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The 1962 He³ Scale of Temperatures. II. Derivation*

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ABSTRACT

An Experimental Thermodynamic Equation (ETE) temperature scale valid from 0.2 to 2.0°K has been calculated for He³. The scale is based on new comparisons, (P₃, P₄), of He³ and He⁴ vapor pressures above 0.9°K; on the 1958 He⁴ temperature scale; and on the best available data for several thermodynamic properties of He³ from 0.2 to 2.0°K.

The T₆₂ Full-range Working Equation (FWE) scale,

$$\begin{aligned} \ln P_3 = & -2.49174/T + 4.80386 - 0.286001 T \\ & + 0.198608 T^2 - 0.0502237 T^3 + 0.00505486 T^4 \\ & + 2.24846 \ln T \end{aligned}$$

fits the ETE scale and the (P₃, T₅₈) data and is therefore valid for use from 0.2 to the critical point, 3.324°K. The maximum deviation from the ETE scale is 0.4 mdeg and the standard deviation from the input data is 0.25 mdeg. The fit to the seven recalculated isotherms of Keller in the range of the 1962 He³ scale can be determined by converting Keller's P₄'s to equivalent P₃'s, using direct P₄ to P₃ interpolation equations. The fit of the 1962 He³ scale is as good as the fit of the 1958 He⁴ scale to the same isotherms, the average displacements of the two scales both being 1.5 mdeg below the isotherms. The average standard deviation for (T₆₂ - T_{iso}) and for (T₅₈ - T_{iso}) is 1.2 and 1.0 mdeg, respectively, for these seven isotherms.

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Introduction

At the time He^3 was first liquefied³⁹ a comparison of its vapor pressures, P_3 , with those of He^4 , P_4 , was given, along with a careful determination of its critical pressure, P_c , which was found to be 875 mm Hg. Subsequently, Abraham, Osborne and Weinstock (AOW) presented⁵ more accurate (P_3, P_4) comparisons, additional critical point data in agreement with Ref. 1, and an empirical temperature scale, T_K , based on a modification⁵ of the 1948 He^4 scale.⁴¹ They found it possible to fit all their data, ranging from $0.011 P_c$ (1.02°K) up to P_c , to an equation for $\log(P/T^{5/2})$ having only three fitted powers of T .

In 1953, Chen and London⁴² criticised the form of the T_K equation and attempted to fit the same (P_3, P_4) data of AOW to an equation having a proper theoretical form for extrapolation to 0°K . Although they fitted coefficients of 5 powers of T , and omitted data at pressures above $0.4 P_c$, the fit of their equation was not satisfactory above $0.1 P_c$.

In 1957 Sydoriak and Roberts⁹ extended the measured range of pressures down to $0.000074 P_c$ (0.45°K), using two different paramagnetic salts calibrated against He^3 above 1° for the temperature measurement. At that time two newer He^4 temperature scales were in use, so two He^3 scales were calculated: the T_E He^3 scale, based on the 1955E He^4 scale,⁴³ and the T_L He^3 scale, based on the 1955L He^4 scale.²⁵

Only by abandoning attempts to retain an analytical expression of proper theoretical form for extrapolation to 0°K did Sydoriak and Roberts

find it possible to get a good fit to their data in combination with those of AOW. To cover the four orders of magnitude range in P_3 only four fitted powers of T were needed.

Following the adoption of the 1958 He^4 scale ² the present authors made a proposal ¹ to the VIIth. International Conference of Low Temperature Physics held at the University of Toronto in 1960 for a new He^3 vapor pressure scale to be based on the 1958 He^4 scale and on various thermodynamic properties of He^3 . The proposed procedure was similar to that used for the existing T_E and T_L scales except that newly available specific heat data ^{3,4} could now be included instead of using a calculated "spin entropy" ⁴⁴ term. In addition, a different magnetic temperature conversion was being studied for the paramagnetic salt data ⁹ intended to be used to extend the scale below 1°K .

The proposal was favorably received by members of the conference, with some reservations as to the feasibility of including vapor pressure data obtained with an iron alum thermometer.

