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PHOTOPRODUCTION OF π^+ MESONS FROM A
POLARIZED PROTON TARGET AT 5 AND 16 GeV

Charles Conover Morehouse

August 1970
(Ph.D. Thesis)

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ABSTRACT

Described in this thesis is the measurement of the asymmetry in the cross section for the reaction $\gamma p \rightarrow \pi^+ n$ between the two states of polarization of the initial proton normal to the plane of scattering. The laboratory photon energies, k , were 5 GeV and 16 GeV, and the regions of momentum transfer, t , covered were $0.14 \leq \sqrt{-t} \leq 1.01$ GeV/c and $0.14 \leq \sqrt{-t} \leq 0.78$ GeV/c respectively. A butanol polarized target was used with the SLAC 20 GeV/c magnetic spectrometer. The data show a sizeable asymmetry at both 5 GeV and 16 GeV. The 16 GeV data peak at $\sqrt{-t} \approx 0.30$ GeV/c with an asymmetry of about -0.70, and 5 GeV data peak at $\sqrt{-t} \approx 0.80$ GeV/c with an asymmetry of about -0.70. (The direction of our normal to the scattering plane is along (photon in) $\times$ (pion out)).

I. INTRODUCTION

In the study of particle physics the production of pions by the collision of energetic photons and nucleons occupies a curious position. The interaction most certainly lies within the realm of the electromagnetic interactions, involving as it does a photon. At the same time the fabulously successful theory of quantum electrodynamics, used to describe electromagnetic processes, has found only limited applicability in photoproduction of pions.¹

There are four single pion photoproduction reactions:

Notice that the final states are purely hadronic and can be formed by the elastic scattering of the hadrons shown. Thus one might expect similarities between certain photoproduction processes and pion-nucleon scattering, and indeed this is the case. In particular the low-energy photoproduction and pion-nucleon total cross sections show similar resonance behavior.² The usefulness of pion photoproduction in the study of the strong interactions is abundantly clear in that the first pion-nucleon resonance state [called now $\Delta(1238)$] was first proposed to explain π^+ and π^0 photoproduction rates.³ Properties of others of the pion-nucleon resonances have been discovered or confirmed in photoproduction experiments.⁴ (This is not to say that the study of one set of reactions must be justified by its pertinence to another set of reactions, but the fact that there exist similarities between the two sets heightens the interest in them.)

The construction of electron accelerators of high energy and the availability of photon beams at these accelerators has allowed the study of single pion photoproduction at laboratory photon

energies up to 18 GeV,⁵ comparable to the highest energies available for pion-nucleon scattering. As with the low-energy photoproduction, the high energy photoproduction ($E_\gamma > 5$ GeV) has shown similarities with purely hadron processes, especially in that Regge pole theory has been applied with some success to photoproduction.⁵

At this time the study of reactions I-1a. -d. has progressed beyond differential cross section measurements to the measurements of some of the spin dependences of the reactions. In particular at low energies ($E_\gamma < 3$ GeV) the recoil nucleon polarizations have been measured over limited angular regions for reactions I-1a. -c. and are seen to be sizeable in some instances.⁶ In addition beams of linearly polarized photons have been constructed and the photon asymmetries have been measured at low energies ($E_\gamma < 4$ GeV) for reactions I-1a. -c. over limited energy regions.^{5,6} The results of these experiments have encouraged an energetic theoretical effort which nevertheless falls short of complete understanding of the photoproduction processes.⁷ Further investigation into the spin dependence of single pion photoproduction is required to provide a more complete description of the process. In particular until this experiment no measurement of the dependence of any of the processes I-1a. -d. on either initial or final nucleon spins had been made for laboratory photon energies above 3 GeV. The need for such measurements and the availability of our polarized proton target at the Stanford Linear Accelerator Center resulted in the experiment to be discussed here.

II. EXPERIMENT

A. General

The purpose of this experiment was to measure the dependence of the photoproduction of positively charged pi mesons from protons upon the spin of the protons. With a polarized proton target, the spins of the protons can be oriented either "up" or "down" (to be defined more precisely below), and the cross sections for pi photoproduction can be measured for both orientations of the proton spin. A difference in these cross sections indicates that the photoproduction process under study is dependent upon the orientation of the target proton spin.

The possible spin dependence is parametrized in terms of the differential photoproduction cross section on unpolarized protons, $I_0(t)$, (t is the square of the invariant momentum transfer), the target polarization, $\vec{P}_T$, the normal to the production plane, $\hat{n} = \vec{k} \times \vec{p} / (|\vec{k} \times \vec{p}|)$ ($\vec{k}$ is the momentum of the initial photon, $\vec{p}$ is the momentum of the produced pion), and the asymmetry parameter, $A(t)$. The expression for the differential photoproduction cross section on polarized protons is given by:

$$I(t) = I_0(t) [1 + A(t) \vec{P}_T \cdot \hat{n}]. \quad (\text{II-1})$$

Thus the physics of the spin dependence of the photoproduction is described by the asymmetry parameter, $A(t)$. The magnitude of $A(t)$ is constrained to be less than or equal to 1 and a value of $A(t) = 0$ means no dependence of the photoproduction on the orientation of target proton spins.

In this experiment the proton spins could be oriented "up" (positive polarization, parallel to $\hat{n}$), or "down" (negative polarization, antiparallel to $\hat{n}$). If we denote by $I_+(t)$ [$I_-(t)$] the production cross section for target polarization along $\hat{n}$ (opposite to $\hat{n}$), then we experimentally measure the quantity $\epsilon(t)$:

$$\epsilon(t) = \frac{I_+(t) - I_-(t)}{I_+(t) + I_-(t)} . \quad (\text{II-2})$$

If the target were pure protons and if it were polarized to the same magnitude for positive and negative polarization then the relationship between $\epsilon(t)$ and $A(t)$ is:

$$A(T) = \frac{\epsilon(t)}{P_T} . \quad (\text{II-3})$$

This experiment was performed at the Stanford Linear Accelerator Center. A butanol polarized proton target⁸ intercepted a high energy ($E_\gamma = 5$ GeV and 16 GeV) photon beam and the photo-produced pions were detected in the 20 GeV/c spectrometer⁹ which measured the angle and momentum of the pions. The conjugate neutron in the reaction $\gamma p \rightarrow \pi^+ n$ was not detected. The plane of production defined by the beam and spectrometer was horizontal and the protons were polarized normal to this plane.

This experiment had several critical areas that had not been a part of photoproduction experiments in the past. The two potentially most serious problems were radiation damage of the target by the beam and isolation of photoproduction from hydrogen in the target from heavy elements in the target. The first problem was overcome by use of a target originally designed for use in intense electron beams.¹⁰ Some interesting effects appear in the substitution of a photon beam for an electron beam, however. (See Section II-C). The second problem of hydrogen isolation was actually overcome by a procedure to be described in Section III-D. We were helped greatly in this effort by the fact that positive pions of proper kinematics can only come from the reaction $\gamma p \rightarrow \pi^+ n$. Thus the neutrons in the heavy nuclei in the target don't contribute to the background. In addition a pion produced inside a nucleus has a high chance of being absorbed and so the protons in the heavy nuclei are not as efficient as free protons in photoproduction.¹¹

B. Beam

The photon beam used was the SLAC End-Station-A, high-power photon beam used previously by Boyarski et al.,^{9,11} with one modification to be discussed below. The primary SLAC electron beam, momentum analyzed to better than 0.5%, was focused onto a 0.25 cm Al radiator, mounted on TC 20 (see Fig. 1). The resulting bremsstrahlung beam passed through a series of bending magnets, B23-B26 (indicated by B24 on Fig. 1), whose purpose it was to deflect the electrons remaining in the beam into a beam dump. The photon beam passed through an adjustable collimator, (C10), and a fixed beam scraper (Slide Collimator), each followed by a sweeping magnet, before striking the polarized target. A beam of 2×10^{11} electrons in a SLAC pulse incident on the radiator yielded about 4×10^9 equivalent quanta* (EQQ) at the target, for a transmission factor of about 2×10^{-2} EQQ/electron.[†] During the experiment the average repetition rate was 180 pulses per second and the pulse length varied from 1 to 2 μ sec, averaging about 1.4 μ sec. The beam intensity varied depending upon the limits set by count rate and target damage rate, but ranged from 0.5 to 4.0×10^9 EQQ/pulse at 180 pps.

The system used to monitor the photon beam intensity is described in detail elsewhere.¹² Briefly it consists of three major elements: two interchangeable beam-destructing absorption monitors, and a non-destructive transmission monitor. The absorption monitors, a calorimeter and a secondary-emission-quantameter (SEQ), were mounted on rails downstream of the

*The number of equivalent quanta in a bremsstrahlung beam is calculated by dividing the total energy contained in the photon beam (as measured, say, by a calorimeter) by the energy of the electrons producing the photons.

[†]This transmission factor is a function of radiator thickness and collimator dimensions.

E47 BEAMLINE SCHEMATIC

PION DETECTION SYSTEM

$$\text{EVENT TRIGGER} = (\text{TRI}) (\text{TR2}) (\text{TR3}) (\text{R1}) \bar{S}$$

XBL 702-2360

Fig. 1. Beam and Spectrometer Detection System.

polarized target so that either could be remotely positioned in the path of the beam transmitted by the polarized target. The calorimeter was the primary calibration device but was not used in the beam during normal data taking as its response time is too long to register pulse to-pulse fluctuation in beam intensity. The SEQ was employed during the normal data taking as the primary beam monitor, along with a non-destructive Cerenkov monitor positioned upstream of the polarized target. The Cerenkov monitor operated by converting a small fraction of the photon beam into electron-positron pairs in a thin (0.127-mm aluminum) window and recording the resultant Cerenkov radiation from those pairs. This monitor was also employed as the intermediate standard during the calibration of the SEQ with the calorimeter. The monitor calibration, done 2 or 3 times each day was performed in two steps: first, the Cerenkov counter and calorimeter would be exposed to the beam, then the Cerenkov counter and SEQ would be exposed. The calibration of the SEQ to the calorimeter was thus made through the intermediary of the Cerenkov counter. In addition to these monitors, a toroid current monitor was monitoring the electron beam upstream of the radiator, and was used in the analysis only when data from the other monitors were unusable (bad readout, monitor not in beam, etc.). The monitors were almost always internally consistent to better than 3%.

The photon beam, having no charge, obviously cannot be steered, and its direction is fixed by the direction of the electrons incident on the radiator, TC 20 (see Fig. 1). Thus, in order to position the photon beam on the target, we had to aim the electron beam through the radiator at the target, and of course, sweep the electrons remaining after the radiator out of the beam. The positioning of the beam was complicated by the fact that the beam scraper (Slide Collimator) was fixed in position and could not be adjusted. Once a target was installed, it too was fixed in position,

and so the beam line was specified. The beam positioning then consisted of moving the electron beam in position and direction at the radiator until it was aligned with the Slide Collimator and the target. [The magnets (A10, A11) controlled the positioning of the beam on the radiator, and the magnets (A12, B28), located closely enough to TC20 not to affect the positioning of the beam, controlled the direction of the beam.] Then the movable collimator C10 was closed down to the desired aperture. Normally we used an aperture at C10 that defined a beam at the target that was 1 mm larger than the target on all sides. This procedure had to be followed each time a target was installed, since the targets were not reproducibly positioned to ± 1 mm, the collimating margins. The precise positioning of the target was complicated by the ill-defined edge of the bag assembly and the difficulty in firmly securing the target in the cavity against the initial flow of helium gas. But we were able to position the beam reliably on the target and so overcome the difficulties of changes in position from target to target. A major aid in the beam positioning was our ability to view directly the relative beam-target positioning by means of exposing glass slides to the beam transmitted by the target.

To accomplish this we placed a 10-cm square, 3-mm thick piece of glass downstream of the target, intercepting the beam which had passed through the target. With the polarized-target magnet, Zoltan, turned off, the electron-positron pairs produced in the materials in the target (including septum, RF cable, etc.) would leave a distinct exposure in the glass from which we could easily determine the relative positions of the collimators and target. An exposure of about 5 minutes to our 10^{11} EQQ/sec beam was adequate to fix the images in the glass. Figure 2 shows our exposure of a greatly expanded (and uncollimated) photon beam on a target to show the detail that could be seen in such glass slides. The response of the slides was linear enough (linear to within 10%)

Fig. 2. Photon Beam Glass Slide Picture of Target. XBB 6910-6804

to permit a mapping of the beam intensity over the target area. The mapping was done with the collimator in place but with no target in position and a lead converter attached to the upstream side of the glass. These exposures were measured by a light transmission technique. The beam profiles so obtained indicated that the dose at the edges of the target was about 0.80 times the dose at the center of the target. The consequence of this uneven distribution of beams is discussed in Section III-E, Error Analysis.

So far this discussion of the beam has assumed that the natural dimensions of the beam at the target are comparable to the dimensions of the target itself. Unfortunately this is not so. The natural shape of the beam (uncollimated) at the pivot is round and is about 2 cm in diameter at 5 GeV and about 1 cm in diameter at 16 GeV. The target to be illuminated is 2.5 cm. It is not easy to enlarge this spot by blowing up the electron beam at the radiator, as many unknown and undesirable instabilities are introduced. Thus it was necessary to employ a stepped sweeping magnet on the electron beam¹³ to sweep the electron beam across the radiator, causing the resulting photon beam to be swept across the target. The sweeping magnet, in the form of two air core coils oriented with their fields at right angles to one another and both fields perpendicular to the electron beam line, is designated B27 in Fig. 1. The power supplies on the two coils are controlled by a logic unit which would cause a sequence of currents to be applied to the two coils resulting in a patterned sweep of the beam. The pattern was two 12×12 position grids, offset in such a way as to give 288 separate beam positions (see Fig. 3). The control logic would advance from one beam position to the next in between accelerator pulses, provided that the previous accelerator pulse contained a minimum amount of beam. This was to insure that each area of the target received its full dose before moving on to the next position, which was the purpose of the beam sweeping in the first place. The control logic actually provided a stepped divider on an

Fig. 34. Beam Sweep Raster Grid.

Beam Sweep Raster Grid

applied supply voltage, so that the extent of the beam sweep or the size of the sweep grid, was easily adjusted and could be set independently for the vertical and horizontal directions. The grid pattern was centered on the undeflected beam, so that the beam steering was first performed, then the sweep was turned on and adjusted to cover the target. The sweep was adjusted to extend slightly beyond the target in order to illuminate the square target as uniformly as possible with the round beam.

The control logic provided two important outputs for use by the online computer: first a scaler that the computer could read which counted from 1 to 288 indicating which grid position was being occupied, and second an interrupt to the computer generated each time the beam passed out of the top, middle, or bottom thirds of the grid pattern. The interrupt occurred following positions 48, 96, 192, and 240. These interrupts were used in recording separately the data from the top, middle and bottom of the target. (See Sections II-D and III-B).

C. Polarized Target

1. Introduction

The polarized proton target that we used in this experiment attains high proton polarization by means of the method of dynamic nuclear orientation. This technique has been described in detail elsewhere,¹⁴ but a short description is warranted here for continuity.

From statistical mechanics it is straightforward to show that the polarization of spin-1/2 particles with magnetic moment μ in a magnetic field H at temperature T is

$$P = \tanh \frac{\mu H}{kT} \quad (II-4)$$

For protons the Boltzmann factor, $\mu H/kT$, is $\sim 1.0 \times 10^{-7} \frac{H}{T}$, H measured in gauss, T in degrees Kelvin. At readily attainable laboratory conditions of 25,000 G and 1°K the so-called "brute

force" or thermal equilibrium proton polarization is $\sim 1/8\%$, not a useful polarization for targets in scattering experiments. However under the same conditions free electrons, because of their higher magnetic moment, are polarized to $\sim 92\%$. The method of dynamic nuclear orientation is to subject a mixture of free (unpaired spins) protons and electrons to a high field and low temperature and transfer the high electron polarization to the protons by saturating the partially forbidden electron-proton double spin flip transitions with a microwave bath. An essential ingredient in this scheme is that the proton spin relaxation time be much longer than the electron spin relaxation time. This condition is true in our target material where the electron spin relaxation time is of the order of milliseconds and the proton spin relaxation time is of the order of a hundred seconds.⁸ We are able to enhance the "brute force" proton polarization from its thermal equilibrium (TE) value of $1/8\%$ to about 30% . This is considerably less than the theoretically maximum value of 92% and depends upon many factors including the spin relaxation times of the protons and electrons.¹⁵

The target material was a butanol-water-porphyrin mixture developed by Borghini and co-workers at CERN.⁸ This material was used instead of the more traditional lanthanum magnesium nitrate crystals primarily because of its resistance to radiation damage. We used a 92% butanol, 8% water mixture saturated then to $\sim 2\%$ with the free radical, porphyrin. This mixture was then sealed in thin-ribbed bags constructed from 12-micron-thick ($1/2$ mil) FEP Teflon. Further details on the preparation of the target can be found elsewhere.¹⁰

The target was mounted in a microwave-tight box which served also to contain the liquid helium bath and act as the coil in the nuclear magnetic resonance circuit reading the proton polarization. This box was contained in a horizontal, continuous flow, Roubau-type cryostat.¹⁶ (For a more detailed description of the

cryostat and its operation see Ref. 13). The cryostat was mounted so that the target was in a vertical magnetic field of 24.8 kG, and could be maintained at a temperature of 1°K for a period of several days if necessary. The effective length of the magnet was such that a charged particle produced at the center of the target would be subject to 154 MeV/c lateral deflection in momentum, a fact corrected for in the analysis programs.

