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NUCLEAR DIVISION

REPORT NO. RN-S-0486

VOLUME II

To

AEC-NASA Space Nuclear Propulsion Office

XE-1 CAM EQUATION DERIVATION (U)

NERVA Program

Contract SNP-1

October 1968

NUCLEAR ROCKET OPERATIONS

AEROJET-GENERAL CORPORATION

SACRAMENTO, CALIFORNIA

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REPORT NO. RN-S-0486

Volume II

XE-1 CAM EQUATION DERIVATION (U)

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FOREWORD

The Common Analog Model for XE-1 is a coordinated effort between Aerojet-General and Westinghouse Electric to develop a mathematical model of the XE-1 engine system suitable for analysis with analog computer facilities. The model includes representation of the engine system, controllers, control mechanisms, and test facilities. Aerojet is responsible for the AGC-supplied components of the engine system, the test facility, and emergency systems; and Westinghouse for the model of the nuclear subsystem.

This report is being published in two separate volumes. Volume I contains the unclassified portion of the technical discussion, Section II.A., Engine Feed System; Volume II contains Section II.B., Nuclear Subsystem, and is classified.

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B. NUCLEAR SUBSYSTEM

The second part of this report has two sections: (1) a discussion of the five basic equations used in the CAM XE-1 nuclear subsystem model and (2) a detailed list of equations and constants that constitute the model.

1. Discussion of Basic Equations

Five basic equations are used in the CAM XE-I nuclear subsystem model. These equations are descriptions of the following standard physical laws:

- a. The conservation of mass
- b. The conservation of momentum
- c. The equation of state
- d. Conservation of energy
- e. Conservation of Neutron density

Each will be discussed from a rather basic standpoint and the simplifications which are used to arrive at the CAM equations are described in detail.

a. The Conservation of Mass Equation

Consider a cylindrical pipe of length Δx and radius r through which fluid is flowing in the x direction. The rate of accumulation of mass is equal to the mass flow in minus the mass flow out. This is*

$$\pi r^2 \Delta x \frac{\partial}{\partial t} \rho = \pi r^2 \left[(\rho v_x) \Big|_x - (\rho v_x) \Big|_{x + \Delta x} \right]$$

where:

- ρ = density, lb/ft³
 v_x = fluid velocity in x direction, ft/sec
 t = time, seconds

*See nomenclature in Table 1, page 18.

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Using the definition $v_x = W/\rho\pi r^2$, where W is the mass flow rate in lb/sec, the above equation reduces to:

$$\Delta x \pi r^2 \frac{\partial}{\partial t} \rho = W \Big|_x - W \Big|_{x+\Delta x}$$

Dividing both sides of the above equation by Δx and taking the limit as $\Delta x \rightarrow 0$, one has:

$$\frac{\partial}{\partial t} \rho = - \frac{1}{\pi r^2} \frac{\partial}{\partial x} W.$$

The above equation is the basis for all the mass balance equations written in the CAM XE-I model. Equations 10 and 20 are simplifications of this equation.

In writing the CAM mass balance equations, the following assumptions have been made:

(1) There is no significant difference between the inlet and outlet temperatures in the plenums.

(2) The volume of the reflector flow paths and the reflector exit plenum is small compared to the dome void volume and that there is a minor change in density in the reflector, such that the estimate of the mass of fluid in the reflector, reflector exit plenum, and dome is satisfactorily accurate. Mathematically,

$$\rho_r V_r + \rho_{re} V_{re} + \rho_6 V_6 \approx \rho_6 (V_r + V_{re} + V_6).$$

As an illustration, design for XE-I gives the following results:

$$(0.75)(2.65) + (0.5)(3.93) + (0.5)(2.12) \approx 0.5 (8.7) \\ 5.01 \approx 4.35$$

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(3) The void volume of the core can be divided between the core inlet plenum and nozzle and that the following relationship is true:

$$\rho_s V_s + \rho_7 V_7 + \rho_c V_c + \rho_{tr} V_{tr} + \rho_8 V_8 \approx \rho_7 \left(V_s + V_7 + \frac{1}{2} (V_c + V_{tr}) \right) + \rho_8 \left(V_8 + \frac{1}{2} (V_c + V_{tr}) \right)$$