We have subsequently abandoned incorporation of any paramagnetic salt data into the scale derivation except for measurements of specific heat using a cerium magnesium nitrate thermometer. ³

An alternative procedure for establishment of the low temperature end of the new He^3 scale has been thoroughly discussed ¹⁰ in a report to the Fourth Symposium on Temperature, Its Measurement and Control in

Science and Industry, Columbus, Ohio, March, 1961. In this method the thermodynamic consistency of the (P,T) data can be examined point by point. The method showed⁴⁵ that the AOW data could not be combined with the 1958 He⁴ scale to yield a thermodynamically consistent scale in the range from 1° to 2°K. A detailed discussion of the inconsistency, which is equivalent to several millidegrees, is given in a companion paper⁷ to this one, hereafter referred to as III. Because of this inconsistency new (P₃,P₄) comparisons were undertaken⁴⁵ in an improved apparatus designed to minimize errors due to He⁴ film reflux. The results are reported in detail in another companion paper⁴⁶ to this one, hereafter referred to as I.

Since the measurements reported in I provide an explanation for the thermodynamic inconsistency, below T_λ, of a scale based on the AOW data, we have used only (P₃,P₄) data given in I in deriving the 1962 He³ scale, reserving the AOW data above T_λ and data¹⁰ of Roberts and Sydoriak near T_λ for the purpose of checking the final scale.

The second and third virial coefficients of He³ are needed to establish the low temperature end of the present scale. For this purpose Keller's isotherm data¹² has been reanalyzed, using the method of multiple variable least squares^{47,48}. For a further discussion of the method and results we refer to III, in which the scale derived below is examined for consistency with isotherm, paramagnetic salt, and latent heat data.

The High Temperature Working Equation Scale

The three steps required to arrive at a full range equation are those discussed in detail for a Method I derivation in Ref. 10.

The first step is to derive a working equation scale by which one can interpolate between the (P_3, P_4) data points of I converted to a (P_3, T_{ss}) table of data. The primary use of this scale is for making small corrections to experimental quantities of second order importance which enter into the thermodynamic treatment of the (P_3, T_{ss}) data. It will suffice to state that we used a working equation scale which fits the (P_3, T_{ss}) data with a variance of 0.25 millidegrees and a maximum deviation of 0.6 millidegrees. (In other He^3 scale derivations a high temperature working scale was considerably more important, since it was also used in the determination of paramagnetic salt calibration equations⁹ and to assign temperatures to He^3 latent heat data¹⁰.)

The Experimental Thermodynamic Scale

The second step is to derive an analytical expression for the thermodynamic vapor pressure equation (see equations (2) and (4) of reference 10) which can be written as follows, putting on the left those terms which can be evaluated from existing thermodynamic data:

$$\ln P_3 + f(T) + f(C_{\text{sat}}, T) - \epsilon + f(V_L, P_3, T) = -\frac{a}{RT} - \frac{b}{R} \quad (1)$$

The values of P_3 and T_{ss} used are given in Table I, which is a portion of the data of I, excluding the lowest three data points, at $P_4 < 40$ microns, because the calculated He^4 film reflux pressure drop was excessive.

The second term is

$$f(T) = -i - (5/2) \ln T \quad (2)$$

where i is the chemical constant, $i = 5.31733$.

For the remaining terms on the left we write the thermodynamic function and its empirical equivalent as follows. The calculable part of the specific heat term is

$$f(C_{\text{sat}}, T) = \frac{1}{RT} \int_{T_m}^T dT' \int_{T_m}^{T'} (C_{\text{sat}}/T'') dT'' , \quad (3a)$$

where C_{sat} is the specific heat of saturated liquid He^3 . Smoothed values of the C_{sat} data of Brewer, Sreedhar, Kramers, and Daunt³ and data points of Weinstock, Abraham, and Osborne⁴ are shown in Table II, and are used to obtain an empirical equation for C_{sat} ,

$$C_{\text{sat},x}/R = 0.25154 + 0.47485 T - 0.54064 T^2 + 0.406356 T^3 - 0.082729 T^4 \quad (3b)$$

for $0.2 < T < 2^\circ$.

