 + J_2^2/I_2 + J_3^2/I_3)/2kT] \\ &\quad \cdot dJ_1 dJ_2 dJ_3 \sin\theta d\theta d\phi d\psi / h^3, \\ &= (2\pi I_1 kT)^{1/2} (2\pi I_2 kT)^{1/2} (2\pi I_3 kT)^{1/2} (8\pi^2/h^3). \end{aligned} \quad (19)$$

⁸Wilson, *Op. Cit.*, p. 143.

Diatomic and Linear Molecules

Diatomic molecules (such as N-N), and polyatomic molecules having linear chain structure (such as O-C-O), have so small a moment of inertia about the axis of rotational symmetry that two of the three quantum excitation thresholds shown in Equation (15) typically lie well up near ionization temperature. At ordinary temperatures, quantum threshold effects thus strongly inhibit rotation of linear molecules about the axis of symmetry. Consequently the rotational Hamiltonian (11) and the square of the angular momentum (12), constrained to span only accessible degrees of excitation freedom ($p_\psi = 0$, $I_1 = I_2 = I$), degenerate to the linear top form,

$$\underline{J} \cdot \underline{I}^{-1} \cdot \underline{J} / 2 = (p_\theta^2 + p_\phi^2 \csc^2 \theta) / 2I = \underline{J} \cdot \underline{J} / 2I. \quad (20)$$

The angular momentum quantization given by Equations (13-14) remains relevant, and the rotation partition function becomes simply

$$\begin{aligned} \langle \exp(-\underline{J} \cdot \underline{I}^{-1} \cdot \underline{J} / 2kT) \rangle &= \sum_{J=0}^{\infty} \sum_{M=-J}^J \exp[-\hbar^2 J(J+1) / 2IkT], \\ &= \sum_{J=0}^{\infty} (2J+1) \exp[-\hbar^2 J(J+1) / 2IkT], \\ &\doteq \int_0^{\infty} (2J+1) \exp[-\hbar^2 J(J+1) / 2IkT] dJ, \\ &= 2IkT / \hbar^2. \end{aligned} \quad (21)$$

The discrete sum here is well represented by the integral at temperatures well above the quantum excitation threshold $T = \hbar^2 / 2Ik$. Representative

thresholds⁹ range from far below liquification temperature for non-hydrogenous molecules (.5476 °K for O-C-O, 2.080 °K for O-O, 2.892 °K for N-N) to 84.971 °K for H-H.

Monatomic Molecules

For monatomic molecules (such as He), all three quantum thresholds (15) are inaccessible at ordinary temperatures. Frozen thus in the ir-rotational ($\underline{J} = 0$) state, the rotational partition function for monatomic molecules collapses to

$$\langle \exp(-\underline{J} \cdot \underline{I}^{-1} \cdot \underline{J}/2kT) \rangle = 1. \quad (22)$$

Vibration

The harmonic oscillator representation of a vibrational normal mode quantizes in the form¹⁰

$$(P^2 + \omega^2 Q^2)/2 = \hbar\omega(\lambda + 1/2); \lambda=0,1,\dots,\infty. \quad (23)$$

The corresponding partition function becomes

$$\begin{aligned} \langle \exp[-(P^2 + \omega^2 Q^2)/2kT] \rangle &= \sum_{\lambda=0}^{\infty} \exp[-\hbar\omega(\lambda + 1/2)/kT], \\ &= \exp(-\hbar\omega/2kT) / [1 - \exp(-\hbar\omega/kT)], \end{aligned}$$

⁹ G. Herzberg, L. Herzberg, "Constants and Energy Levels of Diatomic and Polyatomic Molecules", ch. 7i-7j, "American Institute of Physics Handbook", McGraw-Hill, New York, 1957.

¹⁰ P. A. M. Dirac, "Principles of Quantum Mechancis", pp. 136-139, Third Edition, Clarendon Press, Oxford, 1947.

$$= (1/2) \operatorname{csch}(\hbar\omega/2kT). \quad (24)$$

Summing the Hamiltonian further over all normal modes n having characteristic frequencies $\omega_n/2\pi$ now yields the molecular vibration partition function,

$$\langle \exp[-\sum_n \{P_n^2 + \omega_n^2 Q_n^2\}/2kT] \rangle = \prod_n [(1/2) \operatorname{csch}(\hbar\omega_n/2kT)]. \quad (25)$$

Molecular vibration typically creates large temperature-variant specific heat contributions within the intermediate temperature range of key interest in gas dynamics. Representative vibration threshold temperatures⁹ $T = \hbar\omega/k$ near which specific heats per molecule rise by k are (960.1, 1997, 3380) °K for O-C-O, 2274 °K for O-O, (2295, 5262, 5433) °K for H-O-H, and 3395 °K for N-N.

Ground State Degeneracy

Multiplying nuclear spin degeneracies $2s+1$ by electronic ground state degeneracy g and dividing by the number of indistinguishable molecule orientations σ due to indistinguishable atoms yields for the degeneracy w of the molecule ground state,

$$w = [\prod_s (2s+1)]g/\sigma. \quad (26)$$

The nuclear spin product here extends over all nuclei in the molecule, with justifiable neglect of hyperfine splitting for all gases.

(Hyperfine $T = \Delta E/k$ range from .1 °K to 1.5 °K, according to Landau¹¹.)

¹¹ Landau, Ref. 5, p. 136 footnote +.

This uncoupled treatment of nuclear spin and molecular symmetry fails only for low temperature hydrogenous molecules, according to Mayer¹².

Thermostatics

Assembling the complete partition function (8), from its translational part (10), its rotational part (19, 21, 22), its vibrational part (25), and its ground state part (26), and replacing $N_m!$ by its Stirling asymptotic form $(N_m/e)^{N_m}$, now yields

$$Z(V,T) = \prod_m [(eV/N_m)(2\pi mkT/h^2)^{3/2}]^{N_m} \cdot w_m \left[\begin{array}{l} 1, \text{ monatomic} \\ 2I_m kT/\hbar^2, \text{ linear} \\ \{2(\pi I_1 I_2 I_3)_m\}^{1/3} kT/\hbar^2\}^{3/2}, \text{ nonlinear} \end{array} \right] \cdot \prod_n (1/2) \text{csch}(\hbar\omega_{mn}/2kT)]^{N_m}. \quad (27)$$

All other thermostatic functions are derivable from this partition function by appropriate differentiation. Since the partition function is expressed in terms of volume V and temperature T , the appropriate generator is the Helmholtz potential

$$A = - kT \ln[Z(V,T)] \quad (28)$$

whose total differential

$$dA = - p dV - S dT \quad (29)$$

generates the pressure $p(V,T)$ and entropy $S(V,T)$ equations of state.

¹² Mayer, *Op. Cit.*, Sec. AIX, pp. 440-454.

The pressure falls out as

$$p = - (\partial A / \partial V)_T = kT [\partial \ln(Z) / \partial V]_T = kT \frac{\sum_m N_m}{V} = NkT/V, \quad (30)$$

where $N = \sum_m N_m$ is the total number of molecules. In compressible fluid flow work, it is convenient to replace volume by density with the maneuver

$$p = (k/\bar{m})(N\bar{m}/V)T = r\rho T, \quad (31)$$

where $\bar{m} = \sum_m m N_m / N$ is the mean molecular mass, $r = k/\bar{m}$ is the gas constant, and $\rho = N\bar{m}/V$ is the density (mass/volume).

The two-term structure of the entropy,

$$S = - (\partial A / \partial T)_V = [\partial \{kT \ln(Z)\} / \partial T]_V = kT [\partial \ln(Z) / \partial T]_V + k \ln(Z), \quad (32)$$

suggests generating next the internal energy

$$U = TS + A = kT [\partial \ln(Z) / \partial \ln T]_V, \\ = kT \sum_m N_m \left[\begin{array}{l} 3/2 + \left[\begin{array}{l} 0, \text{ monatomic} \\ 1, \text{ linear} \\ 3/2, \text{ nonlinear} \end{array} \right] \end{array} \right] + \sum_n (\hbar\omega_{mn}/2kT) \coth(\hbar\omega_{mn}/2kT). \quad (33)$$

Converting to internal energy/mass $u = U/N\bar{m}$ and compacting the notation leads to

$$u(T) = rT \sum_m (N_m/N) [3/2 + R_m/2 + \sum_n (T_{mn}/2T) \coth(T_{mn}/2T)], \quad (34)$$

where N_m/N is the fraction of molecules having rotational degrees of excitation freedom $R_m = (0, 2, 3)$ and vibrational mode characteristic temperatures $T_{mn} = \hbar\omega_{mn}/k$. With the help of Equation (31), the enthalpy/mass $h = u + p/\rho$ now becomes

$$h(T) = u(T) + rT, \quad (35)$$

with both internal energy/mass and enthalpy/mass dependent upon temperature only.

Differentiating the last two equations to obtain the specific heat/mass at constant volume $c_V = (\partial u / \partial T)_V$ and the specific heat/mass at constant pressure $c_P = (\partial h / \partial T)_P$ yields

$$c_V(T) = r \sum_m (N_m / N) [3/2 + R_m / 2 + \sum_n \{ (T_{mn} / 2T) \operatorname{csch}(T_{mn} / 2T) \}^2], \quad (36)$$

$$c_P(T) = c_V(T) + r. \quad (37)$$

Subsequent gas flow logic makes frequent use also of the specific heat ratio γ , the sonic speed c , and the first and second derivatives of the specific heats/mass, in the form

$$\gamma(T) = c_P(T) / c_V(T), \quad (38)$$

$$c^2(T) = rT\gamma(T), \quad (39)$$

$$\begin{aligned} c_V'(T) = c_P'(T) = (2r/T) \sum_m (N_m / N) \sum_n [(T_{mn} / 2T) \operatorname{csch}(T_{mn} / 2T)]^2 \\ \cdot [(T_{mn} / 2T) \coth(T_{mn} / 2T) - 1], \end{aligned} \quad (40)$$

$$\begin{aligned} c_V''(T) = c_P''(T) = (4r/T^2) \sum_m (N_m / N) \sum_n [(T_{mn} / 2T) \operatorname{csch}(T_{mn} / 2T)]^2 \\ \cdot [\{ (T_{mn} / 2T) \coth(T_{mn} / 2T) - 1 \}^2 - \{ (T_{mn} / 2T) \coth(T_{mn} / 2T) - 1/2 \} \\ + (1/2) \{ (T_{mn} / 2T) \operatorname{csch}(T_{mn} / 2T) \}^2]. \end{aligned} \quad (41)$$

With the help of Equation (34), the entropy/mass $s = S / N\bar{m}$ now can be read off Equations (32) and (27) as

$$s = u/T + (r/N) \ln(Z),$$

$$\begin{aligned}
&= r \sum_m (N_m/N) [3/2 + R_m/2 + \sum_n (T_{mn}/2T) \coth(T_{mn}/2T) + \ln\{(\bar{e}mN/N_m \rho) (2\pi mkT/h^2)^{3/2}\} \\
&+ \ln(w_m) + (R_m/2) \left[\begin{array}{l} 0, \text{ monatomic} \\ \ln(2T/T_m), \text{ polyatomic} \end{array} \right] - \sum_n \ln\{2 \sinh(T_{mn}/2T)\}], \quad (42)
\end{aligned}$$

where the rotational characteristic temperatures T_m are given by

$$T_m = \left[\begin{array}{l} \hbar^2/I_m k, \text{ linear} \\ \hbar^2/(\pi I_1 I_2 I_3)_m^{1/3} k, \text{ nonlinear} \end{array} \right]. \quad (43)$$

The Helmholtz potential/mass $a = A/N\bar{m}$ and the Gibbs potential/mass $g = G/N\bar{m}$ are obtainable from Equations (34), (35), and (42) in the form

$$a = u - Ts, \quad (44)$$

$$g = h - Ts. \quad (45)$$

The numeric computation sequence summarized in Table 1 helps to convey these ideas to an electronic computer.

Table 1: Equations of State (Subroutine Phase)

Input: Density ρ , temperature T . Molecule masses m , fractions N_m/N , degeneracies w_m ,

rotations R_m , characteristic temperatures (T_m, T_{mn}) .