At design, the above relation yields:

$$\begin{aligned} & (0.5)(1.29) + (0.5)(8.06) + (0.1)(6.6) \\ & + (0.27)(0.27) + (10)(0.025) \approx (0.5)(12.65) + (0.025)(13.6) \\ & (0.644) + (4.03) + 0.66 + 0.1 + 0.25 \approx 6.35 + 0.34 \\ & 5.7 \approx 6.69 \end{aligned}$$

In summary, the estimate of the mass of fluid in the reactor at design is subject to errors of up to 10%. At the lower operating points, the errors could be greater. Errors in the estimate of mass can have dynamic effects at the higher frequencies for dynamic studies.

b. Conservation of Momentum

For a cylinder of radius r and length Δx , the rate of momentum increase is equal to the momentum in minus the momentum out plus the sum of the forces acting on the system. Expressing the law mathematically gives:

$$\Delta x \pi r^2 \frac{\partial}{\partial t} \frac{\rho v_x}{g} = \pi r^2 \frac{\rho v_x^2}{g} \Big|_x - \pi r^2 \frac{\rho v_x^2}{g} \Big|_{x+\Delta x} - \pi r^2 144 P \Big|_x + 144 \pi r^2 P \Big|_{x+\Delta x}$$

$$- \frac{f \rho v_x^2}{4g} \pi r \Delta x,$$

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where:

$$g = 32.2 \text{ ft/sec}^2$$

P = Pressure, psia

f = Friction factor, dimensionless

Dividing both sides of the above equation by πr^2 and taking the limit as $\Delta x \rightarrow 0$, one obtains:

$$\frac{\partial}{\partial t} \rho v_x = - \frac{\partial}{\partial x} \rho v_x^2 - 144g \frac{\partial}{\partial x} P - \frac{f \rho v_x^2}{2D}$$

Substituting $W = \rho A v_x$ in the above equation gives:

$$\frac{\partial}{\partial t} W = - \frac{1}{A} \frac{\partial}{\partial x} \left(\frac{W}{\rho} \right)^2 - 144gA \frac{\partial}{\partial x} P - \frac{f}{2AD} \left(\frac{W}{\rho} \right)^2$$

Writing central-difference approximations to the derivatives in the above equation yields the ordinary differential equation:

$$\frac{d}{dt} W = \frac{1}{V} \left[\frac{W^2}{\rho} \Big|_i - \frac{W^2}{\rho} \Big|_o \right] + \frac{144gA}{L} [P_i - P_o] - \frac{1}{V} \cdot \frac{fL}{2D} \frac{W^2}{\rho}$$

where the subscripts i and o correspond to inlet and outlet, respectively.

Usually, the pressure drop, due to restrictions at the inlet and exit of the flow channels, are comparable to the momentum and friction pressure drop. In these cases, the above general equation reduces to:

$$P_i - P_o = \frac{W_o^2}{\rho_o} \frac{L}{V144gA} = \frac{W_o^2}{\rho_o} \frac{1}{144A^2g} C$$

where in these cases the subscripts i and o are not as above, but represent the inlet and outlet conditions across the restriction or orifice. C is an empirical coefficient. If these expressions are substituted into the general equation and the total pressure drop is considered, the following general equation is obtained:

$$\frac{d}{dt} W = \frac{144Ag}{L} (P_i - P_o) - \frac{W^2}{V} \left[\frac{(1 + C_o)}{\rho_o} + \frac{(C_i - 1)}{\rho_i} + \frac{fL}{2D} \frac{1}{\rho} \right]$$

In the CAM model for XE-I, no compressibility is taken into account within the components themselves such that $W_o = W_i = W$. With this assumption,

$$\frac{d}{dt} W = \frac{144Ag}{L} (P_i - P_o) - \frac{W^2}{V} \left[\frac{(1 + C_o)}{\rho_o} + \frac{(C_i - 1)}{\rho_i} + \frac{fL}{2D} \frac{1}{\rho} \right]$$

Equations 6, 16, 38, and 62 represent various simplifications to this expression. For the reflector and shield, all of the pressure drop is assumed to occur at the exit of the component. This has been found to be adequate in test simulations. For the reactor core and tie rod channels, a significant pressure drop at the inlet occurs so that this inlet term is retained. Again, pressure calculations in comparison with data have yielded satisfactory results.

c. The Equation of State

For the propellant, the relation between pressure, temperature, and density is defined at each part along the path of flow. In the CAM XE model, the ideal gas law is used. Equations 9, 19, and 63 represent these equations.