2. Polarization Measurement

The measurement of nuclear polarization usually consists of detecting the change in impedance of a tuned radio frequency (rf) circuit due to the presence of the polarized nuclei in the circuit. This is accomplished by positioning the polarized nuclei in a coil which is part of a tuned resonance circuit. The coil in our case is the microwave-tight cavity in which we mount our target. This cavity is a rectangular aluminum box, copper plated except where the beam passes through it, 4" long, 1-1/4" high, and 1-1/2" deep along the beam direction (see Fig. 4). A 2"-long aluminum septum runs from top to bottom of the box along the 4" direction of the box dividing it into two sections 3/4" deep along the beam direction. The septum is soldered to the bottom of the box and is insulated along the top. A small piece of the septum, the tab, projects through the top of the box, insulated from the top, so that electrical connection can be made with the septum. By introducing the rf voltage on the center conductor of a coaxial cable to the tab of the septum and connecting the coaxial shield to the top of the cavity, we have arranged the cavity with the septum to be an effective single turn coil. (We call this arrangement the "septum drive.")

The coil is now part of the parallel resonant circuit represented in Fig. 5. The resonant circuit is connected through a high impedance to an oscillator whose frequency is controlled by

XBL702-2353

Fig. 4. Polarized Target Cavity Layout.

Fig. 5. Polarized Target rf System.

a dc input voltage. The rf field in the coil is oriented perpendicularly to the proton spins which are aligned along the direction of the large applied magnetic field, H_0 . In this configuration it can be shown that the presence of the nuclear spins alters the inductance of the coil in a simple way:¹⁷ $L = L_0(1 + X' - iX'')$, where L_0 is the inductance of the coil in the absence of the spins. It can be further shown that X'' is proportional to the target polarization, P_T .¹⁸

If we now write the impedance of the coil containing the polarized protons we can see how this amplitude variation appears:

$$Z_{\text{coil}} = i\omega L_0(1 + X' - iX'') = i\omega L_0(1 + X') + \omega L_0 X''. \quad (\text{II-5})$$

Thus the X'' term appears as a purely resistive impedance in the coil and it is capable of lowering or raising the rf voltage (depending upon the sign of the proton polarization) as the rf frequency is swept through the proton resonance frequency. Since the resonant circuit is coupled through a high impedance to the rf oscillator, the current in the resonant circuit will be essentially independent of the impedance of the circuit. Thus a measurement of the rf voltage as the frequency is swept over the proton resonance is equivalent to a measurement of the impedance and, through Eq. II-5, X'' .

Now we consider how X' appears in the measurement of the impedance of the parallel resonant circuit in Fig. 5. Physically the coil is mounted in the cryostat, connected by a two-wavelength-long transmission line to the remainder of the resonant circuit which is contained within the System-1260.¹⁹ Other elements in the System 1260 are indicated within the labeled border on Fig. 5. Thus the elements of the resonant circuit are the inductance of the coil, L , the distributed resistance in the transmission line and coil, R_1 , the variable capacitor for tuning the circuit, C , and the amplifier input resistance, R_2 . The impedance of this parallel network is given by

$$\frac{1}{Z} = \frac{1}{R_1 + i\omega L} + i\omega C + \frac{1}{R_2}. \quad (\text{II-6})$$

The System 1260 has an unusual feature in that as the oscillator frequency is changed by the input control voltage, an adjustable amount of capacity is added or subtracted from the circuit so that the resonant circuit remains tuned throughout the frequency sweep range of the oscillator. The adjustable capacity is provided in the Input Tracking Circuit by a varicap system which adds to the primary capacitor, C , to maintain the circuit in tune with the oscillator frequency.¹⁹ This tracking of the oscillator by the input circuit is done to eliminate the natural variation in impedance with driving frequency of a fixed tuned circuit. The effect of the tracking, which is adjusted to maximize $|\frac{1}{Z}|$ at each frequency with no polarized target in the coil, is to allow us to rewrite Eq. II-6:

$$\frac{1}{Z} = \frac{1}{R_1 + i\omega L_0 (1 + X' - iX'')} + \frac{1}{R_2} + \frac{i\omega L_0}{R_1^2 + \omega^2 L_0^2} \quad (\text{II-7})$$

With the introduction of the quality factor $Q_1 = \omega L_0 / R_1$ in the cavity branch of the resonance circuit we can again rewrite Eq.

II-7

$$\frac{1}{Z} = \frac{1}{R_1 (1 + iQ_1 (1 + X') + Q_1 X'')} + \frac{1}{R_2} + \frac{iQ_1}{R_1 (1 + Q_1^2)} \quad (\text{II-8})$$

Now we make use of our knowledge of some of the parameters in Eq. II-8. First we note that the enhanced signal changes the rf voltage by about 10% so if we carry our approximations to second order in X' and X'' we will be isolating terms to approximately the 1% level. In addition the Q factor for the entire system has been measured to be ~ 104 so that the Q_1 in the cavity branch is of the order of 100 or larger. We need keep only leading order in Q_1 to obtain a reasonable approximation in the expansion of Eq. II-8. Thus to second order in X' and X'' and to leading order in Q_1 we have:

$$\left| \frac{1}{Z} \right| \approx \left(\frac{1}{R_2} + \frac{1}{R_1 Q_1^2} \right) + \frac{1}{R_1 Q_1} \left[X''(1-2X') - \frac{2X'}{Q_1} + \frac{3(X'^2 - X''^2)}{Q_1} \right]. \quad (\text{II-9})$$

The presence of terms linear in X' do not represent a significant contribution to the signal obtained from integrating over the proton spin resonance line. This is because the function X' , related to X'' by a dispersion relation, is an odd function of frequency about the center frequency of the proton signal and since all of our measurements of the target polarization (to be discussed below) are integrals over the entire proton line the terms linear in X' vanish to first order. (In the language of nuclear magnetic resonance, the X' term is proportional to the dispersive part of the proton signal, while the X'' term is proportional to the absorptive part.) Thus we can write, accurate to about 1%,

$$\left| \frac{1}{Z} \right| \approx \alpha + \beta X'' = \alpha + \beta' P_T. \quad (\text{II-10})$$

We now have an expression linking the target polarization and the magnitude of the detection circuit impedance which itself is proportional to the rf voltage in the coil. The rf amplifier-diode detector-voltage to frequency converter chain in the System 1260 (see Fig. 5) provides a pulse train, 0 to 100 kHz, proportional to the rf voltage across the detector circuit. In order to measure the target polarization we must effect a frequency sweep over the proton signal and measure the rf voltage by means of the voltage-to-frequency converter (VFC) output. By Eq. II-10 the integral over frequency of the rf voltage is equal to a background constant plus a term proportional to the target polarization. A background subtraction technique, to be described below, leaves us with a signal proportional to the target polarization.

The proportionality constant, β' in Eq. II-10, is in principle calculable, but so many factors enter into the calculation that a calibration technique is used instead to determine its value. The

signal measured to calibrate the system is the "brute force" polarization signal whose actual value is calculated using Eq. II-4. As mentioned above the magnitude of the "brute force" polarization (which we call the thermal equilibrium, or TE signal) is very small. Therefore in order to measure the target polarization accurately we must measure this small TE signal accurately. This is the central problem of the measurement of the target polarization.

The integral over frequency, leading to a number proportional to the target polarization, is performed by a PDP-5 computer and an interface electronics device referred to here as the Polarization Digitizer (PD). In performing the integral, the computer through the intermediary of the PD, sets the rf oscillator in the System 1260 to the desired frequency and then requests the PD to digitize the rf voltage. When the PD has completed the digitization the computer reads and stores this number and moves on to another point.

To help understand the specifications of the PD, consider the signals it detects. The enhanced polarization signal generates an rf voltage change of about 10% of the carrier rf voltage. The carrier voltage is usually set so that the VFC input voltage is 60% of the 1 volt maximum. Thus the enhanced signal represents a 6% of full scale deflection. This enhanced signal is in turn roughly 300 times the size of the TE calibration signal. The TE signal then represents a deflection roughly 0.02% of full scale, or about 200 μ volts at the input of the VFC. If we want the error on each digitization of the TE signal to be less than 10% we will have to be accurate to better than 20 μ volts, 2 parts in 10^5 of the VFC range. In addition we want to measure the inverse of the rf voltage, as this is the quantity which is most linear in the target polarization (a consideration of importance for enhanced signals only).

In order to digitize the inverse of the rf voltage to the accuracy necessary, we allow a scaler to count the output from a

100 MHz crystal oscillator during the duration of 100 cycles of the VFC output. Thus for a 1-volt input to the VFC which corresponds to 100 kHz output frequency the gate will be open for 1 msec and the scaler will read 100,000. For a 0.6-volt input to the VFC the output frequency is 60 kHz and the gate will be open for $\sim 1.67 \mu\text{sec}$ and the scaler will read $\sim 167,000$. We see that this scheme indeed digitizes the inverse of the rf voltage and provides least count accuracy of at least 1 part in 10^5 . The contents of this scaler can be read directly by computer and stored for analysis.

In addition to the 100 MHz gated scaler system the PD contains two 12-bit digital-to-analog-converters (DAC1 and DAC2). DAC1 is used to control the frequency of the rf oscillator in the System 1260, and is the device which allows the computer to generate a sweep of the oscillator frequency over the proton signal. In order to do this the computer needs a table of numbers, DAC1 inputs, which corresponds to a specific sequence of frequencies making up the sweep. We actually use the PD in an auxiliary mode to construct this frequency table. To do this we substitute a 10 kHz crystal clock for the VFC output (0 - 100 kHz) and switch the 100 MHz scaler to count the 105-106 MHz output from the monitor output of the rf oscillator in the System 1260. With the PD so arranged the computer can be programmed to search for the DAC1 inputs which yield the desired frequency sequence for the sweep.

The second digital-to-analogue-converter, DAC2, controls the frequency of the microwaves used to polarize the target. The output from DAC2 goes to the control circuitry in the high-voltage power supply which operates the microwave tube, and enables reliable, reproducible, and rapid changes in the target polarization. A schematic of the Polarization Digitizer and its several interconnections is shown in Fig. 6.

The actual frequency sweep made by the computer during the polarization measurement is not monotonic in frequency. The

Fig. 6. Polarized Target rf Digitization System.

reason for this is that we wish to eliminate the constant term; α , in Eq. II-10 as we make the sweep. We make an interpolation linear in frequency of the constant α for each frequency in the sweep by measuring $1/V_{rf}$ in the pattern shown in Fig. 7. If we are trying to measure the proton signal at a frequency ν_0 , we first measure $h_0 = \frac{1}{V_{rf}}$ at ν_0 . Then we measure $h_+ = \frac{1}{V_{rf}}$ at frequency $\nu_0 + \Delta$ (Δ is the width of the final display), and $h_- = \frac{1}{V_{rf}}$ at frequency $\nu_0 - \Delta$, and finally h_0 again at ν_0 . The four readings are combined, $h_0 - h_+ - h_- + h_0$, to give $2\delta h$, twice the actual proton signal at ν_0 . (This factor of two appears in all readings and cancels in the calculation of the polarization.) The combined readings, which represent the proton signal alone, are displayed on the PDP-5 display oscilloscope for visual monitoring. The sum of the $2\delta h$ from the complete sweep is used by the computer to calculate in real time the value of the polarization. Each complete sweep takes about one second.

The entire procedure for measuring the target polarization is the following. First a frequency table is constructed. The program which does this requires as input a center frequency (the frequency about which the proton signal is to be centered) and the frequency interval between steps in the sweep. With the Polarization Digitizer in the "frequency table mode" the computer will construct and store a sequence of DAC1 input constants that will generate the background subtracting sweep. Second we measure the TE calibration signal. With the polarizing microwaves off we allow the protons to come into thermal equilibrium with the helium bath. Then we read the TE polarization in terms of the area of the integrated proton signal. Unfortunately the TE signal is present on a curved background. We measure the magnitude of this background by shifting the 25 kG magnetic field by 1% which moves the TE signal out of range of the rf sweep. This background is subtracted from the TE signal to give the "TE area". (A typical butanol TE with

Proton NMR Signal
Showing Background Subtraction Sequence

Background-Subtracted Signal
Displayed on PDP-5 Oscilloscope

Fig. 7.

background superimposed is shown in Fig. 8). From a measurement of the helium temperature (with carbon resistors) we can calculate the actual TE polarization and from that and the "TE area" we calculate and store in the computer the "TE polarization per unit area". We now can calculate enhanced polarizations by measuring the "enhanced area" and multiplying by the "TE polarization per unit area".

A final word on the polarization measurements concerns the TE's. The TE signal from a single sweep shows noticeable noise. We alleviated this problem by actually summing eight sweeps (octal 10) for each TE or background reading. The signals in Fig. 8 consist of the sum of eight sweeps on both the TE and background traces. In Fig. 9 are shown for comparison single TE and background sweeps, normal TE and background signals, and superimposed positive and negative enhanced signals. The target material used in these illustrations is 1-2-Propanediol, with a Cr^{V} complex for the free radical.²⁰

3. Target Performance--Radiation Damage

The maximum polarization in small samples (a few cubic mm) of our butanol mixture is around 40%.⁸ Our targets were typically 25 cc and would usually polarize initially to just over 30%, occasionally 35% and sometimes 25%. The targets suffered radiation damage from the beam which reduced their polarization continuously during the data taking. The behavior of the polarization was roughly exponential with dose, $P = P_0 \exp(-\phi/\phi_0)$ where ϕ_0 is approximately 7×10^{15} EQQ/cm² for a 3.8-cm-long target. (More about ϕ_0 will be said shortly. At the beam intensities we normally maintained ($\sim 4 \times 10^{11}$ equivalent quanta/sec) the useful life of the target was about 5 hours. Fortunately it was possible to anneal out much of the radiation damage and thus we were enabled to extend the useful life of the target to at least one day. Figure 10 shows

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Fig. 8. Butanol TE with Background Superposed.

a. Single-Sweep TE

b. Normal TE with Background Superposed

XBB 707-2982

c. Enhanced Positive and Negative Polarization

Fig. 9.

the polarization history of a target that we annealed eight times. We believe now that we did not anneal the targets often enough during the experiment (on the average only 3 or 4 times). Thus we were spending unnecessary time changing targets. We changed targets about once a day using a total of 16 polarized targets in 24 days of accelerator time. (Several of the 24 days were taken up by tuning beam and electronics and by background measurements.)

One feature that shows on Fig. 10 is that the polarization is not a pure exponential function of dose. In fact it appears that the function could be written $P = P_0 \exp(-\phi/\phi')$ where ϕ' is itself a function of dose. This point has been studied in Ref. 10 and the effect can be explained in terms of a target whose dose increases linearly from front to back along the beam line. This depth-dependent dose is a good approximation of the actual ionizing dose in the target due to the photon beam. The photons must interact (Compton scatter for low energy photons, pair production for high energy) before the target can be damaged. Thus the upstream part of the target receives less ionizing radiation than the downstream part. This front-back effect can account for the deviation from purely exponential behavior of target polarization in the photon beam. (Also this effect suggests the use of thin targets and intense beams whenever possible in order to take advantage of the need for the photon to convert before damaging the target.) In Fig. 10 the value for ϕ_0 given is the slope of the polarization curve at the beginning of the exposure. The values for ϕ_0 quoted in this thesis are for the average ϕ_0 throughout the exposure, a number more useful to someone using a target in a photon beam.

Under careful scrutiny, the polarization vs dose showed an unusual effect. In Fig. 11 where we have plotted both positive and negative polarization vs dose, one can see an unmistakable difference in the behavior of positive and negative polarization. The fact that the positive polarization is slightly more affected by radi-

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Fig. 10. History of Target Polarization vs Dose.

XBL 707-1339

Fig. 11. Polarization vs Dose for Positive and Negative Polarization.

ation than the negative indicates that the photon beam is causing a change in the effective electron spin resonance line shape. In fact the target data suggest that the electron line is broadened asymmetrically to the low frequency (or high field) side. That this broadening can lead to our observations is seen as follows.

First we must note that the proton nuclear magnetic resonance (NMR) line width is very narrow (~ 60 kHz at 25 kG) compared to the NMR frequency (~ 105 MHz) and the electron spin resonance (ESR) frequency (~ 70 GHz). Then we note that the proton polarization as a function of the polarizing microwave frequency contains the overlap of the electron and proton line shapes²¹ but since the proton line is so narrow, the shape in frequency will be due to the shape of the electron ESR line. If the electron line width is narrow compared with the proton NMR frequency itself, then a measurement of the proton polarization as a function of the polarizing microwave frequency will reveal two separate humps of opposite sign each with the shape of the ESR line,²² the maxima of the humps separated by twice the proton NMR frequency (~ 210 MHz) and the two humps lying on either side of the ESR frequency (~ 70 GHz). In this case the proton enhancements are completely resolved. If however the ESR line width is comparable to the separation of the two humps (as is the case in Porphyrin, ~ 200 MHz) then a more complicated situation exists. In Fig. 12a we show the two broad humps whose width is slightly greater than their separation, which when superposed yield the observed curve.⁸ The effect of this superposition of unresolved "electron lines" is to shift the optimum polarizing microwave frequencies away from those calculated assuming the electron line to be narrow.

Now if we allow the electron line to be broadened to the low frequency side by the radiation damage, the situation in Fig. 12b. is present. Clearly the positive polarization will be reduced relative to the negative polarization, exactly the result we observe.

Electron Spin Resonance Lines in Terms of Proton Polarization

Fig. 12a. Before Radiation Damage.

Fig. 12b. After Radiation Damage.