The lower limit of the fit, 0.2°K , was arbitrarily selected within the range of measured values of C_{sat} . By this choice the lower limit of reliability of the 1962 He^3 scale is chosen to be 0.2°K . By inserting equation (3b), the exact theoretical expression (3a) is converted to the experimental interpolation equation for $T_m = 1.0^\circ$ and $0.2 < T < 2^\circ$:

$$f_x(C_{\text{sat}}, T) = \frac{0.39332}{T} - 0.57013 + 0.237426 T - 0.090344 T^2 + 0.033863 T^3 - 0.0041364 T^4 + 0.25154 \ln T. \quad (3c)$$

For convenience we have taken T_m to be 1.0°K , although any other temperature in the range of equation (3b) could as well have been selected. The effect of the specific heat term on the ETE scale below 1°K is easier to identify with this choice of T_m .

The vapor volume term is

$$\epsilon = \ln (P_3 V_G / RT) - \frac{2B}{V_G} - \frac{3C}{2V_G^2} \quad (4a)$$

For the vapor volume, V_G , we used the inverse volume expansion form of the equation of state

$$PV_G = RT \left(1 + \frac{B}{V_G} + \frac{C}{V_G^2} \right). \quad (4b)$$

For the second and third virial coefficients of He^3 we used the equations found in reference III in the multiple parameter least squares analysis of Keller's isotherm data¹²,

$$B = 4.942 - \frac{270.986}{T} \quad \text{cc/mole} \quad (4c)$$

and

$$C = 2866/\sqrt{T} \quad \text{cm}^6/\text{mole}^2. \quad (4d)$$

Since a high speed calculator was available it was possible to use the implicit form (4a), with P_3 being taken from the (P_3, T_{B8}) data of Table I. Table III shows values of the term and of its component parts.

The liquid volume term is

$$f(V_L, P_3, T) = - \frac{1}{RT} \int_0^{P_3} V_L dP_3 \quad (5a)$$

or

$$f_x(V_L, P_3, T) = - 0.005554 T^3 - 0.000163 T^4, \quad (5b)$$

$T < 2^\circ \text{K}.$

The coefficients of (5b) were evaluated by fitting to the second last column of Table IV, which is calculated from the smoothed⁴⁹ V_L data

of Taylor and Kerr⁵⁰ and of Sherman and Edeskuty⁵¹ and from the working equation (for ΔP_3).

On the right side of equation (1) we have those terms of the thermodynamic equation which cannot be adequately calculated from existing data on He³:

$$a = L_0 - \int_0^{T_m} C_{\text{sat}} dT \quad (6)$$

and

$$b = S_L(T_m) \quad (7)$$

where L_0 is the value of the latent heat of vaporization at absolute zero, and $S_L(T_m)$ is the liquid entropy at T_m .

Using the ordinary method of least squares analysis and weighting each data point equally in this step of the derivation of the 1962 He³ scale, we find

$$a/R = 2.09842 \pm 0.00070 \text{ for } T_m = 1.0^\circ \quad (8a)$$

and

$$b/R = 1.08360 \pm 0.00046 \text{ for } T_m = 1.0^\circ. \quad (8b)$$

By combining all the above functions we obtain an experimental thermodynamic equation scale (ETE) which is valid from 0.2°K to 2.0°K.

Because of the complexity of the ETE equation, and the fact that it is implicit in the pressure, iterative solutions were obtained with the aid of an electronic digital computer. A table in steps of one millidegree was prepared for comparison with the working equation scale.

The 1962 He³ Full-range Working Equation Scale

To obtain an expression valid over the full range from 0.2°K to the critical point we now fit selected portions of equation (1) to a power series in T^n , using as input pressures all of the (P_3, T_{58}) data of Table I. Using the method of multiple variable least squares analysis^{47,48} we fit

$$\ln P_3 + f(T) + f_x(C_{\text{sat}}, T) + \frac{a}{RT} + \frac{b}{R} = \sum_{n=1}^4 d_n T^n \quad (9a)$$

Note that in this fitting the vapor and liquid volume terms are expected to be fitted by the power series. To be acceptable it will therefore be necessary to demonstrate not only that the scale fits the input data but also that the scale of (9a) agrees with the ETE scale below the range of the input data.

The solution of the analysis, combining coefficients of identical powers of T , is the full range working equation scale, FWE,

$$\begin{aligned} \ln P_3 = & -2.49174/T + 4.80386 - 0.286001 T + 0.198608 T^2 - 0.0502237 T^3 \\ & + 0.00505486 T^4 + 2.24846 \ln T \quad 0.2 < T < 3.324. \end{aligned} \quad (9b)$$

The upper limit is the critical point temperature consistent with the redetermination of the critical pressure found in I to be at 873.0 mm Hg.