This expression is accurate in cases where the temperature is above 125°R and below 4000°R, at the pressures encountered in normal NRX and engine tests. No problem is encountered in the two plenums or at the tie rod exit of going above 4000°R. However, on the normal operating

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line, when the chamber temperature gets below 2000°R, temperatures lower than 125°R should be encountered in the plenums. Along the normal profile, the following representative accuracies are obtained:

<u>Temperature</u>	<u>Pressure</u>	<u>Actual Density</u>	<u>Calculated Density</u>	<u>Percent Error</u>
150	400	0.51	0.501	1.6%
125	300	0.5	0.452	10 %
100	200	0.41	0.376	8.8%
80	150	0.4	0.353	11 %
60	100	0.391	0.313	12.5%

In cases where the pressure is high compared with the pressure normally encountered in the operating line, the following values are obtained:

<u>Temperature</u>	<u>Pressure</u>	<u>Actual Density</u>	<u>Calculated Density</u>	<u>Percent Error</u>
150	1000	1.26	1.27	----
125	800	1.38	1.21	11%
100	600	1.38	1.13	22%
80	500	1.90	1.18	61%
60	400	3.5	1.26	280%

Where the pressure is low, the following table can be constructed:

<u>Temperature</u>	<u>Pressure</u>	<u>Actual Density</u>	<u>Calculated Density</u>	<u>Percent Error</u>
150	100	0.127	0.126	----
125	80	0.127	0.120	-6 %
100	60	0.116	0.113	-2.5%
80	50	0.123	0.117	-5 %
60	40	0.135	0.125	-8 %

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In summary, below 125°R, the CAM model is only approximately +10% accurate and significant errors are obtained when the pressure is high compared with that normally encountered along the engine operating line.

d. Conservation of Energy

The rate of change of internal energy is equal to the rate at which heat is picked up or lost to the surroundings minus the rate at which work is done plus the rate at which heat is generated. This is:

$$\frac{D}{Dt} U - \frac{D}{Dt} Q - \frac{1}{J'} \frac{D}{Dt} W + G$$

where:

U = Internal energy, Btu/lb

Q = Heat lost or gained from surroundings, Btu/lb

W = Work, ft

G = Heat generation rate, Btu/lb-sec

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} = \text{substantial derivative}$$

J' = Mechanical equivalent of heat ft-lb/Btu

Neglecting mechanical work, various forces and the effects of gravity, for a chemical process,

$$\frac{D}{Dt} U = \frac{D}{Dt} Q - \frac{1}{J} P \frac{D}{Dt} \hat{V} + G$$

where $\hat{V}$ is specific gravity in ft³/lb, and J is in in.-lb/Btu.

$$\text{Using } U = E - \frac{P\hat{V}}{J'},$$

$$\frac{D}{Dt} E = \hat{V} \frac{1}{J} \frac{D}{Dt} P + \frac{D}{Dt} Q + G$$

where E is enthalpy in Btu/lb.

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For the CAM model convection and conduction are considered with radiation being neglected so that:

$$\frac{D}{Dt} Q = \frac{h \cdot \alpha}{M} (\phi - T) - \left[\frac{\partial}{\partial x} k \frac{\partial T}{\partial x} \right] \frac{1}{\rho} - \left[\frac{\partial}{\partial y} k \frac{\partial T}{\partial y} \right] \frac{1}{\rho}$$

where:

h = Overall heat transfer coefficient, Btu/ft² - °R - sec

α = Heating surface, ft²

M = Mass of fluid = ρV = lb

ϕ = Material temperature, °R

T = Fluid temperature, °R

k = Thermal conductivity, Btu/ft°R-sec

y = Direction normal to fluid flow, ft

Also, in the reactor, heat is generated as $k \cdot S$, where S is the prompt and delayed power. Using the substantial derivative definition, one obtains:

$$\begin{aligned} \frac{\partial}{\partial t} E = & - \frac{W}{\rho A} \frac{\partial}{\partial x} E + \frac{1}{\rho J} \frac{\partial}{\partial t} P + \frac{W}{\rho A J} \frac{\partial}{\partial x} P + \frac{h \cdot \alpha}{\rho V} (\phi - T) - \frac{1}{\rho} \frac{\partial}{\partial x} \frac{\partial}{\partial x} T \\ & - \frac{1}{\rho} \frac{\partial}{\partial y} k \frac{\partial}{\partial y} T + \frac{KS}{\rho V} \end{aligned}$$