Logic: $\bar{m} = \sum_m (N_m/N)$, $r = k/\bar{m}$, $p = r\rho T$,

$$u(T) = rT \sum_m (N_m/N) [3/2 + R_m/2 + \sum_n (T_{mn}/2T) \coth(T_{mn}/2T)],$$

$$a(\rho, T) = -rT \sum_m (N_m/N) [\ln\{(emN/N\rho)(2\pi mkT/h^2)^{3/2}\} + \ln(w_m) + (R_m/2) \ln(2T/T_m) \\ - \sum_n \ln\{2 \sinh(T_{mn}/2T)\}],$$

$$h(T) = u + rT, \quad g(\rho, T) = a + rT, \quad s(\rho, T) = (u - a)/T,$$

$$c_V(T) = r \sum_m (N_m/N) [3/2 + R_m/2 + \sum_n \{(T_{mn}/2T) \operatorname{csch}(T_{mn}/2T)\}^2],$$

$$c_p(T) = c_V + r, \quad \gamma(T) = c_p/c_V, \quad c(T) = (r\gamma T)^{1/2},$$

$$c_V^1(T) = (2r/T) \sum_m (N_m/N) \sum_n [(T_{mn}/2T) \operatorname{csch}(T_{mn}/2T)]^2 [(T_{mn}/2T) \coth(T_{mn}/2T) - 1],$$

$$c_V''(T) = (4r/T^2) \sum_m (N_m/N) \sum_n [(T_{mn}/2T) \operatorname{csch}(T_{mn}/2T)]^2 \{[(T_{mn}/2T) \coth(T_{mn}/2T) - 1]^2 \\ - \{(T_{mn}/2T) \coth(T_{mn}/2T) - 1/2\} + (1/2)\{(T_{mn}/2T) \operatorname{csch}(T_{mn}/2T)\}^2\}].$$

Output: $r, p, u, a, h, g, s, c_V, c_p, \gamma, c, c_V^1, c_V''$.

SHOCKS

Conservation of mass, energy, and momentum in any non-accelerated reference frame implies that traveling-or-static oblique-or-normal shocks all can be generated simply by viewing a static normal shock from a moving reference frame.

Static Normal Shocks

Conservation of mass, energy, and momentum in the static-normal-shock reference frame,

$$\nabla \cdot \rho \underline{v} = 0, \quad (46)$$

$$\nabla \cdot \rho \underline{v} [h(T) + v^2/2] = 0, \quad (47)$$

$$\nabla \cdot (\rho \underline{v} v + r \rho T) = 0, \quad (48)$$

implies the static normal shock continuity integrals,

$$\rho v = \rho_1 v_1, \quad (49)$$

$$h(T) + v^2/2 = h(T_1) + v_1^2/2 \equiv h_o, \quad (50)$$

$$v + rT/v = v_1 + r_1 T_1/v_1 \equiv v_o, \quad (51)$$

relating the exit state (ρ, T, v) to the entry state (ρ_1, T_1, v_1) . Despite the formidable complexity of the enthalpy/mass $h(T)$ as given by Equations (34-35), the shock solution $(\rho, T, v) \neq (\rho_1, T_1, v_1)$ is obtainable numerically by iterating second order expansion logic to solid convergence.

Solving the mass and momentum Equations (49,51) for density and temperature respectively,

$$\rho = \rho_1 v_1 / v, \quad T = (v_o - v)v/r, \quad (52)$$

and constructing a second order velocity expansion of the energy Equation (50), yields the iterative sequence,

$$\delta h \equiv h(T) + v^2/2 - h_o, \quad (53)$$

$$\begin{aligned} (\delta h)' &\equiv d(\delta h)/dv = [dh(T)/dT] \cdot dT/dv + v, \\ &= c_p(T) \cdot (v_o - 2v)/r + v, \end{aligned} \quad (54)$$

$$\begin{aligned} (\delta h)'' &\equiv d(\delta h)'/dv = c_V'(T) \cdot [(v_o - 2v)/r]^2 - 2c_p(T)/r + 1, \\ &= c_V'(T) \cdot [(v_o - 2v)/r]^2 - [1 + 2c_V(T)/r]. \end{aligned} \quad (55)$$

Solving the energy-balance expansion,

$$\delta h + (\delta h)'(\delta v) + (\delta h)''(\delta v)^2/2 = 0, \quad (56)$$

for the velocity refinement δv yields two roots,

$$\delta v = - (\delta h)' / (\delta h)'' \pm [\{ (\delta h)' / (\delta h)'' \}^2 - 2(\delta h) / (\delta h)'']^{1/2}. \quad (57)$$

Occurrence of this $\pm$ sign choice here is quite essential. On the first pass, with the entry state (ρ_1, T_1, v_1) taken as the initial iterate for the exit state (ρ, T, v) , one of the two roots here must be $\delta v = 0$, since the entry state satisfies the continuity Equations (49-51). Consequently, on the first pass, the $\pm$ sign giving the largest $|\delta v|$ must be selected, to jump from the entry-state Riemann branch sheet to the shock-state Riemann branch sheet. After the first pass, the $\pm$ sign giving the smallest $|\delta v|$ must be selected, to remain on the shock-state Riemann branch sheet until converged.

The refined velocity then is formed as

$$v = v + \delta v, \quad (58)$$

and the iteration logic recycles through Equations (52-58) until converged upon the shock state $(\rho, T, v) \neq (\rho_1, T_1, v_1)$, with $\delta v = 0$ and $\delta h = 0$.

Weaving subroutines into a machine analysis begins now by noting that the enthalpy/mass $h(T)$ and its derivatives as needed here are to be obtained by calling Subroutine Phase shown in Table 1, which returns also all

thermostatic functions likely to be useful ($r, p, u, a, h, g, s, c_V, c_p, \gamma, c, c'_V, c''_V$), all of which thus materialize automatically for the converged static shock state.

Traveling Oblique Shocks

Viewed from a moving prime reference frame, the static-normal-shock-frame entry velocity $v_1 \underline{\Omega}$ and exit velocity $v \underline{\Omega}$ transform respectively to

$$\underline{v}'_1 = v_1 \underline{\Omega} + \underline{v}'_S, \quad \underline{v}' = v \underline{\Omega} + \underline{v}'_S, \quad (59)$$

where $\underline{\Omega}$ is the unit shock normal (along the flow in the static-normal-shock frame) and $\underline{v}'_S$ is the shock velocity. Fluid density, fluid temperature, and all other such thermostatic quantities defined in the local fluid rest frame are, of course, intrinsically invariant.

Shifting to an intrinsically invariant characterization of a traveling oblique shock, in terms of its static-normal-shock-frame entry Mach vector $M_{N\underline{\Omega}}$, leads to rearranging Equations (59) as the numeric computational sequence,

$$v_1 = c_1 M_N, \quad (60)$$

$$\underline{v}'_S = \underline{v}'_1 - c_1 M_{N\underline{\Omega}} \quad (61)$$

$$\underline{v}' = \underline{v}'_1 + (v - c_1 M_N) \underline{\Omega}. \quad (62)$$

The square of the exit velocity can be read off the last equation as

$$(\underline{v}')^2 = (\underline{v}'_1)^2 + (v - c_1 M_N)^2 + 2(v - c_1 M_N) v'_1 \underline{\Omega}'_1 \cdot \underline{\Omega}, \quad (63)$$

where $\underline{\Omega}'_1 \cdot \underline{\Omega} = \cos \theta_N$ is the cosine of the angular tilt of the shock exit normal $\underline{\Omega}$ from the entry flow direction $\underline{\Omega}'_1$.

Smooth joining of entry and exit streamtubes at an oblique shock interface requires that the intercepted shock area A project both upstream as the

entry streamtube area A'_1 and downstream as the exit streamtube area A' , in the form

$$A'_1 = A \frac{\Omega \cdot \Omega'_1}{\Omega'_1 \cdot \Omega'_1}, \quad A' = A \frac{\Omega \cdot \Omega'}{\Omega' \cdot \Omega'} \quad (64)$$

By reading the exit flow direction Ω' off Equation (62), the exit/entry area ratio now can be computed as

$$\begin{aligned} A'/A'_1 &= \frac{\Omega \cdot \Omega'}{\Omega'_1 \cdot \Omega'_1} = \frac{(\Omega \cdot \underline{v}_1' + v - c_1 M_N)}{v' \Omega'_1 \cdot \Omega'_1}, \\ &= [v_1' + (v - c_1 M_N)/\cos \theta_N]/v'. \end{aligned} \quad (65)$$

The exit/entry mass flow ratio for a traveling oblique shock then can be evaluated, in the form

$$\begin{aligned} (\rho v A)' / (\rho v A)'_1 &= (\rho/\rho_1) (v'/v'_1) (A'/A'_1), \\ &= (\rho/\rho_1) [1 + (v - c_1 M_N)/v'_1 \cos \theta_N]. \end{aligned} \quad (66)$$

It is informative to note that the exit/entry area ratio reduces to unity for a traveling-or-static normal shock ($\cos \theta_N = 1$) and that the exit/entry mass flow ratio reduces to unity for an oblique-or-normal static shock

$$[M_N = (v'_1 \cos \theta_N)/c_1].$$

The exit impulse/mass v'_o , exit adiabatic-stagnation enthalpy/mass h'_o , and exit Mach ratio M' are calculated, finally, directly from their definitions, as

$$v'_o = v' + rT/v', \quad (67)$$

$$h'_o = h(T) + (v')^2/2, \quad (68)$$

$$M' = v'/c. \quad (69)$$

Table 2 summarizes this complete traveling-or-static oblique-or-normal shock construction in computer lingo, with primed transforms replaced by equivalent dynamic redefinition to minimize storage demand.

Table 2: Traveling-Or-Static Oblique-Or-Normal Shock (Subroutine Shock)

Input: M_N , $\cos\theta_N$, entry state(1). Static shock implies $M_N = M_1 \cos\theta_N$.

Static Normal Shock: $v_N = c_1 M_N$, $v_o = v_N + r_1 T_1 / v_N$, $h_o = h(T_1) + v_N^2 / 2$.

Solve $\begin{cases} v + rT/v = v_o \\ h(T) + v^2/2 = h_o \end{cases}$ for $(v, T) \neq (v_N, T_1)$ by second order iteration.

$$v = v_N.$$

Step 1: $\rho = \rho_1 v_N / v$, $T = (v_o - v)v / r$,

$$\delta h = h(T) + v^2/2 - h_o,$$

$$(\delta h)' = d(\delta h)/dv = c_p(T) \cdot (v_o - 2v)/r + v,$$

$$(\delta h)'' = d(\delta h)'/dv = c_V'(T) \cdot [(v_o - 2v)/r]^2 - [1 + 2c_V(T)/r].$$

Solve $\delta h + (\delta h)'(\delta v) + (\delta h)''(\delta v)^2/2 = 0$ for δv .

$$\delta v = -(\delta h)' / (\delta h)'' \pm [\{ (\delta h)' / (\delta h)'' \}^2 - 2(\delta h) / (\delta h)'']^{1/2}.$$

On first pass, use $\pm$ giving largest $|\delta v|$, to get shock.

After first pass, use $\pm$ giving smallest $|\delta v|$, to home in.

$$v = v + \delta v.$$

Return to Step 1 until converged ($\delta v = 0$, $\delta h = 0$).

Traveling Oblique Shock: $\delta v_N = v - v_N$,

$$v = [v_1^2 + (\delta v_N)^2 + 2(\delta v_N)v_1 \cos\theta_N]^{1/2},$$

$$A = A_1 (v_1/v) [1 + (\delta v_N)/(v_1 \cos\theta_N)],$$

$$\rho v A = (\rho_1 v_1 A_1) (\rho/\rho_1) [1 + (\delta v_N)/(v_1 \cos\theta_N)],$$

$$v_o = v + rT/v, \quad h_o = h(T) + v^2/2, \quad M = v/c.$$

Output: $A, \rho v A, v_o, h_o, s, M, v, \rho, T, p, u, h, a, g, c_V, c_p, \gamma, c, c_V', r, c_V''$.

NOZZLES

Striving for machine analysis versatility leads to generalizing the physics of a Laval nozzle to include any streamtube segment explicitly characterizable by its mass, energy, and entropy balance, in the form

$$\text{Mass Balance: } \nabla \cdot \rho \underline{v} = Q_1, \quad (70)$$

$$\text{Energy Balance: } \nabla \cdot \rho \underline{v} (h + v^2/2) = Q_2, \quad (71)$$

$$\text{Entropy Balance: } \nabla \cdot \rho \underline{v} s = Q_4. \quad (72)$$

The piecewise integrable source-sinks (Q_1, Q_2, Q_4) provide for flexible transfer of mass, energy, and entropy, none of which need be conserved by the nozzle. Converted to surface integrals, to eliminate any need for fine mesh subdivision of the streamtube segment, the mass, energy, and entropy balance equations become

$$\rho v = \phi_o, \quad (73)$$

$$h(T) + v^2/2 = h_o, \quad (74)$$

$$s(\rho, T) = s_o, \quad (75)$$

where entry surface integrals and source-sink volume integrals have been combined to obtain the exit mass flux ϕ_o , the exit adiabatic-stagnation enthalpy/mass h_o , and the exit entropy/mass s_o .