We must point out that this "explanation" is only based on our observations of the target polarization as a function of dose and conversations with people in the EPR field.²³ However this explanation could be confirmed by directly observing the electron line shape (with an ESR measuring system) of our target mixtures before and after exposure to ionizing radiation.

Finally we give the values of ϕ_0 for the several conditions above. The data are averaged over many annealings of the target and over different beam intensities.

Table I. Depolarizing dose, ϕ_0 for a 3.8 cm long target.

	$E_\gamma = 16 \text{ GeV}$	$E_\gamma = 5 \text{ GeV}$
Positive polarization	$7 \pm 1^*$	$9 \pm 1^*$
Negative polarization	$8 \pm 1^*$	$10 \pm 1^*$

* Units of 10^{15} equivalent quanta/cm².

D. Detector System

The photoproduced pions from the polarized target were detected in the 20 GeV/c spectrometer. The spectrometer consists of a thirteen element magnet system and shielded counter detection system mounted on separate carriages which are simultaneously positioned to the desired laboratory production angle by remote control. The spectrometer magnet currents are also under remote control, allowing the experimenter to select any angle and momentum transfer combination desired. In the reaction studied, $\gamma p \rightarrow \pi^+ n$, we detected only the final state pion in the spectrometer, measuring its angle and momentum, and ignored the conjugate neutron.

At each angle the spectrometer momentum was set to select only those pions satisfying single pion photoproduction kinematics. After momentum selection by the magnet system, the pions enter a shielded blockhouse containing the actual active detection apparatus. This apparatus consists of (see Fig. 1): three trigger counters, TR1, TR2, TR3; two hodoscopes, each of crossed arrays of scintillation counters, (ϕ, X) and (θ, P) ; a threshold (G) and a differential (C) Cerenkov counter; a lead plate-plastic scintillator sandwich shower counter (S) and a ten-component (R1 - R10) scintillator-iron block range counter. The usual event trigger consisted simply of (TR1) (TR2) (TR3) (R1). At small momentum transfers, $-t \leq 0.08 (\text{GeV}/c)^2$, positrons from pair production gave large counting rates. At $E_Y = 16 \text{ GeV}$, $-t = 0.08 (\text{GeV}/c)^2$ the shower counter was included in anticoincidence in the trigger, and at both $E_Y = 5 \text{ GeV}$ and 16 GeV , $-t = 0.02 (\text{GeV}/c)^2$, two radiation lengths of lead intercepted the beam in the spectrometer, suppressing the positron to pion rates to an acceptable level ($\sim 1/4$).

Since the event trigger was so simple, events other than single pion photoproduction were measured. However information from the two Cerenkov monitors, shower counter pulse height, and the ten range counters allowed separation of the triggered events by the analysis software into several categories. The basic identifications and typical simultaneous per cent yields are given for $-t = 0.16 (\text{GeV}/c)^2$, $E_Y = 5 \text{ GeV}$ and 16 GeV , in the following table:

Table II. Simultaneous yields in spectrometer

Photon energy	Positrons	Pions	Kaons	Muons	Other hadrons and ambiguous events
5 GeV	10%	25%	not detected simultaneously	2%	63%
16 GeV	10%	33%	5%	1%	51%

The major contributors to the ambiguous events were multiple or missing hodoscope counts ($\sim 15\%$ at both energies) and events which satisfied all criteria except range (15% at 5 GeV, 5% at 16 GeV). The range counter requirement (count in R4) assured us that all the elements in the particle signature were transversed.

Once a particle is identified as a pion by the analysis software, its momentum is calculated from hodoscope information and knowledge of the spectrometer magnet currents, and the identification of a kinematically acceptable pion can be made. (This procedure, a bremsstrahlung end point technique, is described below.)

Data were accumulated, logged, and analysed in the SLAC End Station A on-line computing facility. (For a more detailed description, see Ref. 24). This consists of a SDS 9300 computer with line printer, CRT display, card reader, magnetic tape drives and a 16 core-load capacity memory disc. The central computer itself is a 32,000-word, 24-bit word machine with a 64-channel priority-interrupt system and fast data transfer through a multiplex system. Data acquisition takes place in two steps. First an interrupt is generated by the data to be transferred at an interrupt level predetermined by the experimenter and associated with the importance and rate of the data. Second, after all higher interrupts are serviced, a subroutine appropriate to the outstanding interrupt is called which services the interrupt, transferring and logging the data and performing the desired calculations. One very powerful feature of the computer is its ability to store interrupt servicing subroutines on the disc so that for some of the low priority or low rate data appropriate subroutines could be stored on the disc and not waste valuable core on low-use routines. About one quarter of the core was allocated for specific analysis programs and about one half of this was designated as transient space or overlay space. The disc contained about ten different overlays, any one of which could at a particular time reside in the core. Among the highest level interrupts, naturally, was the event pro-

cessing interrupt, and an example of a lower level interrupt was the spectrometer magnet current regulator interrupt. The target polarization was transmitted also via an interrupt to the SDS 9300 computer from the PDP-5 computer which computed it. (See Section II-C.)

Data read into the computer memory from the multiplexer was stored in a buffer in the order in which it was read. Along with each block of data was stored a word identifying the type of data and a word specifying the length of the following data record. When this buffer became full it was written on magnetic tape. The use of two buffers, one that could be filling while the other was going on tape, assured that every event that the computer recognized was logged on tape. The only event loss occurred because of the fact that only one event per pulse could be processed by the on-line system. Thus we always maintained a data rate of less than one event in five pulses, and the average event rate was closer to one event in ten pulses, which kept the loss rate below 5%, a number which compares favorably with that expected from Poisson statistics.

The size and speed of the SDS 9300 and the data rate we maintained allowed a significant analysis to be performed on-line on most of the events. The analysis could take place in stages so that only pions with proper kinematic characteristics would be fully analyzed, the other events being identified and discarded from the on-line analysis along the way. We were able to perform complete analysis on more than 80% of the data as it was being taken.

In order to gain an understanding of the data accumulation, consider the three primary data source interrupts: the event interrupt, the polarization interrupt, and the beam sweeping or raster interrupt. The event interrupt occurred each time the trigger detected an event, and could therefore occur only during an accelerator pulse, and because of the trigger electronics, at most once per pulse. This interrupt caused the data buffers containing the

hodoscope information, range counter information, shower and Cerenkov counter pulse heights, and raster scaler number and Helmholtz coil currents to be read. This data was transferred to the software tape buffer and an interrupt of low level was generated requesting that the event be analyzed. The event interrupt took about 100 μ sec to service, and the analysis interrupt which is a much lower level took about a 10 msec to service. This would suggest the possibility of analyzing up to 100 events per second, but the existence of higher level interrupts meant that on-line analysis was only 90% complete even at a rate of 10 events/second. By comparing a hardware scaler counting the number of event triggers with a software scaler counting the number of events logged on magnetic tape, we could determine if any events were not being logged. Every trigger resulted in an event being logged on tape during this experiment.

The basic time scale of the experiment was set by the accelerator pulse rate. The timing was accomplished with the beam-sweeping raster interrupt which would interrupt the computer each time the beam passed out of the top, middle, or bottom third of the target. The smallest well-defined increment of data-taking time, called a subrun, was defined to be an integral number of complete sweeps of the beam over the target. This was monitored by the raster interrupt. The raster interrupt, linked as it was with the passage of the beam out of certain areas of the target, allowed us to accumulate the data on the top, middle and bottom thirds of the target independently.

Thus, at each raster interrupt the position of the beam was checked, the monitors were read and, along with the pion counts, accumulated with the other data from the region just illuminated by the beam. The servicing of this interrupt took about 500 μ sec normally, but at the end of a subrun it took about 2 msec, just enough time to finish before another beam pulse would arrive (every 2.7 msec at 360 pulses per sec). Also at the end of a sub-

run the raster interrupt-servicing program would request, via an interrupt, that the end-subrun servicing subroutine be called which would perform the bookkeeping required at the end of a subrun. This interrupt, at a lower level than the event or raster interrupts but higher than the analysis interrupt, took several msec to service, but was easily serviced within a beam pulse or two and so did not prevent very many events from being analyzed on-line.

The other primary data collected was the target polarization. The target polarization measurement was controlled by a PDP-5 computer and was sent by an interrupt to the SDS 9300 computer. This data, sent about once per second, was written in the magnetic tape buffer and accumulated in each subrun for on-line calculation of the asymmetry. The slow rate of data meant that a relatively low priority could be assigned to this interrupt because there was a full second in which we could process this data and not have it lose its meaning. This is an example of important but low-rate data that can be assigned a low level of interrupt, but still be processed safely.

The first calculation made on an event was the calculation of the mass of the undetected baryon. With the kinematical quantities defined in Fig. 13, the missing mass squared of the undetected baryon, M_x^2 , is:

$$M_x^2 = M^2 + 2M(k-E) + [m^2 - 2kE(1 - \frac{p}{E} \cos\theta)]. \quad (\text{II-11})$$

This formula would give a unique missing mass from each pion event if the photon energy, k , were known. The bremsstrahlung beam with its spectrum of energies complicates this calculation and a discussion of this complication and how it is overcome is useful.

Imagine first a monoenergetic beam of photons ($k = k_0$) striking a proton target, with the pion angle and momentum mea-

Fig. 13. Relevant Photoproduction Kinematical Quantities

sured. Then if the spectrometer momentum acceptance is wide enough ($\sim 1\%$ at 20 GeV) pions from both reactions $\gamma p \rightarrow \pi^+ n$ and $\gamma p \rightarrow \pi^+ \Delta^0$ will be detected in the system and calculations of missing mass will yield two distinct values, M_n^2 and M_Δ^2 , and a plot of counts vs M_x^2 will show two spikes. (The heights of the spikes are not intended to agree here with the cross sections.)

Fig. 14. Missing-Mass Spectrum for Monoenergetic Beam

* We assume here that the Δ has a narrow width.

Now if someone slips some photons into the beam with energies less than the energy of the primary beam, $k_0 - \delta k$, those photons would produce pions in conjugate with neutrons and deltas that would have lower momenta than the original pions. If we were to calculate a missing mass assuming these were produced by fully energetic photons it would appear that particles of mass higher than the neutron and delta were being produced in conjugate with these pions. Thus our plot would look like:

Fig. 15. Missing-Mass Spectrum for Mixed Energy Beam

In reality the photon spectrum in a bremsstrahlung beam is a continuum, extending down in energy from a well-defined maximum. The number of photons of energy k in an interval Δk in a bremsstrahlung spectrum is $N(k) = B \frac{\Delta k}{k}$, where B can be approximated in most cases as a step function (of unit height for one equivalent quantum). This means that we would expect to see the following missing mass spectrum in the presence of a bremsstrahlung beam and a detection system with perfect resolution.

Fig. 16. Missing-Mass Spectrum for Bremsstrahlung Beam

The detailed shape of the bremsstrahlung endpoint (the function B above) and the spectrometer resolution cause the steps shown here to lose their sharpness. This together with the multi-pion background which comes in strongly above the Δ^0 step, causes the missing mass spectrum to appear as in the upper part of Fig. 17.

One can see that identifying events as the $\gamma p \rightarrow \pi^+ n$ type is not easy except in the region of M_x^2 less than about 1.25 GeV^2 , where it is kinematically impossible for other reactions to contribute counts. Thus we can call any particle with the detection signature of a pion and calculated conjugate missing mass less than 1.25 GeV^2 a pion to be included in our asymmetry calculation. We do not have any way of distinguishing between events from the "free" (and polarizable) protons and the "bound" (and not polarizable) protons in the heavy nuclei in the target and the procedure for this correction is described in Section III-D, Hydrogen Correction.

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Fig. 17. Missing Mass Spectra, Hydrogen and Butanol Targets.

III. ANALYSIS

A. Running Sum Technique

The basic data accumulated under one kinematic condition is the polarization of the target, the number of pions produced, and the cumulative intensity of the beam. In order to understand how the asymmetry parameter is calculated from these data, consider an experiment consisting of a polarized target, a monitor, and a single counter sensitive only to pions. During the counting interval, i , the target is polarized to the value P_i , the monitor counts M_i counts, and the pion counter counts N_i counts. The N_i counts come both from the target protons bound in the heavy nuclei in the target (carbon, oxygen, aluminum, etc.) and from the protons in hydrogen atoms which we can polarize. If we write the number of events from bound protons during the counting interval i as B_i , then the number of counts from free protons per monitor in the counting interval i can be written as

$$H_i = \frac{N_i - B_i}{M_i} \quad (\text{III-1})$$

But we also can write H_i in terms of the production intensity from unpolarized protons, I_0 , the asymmetry parameter, A , and the target polarization, P_i :

$$H_i = I_0 (1 + P_i A). \quad (\text{III-2})$$

In order to extract A from our data we will perform a fit of the H_i in III-1 to H in III-2. The fit is accomplished by the least squares method and involves minimizing (with respect to I_0 and P) the quantity J :

$$J = \sum_i M_i [H_i - I_0 (1 + P_i A)]^2. \quad (\text{III-3})$$

At this point we wish to specify B_i in terms of quantities that will allow us to perform the least squares reduction. B_i

arises from protons in the nuclei of carbon, oxygen and other heavy nuclei in the target. These protons are not polarizable, so they will be contributing equally to the counting rate for both signs of the target polarization. In fact we can relate the rate B_i/M_i to the unpolarized hydrogen counting rate, I_0 , with a constant η . Thus we write

$$\frac{B_i}{M_i} = \eta I_0. \quad (\text{III-4})$$

The procedure for determining this constant will be described shortly. With this definition we can rewrite III-3 as

$$J = \sum_i M_i \left[\frac{N_i}{M_i} - I_0 (1 + \eta + P_i A) \right]^2 = \sum_i M_i \left(\frac{N_i}{M_i} - I_0 \mathcal{K} - I_1 P_i \right)^2. \quad (\text{III-5})$$

Here we have defined $1 + \eta \equiv \mathcal{K}$ which we will refer to as the hydrogen correction factor, and $I_1 = I_0 A$ for ease in the minimization. A word at this point might be worthwhile about the weighting of the bracketed quantity in the sum III-5. The fitting procedure described here is a least squares fit procedure. The weight factor, M_i , in Eqs. III-3 and III-5 is reasonable as it serves to normalize the counting intervals to equal amounts of beam. However it can be shown that this weight factor is obtained from a χ^2 minimization procedure under the conditions of small observed counting asymmetries and small random fluctuations in M_i and P_i ,²⁵ conditions satisfied in this experiment.

The minimization of III-5 with respect to I_0 and I_1 means:

$$\frac{dJ}{dI_0} = 0 \rightarrow \sum_i N_i - I_0 \mathcal{K} \sum_i M_i - I_0 A \sum_i M_i P_i = 0, \quad (\text{III-6})$$

$$\frac{dJ}{dI_1} = 0 \rightarrow \sum_i N_i P_i - I_0 \mathcal{K} \sum_i M_i P_i - I_0 A \sum_i M_i P_i^2 = 0. \quad (\text{III-7})$$

Eliminating I_0 and solving for A, we get

$$A = \mathcal{K} \left[\frac{(\sum_i N_i P_i) (\sum_i M_i) - (\sum_i N_i) (\sum_i M_i P_i)}{(\sum_i N_i) (\sum_i M_i P_i^2) - (\sum_i N_i P_i) (\sum_i M_i P_i)} \right] \quad (\text{III-8})$$

At this point it should be noted that A is expressed in terms of the sums of N_i , $N_i P_i$, M_i , $M_i P_i$, and $M_i P_i^2$. Thus it is possible to use these "running sums" to calculate A as data is being taken, as long as M_i , N_i and P_i are recorded. This fact allowed us to perform on-line checks of the data as will be explained shortly.

We can define several quantities which allow us to write III-8 in a more compact form:

$$\langle P \rangle = \sum_i M_i P_i / \sum_i M_i, \quad (\text{III-9})$$

$$\langle P^2 \rangle = \sum_i M_i P_i^2 / \sum_i M_i. \quad (\text{III-10})$$

Then define

$$Q_i = P_i - \langle P \rangle \quad (\text{III-11})$$

so that

$$\langle Q^2 \rangle = \langle P^2 \rangle - \langle P \rangle^2. \quad (\text{III-12})$$

Thus we can write III-8 as

$$A = \frac{\epsilon}{1 - \langle P \rangle \epsilon} \equiv \mathcal{K} a, \quad (\text{III-13})$$

where

$$\epsilon = \frac{\sum_i N_i Q_i}{\langle Q^2 \rangle \sum_i N_i}. \quad (\text{III-14})$$

Note that the quantity $\mathcal{K}$ appears as a factor in the expression for A, Eq. III-13. $\mathcal{K}$ is exactly the factor one would expect to have to use to scale up an asymmetry from a target made up of

polarized protons and unpolarized background. Indeed from III-5 and using III-4 we see that

$$\mathcal{K} = 1 + \eta = \frac{I_0 + \eta I_0}{I_0} = \frac{H_i + B_i/M_i}{H_i}. \quad (\text{III-15})$$

For instance, if the target were composed of 1/2 polarizable protons and 1/2 unpolarizable protons then $\mathcal{K}$ would be 2 and any asymmetry seen from this target would have to be multiplied by 2 to give the asymmetry from polarized protons. The procedure for measuring $\mathcal{K}$ in this experiment is described in Section III-D.