In the CAM model, the flow work terms on the right side of the above equation are neglected. For the propellant, the last three terms are also neglected. The equation becomes:

$$\rho \frac{\partial}{\partial t} E = - \frac{W}{A} \frac{\partial}{\partial x} E + \frac{h \alpha}{V} (\phi - T)$$

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In simplifying the equation further, a linear distribution of enthalpy with direction of flow is assumed. The above equation becomes:

$$\frac{d}{dt} E = - \frac{W}{\rho V} \left[E_o - E_i \right] + \frac{h \cdot \alpha}{\rho V} (\phi - T) = \left\{ 2W \left[E_i - E \right] + h \cdot \alpha (\phi - T) \frac{1}{\rho V} \right\}$$

Also, in the CAM equations, an average density is assumed in each component, i.e., $\rho V = C$. Equations 1, 11, 21, 22, and 23 represent the fluid equation. In Equation 58, for the tie rod gas temperature, the mass of fluid is neglected.

For the calculation of fluid temperature from enthalpy and pressure,

$$E = f(T, P), \text{ and}$$

$$dE = \left(\frac{\partial E}{\partial T} \right) \bigg|_P dT + \left(\frac{\partial E}{\partial P} \right) \bigg|_T dP$$

$$\text{OR}$$

$$E_2 - E_1 = \int_{T_1}^{T_2} c_p dT + \int_{P_1}^{P_2} \left(\frac{\partial E}{\partial P} \right) \bigg|_T dP$$

Now,

$$E_2 - E_1 = c_p \bigg|_c \left[T_2 - T_1 \right] + \left(\frac{\partial E}{\partial P} \right) \bigg|_T \bigg|_c \left[P_2 - P_1 \right]$$

where the c subscript indicates, the value is held constant. Rearranging the above and denoting the general variable state without subscripts,

$$E = \left[E_1 - c_p \bigg|_c T_1 - \left(\frac{\partial E}{\partial P} \right) \bigg|_T \bigg|_c P_1 \right] + c_p \bigg|_c T + \left(\frac{\partial E}{\partial P} \right) \bigg|_T \bigg|_c P.$$

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Equations 2, 8, 12, and 18 in the second section are rearrangements of the above expression. Equations 24 and 26 in the reactor core are simplifications of the above expression where $(\partial E / \partial P)_T = 0$. Equations 28 and 30 are representative of:

$$E = f(T) \text{ in general form.}$$

For the enthalpy of the solid material, H, the general form of the energy equation reduces to*:

$$\frac{\partial}{\partial t} H = - \frac{h \cdot \alpha}{\rho_s V_s} (\phi - T) - \frac{1}{\rho_s} \frac{\partial}{\partial x} k \frac{\partial}{\partial x} \phi + \frac{KS}{\rho_s V_s}$$

where the subscript s denotes solid material.

Equations 3, 14, 34, 35, and 36 represent the above equation with the conduction normal to flow assumed to be infinite. The overall heat transfer coefficients h are represented by

$$h = (A + BT) \cdot W^{0.8}$$

Equations 55, 56, and 61, represent the above equation with the first term on the right omitted and with

$$H = c_p \phi$$

The same applies to Equations 57 and 59, but with the convective term included.

*In the tie-rod model, cylindrical coordinates are used.

Equations relating enthalpy and temperature of the solid use the relationship

$$H = \int_0^{\phi} c_p d\phi \cong \sum_{i=1}^n c_{p_i} \Delta\phi_i$$

where it is implied that $H = 0$ at $\phi = 0$.

d. Reactor Kinetics Equations

The reactor kinetics equations used in the CAM models are generally referred to as the point reactor kinetics. These equations can be derived from the non-equilibrium diffusion equation of neutrons in a bare (without shield and/or reflector) thermal reactor. In this derivation, a number of important simplifying assumptions are made. However, the resulting relationships are accurate for large thermal reactors such as the NERVA type. Among the most basic limitations are:

(1) Space-dependent neutron flux shape changes during transients due to reflected neutrons, changes in core material nuclear cross sections, (a material parameter which is indicative of the probability that a neutron will interact in a given process with the given material), or core geometrical changes due to thermal expansion or contractions are neglected.

(2) Energy dependency of delayed neutron precursors, and nuclear cross-sections are neglected. Generally the generation time, l , of a neutron for one generation in a given type reactor is a function of the mean neutron energy in the core. Although this energy may change in a transient, l is assumed fixed in the point kinetics model.