Subsonic or Supersonic Nozzles

These three analytic streamtube integrals are solved for the exit thermodynamic field (ρ, T, v) by iterating second order expansion logic to solid convergence. Observing in Equation (42) that the entropy/mass separates into a complicated function of temperature T plus a simple function of density ρ , in the explicit form

$$s(\rho, T) = s(1, T) - r \ln(\rho), \quad (76)$$

permits solving the entropy balance Equation (75) for the density $\rho(T)$ as

$$\rho(T) = \exp[\{s(l,T) - s_o\}/r], \quad (77)$$

and then the velocity $v(T)$ falls out of the mass balance Equation (73) as

$$v(T) = \phi_o \exp[\{s_o - s(l,T)\}/r]. \quad (78)$$

Constructing a second order temperature expansion of the energy balance Equation (74) now generates the iterative sequence,

$$\delta h \equiv h(T) + v^2(T)/2 - h_o, \quad (79)$$

$$\begin{aligned} (\delta h)' &\equiv d(\delta h)/dT = c_p(T) + v(T)dv(T)/dT, \\ &= c_p(T) - [v^2(T)/rT]c_v(T), \text{ using } c_v = T(\partial s/\partial T)_v, \end{aligned} \quad (80)$$

$$(\delta h)'' \equiv d(\delta h)'/dT = [1 - v^2(T)/rT]c_v'(T) + [2c_v(T)/r + 1][v^2(T)/rT^2]c_v(T). \quad (81)$$

Solving the energy balance expansion,

$$\delta h + (\delta h)'(\delta T) + (\delta h)''(\delta T)^2/2 = 0, \quad (82)$$

for the temperature refinement δT yields two roots,

$$\delta T = - (\delta h)' / (\delta h)'' \pm [\{ (\delta h)' / (\delta h)'' \}^2 - 2(\delta h) / (\delta h)'']^{1/2}. \quad (83)$$

The subsonic or supersonic nozzle logic clings to the entry Riemann branch sheet, both by taking the entry temperature T_1 as the initial iterate for the exit temperature T , and by continually selecting the $\pm$ sign giving the least $|\delta T|$ here. (Transonic nozzles can be handled with this logic, simply by splitting them into subsonic and supersonic segments.) The refined temperature then is formed as

$$T = T + \delta T, \quad (84)$$

and the iteration logic recycles through Equations (77-84) until converged, with $\delta T = 0$ and $\delta h = 0$.

The functions needed here $[s(l,T), h(T), c_v(T), c_p(T), c'_v(T)]$ are generated by calling Subroutine Phase shown in Table 1, which returns also the full thermostatic field $(r,p,u,a,h,g,s,c_v,c_p,\gamma,c,c'_v,c''_v)$ for the converged state. To complete the thermodynamic field, the exit Mach ratio M and the exit impulse/mass v_o are computed directly from their definitions as

$$M = v/c, \quad v_o = v + rT/v. \quad (85)$$

Table 3 summarizes the nozzle logic as a numeric computation sequence.

Table 3: Subsonic or Supersonic Nozzle (Subroutine Nozzle)

Input: Streamtube transverse area A , mass flow rate $\rho v A$,
 adiabatic stagnation enthalpy/mass h_o , entropy/mass s .
 Temperature T estimate (on branch sheet).

Logic: Solve
$$\left\{ \begin{array}{l} \rho v = (\rho v A)/A \\ h(T) + v^2/2 = h_o \\ s(1,T) - r \ln(\rho) = s \end{array} \right\} \text{ for } (\rho, T, v).$$

Step 1: $\rho(T) = \exp[\{s(1,T) - s\}/r]$, $v(T) = (\rho v A)/A\rho(T)$,

$$\delta h = h(T) + v^2(T)/2 - h_o,$$

$$(\delta h)' = d(\delta h)/dT = c_p(T) - [v^2(T)/rT]c_v(T),$$

$$\begin{aligned} (\delta h)'' = d(\delta h)'/dT = [1 - v^2(T)/rT]c_v'(T) \\ + [2c_v(T)/r + 1][v^2(T)/rT^2]c_v(T). \end{aligned}$$

Solve $\delta h + (\delta h)'(\delta T) + (\delta h)''(\delta T)^2/2 = 0$ for δT .

$$\delta T = -(\delta h)' / (\delta h)'' \pm \{[(\delta h)' / (\delta h)']^2 - 2(\delta h) / (\delta h)''\}^{1/2}.$$

Use $\pm$ sign giving least $|\delta T|$, to cling to entry branch sheet.

$$T = T + \delta T.$$

Return to Step 1 until converged ($\delta T = 0$, $\delta h = 0$).

$$M = v/c, v_o = v + rT/v.$$

Output: $A, \rho v A, v_o, h_o, s, M, v, \rho, T, p, u, h, a, g, c_v, c_p, \gamma, c, c_v', r, c_v''$.

PIPES

Striving for machine analysis versatility leads to generalizing the physics of a simple pipe to include any parallel-wall streamtube segment explicitly characterizable by its mass, energy, and momentum balance, in the form

$$\text{Mass Balance: } \nabla \cdot \rho \underline{v} = Q_1, \quad (86)$$

$$\text{Energy Balance: } \nabla \cdot \rho \underline{v} (h + v^2/2) = Q_2, \quad (87)$$

$$\text{Momentum Balance: } \nabla \cdot (\rho \underline{v} \underline{v} + r \rho T) = Q_3. \quad (88)$$

The piecewise integrable source-sinks (Q_1, Q_2, Q_3) provide for flexible transfer of mass, energy, and momentum respectively, none of which need be conserved by the pipe. Converted to surface integrals, to eliminate any need for fine mesh subdivision of the streamtube segment, the mass, energy, and momentum balance equations become

$$\rho v = \phi_o, \quad (89)$$

$$h(T) + v^2/2 = h_o, \quad (90)$$

$$v + rT/v = v_o, \quad (91)$$

where entry surface integrals and source-sink volume integrals have been combined to obtain the exit mass flux ϕ_o , the exit adiabatic-stagnation enthalpy/mass h_o , and the exit impulse/mass v_o .

Subsonic or Supersonic Pipes

These three analytic streamtube integrals are solved for the exit thermodynamic field (ρ, T, v) by iterating second order expansion logic to solid convergence.

Solving the mass and momentum Equations (89,91) for density $\rho(v)$ and temperature $T(v)$ respectively,

$$\rho(v) = \phi_o/v, \quad T(v) = (v_o - v)v/r, \quad (92)$$

and constructing a second order velocity expansion of the energy Equation (90), generates the iterative sequence,

$$\delta h \equiv h(T) + v^2/2 - h_o, \quad (93)$$

$$\begin{aligned} (\delta h)' &\equiv d(\delta h)/dv = [dh(T)/dT] \cdot dT/dv + v, \\ &= c_p(T) \cdot (v_o - 2v)/r + v, \end{aligned} \quad (94)$$

$$\begin{aligned} (\delta h)'' &\equiv d(\delta h)'/dv = c'_V(T) [(v_o - 2v)/r]^2 - 2c_p(T)/r + 1, \\ &= c'_V(T) \cdot [(v_o - 2v)/r]^2 - [1 + 2c_V(T)/r]. \end{aligned} \quad (95)$$

Solving the energy-balance expansion,

$$\delta h + (\delta h)'(\delta v) + (\delta h)''(\delta v)^2/2 = 0, \quad (96)$$

for the velocity refinement δv yields two roots,

$$\delta v = - (\delta h)' / (\delta h)'' \pm [\{ (\delta h)' / (\delta h)'' \}^2 - 2(\delta h) / (\delta h)'']^{1/2}. \quad (97)$$

The subsonic or supersonic pipe logic clings to the entry Riemann branch sheet, both by taking the entry velocity v_1 as the initial iterate for the exit velocity v , and by continually selecting the $\pm$ sign giving the least $|\delta v|$ here. (Transonic pipes can be handled with this logic, simply by splitting them into subsonic and supersonic segments.) The refined velocity next is formed as

$$v = v + \delta v, \quad (98)$$

and then the iteration logic recycles through Equations (92-98) until converged, with $\delta v = 0$ and $\delta h = 0$.

The functions needed here $[h(T), c_p(T), c_V(T), c'_V(T)]$ are generated by calling Subroutine Phase shown in Table 1, which returns also the full thermostatic field $(r, p, u, a, h, g, s, c_V, c_p, \gamma, c, c'_V, c''_V)$ for the converged state. To complete the thermodynamic field, the exit Mach ratio M is formed from its definition as

$$M = v/c. \quad (99)$$

Table 4 summarizes the pipe logic as a numeric computation sequence.

Table 4: Subsonic or Supersonic Pipe (Subroutine Pipe)

Input: Streamtube transverse area A , mass flow rate $\rho v A$,
 adiabatic stagnation enthalpy/mass h_o , impulse/mass v_o .
 Velocity v estimate (on branch sheet).

Logic: Solve
$$\left\{ \begin{array}{l} \rho v = (\rho v A)/A \\ h(T) + v^2/2 = h_o \\ v + rT/v = v_o \end{array} \right\} \text{ for } (\rho, T, v).$$

Step 1: $\rho(v) = (\rho v A)/Av$, $T(v) = (v_o - v)v/r$,

$$\delta h = h(T) + v^2/2 - h_o,$$

$$(\delta h)' = d(\delta h)/dv = c_p(T) \cdot (v_o - 2v)/r + v,$$

$$(\delta h)'' = d(\delta h)'/dv = c_v'(T) \cdot [(v_o - 2v)/r]^2 - [1 + 2c_v(T)/r].$$

$$\text{Solve } \delta h + (\delta h)'(\delta v) + (\delta h)''(\delta v)^2/2 = 0 \text{ for } \delta v.$$

$$\delta v = -(\delta h)' / (\delta h)'' \pm [\{ (\delta h)' / (\delta h)'' \}^2 - 2(\delta h) / (\delta h)'']^{1/2}.$$

Use $\pm$ sign giving least $|\delta v|$, to cling to entry branch sheet.

$$v = v + \delta v.$$

Return to Step 1 until converged ($\delta v = 0$, $\delta h = 0$).

$$M = v/c.$$

Output: $A, \rho v A, v_o, h_o, s, M, v, \rho, T, p, u, h, a, g, c_v, c_p, \gamma, c, c_v', r, c_v''.$

BLOWERS AND TURBINES

Blowers and turbines consist of rotating arrays of nozzles. Blower and turbine analysis logic consequently can be generated by rotary transform of nozzle analysis logic.

Within a streamtube passing through a spinning rotor, the mass balance, energy balance, and angular momentum balance combine cleanly as the mass and energy-work-heat balance integrals,

$$\Delta(\rho v A) = 0 \quad (100)$$

$$\rho v A \cdot \Delta(h + v^2/2 - \omega R v_\phi) = \rho v A \cdot \Delta_o q, \quad (101)$$

where ρ is fluid density (mass/volume), v is fluid velocity, A is streamtube transverse area, h is fluid enthalpy/mass, ω is rotor angular velocity, R is radial distance from rotor axis, and v_ϕ is fluid azimuthal velocity. Good design strives to minimize the parasitic heat/mass $\Delta_o q$ flowing into (or out of) the working fluid, by matching wall temperatures to fluid adiabatic stagnation temperatures, but performance predictions prove to be quite sensitive to inclusion of its design residual value in the analysis logic. Good design strives also for isentropic flow, by shaping the rotor blades to minimize turbulence, but here too the sensitivity of performance predictions leads to casting the entropy balance in the generalized form,

$$\rho v A \cdot \Delta s = \rho v A \cdot \Delta_o s, \quad (102)$$

where s is fluid entropy/mass and $\Delta_o s$ is the design departure from isentropic flow. In all three equations, Δ refers to the difference between exit and entry conditions. Both the heat/mass transfer $\Delta_o q$ and the entropy/mass transfer $\Delta_o s$, in practice, must be extrapolated from previous relevant design experience and viewed as tentative until firmed up by suitable prototype measurements.

Rotary Transform

Whether computing by hand or by machine, the easy way of solving these three blower or turbine Equations (100-102) for the exit thermodynamic field (ρ, T, v) is to shift to a frame of reference rotating with the rotor by use of the velocity transformation

$$\underline{v} = \omega R \underline{u}_\phi + v' \underline{\Omega}', \quad (103)$$

where $\underline{v}$ is the fluid velocity vector in the stationary frame, $\underline{u}_\phi$ is an azimuthal unit vector, and $\underline{v}' = v' \underline{\Omega}'$ is the fluid velocity vector in the rotary frame (along the unit vector direction $\underline{\Omega}'$ down the axis of a streamtube passage through the rotor). Transforming the energy-work-heat balance Equation (101) to the rotary frame, by reading relevant terms off Equation (103) as

$$\begin{aligned} v^2/2 - \omega R v_\phi &= [\omega^2 R^2 + 2\omega R v' \underline{\Omega}' \cdot \underline{u}_\phi + (v')^2]/2 - \omega R (\omega R + v' \underline{\Omega}' \cdot \underline{u}_\phi), \\ &= (v')^2/2 - \omega^2 R^2/2, \end{aligned} \quad (104)$$

generates the rotary frame nozzle equation

$$\Delta[h + (v')^2/2 - \omega^2 R^2/2] = \Delta_o q, \quad (105)$$

which differs from the stationary frame nozzle equation only by the centripetal potential energy/mass term $-\omega^2 R^2/2$.