As shown in Fig. 1, a comparison of the two tables we generate from the ETE scale and the FWE scale shows excellent agreement: nowhere below 2°K do the scales differ by more than 0.4 millidegrees. Equation (9b) is therefore in effect an experimental thermodynamic scale from 0.2°K to 2.0°K and an empirical scale above 2°K.

A comparison of the fit of the 1962 He³ scale to the input (P_3 , T_{58}) data is given in Table I and Fig. 1. The standard deviation of the data from the scale is 0.25 mdeg.

The He⁴ lambda point occurs at $P_{4,\lambda} = 37.80$ mm Hg at 0°C and standard gravity. On the 1958 He⁴ scale this corresponds to 2.1720°K. In I a direct interpolation procedure is described by which the value of P_3 which corresponds to $P_{4,\lambda}$ is 203.25 mm Hg. Hence $T_{62,\lambda} = 2.1721^\circ\text{K}$, for $P_3 = 203.25$ in good agreement with the value of this fixed point on the 1958 He⁴ scale of temperatures.

The most fundamental test which can be given to the 1962 He³ scale is its fit to He³ and He⁴ isotherm data, of which those of Keller^{11,12} are the most complete and accurate. Although Keller used a He⁴ thermometer in most of his isotherms, his observed P_4 's can be related to P_3 's by the direct interpolation equations described in I. In reference III Keller's isotherms have been reanalyzed by the method of multiple variable least squares. Results of the comparison with the 1962 He³ and with the 1958 He⁴ scales are shown in Fig. 2. We note that the weighted average of $T_{N,62}$ is 1.52 mdeg below T_{iso} whereas for the same isotherms $T_{N,58}$ averages 1.50 mdeg below T_{iso} . Since for these isotherms $T_{N,62} - T_{\text{iso}}$ and $T_{N,58} - T_{\text{iso}}$ have average standard deviations of 1.2 mdeg and 1.0 mdeg respectively, we conclude that not much would be gained by basing a He³ scale more directly on the isotherm data. In other words, the 1958 He⁴ scale, which is itself fundamentally based on these isotherm data,

has evidently been an adequate interpolation parameter for fitting the He^3 data to these isotherms.

Table V shows vapor pressures calculated from equation (9b) in steps of 10 millidegrees. In Table VI we show dP/dT and $T_{62} - T_x$ in 0.1 degree steps, where T_{62} corresponds to equation (9b) and T_x to the ETE scale and to the various He^3 scales in use in the past. We also have included on the far right the two He^4 scale difference ($T_{L55} - T_{58}$) and ($T_{55E} - T_{58}$), as given in the N.B.S. Monograph 10 "The 1958 He^4 Scale of Temperatures" ². In a few recent publications an attempt has been made to "correct" the T_L and T_E He^3 scales by adding these He^4 scale "correction" terms. To convert these temperatures to the 1962 He^3 scale it is necessary to apply the sum of columns 3 and 6 (or 4 and 7) to the "corrected T_L " or "corrected T_E " scales.

Detailed (P,T) and (T,P) tables have been published ⁸ by the Los Alamos Scientific Laboratory and in paper IV of this series.

In order to make the advantages of He^3 as a vapor pressure thermometer more widely accessible, specially purified He^3 is being made available for purchase* for thermometry through the United States Atomic Energy Commission isotopes program.

* Information may be obtained from Gaseous Isotopes Sales, Monsanto Research Corporation, Mound Laboratory, Miamisburg, Ohio.

Acknowledgements

If it were possible the authors would like to acknowledge in detail the aid given by numerous cryogenists in the course of the development and evaluation of the 1962 He^3 scale of temperatures and the discussions concerning the scale by the Advisory Committee on Thermometry of the International Committee on Weights and Measures. We are especially indebted to Drs. F. G. Brickwedde, B. M. Abraham, H. van Dijk, M. Durieux, R. I. Joseph, W. E. Keller, D. W. Osborne, and J. H. van Vleck. Since our correspondence ran to several dozen letters it would be impractical to specify all of the participants and to elaborate on their various contributions.