More specific to the running sums calculation of the asymmetry is the fact that in the calculation one must form $\langle P \rangle$ (Eq. III-9) and $\langle P^2 \rangle$ (Eq. III-10). These quantities are valuable to know during the course of data taking, $\langle P \rangle$ because one wants to make it small meaning that the same amount of beam passed through positive as well as negative target polarization, and $\langle P^2 \rangle$ the square root of which indicates the average magnitude of target polarization. The computer used during data taking allowed us to make these calculations and display the results continuously. Several errors in the reading of the monitor counters were discovered by sudden large discontinuities in $\langle P \rangle$. Also from Eq. III-13 one can calculate the error in A due to M_i , N_i , and P_i . Because of the on-line computing capacity we could calculate the error in A due to the statistical error in N_i (or more properly, $\sum_i N_i$) and in fact we determined the amount of time spent at any one kinematic condition by the size of the error in A.

B. Hydrogen Event Isolation

The discussion so far on the calculation of A by the running-sum technique has assumed for simplicity one pion counter and one monitor. In reality there were on the order of one hundred counters in the detection system, but we did not keep the running sums

for all of these. As described in Section II-D, we calculated a missing mass for each pion event. The spectrometer acceptance in momentum $\frac{\Delta p}{p}$ is nominally $\pm 2\%$ and the resolution is nominally 0.5%, which gives forty distinct momentum bins in which to accumulate events. Many of the edge bins in both angle and momentum have poor efficiency for well-defined particle signature or have rapidly varying solid angles and are not desirable in the event accumulation. Discarding these edge bins reduces the number of missing-mass bins to twenty. Thus for every pion identified in the "active area" of the hodoscope, we register a count in one of twenty software counters corresponding to the missing mass calculated for that pion. These twenty counters now take on the roles of the "pion counter" in the discussion of the running sums above, and we likewise maintain the running sums, $\sum_i N_i P_i$, for all twenty missing mass bins. In addition to this, since the position of the sweeping beam was recorded with each event, we also recorded separately the events from the top, middle, and bottom thirds of the target. This was done because the missing-mass calculation depends critically on the vertical positioning of the beam, and the one-inch vertical sweep of the beam actually spanned three missing-mass bins at one angle and momentum at 16 GeV. Even though the position of the beam was recorded with each event, the beam monitors could not be kept for each position of the beam, so the beam monitors and missing-mass counters were kept separately for the three vertical target regions. Actually this was not essential, since we did know how to correct each event for vertical beam position, but by keeping the running sums for the three target regions we were able to calculate the asymmetry independently for each region and check the asymmetries calculated for the three regions were consistent. In every case they were.

The spectrometer acceptance and resolution are fixed fractions of the central momentum, and so the missing-mass accept-

ance and resolution are fixed fractions of the central missing mass (see Eq. II-11). At photon energy, k , of 5 GeV the twenty missing-mass bins spanned are from 0.95 GeV^2 to 1.25 GeV^2 , in steps of 0.15 GeV^2 . At k of 16 GeV the missing-mass bins ranged from 0.60 GeV^2 to 1.60 GeV^2 in steps of 0.50 GeV^2 . Now the neutron step occurs at 0.88 GeV^2 and the Δ^0 step occurs at 1.54 GeV^2 .

The task at hand is to identify those events from the reaction $\gamma p \rightarrow \pi^+ n$ from the events from $\gamma p \rightarrow \pi^+ \Delta^0$. From the discussion in Section II-D we see that any event with $M_x^2 < 1.54 \text{ GeV}^2$ must have come from $\gamma p \rightarrow \pi^+ n$. Actually the width of the Δ^0 and the finite resolution of the spectrometer system smears the Δ^0 step from hydrogen such that one must limit his M_x^2 to less than 1.25 GeV^2 to avoid Δ^0 events from hydrogen (see Fig. 17).

It is true that the Fermi motion of the protons in the heavy nuclei will cause the Δ^0 step from these protons to extend to lower M_x^2 , but since these events should have no asymmetry we will not exclude them from our asymmetry calculation. (We must correct for the dilution effect of these events, of course.) Thus we chose $M_x^2 = 1.25 \text{ GeV}^2$ as the upper limit for identifying pion events from $\gamma p \rightarrow \pi^+ n$. Of course this means that we are counting events caused by photons of energy less than the cutoff energy, but a photon which creates a neutron of apparent $M_x^2 = 1.25 \text{ GeV}^2$ has energy only 1% less than the cutoff energy at 16 GeV. Thus we will continue to call these photons 16-GeV photons. At 5 GeV the 1.25-GeV^2 neutrons came from 3% less energetic photons but again we will call these photons 5 GeV. [Actually we helped ourselves at both energies by setting the bremsstrahlung end point (electron beam energy) about 1/2% higher than 16 GeV and about 1% higher than 5 GeV. This meant that the energy that we quote for the photon is almost exactly the average energy of photons producing events we accept.]

There is no need to set a lower limit on the missing mass of the neutrons from the standpoint of confusions with unwanted

events. But a lower limit is forced upon us by the hydrogen correction factor, $\mathcal{K}$. This comes about because the Fermi motion in the heavy nuclei will smear the neutron step to lower missing mass than can be achieved by the free protons, so in Eq. III-15 one has the numerator finite and the denominator infinitesimal and $\mathcal{K}$ becomes meaningless. A rather conservative limit of 1.05 GeV^2 was originally chosen as the lower limit for M_x^2 . This limit was in use by our collaborators at SLAC and was used because at this M_x^2 one was up on the neutron step, where a good average height could be determined.²⁶ We relaxed this limit slightly at 16 GeV (see Section III-D for details) but for the moment 1.05 GeV^2 is good enough.

Thus we have two limits on the missing mass, the upper limit, $M_x^2 = 1.25 \text{ GeV}^2$, is set by our desire to avoid $\gamma p \rightarrow \pi^+ \Delta^0$ events, and the lower limit, $M_x^2 = 1.05 \text{ GeV}^2$, is set by the necessity to have a reasonably smooth hydrogen correction factor. A pion with a missing mass between these two limits can be considered safely to have come from the reaction $\gamma p \rightarrow \pi^+ n$. Since we calculate the asymmetry missing-mass bin by missing mass-bin, we have to average the asymmetries between our chosen limits in order to calculate the asymmetry in the reaction.

C. False Asymmetries

In the running-sum technique there arises the problem of deciding on the length of the counting interval. Because of the running sums, $\sum_i N_i P_i$, $\sum_i M_i P_i$, and $\sum_i M_i P_i^2$, the interval should be chosen so that the P_{i_1} , the target polarization, represents the average target polarization during the interval. If the target polarization actually did not fluctuate at all, the interval would last for the length of time that the target remained polarized with one sign of polarization. However, since this time was typically four minutes and the times over which the target could change its polar-

ization were of the order of seconds, the basic time interval of the experiment, called the subrun, was taken to be about 10 sec. One subrun actually consisted of an integral number of complete sweeps of the target by the beam and was started and stopped by the raster interrupt. The running sums then were added to at the end of each subrun, usually about five to ten sweeps of the target.

The next basic unit of time in the experiment was the length of time that the target was left in one state of polarization, called a sign run. In order to minimize effects due to possible drifts in beam intensity, monitor stability, magnet currents, etc., we wanted to change the polarization often. But, the process of changeover takes from 20 to 40 sec, and so one is being quite inefficient if he is initiating a change of polarization every minute or two. We chose to change the polarization about every five minutes, meaning that we were spending about 10% of the running time actually changing the polarization. Now comes the observation that makes the running sum calculation of the polarization especially valuable: the asymmetry is being calculated by a continuous accumulation of running sums, each being added to at the end of a 10-sec subrun. Many subruns comprise a sign run which corresponds to about four minutes of one sign of target polarization, which is alternated about every five minutes. If one were to artificially alternate the signs of the polarization readings on every subrun, then the asymmetry still could be calculated as before. The value should be zero for the asymmetry calculated in this manner, as the actual target polarization and thus the counting rate were not changing while we artificially changed the sign of the polarization in the running sums. With this concept one can construct many different patterns of these "false asymmetries" that can be calculated. To keep the false asymmetries to a manageable number, we defined two more quantities: a set (several subruns made one set and four sets made one sign run) and a four sum (4 sum), which

was comprised of four sign runs (2 positive and 2 negative). Thus the following false asymmetries were simultaneously calculated along with the real asymmetry:

Asymmetry	Sign Run 1				Sign Run 2				Sign Run 3				Sign Run 4				4Sum Sign Run Set
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	
Real	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	
Also real	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	
False 1	#	#	-	-	-	-	#	#	+	+	-	-	-	-	+	+	
False 2	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	
False 3	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	
False 4	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	
False 5	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	

Fig. 18. Sign of target polarization in various running sums.

The calculations of the false asymmetries gave us a measure of something whose value we knew (namely they should be zero) and provided us with a check on how consistently we could measure this. The real purpose of the false asymmetries is to give us an explicit measure of the random errors within the equipment. A changing gain or the presence of noise, while not necessarily changing systematically the results of the experiment, could still lead to random errors greater than those calculated on the basis of counting statistics only. The fact that our false asymmetries showed deviations from zero quite consistent with statistical fluctuations gives us confidence that the random errors in the whole experiment are representable by the standard statistical errors.

The real and the five false asymmetries are shown in Fig. 19, for the data taken at both 5 and 16 GeV and at each value of

Experimental asymmetry (not corrected for Hydrogen fraction)

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Fig. 19. Real ($\circ$) and False ($\blacksquare$) Asymmetries.

momentum transfer. The false asymmetries are seen to be statistically consistent with zero and so we avoid the problem of discovering why they are not! Also the simultaneous display of the real and false asymmetries gives us confidence that we were detecting a true counting asymmetry and not just measuring a built-in bias of our system to measure a non-zero real asymmetry.

With a clearly visible real asymmetry, we have now only to calculate the hydrogen correction factor to have the asymmetry parameter, $A(t)$.

D. Hydrogen Correction Factor

In this experiment we detected only one final-state particle, a positive pion, and ignored the conjugate baryon state. This together with the fact that the beam was not monoenergetic meant that the kinematics of the event sample were not well enough specified to cause the events from hydrogen in the target to stand out clearly from the events from the heavy nuclei. As an example of this let us compare the photoproduction case with a recent experiment using the polarized target in measuring the asymmetry parameter in $np \rightarrow pn$.²⁷ In this experiment, as in the photoproduction, the beam was not monoenergetic. But each event was well constrained since the angle and momentum of the final state proton as well as the angle (and roughly the energy) of the final-state neutron were measured. In addition final states containing more than one charged particle and one neutron were discarded before analysis. Thus it was possible to require that every event satisfy precise two-body kinematics before being accepted in the analysis. An example of the procedure used to select hydrogen events in Fig. 20. Here is plotted for one final proton scattering angle, the number of neutrons vs neutron angle. The events from heavy nuclei will not exhibit unique neutron angles since the nucleons in the heavy nuclei are in motion (Fermi motion). However the events

Fig. 20. Elastic Hydrogen
Event Contribution
in Well-Constrained
Case

from hydrogen will fall at a unique angle and separation of the events is straightforward. The shaded area represents the events from hydrogen.

Now consider the photoproduction case where we measure only the momentum and angle of the pion. The missing-mass spectrum of events measured in our system is explained in Section II-E. The spectrum from the butanol target will consist of the superposition of a step from the hydrogen and a step from the heavy nuclei. The hydrogen step is more sharp than the alcohol step (this corresponds to the sharp hydrogen peak compared to the rather flat background in the $np \rightarrow pn$ experiment) but this difference in the shape of the steps is not distinct enough to allow separation of the events on this basis. This can be seen in Fig. 21 where the hydrogen contribution to the alcohol spectrum has been shaded in the missing mass spectrum.

If the spectra cannot be separated by fitting, a different method of determining the dilution factor must be devised. Recall that the factor we are trying to measure is the number of pions per unit monitor from the entire target divided by the number of pions per unit monitor from just the hydrogen in the target. If we write

Fig. 21. Hydrogen Contribution in Poorly Constrained Case
(Bremsstrahlung Beam-Single Arm Spectrometer)

the number of pions per unit monitor from the hydrogen as h and the number of pions per unit monitor from the butanol target (hydrogen and background) as $h + b$, then the hydrogen correction factor is

$$\mathcal{C} = \frac{h + b}{h} . \quad (\text{III-16})$$

We actually can estimate this number from known quantities, and it is useful to do this to give a feeling for the size of correction to be made. The fraction of hydrogen in butanol is 14%, but when the target is considered in position in the cavity the fraction of hydrogen to nearby materials as viewed by the beam is about 10%. Thus we can estimate that

$$\mathcal{C} = \frac{h + b}{h} \text{ is roughly } \frac{10\% + 90\%}{10\%} \rightarrow \frac{10 + 90}{10} . \quad (\text{III-17})$$

Now we observe that in the heavy elements only the protons can make positive pions so that the number 90 goes to 45. Then we

note that the heavy nuclei will absorb pions created inside them¹⁴ and so reduce the effective number of protons in the heavy nuclei by about 2/3. So we have the estimation:

$$\mathcal{K} \text{ is roughly } \frac{10+90}{10} \xrightarrow[\text{protons only}]{} \frac{10+45}{10} \xrightarrow[\text{absorption}]{} \frac{10+30}{10} = 4. \quad (\text{III-18})$$

We actually observed $\mathcal{K}$ to be roughly 3.8 at most momentum transfers, so our estimate was very close.

In order to measure $\mathcal{K}$, we had to measure two quantities: the number of pions per unit monitor from both the butanol target itself and the hydrogen in the target. These quantities can vary with momentum transfer, so we will start now to include t in the expressions for $\mathcal{K}$:

$$\mathcal{K}(t) = \left[\frac{(\pi \text{ counts/monitor})_{\text{Butanol target}}}{(\pi \text{ counts/monitor})_{\text{Hydrogen in target}}} \right]_{\text{both at same } t} \quad (\text{III-19})$$

Since we know the chemical composition of the target, we can measure, by weighing the target, how much hydrogen is in a target. Thus we can calculate, knowing the beam intensity, target area, solid angle of detector, and hydrogen in target, a quantity much like a cross section:

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{butanol}} = \frac{\pi \text{ counts from butanol target}}{(\text{beam on target}) \times (\text{solid angle}) \times \frac{\text{hydrogen in butanol target}}{\text{target area}}} \quad (\text{III-20})$$

The actual photoproduction cross section from a hydrogen target can be written similarly:

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{hydrogen}} = \frac{\pi \text{ counts from hydrogen target}}{(\text{beam}) (\text{solid angle}) \frac{\text{hydrogen}}{\text{area}} \text{ in target}} \quad (\text{III-21})$$

The hydrogen cross section of course has been measured and by combining equal amounts of positive and negative polarized target

running we can calculate $\left(\frac{d\sigma}{d\Omega}\right)$ butanol. The quotient of these two quantities is $\mathcal{K}(t)$:

$$\mathcal{K}(t) = \frac{\left(\frac{d\sigma}{d\Omega}\right)_{\text{butanol}}}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{hydrogen}}} \quad (\text{III-22})$$

However, there is a problem here and it is concerned with the beam flux on the target in the two measurements. In the hydrogen cross section measurement the hydrogen target fully intercepted the beam and so the product of beam flux times number of target hydrogen is simply total beam times target hydrogen per unit area. The beam flux on the polarized target is a different matter, however, since we intentionally sweep the beam beyond the limits of the target so that the target is uniformly illuminated. This means that some of the beam does not encounter any target at all and so the effective flux of the beam on the target is more difficult to calculate than in the hydrogen-target case. In fact we felt that the correction of the flux on the butanol target to the equivalent flux on a hydrogen target to be uncertain enough that we essentially remeasured the hydrogen cross section but with the time-honored " CH_2 " minus C method. The hydrogen cross section measured by this subtraction method might be expected to differ from the standard hydrogen cross section, but because the difference is in the beam flux we expected the ratio of the two measurements of cross sections to be a constant independent of the momentum transfer.

The essential feature of the subtraction measurement was attempting to match the butanol target running conditions as closely as possible. We first prepared a polyethylene block which was 2.54 cm square on the face presented to the beam. Its length along the beam was 1.92 cm, chosen to match this block in radiation lengths to the 3.8-cm butanol targets. A block of carbon was prepared with the same cross sectional area and length along the beam

of 0.91 cm, a length chosen to provide the same amount of carbon as was present in the polyethylene block. We then collected data with the polyethylene target mounted in the microwave cavity just as the butanol target would be, with the spectrometer and monitors in the same conditions as butanol running, and with the same beam sweeping and collimation as used with the butanol target running.

Then under these same conditions data with carbon were taken and, because of the preparation of the carbon target, subtracted from the polyethylene just as "target empty" data would be. By providing the program with the proper hydrogen density we were able to calculate by this subtraction procedure a hydrogen cross section.

Data were taken at five values of momentum transfer at 16 GeV and four values of momentum transfer at 5 GeV. The ratios of these cross sections to the published hydrogen cross sections are displayed in Fig. 22. At 16 GeV the data are certainly consistent with the value, 1, and at 5 GeV the data are also consistent with 1. The ratio of 1 that we see actually tells us that we are making a reasonable measurement of the flux in the experiment. Since we expected that the ratio was going to be a constant we did not attempt to get spectacular statistics on any one point, but contented ourselves with more points and hoped for consistency. Actually the 5-GeV ratios could be better fit by a sloped line, but we really believe that at each momentum transfer we are measuring the same ratio and so we attempt to show consistency with a constant ratio.

With the ratios described above we feel comfortable in using the published hydrogen cross sections in the denominator of Eq. III-22. The numerator, as mentioned above, is properly calculated in runs with even numbers of sign runs. Thus we are able to calculate $\mathcal{H}(t)$. Figure 23 shows the results of this calculation. As mentioned above in the rough calculation of $\mathcal{H}(t)$ the values of

$\frac{d\sigma}{dt}$ from $\text{CH}_2\text{-C}$ vs. published $\frac{d\sigma}{dt}$ H

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Fig. 22. $\text{CH}_2\text{-C}$ Cross Sections from this Experiment. (E-47)² Compared with Hydrogen Cross Sections.