Completing the nozzle equations by noting that the rotor exit face area A must project upstream as the rotor streamtube transverse exit area A' yields

$$A' = A \underline{\Omega}_A \cdot \underline{\Omega}', \quad (106)$$

where $\underline{\Omega}_A$ is the unit normal of the rotor exit face. Making use of mass conservation, finally, leads to casting these rotary frame blower or turbine

equations in precisely the same functional form as the stationary frame nozzle Equations (73-75),

$$\rho v' = \phi'_0 \equiv (\rho v A)_1 / A', \quad (107)$$

$$h(T) + (v')^2/2 = h'_0 \equiv h_{01} - \omega R_1 v_{\phi 1} + \omega^2 R^2/2 + \Delta_0 q, \quad (108)$$

$$s(\rho, T) = s'_0 \equiv s_1 + \Delta_0 s, \quad (109)$$

with subscript 1 referring to entry conditions. It is important to note here that all thermostatic fields, such as (ρ, T) , are intrinsically invariant under rotary transformation, since they are defined in the local fluid rest frame.

The three rotary frame blower or turbine nozzle Equations (107-109) now are solved for the exit thermodynamic field (ρ, T, v') by use of the nozzle logic shown in Table 3 (Subroutine Nozzle), which also automatically completes the full exit thermodynamic field $(A', \rho v' A', v'_0, h'_0, s, M', v', \rho, T, p, u, h, a, g, c_v, c_p, \gamma, c'_v, r, c_v^u)$ in the rotary frame.

Inverse Rotary Transform

Transforming all velocity-dependent exit fields from the rotary frame ('') back to the stationary frame (''), with the help of Equation (103), generates the computational sequence,

$$v'' = [\omega^2 R^2 + 2\omega R v' \underline{\Omega}' \cdot \underline{u}_\phi + (v')^2]^{1/2}, \quad (110)$$

$$h''_0 = h + (v'')^2/2, \quad (111)$$

$$v''_0 = v'' + rT/v'', \quad (112)$$

$$M'' = v''/c, \quad (113)$$

$$A'' = (\rho v' A') / \rho v''. \quad (114)$$

Table 5 cascades the preceding logic to provide multi-stage blower and turbine analysis versatility, with primed transforms replaced by equivalent dynamic redefinition to minimize storage demand.

Table 5. Multi-Stage Blower or Turbine (Entries: Before, After)

Input: Entry state(1), whirl ωR_1 , tilt $\theta_1 = \cos^{-1} (\underline{\Omega}_1 \cdot \underline{u}_\phi)$.

Parasitic inflow heat/mass $\Delta_o q$, entropy/mass $\Delta_o s$.

Exit whirl ωR , tilt $\theta_2 = \cos^{-1} (\underline{\Omega}' \cdot \underline{u}_\phi)$.

Exit face area A, tilt $\theta_3 = \cos^{-1} (\underline{\Omega}' \cdot \underline{\Omega}_A)$.

Before: Do rotary transform.

$$A = A \underline{\Omega}' \cdot \underline{\Omega}_A,$$

$$h_o = h_{o1} - \omega R_1 v_{11} \underline{\Omega}_1 \cdot \underline{u}_\phi + \omega^2 R^2 / 2 + \Delta_o q,$$

$$s = s_1 + \Delta_o s.$$

[Stage: Do nozzle logic (Table 3).]

After: Undo rotary transform.

$$v = [\omega^2 R^2 + 2\omega R v \underline{\Omega}' \cdot \underline{u}_\phi + v^2]^{1/2},$$

$$h_o = h + v^2 / 2,$$

$$v_o = v + rT/v,$$

$$M = v/c,$$

$$A = (\rho v A) / \rho v.$$

Output: A, $\rho v A$, v_o , h_o , s , M , v , ρ , T , p , u , h , a , g , c_v , c_p , γ , c'_v , r , c''_v .

SPIRAL VORTEX

For shockless spiral vortex flow with uniform entry conditions, the mass, energy, angular momentum, and entropy balance take the integrated form

$$\rho v_R R = (\rho v_R R)_1, \quad (115)$$

$$h + (v_R^2 + v_\phi^2)/2 = h_{o1}, \quad (116)$$

$$R v_\phi = (R v_\phi)_1, \quad (117)$$

$$s = s_1, \quad (118)$$

where ρ is fluid density (mass/volume), v_R and v_ϕ are radial and azimuthal fluid velocity components, R is radial distance from the vortex axis, h is fluid enthalpy/mass, s is fluid entropy/mass, h_o is adiabatic stagnation enthalpy/mass, and subscript 1 refers to entry conditions.

Spiral Vortex Transform

Eliminating the azimuthal velocity v_ϕ by use of the angular momentum Equation (117),

$$v_\phi = (R v_\phi)_1 / R, \quad (119)$$

transforms these two-dimensional spiral vortex equations to precisely the same functional form as the one-dimensional nozzle Equations (73-75),

$$\rho v_R = (\rho v_R R)_1 / R, \quad (120)$$

$$h(T) + v_R^2/2 = h_{o1} - [(R v_\phi)_1 / R]^2/2, \quad (121)$$

$$s(\rho, T) = s_1. \quad (122)$$

Solving these three transcendental equations for their exit fields (ρ, T, v_R) by calling Subroutine Nozzle (Table 3) generates also the exit thermodynamic fields $(A, \rho v_R A, v_o', h_o', s, M_R, v_R, \rho, T, p, u, h, a, g, c_v, c_p, \gamma, c_v', r, c_v'')$, with

of course

$$v'_o = v_R + rT/v_R, \quad h'_o = h_o + v_R^2/2, \quad M_R = v_R/c.$$

Inverse Spiral Vortex Transform

Shifting the velocity-dependent exit fields back to the two-dimensional spiral vortex representation generates the computational sequence,

$$v = [v_R^2 + v_\phi^2]^{1/2}, \quad (123)$$

$$h_o = h + v^2/2, \quad (124)$$

$$v_o = v + rT/v, \quad (125)$$

$$M = v/c, \quad (126)$$

$$A = (\rho v_R A)/\rho v. \quad (127)$$

Table 6 cascades the preceding logic to provide multi-stage spiral vortex analysis versatility, with transform notation replaced by equivalent dynamic redefinition to minimize storage demand.

Table 6. Multi-Stage Spiral Vortex (Entries: Before, After)

Input: Entry state(1). Angle θ_1 from outward radius to entry flow.

Circumferential exit area $A_R = 2\pi R z$.

Before: Do spiral vortex transform.

$$h_o = h_o - [(A_1/A_R)v_1 \tan\theta_1]^2/2.$$

[Stage: Do nozzle logic (Table 3).]

After: Undo spiral vortex transform.

$$v = [v^2 + \{(A_1/A_R)v_1 \tan\theta_1\}^2]^{1/2},$$

$$h_o = h + v^2/2,$$

$$v_o = v + rT/v,$$

$$M = v/c,$$

$$A = (\rho v A)/\rho v,$$

$$\theta = \tan^{-1}[(\rho/\rho_1) \tan\theta_1].$$

Output: $A, \rho v A, v_o, h_o, s, M, v, \rho T, p, u, h, a, g, c_v, c_p, \gamma, c, c'_v, r, c''_v$.

IMPLICIT THERMODYNAMIC FUNCTIONS

The complicated temperature dependence of the equations of state, as displayed in Table 1, necessitates the use of temperature T as one of the four independent thermodynamic fields (ρ, T, v, A) throughout the heart of the analysis logic. Choice of the other three as density ρ , velocity v , and streamtube transverse area A stems basically from the structure of the mass, energy, and momentum balance Equations (1-3).

Adequate input-output versatility, however, demands canonical transform freedom in choice of the four independent thermodynamic fields for the input-output interface logic. Inverting the equations of state and the balance equations to generate temperature T as an implicit function of other thermodynamic fields proves to be the only difficult aspect of such field transformation.

The family of four implicit transcendental temperature functions shown in Tables (7,8,9,10) suffice to meet all field transformation flexibility needs thus far encountered or envisioned. The analysis method here involves inverting intricate implicit functions of temperature by iterating suitable second order expansion logic to solid convergence. The explicit functions needed here are generated by calling Subroutine Phase shown in Table 1, which returns also the full thermostatic field ($r, p, u, a, h, g, s, c_v, c_p, \gamma, c, c'_v, c''_v$) for the converged state.

Table 7. Subroutine Stag: Isentropic Stagnation Fields (h_o, s)

Input: Adiabatic stagnation enthalpy/mass h_o , entropy/mass s .

Optional initial estimate of adiabatic stagnation temperature T_o .

Logic: Solve $h(T_o) = h_o$ for T_o by second order iteration.

Then $\rho_o = \exp[\{s(l, T_o) - s\}/r]$.

Option: If $T_o = 0$, set $T_o = h_o/c_p(T)$ as initial (low) estimate.

Step 1: $\delta h = h(T_o) - h_o$,

$$(\delta h)' = d(\delta h)/dT_o = c_p(T_o),$$

$$(\delta h)'' = d(\delta h)'/dT_o = c_p'(T_o).$$

Solve $\delta h + (\delta h)'(\delta T_o) + (\delta h)''(\delta T_o)^2/2 = 0$ for δT_o .

$$\delta T_o = -(\delta h)' / (\delta h)'' \pm [\{ (\delta h)' / (\delta h)'' \}^2 - 2(\delta h) / (\delta h)'']^{1/2}.$$

Use $\pm$ sign giving least $|\delta T_o|$, since $h(T_o)$ is monotonic.

$$T_o = T_o + \delta T_o.$$

Return to Step 1 until converged ($\delta T_o = 0$, $\delta h = 0$).

$$\rho_o = \exp[\{s(l, T_o) - s\}/r].$$

Output: $h_o, s, T_o, p_o, \rho_o, u_o, a_o, g_o, c_{Vo}, c_{po}, \gamma_o, c_o, c_{Vo}', r, c_{Vo}''$.

Table 8. Subroutine Tps: Temperature T(Pressure p, Entropy/Mass s)

Input: Pressure p, entropy/mass s. Initial estimate for temperature T.

Logic: Solve $s(l,T) - r \ln(p/rT) = s$ for T by second order iteration.

Step 1: $\rho = p/rT$,

$$\delta s = s(\rho, T) - s,$$

$$(\delta s)' = Td(\delta s)/dT = c_p(T),$$

$$(\delta s)'' = Td(\delta s)'/dT = Tc'_V(T).$$

Solve $\delta s + (\delta s)'(\delta \ln T) + (\delta s)''(\delta \ln T)^2/2 = 0$ for $\delta \ln T$.

$$\delta \ln T = -(\delta s)' / (\delta s)'' \pm [\{ (\delta s)' / (\delta s)'' \}^2 - 2(\delta s) / (\delta s)'']^{1/2}.$$

Use $\pm$ sign giving least $|\delta \ln T|$, to cling to entry branch sheet.

$$T = T \cdot \exp(\delta \ln T).$$

Return to Step 1 until converged ($\delta T = 0$, $\delta s = 0$).

Output: $s, \rho, T, p, u, h, a, g, c_V, c_p, \gamma, c, c'_V, r, c''_V$.

Table 9: Subroutine Tmh_o: Temperature T(Mach Ratio M, Head h_o)

Input: Mach ratio M, adiabatic stagnation enthalpy/mass head h_o.

Initial estimate for temperature T.

Logic: Solve $h(T) + M^2 c^2(T)/2 = h_o$ for T by second order iteration.

Step 1: $v(T) = M \cdot c(T)$,

$$\delta h = h(T) + v^2(T)/2 - h_o,$$

$$(\delta h)' = d(\delta h)/dT = c_p(T) + (rM^2/2)[\gamma(T) - rTc'_V(T)/c_V^2(T)],$$

$$(\delta h)'' = d(\delta h)'/dT = c'_V(T) + [rM/c_V(T)]^2[-c'_V(T) + T\{\alpha'_V(T)\}^2/c_V(T) - Tc''_V(T)/2].$$

$$\text{Solve } \delta h + (\delta h)'(\delta T) + (\delta h)''(\delta T)^2/2 = 0 \text{ for } \delta T.$$

$$\delta T = -(\delta h)' / (\delta h)'' \pm [\{ (\delta h)' / (\delta h)'' \}^2 - 2(\delta h) / (\delta h)'']^{1/2}.$$

Use $\pm$ sign giving least $|\delta T|$, to cling to entry branch sheet.

$$T = T + \delta T.$$

Return to Step 1 until converged ($\delta T = 0$, $\delta h = 0$).

$$v_o = v + rT/v.$$

Output: $v_o, h_o, M, v, T, u, h, c_V, c_p, \gamma, c, c'_V, r, c''_V$.