Table I

He³ vapor pressure data^a used in deriving the 1962 He³ scale. P_3 is the measured He³ vapor pressure in mm Hg at 0°C and standard gravity, δP_3 the estimated maximum error in P_3 , and δT_{62} the temperature error equivalent to δP_3 . Temperature T_{58} is obtained from the measured He⁴ vapor pressure, P_4 , and the 1958 He⁴ temperature scale.^b δT_{58} is the estimated maximum error in T_{58} equivalent to the maximum estimated error in the measurement of P_4 , excluding possible errors in the 1958 He⁴ scale itself. The last column showing the deviations of the 1962 He³ scale from the input data, exhibits a random scatter above 2°K. At lower temperatures there appears to be some small regularity in the misfit.

In fitting the coefficients of the equation for the 1962 He³ scale, the weight given to each data point was a function of both δP_3 and δT_{58} .^b As expected, for almost all of the data points $(T_{62} - T_{58}) < (\delta T_{62} + \delta T_{58})$, since the right side of the inequality is the estimated maximum error. On average, $(\delta T_{62} + \delta T_{58})$ was 1.7 times $(T_{62} - T_{58})$.

The last entry in the table is the measured value of the critical point.^a In addition to the pressure measurement error listed, there is an uncertainty of ± 1.5 mm Hg in the location of the critical point.

P_3 (mm Hg)	$\pm \delta P_3$ (mm Hg)	$\pm \delta T_{62}$ (mdeg)	T_{58} (deg)	$\pm \delta T_{58}$ (mdeg)	$T_{62} - T_{58}$ (mdeg)
5.254	0.003	0.11	0.89848	0.78	- 0.2
6.671	0.003	0.09	0.94311	0.56	- 0.2
8.692	0.004	0.10	0.99612	0.39	0.3
10.176	0.004	0.09	1.03014	0.48	0.3
11.041	0.004	0.08	1.04850	0.41	0.2
13.487	0.005	0.09	1.09572	0.42	0.0
20.001	0.008	0.11	1.19772	0.27	0.0
28.395	0.011	0.12	1.30024	0.09	0.1
38.512	0.014	0.13	1.39994	0.68	0.0
50.234	0.018	0.13	1.49553	0.58	0.1
66.069	0.027	0.17	1.60410	0.53	- 0.3
81.978	0.060	0.32	1.69703	0.48	- 0.6
82.515	0.028	0.15	1.69980	0.48	- 0.5
102.75	0.060	0.28	1.80113	0.18	- 0.0
103.65	0.060	0.28	1.80548	0.18	- 0.2
103.81	0.060	0.28	1.80627	0.05	- 0.2
124.23	0.070	0.29	1.89616	0.06	- 0.5
125.86	0.060	0.25	1.90300	0.05	- 0.6
151.88	0.070	0.26	2.00293	0.11	- 0.1
153.49	0.035	0.13	2.00845	0.11	0.2
181.58	0.048	0.16	2.10432	0.11	0.2
196.61	0.045	0.14	2.15174	0.10	0.2
203.29	0.055	0.17	2.17207	0.10	0.1

Table I - continued

P_3 (mm Hg)	$\pm \delta P_3$ (mm Hg)	$\pm \delta T_{62}$ (mdeg)	T_{58} (deg)	$\pm \delta T_{58}$ (mdeg)	$T_{62} - T_{58}$ (mdeg)
205.83	0.055	0.16	2.17945	0.10	0.4
214.47	0.020	0.23	2.20521	0.09	0.0
227.21	0.059	0.17	2.24144	0.09	0.1
248.77	0.080	0.21	2.30010	0.08	- 0.1
288.86	0.090	0.22	2.40069	0.07	- 0.1
333.14	0.090	0.20	2.50157	0.06	0.0
381.81	0.110	0.22	2.60250	0.06	0.3
433.39	0.100	0.18	2.70121	0.05	0.1
491.85	0.110	0.19	2.80389	0.05	0.0
555.00	0.110	0.17	2.90588	0.04	0.0
617.09	0.130	0.19	2.99913	0.04	- 0.3
657.11	0.109	0.16	3.05549	0.03	- 0.1
657.78	0.110	0.16	3.05632	0.07	0.0
658.95	0.110	0.16	3.05792	0.07	0.0
721.41	0.114	0.15	3.14129	0.07	0.1
729.70	0.114	0.15	3.15203	0.07	0.1
738.63	0.116	0.15	3.16356	0.07	0.0
804.98	0.120	0.14	3.24556	0.06	- 0.1
806.35	0.120	0.14	3.24724	0.06	- 0.1
853.73	0.140	0.16	3.30238	0.06	0.3
873.00	0.300	0.34	3.3240	0.47	0.6