Hydrogen Correction $\mathcal{H}(t) = \left(\frac{d\sigma}{dt} \right)_{\text{Alcohol}} / \left(\frac{d\sigma}{dt} \right)_H$

XBL 707-1440

Fig. 23.

$\mathcal{K}(t)$ lie mostly between 3.5 and 4.0. The noticeable dip at 5 GeV in $\mathcal{K}(t)$ in the forward direction ($\sqrt{-t} \leq 0.3$ GeV) is not surprising; it is expected.

The explanation for this dip is "Pauli exclusion suppression" of the pion production from protons in the heavy nuclei. The suppression occurs when the neutron in the final state in the photoproduction reaction, $\gamma p \rightarrow \pi^+ n$, is produced with such low energy that it cannot escape the nucleus or be raised to an excited level. In this case the neutron left behind must make a home in a nucleus already containing neutrons and the exclusion principle prevents the states already occupied from accepting this new neutron. In order to understand the momentum transfers at which this is a problem, consider the expression for the momentum transfer calculated from the proton and neutron four-momenta:

$$t = (P_p - P_n)^2 = M_p^2 + M_n^2 - 2P_p P_n. \quad (\text{III-23})$$

For estimation we write $M_p^2 = M_n^2 = M^2$, $\vec{P}_p = 0$, and $E_n = M + T$ (T is the kinetic energy of the neutron) so that III-23 becomes

$$t = 2M^2 - 2M(M + T) = -2MT. \quad (\text{III-24})$$

Thus kinetic energy of the neutron produced is

$$T = \frac{-t}{2M} \sim \frac{1}{2} (-t \text{ in GeV}^2) \text{ GeV}. \quad (\text{III-25})$$

The neutrons in carbon have energies of roughly 20-30 MeV so that for momentum transfers in the range $-t \leq 0.04 - 0.06 \text{ GeV}^2$ ($\sqrt{-t} \leq 0.2 - 0.25 \text{ GeV}$) we might expect to see the suppression, and we do.

It is slightly surprising that the effect is not also prominent at 16 GeV, but the forward point at 16 GeV on Fig. 23 can be argued to be consistent with a forward dip, lying one standard deviation higher than expected.

There is one final subject to be considered in the discussion of the hydrogen correction. This comes up because the spectrometer at 16 GeV spans a much wider range of missing mass than at 5 GeV, and so a major portion of the events collected is discarded in making the missing-mass cuts described in Section III-B. In an attempt to gain more counts we wanted to relax the lower M_x^2 cut from 1.05 GeV^2 . But in doing this we are introducing a wider variation in the values of $\mathcal{K}(t)$. In order to understand this it is only necessary to realize that both the differential cross sections from the butanol and hydrogen are known missing-mass bin by missing-mass bin, and so the hydrogen correction factor can also be calculated missing-mass bin by missing-mass bin. Figure 24 shows the cross sections for the butanol target and hydrogen on the same missing-mass scale along with their quotient, the hydrogen correction. The variation of $\mathcal{K}(t)$ as a function of missing mass is clearly seen and the sharp rise in $\mathcal{K}(t)$ for low missing mass is to be noted. This rise is due to the fact that the butanol spectrum extends to lower missing mass than the hydrogen due to the Fermi motion of the protons in heavy nuclei.

Since the experimental asymmetry (a in Eq. III-13) is calculated bin by bin also we can calculate the asymmetry $A(t)$ in Eq. III-13 bin by bin using the hydrogen correction in Fig. 24. From the bin-by-bin asymmetry we get the final asymmetry by averaging from the missing mass corresponding to the lowest point on the bin-by-bin $\mathcal{K}(t)$ curve in Fig. 24 to the upper limit in missing mass squared of 1.25 GeV^2 . This resulted in an increase of about 50% in the number of events included in the final asymmetry at each momentum transfer at 16 GeV, and resulted in a reduction of the absolute size of the statistical errors by about 0.04 (roughly 25% of themselves). ~~One~~ would think that a 50% increase in the number of events by any means would be desirable, but here this is not the case. The problem is an increase in the systematic errors brought about because of the bin-by-bin hydrogen correction which almost

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Fig. 24. Bin-by-Bin Calculation of Hydrogen Correction.

cancels the gain in statistics. This systematic error is discussed more fully in Section III-E.

In conclusion the hydrogen correction is calculated by dividing an effective cross section from butanol by the hydrogen cross section. At 5 GeV each of these quantities was averaged over the missing mass squared range of 1.05 GeV^2 to 1.25 GeV^2 before calculating $\mathcal{H}(t)$ and then used to calculate $A(t)$ with a similarly averaged $a(t)$ (see Eq. III-13). We can write (the average is over missing mass)

$$5 \text{ GeV} \quad A(t) = \frac{\left[\frac{d\sigma''(t)}{d\Omega} \text{ butanol} \right]}{\left[\frac{d\sigma(t)}{d\Omega} \text{ hydrogen} \right]} * \overline{a(t)} . \quad (\text{III-26})$$

At 16 GeV, $A(t)$ was calculated one missing-mass bin at a time and then averaged over a range of missing mass squared whose lower limit was determined by the shape of the hydrogen correction and whose upper limit was the canonical 1.25 GeV^2 :

$$16 \text{ GeV} \quad A(t) = \left[\frac{\frac{d\sigma''(t)}{d\Omega} \text{ butanol}}{\frac{d\sigma(t)}{d\Omega} \text{ hydrogen}} * a(t) \right] . \quad (\text{III-27})$$

E. Errors

The major sources of systematic error in the experiment can be identified in the following list:

- 1) Target polarization,
- 2) Unlike sampling of target by beam and NMR,
- 3) False asymmetries,
- 4) Contamination by $\gamma p \rightarrow \pi^+ \Delta^0$ events,
- 5) $\mathcal{H}(t)$ missing-mass-bin misidentification,
- 6) Overall monitor problems.

Consider each of these categories and the contributions they make.

1. Target Polarization

The target polarization is calculated under the following assumption: $P = C \cdot \text{Area}$. (Area is the area described in Section II-C). The calculation is then:

$$P_{\text{enhanced}} = \frac{(\text{Area})_{\text{enhanced}}}{(\text{Area})_{\text{TE}}} P_{\text{TE}} \quad (\text{III-28})$$

The errors in this calculation will enter through the assumption of linearity of the area with polarization and the measurement of the thermal equilibrium (TE) polarization. The TE polarization is given by

$$P_{\text{TE}} = \tanh \frac{\mu_p H}{kT} \approx \frac{\mu_p H}{kT} = \frac{h\nu}{2kT} \quad (\text{III-29})$$

We know ν to the parts-per-million level and so there is essentially no error contributed there. We measure the target temperature with carbon resistors and we have observed them to agree with vapor pressure measurements of the temperature to within 2%. Here is a 2% uncertainty in the TE polarization.

The assumption of linearity is a more difficult question, as several elements enter the problem. A block diagram serves to illustrate these elements.

Working backwards from the computer to the digitizer, we have evidence that this device contributes error only through the last-count or round-off error in the 24-bit binary scaler. The test for

this effect is not easy as one requires test pulse trains which are compatible with the unit and stable themselves to a level lower than the unit under test. The test we performed was merely to run the digitizer in the "frequency table mode" (see Section II-C) where it was reading the frequency of the 1260 oscillator (105 MHz) by means of an internal 10-kHz clock-generated gate. The stability on the reading of this frequency over 10-msec intervals was the round-off in the scaler, $\pm 1/2$ count in one million. Thus we have only to consider round-off in digitization error and this is only a problem in the TE measurement where the round-off error is a larger fraction of the area representing a TE signal. Even there the round-off error is only about 1/3%.

The Voltage-to-Frequency Converter is a commercial model (Hewlett-Packard 2212) with some modifications to reduce the 60 Hz ripple that we observed when originally purchased. This ripple, originally about one part in 10^4 of the output, was reduced about a factor of 10. Deviations from linearity of the output from the input have been measured to be about one part in 10^5 over the entire input range and since we employ only about 10% of the range in measuring the polarization this is not a source of error. There is a zero offset of about 3 parts in 50,000 but this only contributes an error in the TE signal of one part in 10^4 of the TE signal itself, again a completely negligible correction.

The RF Amplifier and Diode Detector section has linear response in both amplitude and frequency. This response can be written in terms of the input rf voltage amplitude V_{in} and the detector dc output voltage V_{out} :

$$V_{out}(\omega) = [\alpha + \beta V_{in}(\omega)] [1 + \gamma (\frac{\omega}{\omega_0} - 1)]. \quad (III-30)$$

The amplitude of the rf voltage into the amplifier has a slight linear increase as a function of frequency. Thus in the absence of a polarized signal we have

$$V_{in}(\omega) \equiv a + b\omega. \quad (III-31)$$

The polarized protons will alter this signal in a way that we can write as

$$V_{in}(\omega)_{polarized} = \frac{V_{in}(\omega)}{1 + x(\omega)P_T}. \quad (III-32)$$

Recall that a point to be added into the sum representing the polarized area is calculated from four readings, with the contribution from the polarized protons appearing only in the first term on the right-hand side,

$$p(\omega_1) = \frac{2}{V_{out}(\omega_1)} - \frac{1}{V_{out}(\omega_1 + \Delta)} - \frac{1}{V_{out}(\omega_1 - \Delta)}. \quad (III-33)$$

Combining Eqs. III-30, III-31, and III-32 with some obvious contraction in notation and substituting in III-33, we get

$$P(\omega_1) = \frac{2(1+x(\omega_1)P_T)}{[\alpha' + \beta'\omega_1][1 + \gamma(\frac{\omega_1}{\omega_0} - 1)]} - \frac{1}{[\alpha' + \beta'\omega_1 + \beta'\Delta][1 + \gamma(\frac{\omega_1}{\omega_0} - 1) + \gamma\frac{\Delta}{\omega_0}]} - \frac{1}{[\alpha' + \beta'\omega_1 - \beta'\Delta][1 + \gamma(\frac{\omega_1}{\omega_0} - 1) - \gamma\frac{\Delta}{\omega_0}]} \quad (III-34)$$

Using the fact that $\gamma(\frac{\omega_1}{\omega_0} - 1)$ and $\gamma\frac{\Delta}{\omega_0}$ are small ($\sim 1\%$) and $\frac{\beta'\Delta}{\alpha' + \beta'\omega_1}$ is small ($\sim 1\%$) we can rewrite III-34 as

$$P(\omega_1) \approx \frac{2(1+x(\omega_1)P_T)}{\alpha' + \beta'\omega_1} [1 - \gamma(\frac{\omega_1}{\omega_0} - 1)] - \frac{1}{\alpha' + \beta'\omega_1} \left\{ (1 - \frac{\beta'\Delta}{\alpha' + \beta'\omega_1}) [1 - \gamma(\frac{\omega_1}{\omega_0} - 1) - \gamma\frac{\Delta}{\omega_0}] \right\} - \frac{1}{\alpha' + \beta'\omega_1} \left\{ (1 + \frac{\beta'\Delta}{\alpha' + \beta'\omega_1}) [1 - \gamma(\frac{\omega_1}{\omega_0} - 1) + \gamma\frac{\Delta}{\omega_0}] \right\} = \frac{2}{\alpha' + \beta'\omega_1} \left\{ x(\omega_1)P_T [1 - \gamma(\frac{\omega_1}{\omega_0} - 1)]^2 - (\gamma\frac{\Delta}{\omega_0})(\frac{\beta'\Delta}{\alpha' + \beta'\omega_1}) \right\}. \quad (III-35)$$

Now using the fact that $\frac{\beta'\omega_1}{\alpha'}$ is small ($\sim 1\%$) we have

$$p(\omega_1) \approx \frac{2}{\alpha'} \left\{ x(\omega_1)P_T [1 - \gamma(\frac{\omega_1}{\omega_0} - 1)] (1 - \frac{\beta'\omega_1}{\alpha'}) - (\gamma\frac{\Delta}{\omega_0}) \frac{\beta'\Delta}{\alpha'} (1 - \frac{2\beta'\omega_1}{\alpha'}) \right\}. \quad (III-36)$$

Finally, uncontracting the notation to give back parameters in Eqs. III-30 and III-31, we get

$$p(\omega_1) \approx \frac{2}{\alpha + \beta a} \left\{ x(\omega_1) P_T \left[1 - \gamma \left(\frac{\omega_1}{\omega_0} - 1 \right) \right] \left(1 - \frac{\beta b \omega_1}{\alpha + \beta a} \right) - \left(\frac{\gamma \Delta}{\omega_0} \right) \frac{\beta b \Delta}{\alpha + \beta a} \left(1 - \frac{2 \beta b \omega_1}{\alpha + \beta b} \right) \right\}. \quad (\text{III-37})$$

Recall that ω_0 (and ω_1) are 105 MHz and that the sweep range of a is about 200 kHz. The frequency variation in Eq. III-30 is measured to be $\sim 3\%$ over the range 105-106 MHz so that $\gamma \frac{\Delta}{\omega_0} \sim 0.006$. The amplitude offset in Eq. III-30 is observed to be $\sim 1.5\%$ so $\frac{\alpha}{\beta V_{in}} \approx \frac{\alpha}{\beta a} \approx 0.03$. The frequency dependence in Eq. III-31 is about 1% over the whole sweep range so that we can write $\frac{b \omega_1}{a} \approx 0.002$. Thus we can write the maximum corrections into Eq. III-37:

$$p(\omega_1) \approx \left(\frac{2}{\alpha + \beta a} \right) [x(\omega_1) P_T (1 - 0.006)(1 - 0.002) - (0.006)(0.002)(1 - 0.004)] \\ \approx \left(\frac{2}{\alpha + \beta a} \right) [x(\omega_1) P_T (1 - 0.008) - 0.000012(1 - 0.004)]. \quad (\text{III-38})$$

The value of xP_T for enhanced polarizations is $\sim 10\%$ so the last term is completely negligible. The TE polarization is about $1/250$ the size of enhanced polarization, so that $(xP_T)_{TE} \approx 0.0004$. This means that the additive correction becomes more important but is still at the 2-3% level. We actually observe a more significant additive contribution to the TE signal in practice, the shape of which suggests that its origin is a non-linear correction which have not been considered here. As described in Section II-C this background is subtracted during the measurement of the TE signal. The error which is a factor of xP_T in Eq. III-38 actually contributes less than the 0.8% shown. This is seen by considering Fig. 25.

If we consider the amplifier gain variation and the carrier rf voltage increase linear in frequency, they will distort the natural polarization signal as shown, where the curves are normalized at the maximum. If the polarization signal is symmetric and the system error is linear, then to first order there is no correction to be made in the measured signal. Since the conditions of symmetry and

Fig. 25. Distortion of NMR Signal by Frequency- Dependent Gain

linearity are not exactly met, a correction could be made, but this will be less than 1%. For safety's sake we will call the error introduced by RF Amplifier and Diode Detector 0.01.

The last element in the target polarization flow block diagram is the detection circuit itself. The relation, Eq. II-10, has been derived relating $|\frac{1}{Z}|$ to the target polarization. The expression is correct to the extent that under integration the integral over the dispersive contribution, X' , vanishes. The accuracy of this assumption is hard to check in our system, but we believe it to be true to the 1% level. Other corrections in Eq. II-10 are of order 1/2 smaller than this or are 1/Q times the magnitude of X' or X'' smaller than the signal itself less than a 1% correction. It should be noted in passing that due to the constant current nature of the detection system that Eq. II-10 and Eq. III-32 are equivalent representations of the target polarization in terms of electrical quantities.

Finally collecting the errors of 2% (temperature), 1/3% (Digitizer), 1% (RF Amplifier and Diode Detector), and 1% (Input

Circuit) and combining in quadrature we get 2.5%. We arbitrarily double this to 5% to be sure to include incalculable errors of prejudice used in taking TE measurements, as some prejudice can enter in the decision about whether or not the target had fully reached thermal equilibrium and whether or not the cryostat had settled to a stable temperature.

2. Unlike Sampling of Target by Beam and NMR

The second systematic error source to be studied is the error introduced by the fact that the beam intensity and the rf field which is reading the target polarization are not distributed over the target in the same way. Ideally we would like a completely uniform distribution of beam and rf field over the target, but in practice this is not achieved.