Table 10. Subroutine TMv: Temperature T(Mach Ratio M, Impulse/Mass v_o)

Input: Mach ratio M, impulse/mass v_o . Initial estimate of temperature T.

Logic: Solve $Mc(T) + rT/Mc(T) = v_o$ for T by second order iteration.

Step 1: $v(T) = M \cdot c(T)$,

$$\delta q = [v(T) + rT/v(T) - v_o][v(T) + rT/v(T) + v_o],$$

$$\gamma'(T) = d\gamma(T)/dT = -rc'_V(T)/c_V^2(T),$$

$$(\delta q)' = d(\delta q)/dT = r[M^2\gamma(T) + 2 + 1/M^2\gamma(T) \\ + \{M^2 - 1/M^2\gamma^2(T)\}T\gamma'(T)],$$

$$\gamma''(T) = d\gamma'(T)/dT = 2r[c'_V(T)]^2/c_V^3(T) - rc''_V(T)/c_V^2(T),$$

$$(\delta q)'' = d(\delta q)'/dT = r[\{M^2 - 1/M^2\gamma^2(T)\}\{2\gamma'(T) + T\gamma''(T)\} \\ + 2\{\gamma'(T)/M\gamma(T)\}^2T/\gamma(T)].$$

Solve $\delta q + (\delta q)'(\delta T) + (\delta q)''(\delta T)^2/2 = 0$ for δT .

$$\delta T = -(\delta q)' / (\delta q)'' \pm [\{(\delta q)' / (\delta q)''\}^2 - 2(\delta q) / (\delta q)'']^{1/2}.$$

Use $\pm$ sign giving least $|\delta T|$, to cling to entry branch sheet.

$$T = T + \delta T.$$

Return to Step 1 until converged ($\delta T = 0$, $\delta q = 0$).

$$h_o = h + v^2/2,$$

Output: $v_o, h_o, M, v, T, u, h, c_V, c_p, \gamma, c, c'_V, r, c''_V$.

BYPASS FLOW

Bypass flow can improve thermodynamic performance by broadening the accessible domain of operating conditions. Bypass design involves matching the bypass exit pressure to the mainstream exit pressure, both to inhibit circulatory flow and to provide a pressure-matched shockless remix. Conservation of mass, energy, and momentum suffice to uniquely determine the remix logic. The entropy of mixing is obtainable, if needed, simply by direct differencing of final and initial states.

Table 11 translates these thoughts into machine language, for isentropic bypass flow. The bypass logic resides in Subroutine Engine, as an "empty engine".

Table 11: Isentropic Bypass Flow With Isopiestic Remix (Subroutine Engine)

Input: Mainstream mass flow rate $\rho v A$ leaving bypass.

Logic: Do pressure-matched shockless remix, conserving mass, energy, and momentum. Prime (') is bypass flow, subscript (1) is entry state.

At Leak: Mainstream: $A = A_1 (\rho v A) / (\rho v A)_1$.

Bypass: $(\rho v A)' = (\rho v A)_1 - (\rho v A)$, $h'_o = h_{o1}$, $s' = s_1$.

At Join: Bypass: $T' = T(p_1, s')$ by calling Subroutine Tps (Table 8),

$$\rho' = p_1 / r' T', \quad v' = [2\{h'_o - h(T')\}]^{1/2},$$

$$A' = (\rho v A)' / \rho' v', \quad v'_o = v' + r' T' / v'.$$

Mainstream: $A = A_1 + A'$, $\rho v A = (\rho v A)_1 + (\rho v A)'$,

$$v_o = [(\rho v A)_1 v_{o1} + (\rho v A)' v'_o] / \rho v A,$$

$$h_o = [(\rho v A)_1 h_{o1} + (\rho v A)' h'_o] / \rho v A.$$

Output: Mainstream $(A, \rho v A, v_o, h_o)$ input to Subroutine Pipe (Table 4) at Leak and at Join.

VARIATIONAL LOGIC

Coupled variation of design specifications to meet performance requirements involves, in general, varying a subset $\underline{x}$ of the design parameters to obtain a subset $\underline{y}(\underline{x})$ of performance predictions which meet a subset of performance requirements $\underline{z}(\underline{y}) = 0$. Mathematically, this means inverting the cascaded vector function equation,

$$\underline{z}(\underline{y}(\underline{x})) = 0, \quad (128)$$

to obtain $\underline{x}$. In the context of machine analysis, $\underline{x}$ consists of some of the input data, $\underline{y}(\underline{x})$ consists of some of the output, and $\underline{z}(\underline{y})$ is a specified design evaluation function which measures the extent to which the design departs from performance requirements.

Consider looping the entire machine analysis, by varying $\underline{x}$ systematically in successive runs by small input-controlled increments $\underline{\delta}(\underline{x})$ to generate a matrix difference ratio representation of the evaluation/design sensitivity gradient $[\underline{\delta}(\underline{x})]^{-1} \cdot [\underline{\delta}(\underline{z})]$, and then expanding the design evaluation Equation (128) in the form

$$\underline{z}(\underline{y}(\underline{x})) + (\underline{dx}) \cdot [\underline{\delta}(\underline{x})]^{-1} \cdot [\underline{\delta}(\underline{z})] \doteq 0. \quad (129)$$

Solving this set of linear algebraic equations, by cascaded matrix inversion, for the design changes $\underline{dx}$ needed to meet performance requirements yields

$$\underline{dx} = - \underline{z}(\underline{y}(\underline{x})) \cdot [\underline{\delta}(\underline{z})]^{-1} \cdot [\underline{\delta}(\underline{x})]. \quad (130)$$

It is to be noted here that existence of the matrix inverse $[\underline{\delta}(\underline{z})]^{-1}$ implies that the design vector $\underline{x}$ and the evaluation vector $\underline{z}$ must have the same dimensionality and that the performance vector $\underline{y}$ must have dimensionality equal to or greater than that common to $\underline{x}$ and $\underline{z}$. Applying the design

refinement $\underline{dx}$ to the original design $\underline{x}$ now generates the recursive refinement algorithm of the variational logic,

$$\underline{x} = \underline{x} - \underline{z}(\underline{y}(\underline{x})) \cdot [\underline{\delta}(\underline{z})]^{-1} \cdot [\underline{\delta}(\underline{x})]. \quad (131)$$

Striving for unrestricted versatility in the variational logic has led to defining both the design vector $\underline{x}$ and the evaluation vector $\underline{z}(\underline{y}(\underline{x}))$ with an input subroutine having two entry points called Give and Take. Control of the variational logic resides in Subroutine Flow, which iteratively calls upon Give(Vary, Zero) for the design vector Vary = $\underline{x}$ and the corresponding evaluation vector Zero = $\underline{z}(\underline{y}(\underline{x}))$, and subsequently feeds back to Take(Vary) the design vector to be stored as $\underline{x} = \underline{\text{Vary}}$ for the next iteration.

Successive inner variations (labeled Look in the output) sequentially vary each component of the design vector $\underline{x}$ by small input-controlled increments $\underline{\delta}(\underline{x})$ to generate the local evaluation/design gradient $[\underline{\delta}(\underline{x})]^{-1} \cdot [\underline{\delta}(\underline{z})]$. Successive outer variations (labeled Leap in the output) then extrapolate the design vector $\underline{x}$ to meet performance requirements, in accordance with Equation (131).

The variational iteration terminates as converged when every component of the design evaluation vector meets the performance requirements, Equation (128), to within an error tolerance [Err(1)] specified in the input. The variational iteration terminates as divergent if the component of the design evaluation vector largest in magnitude fails to show monotonic reduction, if the evaluation matrix $\underline{\delta}(\underline{z})$ is non-invertible, if spill or division by zero occurs, if the flow chokes or is physically unreal, or if the outer iteration limit [Max(1)] specified in the input is reached.

INPUT-OUTPUT LOGIC

Dynamic accrual of day-to-day input in two completely flexible internal data files minimizes both input error and input volume by complete elimination of repetitive bulk input. Bulk data seldom revised, such as basic thermostatic data for each molecular species, reside in a compiled Block Data element called Preset. Bulk data frequently revised, such as perhaps complete input for a dozen reference cases about which to center a design study, reside uncompiled in an Update element called Data. Either of these two internal data files may be overloaded freely with alternate versions developed for specific applications. Input for today's run thus typically may be limited to changes since yesterday.

To provide adequate input versatility and to further minimize input labor, both canonical transform freedom and numeric cross reference freedom are provided at the input interface [by sequential traverse of Subroutines (Theory, Engine, Imply, Before, After)]. It becomes crucially important, therefore, to grasp that intrinsic input for each streamtube segment consists, implicitly or explicitly, of relevant gas mixture and molecule data [mass m , fraction N_m/N , degeneracy w_m , rotational freedom R_m , characteristic temperatures (T_m, T_{mn}) of Table 1], and of either shock intensity and shock tilt data [$M_N, \cos\theta_N$ of Table 2], or else transverse area A and mass-energy-momentum-entropy source-sinks [Q_1, Q_2, Q_3, Q_4] for any three of the four balance Equations (1-4).

The input interface logic may be expanded freely by flexible overload of Subroutine Theory, which is preset to do nothing. Old reliable thermodynamic machines, guaranteed to run well, reside in Subroutine Engine. New machines normally are developed in Subroutine Theory and then placed in stock in Subroutine Engine as their logic matures.

Dynamic cross reference freedom is provided by Subroutine *ImPLY*, which replaces any negative integer entry $-I$ within the positive definite input set $[M_N, A, \rho vA, v_o, h_o, s, T_o, p_o, M]$ with a duplicate of the I -th preceding analogous entry. Analysis versatility is enhanced beyond belief by this simple artifice of admitting symbolic input use of key numbers unknown at input time but destined to have been computed before their entry into the analysis logic materializes. Upon request ($M_N = 0$), Subroutine *ImPLY* also generates the static normal shock entry Mach ratio $M_N = M_1 \cos \theta_N$ by use of the Mach ratio M_1 exiting from the preceding streamtube segment. For a shockless streamtube segment ($M_N = 1$), finally, Subroutine *ImPLY* then applies the canonical transformation logic needed to admit implicit area and source-sink input loaded as any four physically determinant non-zero entries in the eight-fold set $[A, \rho vA, v_o, h_o, s, T_o, p_o, M]$, consisting of streamtube transverse area A , mass flow rate ρvA , impulse/mass v_o , adiabatic stagnation enthalpy/mass head h_o , entropy/mass s , adiabatic stagnation temperature T_o , isentropic stagnation pressure p_o , and Mach ratio M .

Two powerful final stages of unrestricted canonical transformation freedom are provided by traversing Subroutines *Before* and *After*, respectively, just before and just after each streamtube segment integration. Subroutines *Before* and *After* contain the rotary and spiral transforms shown in Tables (5-6), and either subroutine may be further expanded by input overload as the need arises.

Coupled variation of design specifications to meet performance requirements is controlled by flexible overload of Subroutine (*Give*, *Take*) as described in the preceding Variational Logic section.

Line printer output includes all input, iterative design variations (if any), and thermodynamically complete portrayals of final design and performance. Teletype output provides a high-information-density summary of design variations and final design performance. Plotter output provides multi-dimensional phase-space overlay cross plots of all thermodynamic fields, displayed in any coordinate system and with complete canonical transformation freedom by appropriate overload of Subroutine Map, and fully labeled ready for publication by appropriate overload of Subroutine Artist. Subroutine Map is preset as an identity transform to produce linearly scaled Cartesian plots, and Subroutine Artist is preset to generate tics and legends for Cartesian axes.

Exhaustive internal error detection logic may intersperse up to an unlucky thirteen lines of relevant diagnostic information in both line printer and teletype output. The occurrence of any detectable failure (spill, division by zero, non-invertible matrix, flow choke, physically unreal field) promptly terminates the run, with dual output on both line printer and teletype, tabulated first for the design variation which collapsed in chaos, and then for the best available design generated prior to the first sign of trouble. Plotter output is limited to the best available design.