^a Ref. (16) (Paper I of this series)

^b Ref. (16) (Paper III of this series)

Table II

Values at selected temperatures, T , of the specific heat function ^a, $f(C_{\text{sat}}, T)$, appearing in the thermodynamic equation for $\ln P$. The function is evaluated by means of an explicit equation ^b, $C_{\text{sat},x}$, fitted to smoothed C_{sat} data of Brewer, Sreedhar, Kramers and Daunt ^c below 1°K and plotted data points of Weinstock, Abraham and Osborne ^d above 1°K. The misfit of the equation, $\Delta C = C_{\text{sat},x} - C_{\text{sat},\text{obs}}$, is seen to have a random scatter.

T (deg)	$C_{\text{sat},x}$ (cal mole ⁻¹ deg ⁻¹)	$\Delta C_{\text{sat}} \times 10^3$ (mcal mole ⁻¹ deg ⁻¹)	$f_x(C_{\text{sat}}, T)$ (dimensionless)
0.2	0.652 ^b	8 ^c	0.000
0.3	0.706	-8	0.0243
0.4	0.752	-5	0.0665
0.5	0.793	0	0.1109
0.6	0.831	8	0.1539
0.7	0.870	3	0.1946
0.8	0.911		0.2329
0.9	0.957		0.2691
1.0	1.009		0.3034
1.1	1.069	-4, -9 ^{d,e}	0.3362
1.2	1.136	+1, -9, +4	0.3675
1.3	1.211	+6, +4	0.3977
1.4	1.294	+5, -1	0.4269
1.5	1.385	-6	0.4553
1.6	1.482	-13, +14	0.4831
1.7	1.585	+2	0.5104
1.8	1.692	+16, -4	0.5372
1.9	1.800	-20, +2	0.5637
2.0	1.908	+5	0.5899

- a Eq. (3a)
- b Eq. (3b)
- c Ref. (11)
- d Ref. (12). Both T and C_{sat} were adjusted to the T_{SR} scale; e.g., C_{sat} was multiplied by the value of dT_{SR}/dT_K at each data point.
- e Where multiple entries appear they refer to data points nearest to the indicated temperatures.

Table III

The vapor volume term ^a, ϵ , in the equation for $\ln P_3$, as evaluated by an iterative procedure, using a high speed digital computer. For their general usefulness the table also shows solutions of the cubic equation ^b for the vapor volume, V_G , and values of the second ^c and third ^d virial coefficients, B and C, based on reanalysis ^e of the isotherms of Keller^f.

T (deg)	V_G (cc/mole)	B (cc/mole)	C (cc/mole) ²	ϵ
0.2	1.03×10^9	-1350	6409	0.0000
0.3	9.97×10^8	- 898.3	5233	0.0001
0.4	8.87×10^8	- 672.5	4532	0.0008
0.5	1.95×10^8	- 537.0	4053	0.0027
0.6	6.83×10^4	- 446.7	3700	0.0065
0.7	3.123×10^4	- 382.2	3426	0.0122
0.8	1.692×10^4	- 333.8	3204	0.0195
0.9	1.028×10^4	- 296.2	3021	0.0284
1.0	6780	- 266.0	2866	0.0384
1.1	4747	- 241.4	2732	0.0495
1.2	3478	- 220.9	2616	0.0613
1.3	2640	- 203.5	2513	0.0738
1.4	2061	- 188.6	2422	0.0868
1.5	1646	- 175.7	2340	0.1003
1.6	1339	- 164.4	2266	0.1141
1.7	1106	- 154.5	2198	0.1282
1.8	925.6	- 145.6	2136	0.1427
1.9	782.8	- 137.7	2079	0.1574
2.0	667.9	- 130.6	2027	0.1723

- a Eq. (4a)
- b Eq. (4b)
- c Eq. (4c)
- d Eq. (4d)
- e Ref. (16)
- f Ref. (18)

Table IV

The liquid volume term, $f(V_L, P_3, T)$, in the equation for $\ln P$ is calculated by numerical integration at selected temperatures, T , using smoothed values ^a, V_L , of liquid volume data of Kerr and Taylor ^b and Sherman and Edeskuty ^c. For P_3 the high temperature working equation is used. Column 3 shows values ^d calculated from a power series in T^n fitted to column 2. The last two columns show the mismatch and its equivalent in millidegrees.