In order to understand the error introduced by the unequal distributions of beam and rf field, consider a target containing two distinct equal-mass pieces 1 and 2, with beam intensity, ρ , target polarization P , asymmetry A , hydrogen counts for each sign of polarization $N_{\pm}$, and rf field intensity $\langle H^2 \rangle$ all subscripted to refer to each region. Then the asymmetry from the two regions, averaged by weighing based on the errors is given by

$$A_{TOT} = \left\{ \frac{A_1}{\delta A_1^2} + \frac{A_2}{\delta A_2^2} \right\} \frac{1}{\frac{1}{\delta A_1^2} + \frac{1}{\delta A_2^2}} \quad (III-39)$$

But the square of the error from one region is proportional to the total count from that region which is proportional to the beam intensity ($\delta A_i^2 \approx N_{i+} + N_{i-} \approx \rho_i$). Thus we can rewrite III-39:

$$A_{TOT} = \frac{\rho_1 A_1 + \rho_2 A_2}{\rho_1 + \rho_2} \quad (III-40)$$

The relationship seems quite reasonable, that the total asymmetry is proportional to the beam-averaged asymmetries from the two regions. Unfortunately we are unable to measure

A_i from the individual target regions. What we actually calculate for the asymmetry from the target made up of two pieces is

$$A_{\text{measured}} = \frac{1}{\bar{P}} \frac{(N_{1+} + N_{2+}) - (N_{1-} + N_{2-})}{(N_{1+} + N_{2+}) + (N_{1-} + N_{2-})}. \quad (\text{III-41})$$

The target polarization is sampled by the square of the rf field, or the average rf field intensity,¹⁸ so

$$\bar{P} = \frac{\langle H_1^2 \rangle P_1 + \langle H_2^2 \rangle P_2}{\langle H_1^2 \rangle + \langle H_2^2 \rangle}. \quad (\text{III-42})$$

Now we note that

$$A_i = \frac{1}{P_i} \frac{N_{i+} - N_{i-}}{N_{i+} + N_{i-}}. \quad (\text{III-43})$$

Using the relation $N_{i+} + N_{i-} \approx \rho_i$ again we can rewrite Eq. III-41 with the aid of Eqs. III-42 and III-43:

$$A_{\text{measured}} = \left[\frac{\langle H_1^2 \rangle + \langle H_2^2 \rangle}{\langle H_1^2 \rangle P_1 + \langle H_2^2 \rangle P_2} \right] \frac{\rho_1 A_1 P_1 + \rho_2 A_2 P_2}{\rho_1 + \rho_2}. \quad (\text{III-44})$$

The last observation to make is that the actual asymmetry A_i from each of the target regions is really controlled by the physics and should be the same for each region. Thus we factor out the actual asymmetry A (which we are trying to measure):

$$A_{\text{measured}} \bar{P} \left[\frac{\langle H_1^2 \rangle + \langle H_2^2 \rangle}{\langle H_1^2 \rangle P_1 + \langle H_2^2 \rangle P_2} \right] \left(\frac{\rho_1 P_1 + \rho_2 P_2}{\rho_1 + \rho_2} \right) A = \frac{(\bar{P})_{\text{Beam}}}{(\bar{P})_{\text{rf}}} A. \quad (\text{III-45})$$

We now generalize to a target made up of many small regions (of equal mass):

$$A_{\text{measured}} = \left[\frac{\sum_i \langle H_i^2 \rangle}{\sum_i \langle H_i^2 \rangle P_i} \right] \left[\frac{\sum_i \rho_i P_i}{\sum_i \rho_i} \right] A. \quad (\text{III-46})$$

This is the expected result as can be seen by two considerations: one, if the polarization is uniform throughout then any sampling will suffice and Eq. III-46 agrees; two, if the polarization is not uniform, but the beam and $\langle H_{rf}^2 \rangle$ are distributed identically then we measure the asymmetry, A, directly. Non-uniform target polarization and unlike distribution of beam and rf intensity will require a correction to be made. Since we know already that the target polarization is non-uniform due to beam damage we can consider the bracketed terms in Eq. III-46 to be a correction factor.

In evaluating this factor we can contract our notation by noting first that we will consider the number of terms in each of the sums in Eq. III-46 to be the same. Then if we define the quantities f_i^p and f_i^{rf} to be the ratios of ρ_i and $\langle H_i^2 \rangle$ to their respective averages we can write the correction factor, C, as

$$C = \left[\frac{\sum_i \langle H_i^2 \rangle}{\sum_i \langle H_i^2 \rangle P_i} \right] \left[\frac{\sum_i \rho_i P_i}{\sum_i \rho_i} \right] = \frac{\sum_i f_i^p P_i}{\sum_i f_i^{rf} P_i} \quad (\text{III-47})$$

We actually know the dependence of the target polarization on the dose to be roughly exponential. Then making the approximation that the photons convert in the target to ionizing particles linearly with the depth of the target we can write

$$P_i = P_0 \exp \left\{ -f_i^p \frac{x_i}{\ell} \alpha / \phi_0 \right\}, \quad (\text{III-48})$$

where x_i is the position of the i th target element along the beam direction, ℓ is the length of the target along the beam, f_i^p represents the fractional photon flux on the i th element (which is essentially constant along the beam direction in the target), ϕ_0 is the dose necessary to depolarize the element to $1/e$ of its original value, and α is the constant required to give a normalized dose in a

specified time at a specified point in the target. Before the beam comes on the target, the polarization is uniform throughout the target. The maximum nonuniformity occurs after the target has been irradiated, and the irradiation to which we submitted a target before annealing caused α/ϕ_0 in Eq. III-48 to be about 1. Thus for the maximum correction under the assumption of exponential dependence of the target polarization on dose and linear dose along the beam line can be written

$$C_{\max} = \frac{\sum_i f_i^p \exp(-f_i^p x_i/\ell)}{\sum_i f_i^{rf} \exp(-f_i^p x_i/\ell)} . \quad (\text{III-49})$$

We have beam intensity profiles from which we are able to calculate the quantities f_i^p across the face of the target. These measurements were performed by exposing glass slides as described in Section II-B with no target in place but with the normal beam sweeping and collimation. The glass slides were then measured by light transmission with a microdensitometer.²⁸ The profiles so obtained were used to obtain by direct calculation the ratios f_i^p in a plane perpendicular to the beam line. We assume, with negligible error, that this profile is valid at any position in the target along the beam line. The glass slide-densitometer system was checked for linearity and found to be linear well within the limits of the test (~10%). Beam profiles in the vertical and horizontal directions are shown in Fig. 26. The sensitivity of the measuring system is shown nicely in Fig. 27, where the glass slide from an actual target was measured by repeated parallel sweeps, each displaced from the previous sweep both on the slide and on the recording paper. Clearly seen on this perspective recording (Fig. 27) are a lead blob fastened to the cavity (large peak in center of target image) and the rib structure of the target itself (periodic rise and fall across each sweep).

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Fig. 26. Collimated Beam Profiles.

Fig. 27. Densitometer Tracing of Glass Slide of Polarized Target. Rib Structure of Target and Pb Positioning Droplet are Visible.

We do not have a good quality rf field intensity map. We have a field map made on a cylindrical cavity rather than a rectangular cavity and for the purposes of illustration we will consider this map. This map only gives the field intensity variation in the horizontal along a direction perpendicular to the beam. We will consider the variation to remain constant when moving vertically or along the beam direction. The rf profile and beam profiles are given in Table III.

One simplification of Eq. III-49 can be effected by the consideration of constant f_i^{rf} along the beam direction. This amounts to integrating the exponentials to yield

$$C_{\max} \approx \frac{\sum_i [1 - \exp(-f_i^{\rho})]}{\sum_i \frac{f_i^{rf}}{f_i^{\rho}} [1 - \exp(-f_i^{\rho})]} . \quad (\text{III-50})$$

The data of Table III yield a $C_{\max}$ of 0.99, or we can say that our measured asymmetry is systematically low by 1% if we believe the field map used. Since there are other systematic errors around much more significant than 1% we will let this number stand, even though it is poorly known.

One manifestation of a beam-rf sampling difference might be in a correlation of measured asymmetry values with magnitude of target polarization. We have investigated such a possible correlation by examining the distribution of measured asymmetries against the average magnitude of polarization for each run. No such correlation was seen but the poor statistical quality of asymmetries, from individual runs, prevents significant conclusions from being drawn. In addition, a large beam-rf sampling difference could be present without manifesting itself in a correlation of the sort mentioned here.

Table III. Beam and rf field intensity profiles (ratios with respect to average).

Target Element*	<u>Beam</u>		<u>rf Intensity</u>	
	Top to Bottom	Beam's Left to Right	Top to Bottom	Beam's Left to Right
1	0.92	0.85	1.0	0.87
2	0.99	0.93	1.0	0.92
3	1.03	1.00	1.0	1.00
4	1.04	1.05	1.0	1.02
5	1.05	1.06	1.0	1.04
6	1.05	1.05	1.0	1.05
7	1.04	1.02	1.0	1.03
8	1.00	0.99	1.0	1.00
9	0.97	0.90	1.0	0.95
10	0.91	0.80	1.0	0.85
* 2.5 mm spacing				

3. False Asymmetries

The false asymmetries, described in Section III-C, were designed to be a measure of the random errors in the equipment. Such effects as drift of counter efficiency or drift or noise in a beam monitor could introduce random errors, even though they should not be able to introduce a systematic error since those effects would cancel out over a very long running period. Since a number of false asymmetries were calculated, their r.m.s. deviation from zero could be calculated with good accuracy. If their r.m.s. deviation from zero were larger than that expected from the calculation based on counting statistics then we would know there were sizeable random errors from other sources, and the accuracy claimed could be accordingly adjusted. It is gratifying to note that the false asymmetries were consistent with zero, showing 0.80 times the deviation from zero expected from counting statistics alone. Thus, we are able to say with confidence that the random errors in the experiment are very closely given by the statistical errors alone.

The false asymmetries can in some cases detect small systematic effects. One can invent time patterns of, say, beam intensity that will produce non-zero false asymmetries but will not affect the real asymmetry. Such an effect involving an unexpectedly long electronic dead time and a systematic increase in beam intensity during a sign run was detected by means of the false asymmetries during the inelastic electron scattering experiment immediately preceding this photoproduction experiment.¹³

4. Contamination by $\gamma p \rightarrow \pi^+ \Delta^0$ Events

If the process $\gamma p \rightarrow \pi^+ \Delta^0$ itself has a non-zero asymmetry, the fact that we are slightly contaminated by these events could introduce some error into our results. To estimate this error consider the definitions:

$N_N(N_\Delta)$ number of neutron (delta) events per monitor from
free protons in target

$NR_N(NR_\Delta)$ number of raw neutron (delta) events per monitor
 from entire target
 $A_N(A_\Delta)$ asymmetry due to neutron (delta) events from free
 hydrogen in target
 $AR_N(AR_\Delta)$ asymmetry observed due to neutron (delta) events
 from entire target
 $NR_T(AR_T)$ ~~total~~ raw events per monitor (asymmetry) from
 both neutrons and deltas from the entire target.

Then the total raw asymmetry observed is

$$AR_T = AR_N \frac{NR_N}{NR_T} + AR_\Delta \frac{NR_\Delta}{NR_T} \quad (III-51)$$

We normally calculate, using the hydrogen correction:

$$(A_N)_{\text{calculated}} = \frac{NR_T}{N_N} AR_T = AR_N \frac{NR_N}{N_N} + AR_\Delta \frac{NR_\Delta}{N_N} \quad (III-52)$$

Now we note that

$$AR_\Delta = A_\Delta \frac{N_\Delta}{NR_\Delta}, \quad AR_N = A_N \frac{N_N}{NR_N} \quad (III-53)$$

Substituting in Eq. III-52, we get

$$(A_N)_{\text{calculated}} = A_N + A_\Delta \frac{N_\Delta}{N_N} \quad (III-54)$$

The next task is to obtain the free-hydrogen ratio of deltas to neutrons in our data. This was done by means of missing-mass steps used in the calculation of $\gamma p \rightarrow \pi^+ \Delta^0$ cross sections.¹¹ (An example of the curves used with $\gamma p \rightarrow \pi^+ n$ and $\gamma p \rightarrow \pi^+ \Delta^0$ missing mass steps is shown in this reference.) Examination of these steps within the missing-mass limits we were using showed that only at 16 GeV and only for $\sqrt{-t} \leq 0.4$ GeV was there any Δ^0 contamination, and there $N_\Delta/N_N \sim 0.03$. The maximum effect this could have is to change our actual quoted asymmetry by ± 0.03 for the three points with $\sqrt{-t} \leq 0.4$ GeV, and then only if the asym-

metry $A(t)$ in $\gamma p \rightarrow \pi^+ \Delta^0$ at these values of t was ± 1.00 .

5. $\mathcal{K}(t)$ Missing Mass Bin Misidentification

In Section III-D the bin-by-bin calculation of the asymmetry is described. It is also explained that this technique allowed a relaxation of the lower missing mass limit which resulted in an $\sim 50\%$ increase in analizable events and appropriate reduction of the errors by $\sim 25\%$. This gain was short-lived, however. It was called to our attention that slight drifts in the beam switchyard (electron beam momentum definition) and in the spectrometer can cause the photo-production missing-mass step from hydrogen to shift by something like half a missing-mass bin from one experiment to another.²⁹ This means that in calculating the bin-by-bin hydrogen correction there may be half a missing-mass-bin error in the association of butanol and hydrogen missing-mass spectra.

In order to test the possible effect of this mis-identification we artificially shifted the hydrogen step by one missing mass bin up and down before calculating the hydrogen correction and the asymmetry. This shift corresponds to more than the uncertainty in missing-mass bin misidentification but it can give us an idea of the sensitivity of the asymmetry to this kind of shift. Unfortunately this shift produced fairly large shifts in $A(t)$, about ± 0.09 in $A(t)$ itself on the average. Since we know we were testing with shifts about twice the expected maximum possible, we can say that in our data the maximum possible missing-mass bin misidentification can result in shifts of $A(t)$ itself of ± 0.04 . Note that this is well within the statistical errors on the asymmetry points. However this systematic error nearly offsets the gain in statistical error made by the bin-by-bin system of asymmetry calculation. Since the average 16-GeV asymmetry is around 0.50 we can say this systematic error averages 8% of the asymmetry itself at 16 GeV. The bin-by-bin approach was not applied to the 5 GeV data at all as it would result in an insignificant increase in the number of events and a significant

increase in the work.

6. Overall Monitor Problems

As described in Section II-B we employed three beam monitors, an SEQ, Cerenkov cell, and toroid monitor on the electron beam. A beam-absorbing calorimeter was used to calibrate these monitors. During this experiment we intercalibrated the monitors about three times per day and throughout the experiment the monitors agreed on the photon-beam intensity to about 1%. The overall systematic error in the monitor normalization is estimated to be around 7%.¹¹ This error enters through the hydrogen correction, the only point where absolute monitor normalization plays a role. Even here, if the system has an inherent systematic error present during both our polarized-target run and during the hydrogen cross section experiment, the fact that the hydrogen correction is a quotient of these means the effect of the error is reduced. Finally we can rewrite the list at the beginning of this section with the errors from each source included. (See Table IV.)

Combining in quadrature the errors for 5 and 16 GeV we have for 5 GeV, 9% and for 16 GeV, 12%. These errors are to be applied to the data in the form of factors, (1 ± 0.09) for 5 GeV and (1 ± 0.12) for 16 GeV.

Table IV. Systematic Errors.

Source	Systematic Error Contribution
1) Target polarization	5%
2) Beam-rf sampling difference	1%
3) False Asymmetries	0
4) $\gamma p \rightarrow \pi^+ \Delta^0$ contamination	16 GeV, $\sqrt{-t} \leq 0.4$ GeV ~ 5%
5) Missing mass bin shift	16 GeV only 8%
6) Overall monitors	7%

F. Consistency Checks

A few of the checks of the experimental system are worth mentioning here. The most important was concerned with calculating what we called the Miscellaneous Asymmetries. For these asymmetries we kept running sums for various counters that did not correspond to missing-mass bins. A total of ten miscellaneous-counter running sums were kept, the first three were the monitors themselves and the remaining seven were associated with various items in the trigger system, single counts in TR1, total event triggers, accidental triggers, and electron showers, to name a few. During the experiment these Miscellaneous Asymmetries were monitored and were observed to demonstrate no unexpected asymmetries. (Some would be expected to show asymmetries, total triggers and accidentals, for example, while others should show none, monitors and electron showers for example.)

One check we made on each target was to run at a so-called "standard angle." This was made possible by the great ease with which the 20-GeV/c spectrometer could be positioned in angle and adjusted in momentum. The actual asymmetries observed in these "standard" runs had such large statistical errors that only rough qualitative agreement could be checked, but the various counting rates of these runs gave us good checks on the state of health of the counting system.

During the course of the experiment we had the opportunity to observe the effect of the change of the relative directions of the target polarization and the normal to the scattering plane. At one point we moved the spectrometer across the beam line to "negative" angles, leaving the magnetic field at the polarized target unchanged. This flips the direction of the scattering plane normal and we observed the asymmetry measured at positive and negative angles to differ in sign. (At this time the program calculating the asymmetry was not aware of the sign of the scattering angle.) At another time we were forced to reverse the direction of the target magnetic

field and we again took data at a standard angle. This time the program was informed of the change and we observed the asymmetry measured to agree with the standard measurement.

Finally, during the background running with the polyethylene and carbon targets, we continued to accumulate data in running sums, entering a fictitious polarization into the calculation of these running sums. The results of the asymmetries from these background runs were always consistent with zero.

IV. RESULTS

A. Discussion

From the running sums described in Section III-A and the hydrogen correction described in Section III-D we are able to calculate the asymmetry parameters, $A(t)$. Data were taken at photon energies of 5 and 16 GeV, the 5-GeV energy chosen to allow comparison with experiments which may be done at other accelerators, and the 16-GeV energy chosen to coincide with the maximum energy usually studied by our SLAC collaborators. At 5 GeV we measured $A(t)$ for six values of invariant momentum transfer, t , corresponding to approximately equal intervals (~ 0.14 GeV = m_π) in $\sqrt{-t}$. At 16 GeV $A(t)$ was measured for five values of t , again with the spacing as at 5 GeV. The results are tabulated in Table V and displayed in Fig. 28.