INPUT MNEMONICS

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C REAL GAS FLOW WITH MASS, MOMENTUM, ENERGY, AND ENTROPY TRANSFER
C BH DUANE (11DEC68) BATTLE-NORTHWEST PROGRAM GAS-R1 (21DEC69) BNW/TNP

C THEORY AH SHAPIRO COMPRESSIBLE FLUID FLOW I-II (RONALD PRESS 1954)
C THEORY JE MAYER MG MAYER STATISTICAL MECHANICS (JOHN WILEY 1940)

C TELETYPE INPUT LOGIC TYPIFIED BY UNIVAC-1108 EXEC-II INTERNAL TEST RUN
C 1 2 3 4 5 6 7
C 12345678901234567890123456789012345678901234567890123456789012345678

C @ TEL TNP JOB1W1234 TAPES U3463, PLOT GAS 0
C @ RUN 1W1234, CUS123, 05, 100 BH DUANE (946-2430) BNW/R0/1533.. GAS 1
C @ PRL TEL CSC GAS 1
C @X ASG A=U3463, H=(, NH, AT) GAS 2
C @N XQT CUR GAS 3
C IN A GAS 4
C TRI A GAS 5
C @ DAT DATA, 0 GAS 6
C @N XQT GAS GAS 7
C $GAS TYPE=T, NEXT=0, S GAS 8
C @ MSG CALCOMP H=PLOT, PAPER 401, BLACK BALL, MULTIPLE PLOT. GAS 9
C @ FIN GAS10

C SPILL, DIVISION BY ZERO, OR IMAGINARY ABORTS RUN WITH DIAGNOSIS
C $GAS NAMELIST INPUT DEFINITION MODE

C MODES= MODAL TEMPERATURES PER MOLECULE (MIN 1, MAX 50) (PRESET 4) INTEGER
C MOL= NUMBER OF MOLECULAR SPECIES (MIN 1, MAX 50) (PRESET 6) INTEGER
C MIXES= NUMBER OF GAS MIXTURES (MIN 1, MAX 50) (PRESET 1) INTEGER
C NAME(MOL), MOL=1, MOLS NAME(MOLECULE) PRESET HE H-O-H N-N O-O O-C-O AR 6H/MOL
C A(MOL), MOL=1, MOLS MASS(MOLECULE)/(MASS(016)/16)=MOLECULE ATOMIC MASS REAL
C (FN(MOL, MIX), MOL=1, MOLS), MIX=1, MIXES MOLECULE FRACTIONS IN GAS MIX REAL
C RG= (BOLTZMANN CONSTANT)/(MASS(016)/16) PRESET 49734.8079 (F/S)**2/R REAL
C W(MOL), MOL=1, MOLS MOLECULE GROUND STATE DEGENERACY (GREATER THAN 0.) REAL
C ROT(MOL), MOL=1, MOLS MOLECULAR ROTATION DEGREES OF EXCITATION FREEDOM REAL
C (MONATOMIC 0., DIATOMIC OR LINEAR 2., NONLINEAR POLYATOMIC 3.)
C SC=5/2+LN(AMU*(2*PI*AMU*K/H**2)**3/2) SACKUR-TETRODE ENTROPY LIMIT REAL
C (PRESET TO -5.79205676 FOR USE WITH FOOT-SLUG-SECOND-RANKINE UNITS)
C (TC(MODE, MOL), MODE=1, MODES), MOL=1, MOLS CHARACTERISTIC TEMPERATURES REAL
C (ROTATION MODE FIRST, THEN VIBRATION MODES, THEN ZERO)

C MESH= NUMBER OF FLOW STATES (MINIMUM 1, MAXIMUM 500) (PRESET 1) INTEGER
C MIX(N), N=1, MESH GAS MIXTURES INTEGER

C (STATE(N, M), N=1, 10), M=1, MESH FLOW STATES (IN SELF-CONSISTENT UNITS) REAL

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C      -1. FOR POSITIVE-DEFINITE STATE MEANS USE I-TH ANCESTOR (MOD MESH)
C      0. FOR POSITIVE-DEFINITE STATE MEANS DEDUCE STATE FROM GAS THEORY

C DYNAMIC-OR-STATIC OBLIQUE-OR-NORMAL SHOCK GENERATES STATE(N,M), N=3,10
C STATE(1,M)=(VIN-VSN)/C1=(NORMAL SHOCK-ENTRY SPEED)/(ENTRY SONIC SPEED) REAL
C (1. BYPASS, 0. SETS STATE(1,M)=STATE(10,M-1)*STATE(2,M) STATIC SHOCK)
C STATE(2,M)=COS(VIN-VSN,V1)=COSINE(TILT(SHOCK NORMAL, ENTRY VELOCITY)) REAL
C STATE(3,M)=A AREA TRANSVERSE TO FLOW STREAMLINES EXTENSIVE REAL
C STATE(4,M)=RHO*V*A MASS FLOW RATE (DENSITY*VELOCITY*AREA) EXTENSIVE REAL
C STATE(5,M)=V+P/(RHO*V) ((MOMENTUM FLUX)+PRESSURE)/(MASS FLUX) REAL
C STATE(6,M)=H0 ADIABATIC-STAGNATION ABSOLUTE ENTHALPY/MASS REAL
C STATE(7,M)=S FLUID ABSOLUTE ENTROPY/MASS REAL
C STATE(8,M)=T0 ADIABATIC-STAGNATION ABSOLUTE TEMPERATURE REAL

C STATE(9,M)=P0 ISENTROPIC-STAGNATION ABSOLUTE PRESSURE REAL
C STATE(10,M)=M=V/C MACH RATIO (FLOW VELOCITY)/(SONIC SPEED) REAL

C VCUT= ENTRY FLOW VELOCITY ESTIMATE WITHIN PHYSICAL BRANCH SHEET GT.0 REAL
C TCUT= ENTRY FLUID TEMPERATURE ESTIMATE IN PHYSICAL BRANCH SHEET GT.0 REAL

C SUBROUTINE THEORY MAKES SPECTACULAR NEW MACHINES (CAVEAT EMPTOR) FORTRAN
C SUBROUTINE ENGINE MAKES OLD RELIABLE MACHINES (GUARANTEED TO RUN) FORTRAN
C SUBROUTINE IMPLY CONVERTS IMPLICIT INPUT TO EXPLICIT INPUT FORTRAN
C SUBROUTINE BEFORE TRANSFORMS MESH INPUT JUST BEFORE EACH MESH STEP FORTRAN
C ENTRY AFTER TRANSFORMS MESH OUTPUT JUST AFTER EACH MESH STEP FORTRAN

C ISENTROPIC BYPASS FLOW WITH PRESSURE-MATCHED SHOCKLESS REMIX (ENGINE) OPTION
C LEAK= ISENTROPIC-BYPASS MESH POINT (PRESET 0) INTEGER
C JOIN= BYPASS ISOPIESTIC-REMX MESH POINT (PRESET 0) INTEGER
C AT LEAK, LOAD MAINSTREAM (RHO*V*A,V+P/(RHO*V),H0), LOAD AREA A=0
C AT JOIN, LOAD (A,RHO*V*A,V+P/(RHO*V),H0)=0

C MULTI-STAGE BLOWER OR TURBINE ROTARY TRANSFORM LOGIC (BEFORE, AFTER) OPTION
C IMPEL(J),J=1,3 ROTORS DO MESH M=IMPEL(1),IMPEL(2),IMPEL(3) PRESET 0 INTEGER
C TILT(1)= ANGLE FROM STATIC FRAME ENTRY FLOW TO ROTOR WHIRL (PRESET 0) REAL
C TILT(2)= ANGLE FROM EXIT ROTOR STREAMTUBE TO ROTOR WHIRL (PRESET 0) REAL
C TILT(3)= ANGLE FROM ROTOR EXIT PORT NORMAL TO ROTOR STREAMTUBE (IS 0) REAL
C WHIRL(J),J=1,2 ENTRY AND EXIT ROTOR WHIRL VELOCITIES (PRESET 0) REAL
C LOAD ROTOR EXIT PORT AREA IN A, ROTOR EXIT MASS FLOW RATE IN RHO*V*A
C LOAD ROTOR PARASITIC INFLOW (HEAT/MASS DO IN H0, ENTROPY/MASS DS IN S)

C MULTI-STAGE SPIRAL VORTEX TRANSFORM LOGIC (BEFORE, AFTER) OPTION
C LOXOD(J),J=1,3 SPIRALS DO MESH M=LOXOD(1),LOXOD(2),LOXOD(3) (IS 0) INTEGER
C RHUMB(1)= ANGLE FROM OUTWARD RADIUS TO ENTRY FLOW (0 IMPLY) PRESET 0 REAL
C LOAD CIRCUMFERENTIAL EXIT AREA IN A, LOAD OR IMPLY EXIT (RHO*V*A,H0,S)

C VARIATION OF VECTOR VARY(L),L=1,LOTS NULLS VECTOR ZERO(L),L=1,LOTS
C SUBROUTINE GIVE(VARY,ZERO) RETURNS (VARY(L),ZERO(L)),L=1,LOTS FORTRAN

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```

C ENTRY TAKE(VARY) STORES VARY(L),L=1,LOTS FORTRAN
C ERR(1)= VARIATION ERROR LIMIT=AMAX1(ABS(ZERO(L)),L=1,LOTS) IS 2**-18 REAL
C ERR(2)= IMAGINARY FRACTIONAL ERROR TOLERANCE (PRESET 2**-21) REAL
C ERR(3)= TRANSCENDENTAL STATE FRACTIONAL ERROR LIMIT (PRESET 2**-24) REAL
C MAX(1)= VARIATION ITERATION LIMIT (MAXIMUM 32766) (PRESET 6) INTEGER
C MAX(2)= TRANSCENDENTAL STATE ITERATION LIMIT (PRESET 64) INTEGER
C LOTS= VARIATIONAL DEGREES OF FREEDOM (MAXIMUM 50) (PRESET 0) INTEGER
C HOP(L),L=1,LOTS VARIATIONAL-GRADIENT VARY-SHIFT FRACTION (IS 2**-14) REAL
C MAPS= NUMBER OF PHASE-SPACE CROSS PLOTS (MAXIMUM 100) (PRESET 0) INTEGER
C MARK= FIRST PLOT DIAL MARK (MULTI-NEEDLE ANGLE/(PI/4)) (PRESET 1) INTEGER
C ((PLOT(K,L,M),K=1,5),L=1,X,L=2-Y),M=1,MAPS PHASE-SPACE CROSS PLOTS REAL
C PLOT(1,L,M) PLOT(2,L,M) PLOT(3,L,M) PLOT(4,L,M) PLOT(5,L,M)
C END DATA DATA END DATA ITEM END PLOT (INCHES) PLOT END
C ITEM 00 M MESH INDEX
C ITEM 01 (V1N-VSN)/C1=(NORMAL SHOCK-ENTRY SPEED)/(ENTRY SONIC SPEED)
C ITEM 02 COS(V1N-VSN,V1)=COSINE(ANGLE(SHOCK NORMAL, ENTRY VELOCITY))
C ITEM 03 A AREA TRANSVERSE TO FLOW STREAMLINES
C ITEM 04 RHO*V*A MASS FLOW RATE
C ITEM 05 V*P/(RHO*V) ((MOMENTUM FLUX)+PRESSURE)/(MASS FLUX)
C ITEM 06 H0 ADIABATIC-STAGNATION ABSOLUTE ENTHALPY/MASS
C ITEM 07 S FLUID ABSOLUTE ENTROPY/MASS
C ITEM 08 T0 ADIABATIC-STAGNATION ABSOLUTE TEMPERATURE
C ITEM 09 P0 ISENTROPIC-STAGNATION ABSOLUTE PRESSURE
C ITEM 10 M=V/C MACH RATIO (FLOW VELOCITY V)/(SONIC SPEED C)
C ITEM 11 V FLOW VELOCITY
C ITEM 12 RHO FLUID MASS DENSITY
C ITEM 13 T FLUID ABSOLUTE TEMPERATURE
C ITEM 14 P FLUID ABSOLUTE PRESSURE
C ITEM 15 U FLUID INTERNAL ENERGY/MASS
C ITEM 16 H=U+P/RHO FLUID ENTHALPY/MASS
C ITEM 17 A=U-T*S FLUID HELMHOLTZ POTENTIAL/MASS
C ITEM 18 G=H-T*S FLUID GIBBS POTENTIAL/MASS
C ITEM 19 CV(T)=QU(T)/DT SPECIFIC HEAT/MASS AT FIXED VOLUME
C ITEM 20 CP(T)=CV(T)+R=QU(T)/DT SPECIFIC HEAT/MASS AT FIXED PRESSURE
C ITEM 21 CP/CV SPECIFIC HEAT RATIO (FIXED PRESSURE/FIXED VOLUME)
C ITEM 22 C SONIC SPEED (C=C*R*T*CP/CV=DP/DRHO AT FIXED S)
C ITEM 23 DCV/DT RATE OF CHANGE OF CV WITH RESPECT TO TEMPERATURE
C ITEM 24 R=P/(RHO*T)=CP-CV/R*(BOLTZMANN CONSTANT)/(MOLECULAR MASS)
C ITEM 25 DDCV/DTDT SECOND TEMPERATURE DERIVATIVE OF CV
C SUBROUTINE ARTIST DRAWS SCALES AND LABELS PLOTS READY FOR PUBLICATION FORTRAN