T (deg)	$f(V_L, P_3, T)$	$f_x(V_L, P_3, T)$	$f_x - f$	ΔT (mdeg)
0.2	0.0000	0.0000	0.0000	0.0
0.4	0.0000	0.0004	0.0004	0.0
0.6	0.0005	0.0012	0.0007	0.1
0.8	0.0021	0.0029	0.0008	0.1
1.0	0.0052	0.0057	0.0005	0.1
1.2	0.0099	0.0099	-0.0000	-0.0
1.4	0.0163	0.0158	-0.0005	-0.2
1.6	0.0244	0.0238	-0.0006	-0.2
1.8	0.0342	0.0341	-0.0001	-0.0
2.0	0.0459	0.0470	0.0011	0.6

^a Ref. (49)

^b Ref. (50)

^c Ref. (51)

^d Eq. (5b)

Table V. He^3 vapor pressures on the λ scale, at 0°C and standard gravity, 980.665 cm/sec². The units of pressure are microns (10^{-3} mm) of mercury below 1°K and millimeters of mercury at higher temperatures.

λ	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.00	8.842	9.267	9.704	10.156	10.622	11.102	11.597	12.106	12.631	13.170
1.10	13.725	14.295	14.861	15.434	16.102	16.757	17.388	18.056	18.741	19.443
1.20	20.163	20.900	21.655	22.428	23.220	24.029	24.857	25.704	26.571	27.456
1.30	28.360	29.285	30.229	31.193	32.177	33.181	34.206	35.252	36.319	37.407
1.40	38.516	39.646	40.799	41.973	43.169	44.388	45.629	46.893	48.179	49.489
1.50	50.822	52.178	53.598	54.981	56.389	57.840	59.316	60.817	62.342	63.892
1.60	65.467	67.068	68.694	70.345	72.022	73.726	75.455	77.211	78.993	80.802
1.70	82.638	84.501	86.391	88.309	90.254	92.228	94.229	96.258	98.315	100.402
1.80	102.516	104.660	106.833	109.035	111.266	113.527	115.818	118.138	120.489	122.870
1.90	125.287	127.724	130.197	132.701	135.236	137.803	140.401	143.031	145.692	148.386
2.00	151.112	153.870	156.661	159.485	162.342	165.232	168.155	171.112	174.102	177.126
2.10	183.276	186.564	189.893	193.263	196.760	200.296	203.874	207.597	211.364	215.266
2.20	212.673	216.117	219.577	223.113	226.665	230.255	233.881	237.544	241.244	244.982
2.30	248.757	252.570	256.420	260.3 9	264.236	268.202	272.206	276.249	280.331	284.452
2.40	288.613	292.813	297.053	301.333	305.653	310.013	314.414	318.855	323.337	327.861
2.50	332.425	337.031	341.679	346.368	351.100	355.874	360.690	365.549	370.450	375.395
2.60	380.383	385.414	390.489	395.608	400.771	405.978	411.230	416.526	421.978	427.254
2.70	432.626	438.164	443.687	449.296	454.872	460.534	466.282	471.978	477.801	483.651
2.80	489.549	495.495	501.436	507.581	513.622	519.762	525.951	532.189	538.477	544.815
2.90	551.203	557.642	564.131	570.672	577.264	583.907	590.602	597.349	604.149	611.002
3.00	617.907	624.866	631.879	638.945	646.066	653.241	660.472	667.757	675.094	672.476
3.10	689.949	697.459	705.026	712.650	720.332	728.072	735.871	743.728	751.644	759.620
3.20	767.656	775.753	783.910	792.128	800.400	808.750	817.155	825.622	834.153	842.747
3.30	851.406	860.130	868.918	877.773	886.700	895.701	904.776	913.922	923.140	932.430

Table VI

The temperature derivative dP_3/dT_{62} and deviations of various He^3 temperature scales from the present 1962 He^3 scale. Columns 3, 4, and 5 give differences of the T_E , T_L , and T_K scales respectively from the 1962 He^3 scale in the form $(T_{62} - T_x)$ expressed in millidegrees. In Columns 6 and 7 are reproduced the deviations of the 1955 He^4 scales from the accepted 1958 He^4 scale as these deviations have been utilized by the investigators in an attempt to "update" T_E and T_L .