The obvious feature of the data at both energies is a significant negative asymmetry, with the maximum value in both cases about -0.70. This asymmetry at 16 GeV is substantially larger than the asymmetry seen in $\pi^\pm p$, $K^\pm p$, pp , and $\bar{p}p$ elastic scattering at comparable energies.³⁰ Also the asymmetry does not vanish at $\sqrt{-t} \approx 0.8$ GeV (this is arguable at 16 GeV but is obvious at 5 GeV), which means in the traditional Regge pole approach that the ρ trajectory, whose contribution vanishes for $\sqrt{-t} \approx 0.8$ GeV, cannot be making a major contribution in this region.

For $\sqrt{-t} \leq 0.8$ GeV the 5-GeV data are rather featureless, that is, a straight line fits the data very well. This observation is of particular interest since a similar observation was made in a recent measurement of the asymmetry in $np \rightarrow pn$ with a polarized target.²⁷ The $np \rightarrow pn$ and $\gamma p \rightarrow \pi^+ n$ reactions have attracted attention recently as both show narrow peaks in their differential cross sections for $\sqrt{-t} < 0.14$ GeV. When referred to the same normal the asymmetries in the two reactions differ in sign, but

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Fig. 28. Asymmetry in π^+ Photoproduction with a Polarized Target.

Table V. Asymmetry in $\gamma p \rightarrow \pi^+ n$ with a polarized proton target vs. $\sqrt{-t}$ at photon energies of 5 and 16 GeV. The errors shown are statistical only, and the data should include a systematic error in the form of a factor 1.00 ± 0.09 for 5 GeV and 1.00 ± 0.12 for 16 GeV.

$\sqrt{-t}$ (GeV)	A(t)	$\sqrt{-t}$ (GeV)	A(t)
0.14	-0.18 ± 0.12	0.14	-0.25 ± 0.24
0.28	-0.34 ± 0.11	0.28	-0.72 ± 0.13
0.42	-0.38 ± 0.11	0.40	-0.66 ± 0.12
0.57	-0.56 ± 0.14	0.56	-0.59 ± 0.19
0.79	-0.71 ± 0.15	0.78	-0.36 ± 0.20
1.01	-0.40 ± 0.18		

the fact that both share the featureless behavior at similar energies suggests that the two processes share a common mechanism.

Turning our attention to the 16 GeV data we notice that they peak at $\sqrt{-t} \approx 0.3$ GeV. This is in contrast to the momentum transfer where the 5-GeV data peak, $\sqrt{-t} \approx 0.8$ GeV. The possibility of energy dependence is important as we will explore shortly, so a closer look at the data concerning this point is required.

The first test one can make is to check the hypothesis that the difference between the 5- and 16 GeV data is consistent with zero. The data can be compared at five points, and the χ^2 for the hypothesis of zero difference is 10 for 5 degrees of freedom. Thus this hypothesis has a small chance ($\sim 5\%$ confidence level) of being valid.

Another test one can make is to construct a curve that passes through the average of the 5- and 16 GeV asymmetries at each value of t and calculate the χ^2 for the hypothesis that this curve describes both asymmetries. We have not attempted actually to parametrize the resulting curve for reasons to be explained below, but we estimate a minimum of three parameter for this curve. This means 8 degrees of freedom and we calculated

the χ^2 to be again 10. The probability that this hypothesis is valid is more substantial ($\sim 25\%$ confidence level).

We have not attempted to construct the best curve specified by the lowest number of parameters which passes through the data. We felt this would be pointless without a model, and the construction of a model which fits a reasonably large amount of existing single-pion photoproduction data is a truly formidable task. We wish merely to point out that although to the eye the asymmetry data appear to differ at 5 and 16 GeV, the case is not so clear statistically. An average curve can fit the data to about 25% confidence level, but a model with some energy dependence of the asymmetry could result in a much lower χ^2 . The point is that our data can be argued to be consistent with no energy dependence, even though they appear to show such dependence to the eye.

Now a word about the attention devoted to the existence or absence of energy dependence. The differential cross section for the reaction $\gamma p \rightarrow \pi^+ n$ is well described for photon energies 5 GeV and above as a function of t only (a "universal function of t ") multiplied by $(s - M^2)^{-2}$. In other words, no energy dependence of the shape in t of the differential cross section is observed. This means the sum over g_i in Eq. A-19 (see Appendix) is independent of energy, which implies via Eqs. A-16, A-17, and A-20 that the magnitudes of the F_i all have the same, factorizable energy dependence. This has immediate consequences for the polarized-photon asymmetry as can be seen by inspection of Eq. A-29. Here we see that the polarized-photon asymmetry is composed not of interfering F_i amplitudes but of their moduli squared. Thus one expects that over the same range of energies where the differential cross section shows the so-called "universal-function-of- t " behavior, the polarized-photon asymmetry will show no energy dependence at all! (Whatever energy dependence the F_i possess factors out of the modulus squared of each term in both numerator

and denominator in Eq. A-29 leaving no energy dependence of Σ !) The only physics assumption in this prediction is crossing, and the approximations are those of the crossing matrix (Eq. A-17). The photon asymmetries, measured so far ($E_\gamma < 4$ GeV) have shown behavior consistent with no energy dependence, even though the cross sections below 5 GeV do not show such energy independent behavior. However the accuracy of the cross section measurements is such that they can show deviations from the "universal-function-of-t" behavior while the photon asymmetries of less precision may still be consistent with energy independence.

To close out this section we note that to the extent that the energy dependence of the unpolarized cross section suggests factorizable energy dependence in the moduli of the F_i , any energy dependence of the polarized-target asymmetry must come in the phases of the F_i .

B. Relation to Theory

In this section we wish to investigate the possible impact of our results on the theoretical efforts to understand high-energy photoproduction. Only two models have survived the storm of experiments of the past few years, the vector-meson-dominance model (VDM) and the Regge pole model modified by absorption or cuts.^{1, 5, 7} (It is to be understood that evaluations of the applicability of theoretical models or the validity of their results are not necessarily universally accepted and so a degree of bias on the part of the author is present in these remarks.)

The vector-dominance model, invented to explain nucleon form factor behavior,³¹ has been applied to single-pion photoproduction with a fair degree of success. The actual application is quite straightforward and relates single-pion photoproduction through time-reversal invariance to neutral-vector-meson production in pion-nucleon interactions. For VDM to be tested in a particular photoproduction reaction, companion experiments in

vector-meson production must be performed. For example in order to test VDM using our experiment, the recoil nucleon asymmetry in the two reactions $\pi^+ n \rightarrow \rho^0 p$ and $\pi^+ n \rightarrow \omega p$ must be measured. These difficult experiments have not been performed to date. Actually the most accessible test would be one involving two additional assumptions. First one assumes that in photoproduction the recoil nucleon polarization and polarized target asymmetry are identical. This is a reasonable assumption since the F_4 amplitude in Eqs. A-23 and A-26 are usually taken to be zero. (Only the A_1 meson can contribute to the F_4 amplitude in a Regge-pole-only model, and this amplitude is found to make a negligible contribution in most charged pion photoproduction analyses.¹⁾ Then we use the assumption of charge symmetry of the strong interactions to relate $\pi^+ n \rightarrow \rho^0 p$ to $\pi^- p \rightarrow \rho^0 n$ and $\pi^+ n \rightarrow \omega^0 p$ to $\pi^- p \rightarrow \omega^0 n$. Thus we have the sequence

$$A(\gamma p \rightarrow \pi^+ n) \xleftrightarrow{F_4=0} P(\gamma p \rightarrow \pi^+ n) \xleftrightarrow{\text{VDM}} A[\pi^+ n \rightarrow (\rho^0, \omega^0) p] \xleftrightarrow{\substack{\text{Isospin} \\ \text{Invariance}}} A[\pi^- p \rightarrow (\rho^0, \omega^0) n]. \quad (\text{IV-1})$$

Making the additional simplifying (but unfortunately not correct) assumption that the photon in VDM looks like a ρ^0 only (rho dominance) this author has calculated from expressions in the literature³² that:

$$A(\gamma p \rightarrow \pi^+ n) = 2A(\pi^- p \rightarrow \rho^0 n) \quad (\text{IV-2})$$

The difference in π^+ and π^- photoproduction cross sections indicates that one must include ω^0 as well as ρ^0 contributions in VDM calculations. However the difficulties of including both contributions in a prediction of the type IV-2 place that effort beyond the scope of this work. An interesting point here is that Eq. IV-2 suggests an asymmetry in $\pi^- p \rightarrow \rho^0 n$ of generally the magnitude of the other high-energy, purely hadronic asymmetries. The factor 2 in IV-2 is associated with the spin 1 of the ρ^0 , a consideration

not present in the other hadronic reactions mentioned.

A Regge-pole model with absorption-generated cuts by Jackson and Quigg predicted our 16-GeV results.³³ Jackson and Quigg have since pointed out the flexibility of their model and have stressed its ability to accommodate a wide range of asymmetries including the values presented here.¹ The model of Jackson and Quigg uses finite-energy sum rules (FESR) to provide the values of parameters in their absorptive-cut parametrization. The use of FESR requires that the parametrization (in this case, phase shifts) of the low-energy photoproduction data be well known. In fact this is not the case overall, and the great flexibility of the asymmetry in Jackson and Quigg can be traced to poorly known amplitudes in the low-energy processes. More experiments at both high and low energy are needed to clear up the ambiguities.

An observation can be made here about energy dependence in Regge-pole models with cuts. The model of Jackson and Quigg shows distinct energy dependence, the peak of the asymmetry growing in magnitude and moving to smaller momentum transfers as the energy is raised. Our data are consistent with the second of these trends. That this might be a general feature of Regge-cut models is supported by a similar energy dependence in an independent Regge-cut model of Krzywicki and Tran.³⁴ Only time and better experimental and theoretical results will resolve this issue.

C. Additional Data

In addition to the $\gamma p \rightarrow \pi^+ n$ data we have data of poor statistical quality on the reaction $\gamma p \rightarrow \pi^+ \Delta^0$, $\gamma p \rightarrow \pi^- \Delta^{++}$, and $\gamma p \rightarrow K^+ (\Lambda, \Sigma^0)$. These results are shown in Table VI. The data involving the Δ 's is of such poor statistical quality that no definite conclusions can be drawn from them. It would be interesting to investigate these asymmetries further. However these experiments

are difficult, and in the case of $\gamma p \rightarrow \pi^+ \Delta^0$ there is polarized background to deal with in single arm spectrometer experiments in bremsstrahlung beams. The difficulties can be overcome by extended counting in the Δ^0 and n missing mass regions, by use of a monoenergetic photon beam, and/ or by employing coincidence methods to detect the decay products of the Δ^0 .

The K^+ asymmetry is of better statistical quality than the Δ data, but not as good as the π^+ data in general. The reasons for this are not at all difficult to correct within our own experimental technique. We simply did not run in the Λ and Σ^0 missing mass region long enough to get good K^+ data. A problem of a more fundamental nature is the separation of events from $\gamma p \rightarrow K^+ \Lambda$ and $\gamma p \rightarrow K^+ \Sigma^0$. We were unable to effect such a separation so our data in Table VI are the data from the two reactions combined, hence the notation $\gamma p \rightarrow K^+(\Lambda, \Sigma^0)$. To accomplish the separation either a monoenergetic photon beam or high-statistics counting is required. If coincidence techniques were employed alone the polarization of the final baryon might be measured and compared to the polarized target asymmetry. [These two asymmetries should be equal under the (usual) assumption that $F_4=0$ in Eqs. A-23 and A-26].

Table VI. Asymmetries from a polarized proton target in the processes $\gamma p \rightarrow \pi^+ \Delta^0$, $\gamma p \rightarrow \pi^- \Delta^{++}$, and $\gamma p \rightarrow K^+(\Lambda, \Sigma^0)$ at photon energy of 16 GeV. Data were taken at the several momentum transfers shown and the errors are statistical only.

Process	$\sqrt{-t}$ (GeV)	A(t)
$\gamma p \rightarrow \pi^+ \Delta^0$	0.28	-0.06 ± 0.76
	0.56	-1.21 ± 1.09
$\gamma p \rightarrow \pi^- \Delta^{++}$	0.40	$+0.33 \pm 0.34$
$\gamma p \rightarrow K^+(\Lambda, \Sigma^0)$	0.28	$+0.05 \pm 0.18$
	0.40	-0.83 ± 0.33
	0.56	-0.29 ± 0.15

D. Possible Future Experiments

At the conclusion of an experiment one should consider the implications of his results and how they might guide or influence further research efforts. In this section we turn our attention to this task. (Again the comments following are primarily those of this author and are not necessarily universally agreed upon.)

The most important possibility raised by this experiment is that of non-trivial energy dependence of high-energy ($E_\gamma > 5$ GeV) photoproduction. Since it appears that the nucleon asymmetries are the place to look for this effect, not the photon asymmetries (see Section IV-A), this experiment should be repeated with measurements at energies intermediate between 5 and 16 GeV and with good enough statistical precision to determine the energy dependence. Of course the measurements should extend to larger values of t , if for no other reason than to measure whether, and at what value of t , the asymmetry changes sign.

The asymmetry measurements should be extended to the reactions $\gamma p \rightarrow \pi^0 p$ and $\gamma n \rightarrow \pi^- p$, but there are difficulties in detection system for the former and difficulties in the target in the latter. The K^+ photoproduction asymmetries should be measured, and this is probably the easiest experiment after the π^+ reaction. The various Δ asymmetries should be studied but as mentioned above these are not easy experiments. The recoil polarizations have not been mentioned here, but are of course of interest. It would be interesting to compare A and P to investigate the importance of the unnatural parity amplitudes F_2 and F_4 in Eqs. A-23 and A-26. These comparisons will probably be made at lower energies where the rates and analyzing powers allow recoil asymmetries to be conveniently measured.

All of the single-meson photoproduction should be studied with polarized targets in the resonance region in order to help with the description of these processes. To date the recoil-nucleon polarization has been measured as a function of energy at a small

number of angles. The polarized-target asymmetry should be measured at these points for comparison, and also at all accessible angles and energies for the benefit of phase-shift analyses.

Another experiment which our results encourage is the measurement of the polarized-target asymmetry in $np \rightarrow pn$ from 5 to 16 GeV. The interest here is two-fold, first to explore the possibility of similarity with $\gamma p \rightarrow \pi^+ n$, and second to explore the possibility of dis-similarity with the other purely hadronic high energy asymmetries.

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APPENDIX

Helicity Amplitude Expressions for Observables

In this section will be collected the expressions for some of the observables in terms of helicity amplitudes. Some of the following is adapted from an article by Jackson and Quigg.¹

In the reaction $\gamma p \rightarrow \pi^+ n$ the photon and proton in the initial state, each with two possible helicity states, and the neutron in the final state, with two possible helicities, give a total multiplicity of eight possible transitions. Invariance of the electromagnetic interactions under the parity operator means that only four independent transition probabilities describe the photoproduction reaction. If we denote the helicities of the photon and initial nucleon by λ_γ and λ_N , respectively, and helicity of the final nucleon by $\lambda_{N'}$, then the four independent s-channel helicity amplitudes, g_i , coupling states $(\lambda_\gamma, \lambda_N; \lambda_{N'})$ are (following Jackson and Quigg¹)

Definitions	Parity Invariance	Net Helicity Change (small t)
$g_1 \equiv (-1, -1/2; 1/2)$	$(1, 1/2; -1/2) = -g_1$	0
$g_2 \equiv (1, -1/2; -1/2)$	$(-1, 1/2; 1/2) = g_2$	1
$g_3 \equiv (1, 1/2; 1/2)$	$(-1, -1/2; -1/2) = g_3$	1
$g_4 \equiv (1, -1/2; 1/2)$	$(-1, 1/2; -1/2) = -g_4$	2

(A-1)

In terms of the helicity amplitudes the differential cross section for production of a specific set of helicity states, $(1, 2; 3)$, is

$$\frac{d\sigma}{dt}_{1,2;3} = \frac{1}{16\pi(s-M^2)^2} |(1, 2; 3)|^2. \quad (A-2)$$

Then the differential cross section for unpolarized photons and unpolarized nucleons is

$$\frac{d\sigma}{dt} = \frac{1}{32\pi(s-M^2)^2} \sum_{i=1}^4 |g_i|^2. \quad (A-3)$$

In order to calculate the asymmetry from a polarized target, we must define our coordinate system and recall some important conventions concerning helicity amplitudes. Consider the c.m. frame where the laboratory beam particle moves in the positive $\hat{z}$ direction.

Fig. A-1. C.M. - Frame Photoproduction Kinematical Quantities.

We take as the plane of production the x-z plane. In order to write a cross section from a polarized proton target, we must write the state functions for the initial proton oriented along the $\hat{y}$ direction.

In the helicity formalism³⁵ a distinction is made between the particle moving in the positive z-direction (called "particle 1" and described by a state functions:

$$\Psi_{p\lambda_Y\lambda_N} = \Psi_{p,\lambda_Y} \chi_{p,\lambda_N}. \quad (\text{A-4})$$

The "particle 2" state is derived from a "particle 1" state by a rotation.

$$\chi_{p,\lambda_N} = (-1)^{S_N - \lambda_N} \exp[i\pi J_y(2)] \Psi_{p,\lambda_N}. \quad (\text{A-5})$$

An important property of the helicity of a particle is that it is unchanged under a Lorentz boost transformation which brings the particle to rest. For a particle originally moving in the positive

z-direction, the new state function after this boost, $\Psi_{0,\lambda}$, describes a particle at rest with z-component of spin λ . For a particle originally moving in the negative z-direction, the state function after a halting boost, $\chi_{0,\lambda}$, describes a particle at rest with z-component of spin $-\lambda$. The minus sign occurs because positive helicity for a "particle 2" is actually referred to the negative z-direction. The phase convention in Eq. A-5 is chosen such that "particle 1" and "particle 2" states at rest are related by $\Psi_{0,\lambda} = \chi_{0,-\lambda}$. Then if we call the spin up state, $(\uparrow)$, along the positive y-direction and the spin down state, $(\downarrow)$, along the negative y-direction we have

$$\begin{aligned} (\uparrow) &\equiv \frac{1}{\sqrt{2}}(\psi_{0,+1/2} + i\psi_{0,-1/2}) = \frac{1}{\sqrt{2}}(\chi_{0,-1/2} + i\chi_{0,+1/2}), \\ (\downarrow) &\equiv \frac{1}{\sqrt{2}}(\psi_{0,+1/2} - i\psi_{0,-1/2}) = \frac{1}{\sqrt{2}}(\chi_{0,-1/2} - i\chi_{0,+1/2}). \end{aligned} \quad (\text{A-6})$$

Note here that because the initial proton is "particle 2," the formation of up and down states is not the same in terms of helicity states as if the proton were "particle 1."