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C          PLOT DATA TRANSFORM FLEXIBLY VIA SUBROUTINE MAP(Z)          FORTRAN
C          THEN TRANSLATE AND SCALE LINEARLY TO MATCH ENDS
C          OFF-SCALE POINTS PLOT PROJECTED TO NEAREST MARGIN
C          PLUS X-ITEM MEANS DO END-OF-CURVE LOGIC, AFTER PLOT (MARK=MARK+1)
C          MINUS X-ITEM MEANS BYPASS END-OF-CURVE LOGIC, AFTER PLOT
C          PLUS Y-ITEM MEANS DO END-OF-PAGE LOGIC, AFTER PLOT (MOVE PAPER)
C          MINUS Y-ITEM MEANS BYPASS END-OF-PAGE LOGIC, AFTER PLOT
C          PLUS MAPS MEANS DO END-OF-FILE LOGIC, AFTER PLOTS (TEF TRI TAPE H)
C          MINUS MAPS MEANS BYPASS END-OF-FILE LOGIC, AFTER PLOTS
C
C TYPE= TELETYPE=PUNCH (USE PRL CARD, TYPE PUNCH FILE) (PRESLT .TRUE.) LOGICAL
C NEXT= REFERENCE CASE NUMBER IN PROGRAM-COMPLEX-FILE DATA (PRESET 1) INTEGER
C          PROGRAM-COMPLEX-FILE DATA ELEMENT PRESET TO TYPICAL TEST CASE
C          NEXT=0 BYPASSES LOADING REFERENCE DATA FROM PROGRAM COMPLEX FILE
C          -NEXT BYPASSES EXECUTION, THEN LOADS +NEXT CASE
C
C          SOURCE LANGUAGE IS DIALECTAL INTERSECTION OF UNIVAC-1108 FORTRAN-V
C          -CSC AND IBM-7090 FORTRAN-IV-13. OPTIONS ARE FLAGGED UNIVAC/IBM.

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SAMPLE RUN

Formulation of BNW Program Gas was motivated by feasibility studies on supersonic pressurized incandescent gas flow loops, at the general level of complexity illustrated in Figure 1. The design concept here involves rearranging the hardware of an existing subsonic pressurized hot gas loop to provide supersonic flow in the test section. For initial trial runs, the working fluid might be dry air ($.7809 \text{ N}_2 + .2095 \text{ O}_2 + .0093 \text{ Ar} + .0003 \text{ CO}_2$).

With reference to the analysis mesh sketched in Figure 1, low temperature gas is pressurized by paralleled axial-inflow radial-outflow turbopump rotors (01-02) having vaneless spiral vortex radial-outflow diffusers (02-03) followed by circumferential spiral collectors (03-04). After partial flow bypass (04-05) for subsequent cooling, the mainstream is raised to incandescent temperature by an electric heater (06-07). The gas flow then accelerates through the sonic throat (08) of a Laval nozzle (07-09) to feed the supersonic test section (09-10).

A triple-shock supersonic diffuser (10-13) following the test section decelerates the flow to subsonic with minimal entropy rise. The subsonic mixer (13-14) cools the hot gas with bypass flow. The cooler (15-16) then brings the gas back to its initial entry state to close the loop. The adapters (05-06, 14-15, 16-01) are tapered pipes needed to couple existing hardware.

The design variation logic involves: (1) varying the turbopump rotor angular velocity to attain the desired isentropic stagnation pressure at the test section entry; (2-3) varying the two oblique shock angles within the supersonic diffuser to equalize entropy rises (or equivalently,

FIGURE 1: SUPERSONIC PRESSURIZED INCANDESCENT GAS TEST LOOP

isentropic stagnation pressure ratios) across all three shocks: (4) varying the bypass mass flow fraction to match the cooler entry adiabatic stagnation temperature to its design limit; and (5) varying the turbo-pump entry isentropic stagnation pressure to obtain loop-closure match of entry and exit isentropic stagnation pressures. Loop closure quite strongly couples all these design variations.

The sections which follow illustrate both input language and analysis capabilities, by displaying in sequence: the brief teletype input controlling the run; the line printer output listing of both the relevant internal version of the design variation logic [Subroutine (Give, Take)] and the relevant internal input data file [Element Data]; the line printer output tabulation of all input, all design variations, and complete thermodynamic performance for the final design; and plotter output revealing the lobular temperature-entropy cycle characteristic of a subsonic-supersonic closed loop.

Teletype Input

© TEL TNP JOB1W1209 TAPES U3463,PLOT

© RUN 1W1209,CUS189,05,100

ROB-2..BH DUANE(946-2430)R0/1533

©PRL TEL CSC

©X ASG A=U3463,H=(,NH,AT)

©N XQT CUR

IN A

TRI A

©IT FR5,* GIVE/TEST,GIVE/TEST

©ITL UPD,* DATA/TEST,DATA/TEST

© DAT DATA/TEST,0

©N XQT GAS

\$GAS TYPE=T, NEXT=0, \$

© MSG CALCOMP H=PLOT (THEN SCRATCH). PAPER 400. BLACK BALL.

© MSG ORIGIN HALF INCH FROM RIGHT LIMIT. MULTIPLE PLOTS.

© MSG BH DUANE(946-2430)ROB-2 BNW/R0/1533 CUS189 JOB-W1209.

© FIN

ATR TEST

Internal Input Variational Logic: Subroutine (Give, Take)

```

C      VARIATION OF VECTOR VARY(L),L=1,LOTS NULLS VECTOR ZERO(L),L=1,LOTS
C      SUBROUTINE GIVE(VARY,ZERO)      RETURNS (VARY(L),ZERO(L)),L=1,LOTS
C      SUBROUTINE GIVE(VARY,ZERO)
COMMON /INOUT/ HOP(50),IT(2),LOTS,M,MAPS,MARK,MESH,MIX(500),MIXES
1,NEXT,PLOT(5,2,100),STATES(10,500),STATE(25,500),TCUT,TYPE,VCUT
COMMON /GASES/ A(50),FN(50,50),MODES,MOLS,NAME(50),R0,ROT(50),S0
1,TC(50,50),W(50)
COMMON/SET/IMPEL(3),TILT(3),WHIRL(2), LOXOD(3),RHUMB(2), LEAK,JOIN
DIMENSION VARY(50),ZERO(50)
DATA PTEST/38029.9380/,TCOOL/2084.688/

P0 T0
1YESN02
ROTOR
P0/P0-1
COS(VS1)
D(P0/P0)
COS(VS2)
D(P0/P0)
RHOVA
T0/T0-1
ENTRY P0
(DP0)/P0

IMAGIN=2
VARY(1)=WHIRL(2)
ZERO(1)=STATE(9,9)/PTEST-1.
VARY(2)=STATE(2,11)
ZERO(2)=STATE(9,11)/STATE(9,12)-STATE(9,12)/STATE(9,11)
VARY(3)=STATE(2,12)
ZERO(3)=STATE(9,12)/STATE(9,11)-STATE(9,13)/STATE(9,12)
VARY(4)=STATE(4,LEAK)
ZERO(4)=STATE(8,JOIN)/TCOOL-1.
VARY(5)=STATE(9,1)
ZERO(5)=2.*(STATE(9,MESH)-STATE(9,1))/(STATE(9,MESH)+STATE(9,1))
GO TO 500

ENTRY TAKE(VARY)
ENTRY TAKE(VARY)
STORES VARY(L),L=1,LOTS

IMAGIN=2
WHIRL(2)=VARY(1)
STATE(2,11)=VARY(2)
STATE(2,12)=VARY(3)
STATE(4,LEAK)=VARY(4)
STATE(9,1)=VARY(5)
500 CONTINUE

1YESN02
ROTOR
COS(VS1)
COS(VS2)
RHOVA
ENTRY P0

C      IF SPILL, DIVISION BY ZERO, OR IMAGINARY, SAY WHERE, NOW WHAT
C      CALL ERROR($9999,IMAGIN,3,18H GIVE ABORT FLOW)
9999 RETURN
END

```


Internal Input Data: Element Data

```

$GAS LOTS=5, MESH=17, MAX(1)=10, MAPS=14, NEXT=1, TYPE=T, AIR LOOP
MODES=4, MOLS=6, FN(1,1)=0.,0.,.7809,.2095,.0003,.0093, DRY AIR
MIXES=1, MIX(1)=17*1, ATR
IMPEL=2,2,3, TILT=1.57079633,2.00712864,.436332313, WHIRL=0.,999.71710, GE BLOW
LOXOD=3,3,3, RHUMB=0.,0., GE SPIN
LEAK=5, JOIN=14, ATR BYP
VCUT=17.863099, TCUT=659.79736, V CUT T
STATE(01,01)= 1.0000000+00, 1.0000000+00, 5.62592780-1, M C A
1.72672206-1, 0.0000000+00, 0.0000000+00, R V H S
6.59824046+2, 1.9462192+04, 0.0000000+00, T P M
STATE(01,02)= 1.0000000+00, 1.0000000+00, 2.43473431-1, M C A
-1.0000000+00, 0.0000000+00, 0.0000000+00, R V H S
0.0000000+00, 0.0000000+00, 0.0000000+00, T P M
STATE(01,03)= 1.0000000+00, 1.0000000+00, 3.85479777-1, M C A
-1.0000000+00, 0.0000000+00, -1.0000000+00, R V H S
0.0000000+00, 0.0000000+00, 0.0000000+00, T P M
STATE(01,04)= 1.0000000+00, 1.0000000+00, -3.0000000+00, M C A
-1.0000000+00, 0.0000000+00, -1.0000000+00, R V H S
0.0000000+00, 0.0000000+00, 0.0000000+00, T P M
STATE(01,05)= 1.0000000+00, 1.0000000+00, 0.0000000+00, M C A
1.0250816-01, -1.0000000+00, -1.0000000+00, R V H S
0.0000000+00, 0.0000000+00, 0.0000000+00, T P M
STATE(01,06)= 1.0000000+00, 1.0000000+00, 1.40648195-1, M C A
-1.0000000+00, 0.0000000+00, -1.0000000+00, R V H S
0.0000000+00, 0.0000000+00, 0.0000000+00, T P M
STATE(01,07)= 1.0000000+00, 1.0000000+00, -1.0000000+00, M C A
-1.0000000+00, -1.0000000+00, 0.0000000+00, R V H S
2.45968800+3, 0.0000000+00, 0.0000000+00, T P M
STATE(01,08)= 1.0000000+00, 1.0000000+00, 0.0000000+00, M C A
-1.0000000+00, 0.0000000+00, -1.0000000+00, R V H S
0.0000000+00, 0.0000000+00, 1.0000000+00, T P M
STATE(01,09)= 1.0000000+00, 1.0000000+00, 0.0000000+00, M C A
-1.0000000+00, 0.0000000+00, -1.0000000+00, R V H S
0.0000000+00, 0.0000000+00, 3.7388100+0, T P M
STATE(01,10)= 1.0000000+00, 1.0000000+00, -1.0000000+00, M C A
-1.0000000+00, -1.0000000+00, 0.0000000+00, R V H S
0.0000000+00, 0.0000000+00, 0.0000000+00, T P M
STATE(01,11)= 0.0000000+00, 4.6482834-01, 0.0000000+00, M C A
0.0000000+00, 0.0000000+00, 0.0000000+00, R V H S
0.0000000+00, 0.0000000+00, 0.0000000+00, T P M
STATE(01,12)= 0.0000000+00, 6.4770688-01, 0.0000000+00, M C A
0.0000000+00, 0.0000000+00, 0.0000000+00, R V H S
STATE(01,13)= 0.0000000+00, 1.0000000+00, 0.0000000+00, T P M
STATE(01,13)= 0.0000000+00, 1.0000000+00, 0.0000000+00, M C A