T (°K)	dp_3/dT_{62} (mm Hg/°K)	$T_{62}-T_L^a$ (m deg)	$T_{62}-T_E^a$ (m deg)	$T_{62}-T_K^b$ (m deg)	$(T_{L55}-T_{58})^c$ (m deg)	$(T_{55E}-T_{58})^c$ (m deg)
0.2	0.001					
0.3	0.066	-5.0	-6.3			
0.4	0.592	-5.8	-7.4			
0.5	2.283	-6.2	-8.1			
0.6	5.756	-6.2	-8.3			
0.7	11.362	-5.8	-8.2		-1.1	1.0
0.8	19.234	-5.2	-7.8		-1.2	1.1
0.9	29.379	-4.4	-7.3		-1.3	1.2
1.0	41.745	-3.6	-6.7	-4.0	-1.5	1.3
1.1	56.26	-2.9	-6.1	-3.7	-1.6	1.4
1.2	72.84	-2.1	-5.4	-3.2	-1.7	1.5
1.3	91.44	-1.5	-4.9	-2.5	-1.8	1.5
1.4	111.99	-0.9	-4.4	-1.6	-1.9	1.6
1.5	134.45	-0.4	-3.9	-0.7	-2.0	1.6
1.6	158.78	0.0	-3.4	0.2	-2.1	1.5
1.7	184.94	0.3	-3.1	1.1	-2.1	1.4
1.8	212.92	0.5	-2.8	2.0	-2.2	1.1
1.9	242.68	0.7	-2.4	2.7	-2.2	0.8
2.0	274.22	0.9	-2.0	3.4	-2.2	0.6
2.1	307.51	1.1	-1.6	3.9	-2.2	0.6
2.2	342.57	1.4	-1.1	4.3	-2.2	0.8
2.3	379.40	1.6	-0.6	4.5	-2.1	0.5
2.4	418.03	1.9	0.0	4.5	-2.0	0.0
2.5	458.53	2.2	0.6	4.4	-1.9	-0.4
2.6	500.96	2.5	1.3	4.1	-1.8	-0.7
2.7	545.46	2.7	1.9	3.5	-1.57	-0.9
2.8	592.18	2.9	2.4	2.7	-1.3	-1.1
2.9	641.34	2.8	2.7	1.5	-0.9	-1.1
3.0	693.22	2.5	2.9	0.0	-0.6	-1.1
3.1	748.16	1.9	2.6	-2.0	-0.2	-1.1
3.2	806.61	0.6	1.7	-4.7	0.2	-0.9
3.3	869.11	-1.3	0.1	-8.0	0.7	-0.7

a Reference 6

b Reference 2, 3, 4

c Reference 9

Fig. 1. The solid line is the deviation of the 1962 He^3 scale (i.e., the Full-range Working Equation, eq. 9b) from the Experimental Thermodynamic Equation (ETE) scale in the range of validity of the ETE scale. Plotted points are deviations of the 1962 He^3 scale from the (P_3, P_4) input data: $T_{62}(P_3) - T_{58}(P_4)$.

Fig. 2. Deviations of temperature scales from Keller's (ref. 18,24) isotherm temperatures; T_{iso} , as reanalyzed in III (ref. 16). $\odot$ ($T_{62} - T_{iso}$) for He^4 isotherms; $\square$ ($T_{62} - T_{iso}$) for He^3 isotherms; $\circ, \square$ ($T_{58} - T_{iso}$) for He^4 and He^3 isotherms, respectively. To get T_{62} , Keller's He^4 vapor pressure thermometer readings are converted to equivalent P_3 's by means of direct P_4 -to- P_3 interpolation equations derived in I (ref. 17). Lengths of bars for the 1958 He^4 scale deviations are equal to the standard deviation for T_{iso} as calculated in the analysis of the isotherm data. For 1962 He^3 scale deviations we add on the standard deviation of the conversion from P_4 to equivalent P_3 . The T_{62} and T_{58} bars terminate in solid and open triangles respectively.

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Additional references will be added when papers III and IV are submitted.