The polarized target asymmetry is defined by $(\uparrow, \downarrow)$ refer to target spin)

$$A = \frac{\frac{d\sigma}{dt}(\uparrow) - \frac{d\sigma}{dt}(\downarrow)}{\frac{d\sigma}{dt}(\uparrow) + \frac{d\sigma}{dt}(\downarrow)} = \frac{\sum_{\lambda_Y \lambda_{N'}} \left[|(\lambda_Y, \uparrow; \lambda_{N'})|^2 - |(\lambda_Y, \downarrow; \lambda_{N'})|^2 \right]}{\sum_{\lambda_Y \lambda_{N'}} \left[|(\lambda_Y, \uparrow; \lambda_{N'})|^2 + |(\lambda_Y, \downarrow; \lambda_{N'})|^2 \right]}. \quad (\text{A-7})$$

Now substitutions based on Eq. A-6 are made:

$$\begin{aligned} (\lambda_Y, \uparrow; \lambda_{N'}) &= \frac{1}{\sqrt{2}} \left[(\lambda_Y, -1/2; \lambda_{N'}) + i(\lambda_Y, +1/2; \lambda_{N'}) \right], \\ (\lambda_Y, \downarrow; \lambda_{N'}) &= \frac{1}{\sqrt{2}} \left[(\lambda_Y, -1/2; \lambda_{N'}) - i(\lambda_Y, +1/2; \lambda_{N'}) \right]. \end{aligned} \quad (\text{A-8})$$

Straightforward complex algebra and substitutions from Eqs. A-1 yield the expression for the asymmetry, A, in terms of the s-channel helicity amplitudes:

$$A = \frac{2\text{Im}(g_1 g_2^* - g_3 g_4^*)}{\sum_{i=1}^4 |g_i|^2} \quad (\text{A-9})$$

The recoil nucleon polarization, P, is defined similar to Eq. A-7 but with the spin directions referring to the final nucleon spin:

$$P = \frac{\sum_{\lambda_Y \lambda_N} \left[|(\lambda_Y, \lambda_N; \uparrow)|^2 - |(\lambda_Y, \lambda_N; \downarrow)|^2 \right]}{\sum_{\lambda_Y \lambda_N} \left[|(\lambda_Y, \lambda_N; \uparrow)|^2 + |(\lambda_Y, \lambda_N; \downarrow)|^2 \right]} \quad (\text{A-10})$$

We write the spin-up, spin-down states for the final nucleon similar to those states for the target nucleon. (In the region of interest, small momentum transfers to the nucleon, the final nucleon is moving essentially in the negative z-direction.) Then remembering that final state wave functions are transposed and complex conjugated to initial states, we have analogous to Eqs. A-8:

$$\begin{aligned} (\lambda_Y, \lambda_N; \uparrow) &= \frac{1}{\sqrt{2}} \left[(\lambda_Y, \lambda_N; -1/2) - i(\lambda_Y, \lambda_N; +1/2) \right], \\ (\lambda_Y, \lambda_N; \downarrow) &= \frac{1}{\sqrt{2}} \left[(\lambda_Y, \lambda_N; -1/2) + i(\lambda_Y, \lambda_N; +1/2) \right]. \end{aligned} \quad (\text{A-11})$$

Substitution of Eq. A-11 into Eq. A-10 and making use of Eq. A-1 yields

$$P = \frac{2\text{Im}(g_1 g_3^* - g_2 g_4^*)}{\sum_{i=1}^4 |g_i|^2} \quad (\text{A-12})$$

The last spin asymmetry to be considered here is that for polarized photons. The photon asymmetry of interest is the difference in rates between photon electric field perpendicular and parallel to the scattering plane. With subscripts referring to the direction of the photon electric field relative to the scattering plane, we write the polarized photon asymmetry, Σ , as:

$$\Sigma = \frac{\frac{d\sigma}{dt}(\perp) - \frac{d\sigma}{dt}(\parallel)}{\frac{d\sigma}{dt}(\perp) + \frac{d\sigma}{dt}(\parallel)} = \frac{\sum_{\lambda_N, \lambda_{N'}} \left[|(\perp, \lambda_N; \lambda_{N'})|^2 - |(\parallel, \lambda_N; \lambda_{N'})|^2 \right]}{\sum_{\lambda_N, \lambda_{N'}} \left[|(\perp, \lambda_N; \lambda_{N'})|^2 + |(\parallel, \lambda_N; \lambda_{N'})|^2 \right]}. \quad (\text{A-13})$$

The problem here is to define states of photon polarization in terms of photon helicity. A common convention for $\lambda_\gamma = \pm 1$ for photons in a plane wave³⁵ is $\pm \frac{1}{\sqrt{2}} (\mathbf{e}_x \pm i\mathbf{e}_y)$. Thus we can write the photon states $\perp$ and $\parallel$, and $\lambda_N, \lambda_{N'}$ analogous to Eqs. A-8 we have:

$$\begin{aligned} (\perp, \lambda_N; \lambda_{N'}) &= \frac{i}{\sqrt{2}} \left[(+1, \lambda_N; \lambda_{N'}) + (-1, \lambda_N; \lambda_{N'}) \right], \\ (\parallel, \lambda_N; \lambda_{N'}) &= \frac{-1}{\sqrt{2}} \left[(+1, \lambda_N; \lambda_{N'}) - (-1, \lambda_N; \lambda_{N'}) \right]. \end{aligned} \quad (\text{A-14})$$

Substitution of Eqs. A-14 into Eq. A-13 and again making use of Eq. A-1 we obtain an expression for Σ :

$$\Sigma = \frac{2 \operatorname{Re} \{g_1 g_4^* + g_2 g_3^*\}}{\sum_{i=1}^4 |g_i|^2}. \quad (\text{A-15})$$

Whereas the vector-dominance model provides relations among the s-channel helicity amplitudes, the Regge-pole model and its variations concern themselves directly with the crossed-channel (t-channel) amplitudes describing the reaction $\gamma\pi \rightarrow N\bar{N}$. The t-channel amplitudes, F_i , are related to the s-channel amplitudes,

g_j , by the crossing matrix, X . From Jackson and Quigg¹ the relationship is

$$g = \frac{s-M^2}{2M} \frac{1}{\sqrt{2}} X F \quad (A-16)$$

With the definitions $\nu = s-M^2/2M$ (M is the nucleon mass), μ is pion mass, Jackson and Quigg¹ give X to leading order in $-t/4m^2$ and ν as

$$X = \begin{pmatrix} t/2m & -2M/(\mu^2-t) & 1 & 0 \\ (-t)^{1/2} & 0 & (-t)^{1/2}/2M & 2M(-t)^{1/2} \\ (-t)^{1/2} & 0 & (-t)^{1/2}/2M & -2M(-t)^{1/2} \\ t/2M & 2M/(\mu^2-t) & 1 & 0 \end{pmatrix} \quad (A-17)$$

It is now possible to express the observables considered above in terms of the t -channel amplitudes, F_i . In Table A-1 we have listed these observables and their expressions in terms of the various helicity amplitudes. Also included are the expressions in terms of the invariant Ball amplitudes.³⁶ (From Jackson and Quigg¹ the Ball amplitudes, A_i , and t -channel amplitudes, F_i , are related by:

$$\begin{aligned} F_1 &= -A_1 + 2MA_4, \\ F_2 &= (t-\mu^2)(A_1 + tA_2), \\ F_3 &= 2MA_1 - tA_4, \\ F_4 &= -A_3. \end{aligned} \quad (A-18)$$

The final comment on the F_i amplitudes is that they can be identified with exchange particles in the t -channel. The association with the more well-known mesons is:

<u>t-Channel amplitude</u>	<u>Exchange particles</u>
F_1, F_3 Natural parity	ρ, A_2, π_c (pion conspirator)
F_2	π, B
F_4 } Unnatural parity	A_1

Table A-1. Photoproduction Observables.

Differential Cross Section $\frac{d\sigma}{dt}$

s-Channel helicity amplitudes

$$\frac{1}{32\pi(s-M^2)^2} \sum_{i=1}^4 |g_i|^2 \quad (A-19)$$

t-Channel helicity amplitudes

$$\frac{1}{32\pi} \left[\frac{-t|F_1|^2 + |F_3|^2}{4M^2} \left(1 - \frac{t}{4M^2}\right) + \frac{|F_2|^2}{(\mu^2 - t)^2} - t|F_4|^2 \right] \quad (A-20)$$

Ball amplitudes

Ball amplitudes

$$\frac{1}{32\pi} \left[(|A_1|^2 - t|A_4|^2) \left(1 - \frac{t}{4M^2}\right)^2 + |A_1 + tA_2|^2 - t|A_3|^2 \right] \quad (A-21)$$

Polarized Target Asymmetry A

s-Channel helicity amplitudes

$$\frac{2 \operatorname{Im}(g_1 g_2^* - g_3 g_4^*)}{\sum_{i=1}^4 |g_i|^2} \quad (A-22)$$

t-Channel helicity amplitudes

$$\frac{2\sqrt{-t} \operatorname{Im} \left[\frac{F_1^* F_3}{4M^2} \left(1 - \frac{t}{4M^2}\right) + \frac{F_2^* F_4}{(\mu^2 - t)^2} \right]}{\frac{-t|F_1|^2 + |F_3|^2}{4M^2} \left(1 - \frac{t}{4M^2}\right) + \frac{|F_2|^2}{(\mu^2 - t)^2} - t|F_4|^2} \quad (A-23)$$

Table A-1 (continued)

Ball amplitudes

$$\frac{2\sqrt{-t} \operatorname{Im} \left[A_1 A_4^* \left(1 - \frac{t}{4M^2}\right)^2 + (A_1 + tA_2)^* A_3 \right]}{(|A_1|^2 - t|A_4|^2) \left(1 - \frac{t}{4M^2}\right)^2 + |A_1 + tA_2|^2 - t|A_3|^2} \quad (\text{A-24})$$

Recoil Nucleon Polarization P

s-Channel helicity amplitudes

$$\frac{2 \operatorname{Im}(g_1 g_3^* - g_2 g_4^*)}{\sum_{i=1}^4 |g_i|^2} \quad (\text{A-25})$$

t-Channel helicity amplitudes

$$\frac{2\sqrt{-t} \operatorname{Im} \left[\frac{F_1^* F_3}{4M^2} \left(1 - \frac{t}{4M^2}\right) - \frac{F_2^* F_4}{\mu^2 - t} \right]}{\frac{-t|F_1|^2 + |F_3|^2}{4M^2} \left(1 - \frac{t}{4M^2}\right) + \frac{|F_2|^2}{(\mu^2 - t)^2} - t|F_4|^2} \quad (\text{A-26})$$

Ball amplitudes

$$\frac{2\sqrt{-t} \operatorname{Im} \left[A_1 A_4^* \left(1 - \frac{t}{4M^2}\right)^2 - (A_1 + tA_2)^* A_3 \right]}{(|A_1|^2 - t|A_4|^2) \left(1 - \frac{t}{4M^2}\right)^2 + |A_1 + tA_2|^2 - t|A_3|^2} \quad (\text{A-27})$$

Table A-1 (continued)

Polarized Photon Asymmetry $\sum$

s-Channel helicity amplitudes

$$\frac{2 \operatorname{Re}(g_1 g_4^* + g_2 g_3^*)}{\sum_{i=1}^4 |g_i|^2} \quad (\text{A-28})$$

t-Channel helicity amplitudes

$$\frac{\frac{-t|F_1|^2 + |F_3|^2}{4M^2} \left(1 - \frac{t}{4M^2}\right) - \frac{|F_2|^2}{(\mu^2 - t)^2} + t|F_4|^2}{\frac{-t|F_1|^2 + |F_3|^2}{4M^2} \left(1 - \frac{t}{4M^2}\right) + \frac{|F_2|^2}{(\mu^2 - t)^2} - t|F_4|^2} \quad (\text{A-29})$$

Ball amplitudes

$$\frac{(|A_1|^2 - t|A_4|^2) \left(1 - \frac{t}{4M^2}\right)^2 - |A_1 + tA_2|^2 + t|A_3|^2}{(|A_1|^2 - t|A_4|^2) \left(1 - \frac{t}{4M^2}\right)^2 + |A_1 + tA_2|^2 - t|A_3|^2} \quad (\text{A-30})$$

REFERENCES

1. J. D. Jackson and C. Quigg, Remarks on Pion Photoproduction, UCRL-19410; to be published in Nucl. Phys.
2. For a recent review of low-energy single-pion photoproduction see R. L. Walker, Proc. 4th International Symposium on Electron and Photon Interactions at High Energies, Daresbury, England (1969).
3. K. A. Brueckner and K. M. Case, Phys. Rev. 83, 1141 (1951).
4. R. L. Walker, op. cit.
5. For recent reviews of high-energy single-pion photoproduction see R. Diebold, Proc. Boulder Conference on High Energy Physics, Boulder, Colorado (1969), or K. Lübelmeyer, Proc. 4th International Symposium on Electron and Photon Interactions at High Energies, Daresbury, England (1969).
6. For data and references see R. L. Walker, Phys. Rev. 182, 1729 (1969).
7. For recent theoretical reviews see papers by J. J. Sakurai and H. Harari in Proc. 4th International Symposium on Electron and Photon Interactions at High Energies, Daresbury, England (1969).
8. S. Mango et al., Nucl. Instr. Methods 72, 45 (1969).
9. A. M. Boyarski et al., Phys. Rev. Letters 20, 300 (1968).
10. M. Borghini et al., Polarized Proton Target for Use in Intense Electron and Photon Beams, Nucl. Instr. Methods 84, 168 (1970).
11. A. M. Boyarski et al., Phys. Rev. Letters 23, 1343 (1969).
12. G. E. Fischer and Y. Murata, Nucl. Instr. Methods 78, 25 (1970).

13. S. Rock, Test of Time Reversal Invariance in the Electromagnetic Interaction of Hadrons: Asymmetry in Inelastic Electron Scattering, (Ph.D. thesis), UCRL-20041 (unpublished).
14. C. D. Jeffries, Dynamic Nuclear Orientation (Interscience Publishers, Inc., New York, 1963); A. Abragam, The Principles of Nuclear Magnetism (Clarendon Press, Oxford, 1961); M. Borghini, Polarized Proton Targets, to be published in Methods in Subnuclear Physics, M. Nikolic, ed., Vol. IV, Gordon and Breach, publishers.
15. A. Abragam and M. Borghini, Dynamic Polarization of Nuclear Targets, in Progress in Low Temperature Physics (North-Holland Publishing Co., Amsterdam, 1964), Vol. IV, p. 384.
16. P. Roubeau, Cryogenics 6, 207 (1966) and thesis (Grenoble, 1966).
17. A. Abragam, op. cit., pp. 41, 75.
18. Claude Henry Schultz, Scattering of 250-MeV Positive Pi Mesons from a Polarized Proton Target, (Ph.D. thesis), UCRL-1149 (unpublished), p. 39.
19. Q. A. Kerns, H. W. Miller, A. H. Wolvertson, Chamberlain Proton Spin Resonance RF System 1260, UCRL-17548, Vol. I, Vol. II (unpublished).
20. From recent polarized target development by William Gorn, private communication.
21. C. D. Jeffries, op. cit. Sect. 7.1a.
22. See for example C. D. Jeffries, ibid.
23. Private communication with Dr. Donald Holmes and Prof. C. D. Jeffries. The conclusions reached are those of this author only.
24. A. Boyarski, The On-Line Control Analysis and Display System for the SLAC Multi-GeV/c Spectrometers,

SLAC-PUB-559, and Proceedings of Conference on Computer Systems in Experimental Physics, Skytop, Pennsylvania (1969).

25. Peter R. Robrish, Backward np Scattering from a Polarized Target (Ph. D. thesis), UCRL-20043 (~~unpublished~~).
26. This was necessary for cross-section measurements which had been done previously by our SLAC collaborators, Refs. 9, 11.
27. P. R. Robrish et al., Phys. Letters 31B, 617 (1970).
28. We thank Mr. Michael Lateur of SLAC for these measurements.
29. Private communication, Dr. Robert Diebold and Dr. Stanley Ecklund.
30. M. Borghini et al., (CERN-Orsay-Pisa Collaboration), Phys. Letters 31B, 405 (1970) and to be published.
31. Y. Nambu, Phys. Rev. 106, 1366 (1957).
32. C. Iso and H. Yoshii, Ann. Phys. (N. Y.) 47, 424 (1968).
33. J. D. Jackson and C. Quigg, private communication.
34. A. Krzywicki and J. Tran Thanh Van, Letters al Nuovo Cimento II, 249 (1969).
35. M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).
36. J. S. Ball, Phys. Rev. 124, 2014 (1961).

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