```

0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	R V H S
0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	T P M C A
STATE(01,14)=	1.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	R V H S
0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	T P M C A
0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	R V H S
STATE(01,15)=	1.000000+00,	1.000000+00,	1.40648195-1,	1.40648195-1,	T P M C A
-1.000000+00,	0.000000+00,	-1.000000+00,	-1.000000+00,	-1.000000+00,	R V H S
0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	T P M C A
STATE(01,16)=	1.000000+00,	1.000000+00,	-1.000000+00,	-1.000000+00,	R V H S
-1.000000+00,	-1.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	T P M C A
-1.500000+01,	0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	R V H S
STATE(01,17)=	1.000000+00,	1.000000+00,	-1.600000+01,	-1.600000+01,	T P M C A
-1.000000+00,	0.000000+00,	-1.000000+00,	-1.000000+00,	-1.000000+00,	R V H S
0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	T P M C A
PLOT(1,1,1)=	0.000000+00,	0.000000+00,	0.000000+00,	0.000000+00,	PLOTS
41000., 51000.,	-7., 1., 11.,	0., 4000., +13., 1., 9.,	0., 4000., +13., 1., 9.,	0., 4000., +13., 1., 9.,	S T
41000., 51000.,	-7., 1., 11.,	0., 4000., +8., 1., 9.,	0., 4000., +8., 1., 9.,	0., 4000., +8., 1., 9.,	S T
19000., 39000.,	-9., 1., 11.,	0., 4000., +8., 1., 9.,	0., 4000., +8., 1., 9.,	0., 4000., +8., 1., 9.,	P0 T0
0., 4000.,	-13., 1., 9.,	0., 2., +21., 1., 9.,	0., 2., +21., 1., 9.,	0., 2., +21., 1., 9.,	T CP/CV
0., 18., 0., 3., 21.,	0., 3., 21.,	0., .8, -3., 1., 9.,	0., .8, -3., 1., 9.,	0., .8, -3., 1., 9.,	M A
0., 18., 0., 3., 21.,	0., 3., 21.,	0., .2, -4., 1., 9.,	0., .2, -4., 1., 9.,	0., .2, -4., 1., 9.,	M RHOVA
0., 18., 0., 3., 21.,	0., 3., 21.,	0., 16000., -5., 1., 9.,	0., 16000., -5., 1., 9.,	0., 16000., -5., 1., 9.,	M V+P/RV
0., 18., 0., 3., 21.,	0., 3., 21.,	0., 8000., -7., 1., 9.,	0., 8000., -7., 1., 9.,	0., 8000., -7., 1., 9.,	M S
0., 18., 0., 3., 21.,	0., 3., 21.,	0., 4000., -8., 1., 9.,	0., 4000., -8., 1., 9.,	0., 4000., -8., 1., 9.,	M T0
0., 18., 0., 3., 21.,	0., 3., 21.,	0., 4000., -9., 1., 9.,	0., 4000., -9., 1., 9.,	0., 4000., -9., 1., 9.,	M P0
0., 18., 0., 3., 21.,	0., 3., 21.,	0., 4., -10., 1., 9.,	0., 4., -10., 1., 9.,	0., 4., -10., 1., 9.,	M V/C
0., 18., 0., 3., 21.,	0., 3., 21.,	0., 8000., -11., 1., 9.,	0., 8000., -11., 1., 9.,	0., 8000., -11., 1., 9.,	M V
0., 18., 0., 3., 21.,	0., 3., 21.,	0., 4000., -13., 1., 9.,	0., 4000., -13., 1., 9.,	0., 4000., -13., 1., 9.,	M T
0., 18., 0., 3., 21.,	0., 3., 21.,	0., 4000., +14., 1., 9.,	0., 4000., +14., 1., 9.,	0., 4000., +14., 1., 9.,	M P
0., 18., 0., 3., 21.,	0., 3., 21.,				END CASE

Line Printer Output: Input, Variations, Final Design Performance

```

TIME 1.7773438-02M
REAL GAS FLOW WITH MASS, MOMENTUM, ENERGY, AND ENTROPY TRANSFER
(11DEC68) BATTELL-F-NORTHWEST PROGRAM GAS-R1 (21DEC69)
INPUT INPUT 1
MODES/MOLECULE MOLES/MIX GAS MIXTURES FLOW SEGMENTS VELOCITY/CUT T(TEMPERATURE)/CUT ERR(1) VARY ERR(2) UNREAL
MODES= 4 MOLES= 6 MIXES= 1 MESH= 17 1.7863099+01 6.5979736+02 3.8146973-06 4.7683716-07
ERR(3) STATE OUTER ITERATION LIMIT INNER VARY FREEDOM CROSS PLOTS PLOT SYMBOLS TELETYPE RUN NEXT INPUT
5.9604645-08 MAX(1)= 10 MAX(2)= 64 LOTS= 5 MAPS= 14 MARK= T TYPE= T NEXT= 0
GAS CONSTANT R0 ENTROPY S0
4.9734808+04 -5.7920568+00
A(MOL),MOL=1, 6 MASS(MOLECULE)/(MASS(016)/16)=MOLECULE ATOMIC MASS AR
H-O-H N-N O-C-O
4.0040901+00 1.8020906+01 2.8023629+01 3.2008713+01 4.4021984+01 3.9954877+01
W(MOL),MOL=1, 6 MOLECULE GROUND STATE DEGENERACY O-O O-C-O AR
H-O-H N-N O-O
1.0000000+00 2.0000000+00 4.5000000+00 1.5000000+00 5.0000000-01 1.0000000+00
ROT(MOL),MOL=1, 6 MOLECULAR ROTATION DEGREES OF EXCITATION FREEDOM O-C-O AR
H-O-H N-N O-O
0.0000000 3.0000000+00 2.0000000+00 2.0000000+00 2.0000000+00 0.0000000
(TC(MODE,MOL),MODE=1, 4),MOL=1, 6 CHARACTERISTIC TEMPERATURES (ROTATION MODE, THEN VIBRATION MODES, THEN 0)
HE 0.0000000 0.0000000 0.0000000 0.0000000
H-O-H 5.4975846+01 4.1308000+03 9.4711863+03 9.7787043+03
N-N 1.0411170+01 6.1110201+03 0.0000000 0.0000000
O-O 7.4880657+00 4.0928873+03 0.0000000 0.0000000
O-C-O 1.9713887+00 1.7282024+03 3.5954794+03 6.0843188+03
AR 0.0000000 0.0000000 0.0000000 0.0000000
(FN(MOL,MIX),MOL=1, 6),MIX=1, 1 MOLECULE FRACTIONS IN GAS MIXTURES
MIX MOL HE H-O-H N-N O-O AR
1 0.0000000 0.0000000 7.8089999-01 2.0950000-01 3.0000000-04 9.2999998-03
MIX(M),M=1, 17 GAS MIXTURES(MESH SEGMENTS)
1( 1) 1( 2) 1( 3) 1( 4) 1( 5) 1( 6) 1( 7) 1( 8) 1( 9) 1( 10) 1( 11) 1( 12) 1( 13) 1( 14) 1( 15)
1( 16) 1( 17)
MESH SHOCK MACH SHOCK TILT TRANSVERSE MASS FLOW IMPULSE/MASS ENERGY/MASS ENTROPY/MASS STAGNATION
M (VIN+VSN)/C1 COS(VIN-VSN,V1) AREA A R.TE RHO*V*V V+P/(RHO*V) HEAD H0=H+VV/2 ABSOLUTE S TEMPERATURE T0
1 1.0000000+00 1.0000000+00 5.6259277-01 1.7267220-01 0.0000000 0.0000000 0.0000000 0.0000000 6.5982404+02
2 1.0000000+00 1.0000000+00 2.4347343-01 -1.0000000+00 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
3 1.0000000+00 1.0000000+00 3.8547978-01 -1.0000000+00 0.0000000 -1.0000000+00 -1.0000000+00 0.0000000
4 1.0000000+00 1.0000000+00 -3.0000000+00 -1.0000000+00 0.0000000 -1.0000000+00 -1.0000000+00 0.0000000
5 1.0000000+00 1.0000000+00 0.0000000 1.0250816-01 -1.0000000+00 -1.0000000+00 0.0000000 0.0000000

```

TIME 2.2770182-02M 2

MESH	STAGNATION	MACH	RATIO M=V/C	ROTORS BEGIN	ROTORS END	IMPEL(2)= 2	IMPEL(3)= 3	WHIRL TILT(1)	WHIRL TILT(2)	PORT TILT(3)	ENTRY WHIRL(1)	EXIT WHIRL(2)
6	1.0000000+00	1.4	64819-01	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	0.0000000
7	1.0000000+00	-1.0	0000000+00	-1.0000000+00	-1.0000000+00	-1.0000000+00	-1.0000000+00	-1.0000000+00	-1.0000000+00	-1.0000000+00	0.0000000	2.4596880+03
8	1.0000000+00	1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	0.0000000
9	1.0000000+00	1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	0.0000000
10	1.0000000+00	1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	0.0000000
11	1.0000000+00	4.6482834-01	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
12	1.0000000+00	6.4770687-01	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
13	1.0000000+00	1.0000000+00	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
14	1.0000000+00	1.0000000+00	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
15	1.0000000+00	1.0000000+00	1.4064819-01	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	0.0000000
16	1.0000000+00	1.0000000+00	-1.0000000+00	-1.0000000+00	-1.0000000+00	-1.0000000+00	-1.0000000+00	-1.0000000+00	0.0000000	0.0000000	0.0000000	-1.5000000+01
17	1.0000000+00	1.0000000+00	-1.6000000+01	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	0.0000000	-1.0000000+00	-1.0000000+00	0.0000000

ROTORS BEGIN ROTORS END ROTORS JUMP WHIRL TILT(1) WHIRL TILT(2) PORT TILT(3) ENTRY WHIRL(1) EXIT WHIRL(2)

IMPEL(1)= 2 IMPEL(2)= 2 IMPEL(3)= 3 1.57 7963+00 2.0071287+00 4.3633231-01 0.0000000 9.9971709+02

SPIRALS BEGIN SPIRALS END SPIRALS JUMP YAW RHUMB(1) BYPASS ENTRY BYPASS REMIX

LOXOD(1)= 3 LOXOD(2)= 3 LOXOD(3)= 3 0.0000000 LEAK= 5 JOINE= 14

HOP(L),L=1, 5 VARIATIONAL-GRADIENT VARY-SHIFT FRACTION

6.1035156-05 6.1035156-05 6.1035156-05 6.1035156-05 6.1035156-05

(PLOT(K,L,M),K=1,5),L=1=X,L=2=Y),M=1, 14 PHASE-SPACE CROSS PLOTS

MAP END DATA DATA END DATA ITEM END PLOT (INCHES) PLOT END

1 4.1000000+04 5.1000000+04 -7.0000000+00 1.0000000+00 1.0000000+01 1.0000000+01 X

0.0000000 4.0000000+03 1.3000000+01 1.0000000+00 9.0000000+00 Y

TIME	3.3774414-01M	STAGNATION PRESSURE P0	MACH RATIO M=V/C	FLOW VELOCITY V	MASS/VOLUME DENSITY RHO	STATIC FLUID TEMPERATURE T	STATIC FLUID PRESSURE P	ENERGY/MASS INTERNAL U	ENTHALPY/MASS HEU+P/RHO
12	1.8150780+00	6.6066861-01	2.4600734-02	1.6941144+01	1.3171352-01	4.2524444+03	2.0663229+07	4.9519145+04	2.4596880+03
13	1.8067031+00	1.0000000+00	2.4600734-02	1.6941144+01	1.3171353-01	4.2524444+03	2.0663229+07	4.9904475+04	2.4596881+03
14	1.0000000+00	1.0000000+00	2.6494841-02	1.7267221-01	1.7267221-01	3.7608127+03	1.8028726+07	4.8653310+04	2.0846882+03
15	1.0000000+00	1.0000000+00	1.4064819-01	1.7267221-01	1.7267221-01	1.6749109+04	1.8028726+07	4.8653310+04	2.0846882+03
16	1.0000000+00	1.0000000+00	1.4064819-01	1.7267221-01	1.7267221-01	1.6749109+04	4.7919538+06	4.1293871+04	6.5982402+02
17	1.0000000+00	1.0000000+00	5.6259277-01	1.7267221-01	1.7267221-01	6.6869363+04	8.7919538+06	4.1293871+04	6.5982402+02
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DO PLOT	1	TIME	3.5651042-01M
DO PLOT	2	TIME	3.9057617-01M
DO PLOT	3	TIME	4.2584635-01M
DO PLOT	4	TIME	4.6254883-01M
DO PLOT	5	TIME	4.9282227-01M
DO PLOT	6	TIME	5.2335612-01M
DO PLOT	7	TIME	5.3593750-01M
DO PLOT	8	TIME	5.5174153-01M
DO PLOT	9	TIME	5.6695963-01M
DO PLOT	10	TIME	5.8266601-01M
DO PLOT	11	TIME	5.9858398-01M
DO PLOT	12	TIME	6.1158854-01M
DO PLOT	13	TIME	6.2483723-01M
DO PLOT	14	TIME	6.3977864-01M

PLOTS ALL DONE (PLOT TAPE END-OF-FILED, REWOUND, INTERLOCKED) TIME 6.5844727-01M

GOOD RUN (NO SPILL, NO DIVISION/ZERO, NO IMAGINARY) TIME 6.5922851-01M

Plotter Output: Lobular Temperature-Entropy Cycle

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