(3) The non-equilibrium diffusion equation itself is an approximation to the more rigorous Boltzmann Transport equation for

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neutrons. One of the basic limitations encountered in this approximation is that it is assumed that at any point in time where this equation applies, the reactor is never far from critical.

In deriving the point kinetics model, several choices are available. The simple time dependent equations are then obtained by eigen function expansions and separation of variables using the diffusion equation for neutrons. One can also arrive at these equations via a stochastic derivation with the dependent variables appearing as appropriate time averages. The approach used here is a heuristic derivation with some detail devoted to the meaning of the pertinent parameters of the point reactor model.

The familiar equations of the point reactor kinetics model are:

$$\dot{n} = \frac{\rho - \beta}{\Lambda} n + \sum_{i=1}^m \lambda_i C_i + S \quad (1)$$

$$\dot{C}_i = \frac{\beta_i}{\Lambda} n - \lambda_i C_i \quad (i = 1, m) \quad (2)$$

where the dot signifies time derivative, and where:

n = no. of neutrons (or neutron density, power, etc.)

C_i = no. of delayed neutron precursors (potential neutrons).....
same units as n .

$$\rho = \text{total reactivity} = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}}$$

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Here, k_{eff} = the effective multiplication constant of the reactor, which is a complicated function of the reactor geometry, reactor size, mean neutron energy, and the reactor core materials. It is defined as the number of "particles" (prompt neutrons plus delayed neutron precursors) born in fission per neutron lost by leakage or absorption. When $k_{\text{eff}} = 1$, the reactor is said to be source critical.

β_i = number of delayed neutron precursors of the ith group born per "particle" born in fission.

$$\beta = \sum_{i=1}^m \beta_i; m = \text{number of distinct delayed neutron groups}; 1 - \beta =$$

number of prompt neutrons born per "particle" born in fission. l, l_0 = characteristic times for the neutron population.

Here, l_0 is referred to as the mean neutron lifetime, and l is referred to as the neutron generation time where $l = l_0 / k_{\text{eff}}$. In this equality, where the assumption is made that l is constant, and the reactor is never far from critical i.e., $k_{\text{eff}} \approx 1$.

λ_i = decay constant for delayed neutron precursors of the ith group.

S = the source of neutrons (same units as $\dot{n}$).

The point-reactor model may be visualized by the following diagram:

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Derivation

The equation for n represents the time rate of change of the reactor neutron population.

$$\text{Rate of Change} = \text{Production} + \text{Source} - \text{Loss}$$

The heuristic derivation is now applied

$$\text{Loss} = n/l_o = \text{number of neutrons lost per second}$$

$$k_{eff}n/l_o = \text{number of "particles" born per second}$$

$$k_{eff}(1-\beta) = \text{number of prompt neutrons born per neutron lost}$$

$$k_{eff}(1-\beta)n/l_o = \text{number of prompt neutrons born per second}$$

$$\lambda_i C_i = \text{number of delayed neutrons born per second}$$

$$\sum_{i=1}^m \lambda_i C_i = \text{number of delayed neutrons born per second}$$

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Thus, the total neutron production rate is:

$$\begin{aligned} \text{Production} &= k_{\text{eff}}(1-\beta)n/l_o + \sum_{i=1}^m \lambda_i C_i \text{ and} \\ \text{Source} &= S \end{aligned}$$

The time rate of change of the neutron population = $\dot{n}$. Thus, the above equation becomes:

$$\dot{n} = k_{\text{eff}}(1-\beta)n/l_o + \sum_{i=1}^m \lambda_i C_i + S - n/l_o$$

Collecting Terms

$$\dot{n} = k_{\text{eff}}(1-\beta - 1/k_{\text{eff}}) n/l_o + \sum_{i=1}^m \lambda_i C_i + S - n/l_o$$

By the definitions that $\beta = l_o/k_{\text{eff}}$ and $\rho = \frac{k_{\text{eff}}-1}{k_{\text{eff}}}$ Equation 1 follows:

$$\dot{n} = \frac{\rho-\beta}{1} n + \sum_{i=1}^m \lambda_i C_i + S$$

The rate of change of delayed neutron precursor concentrations follows the same logic.

$$\text{Rate of Change} = \text{Production} - \text{Loss}$$

The derivation is:

$$k_{\text{eff}}\beta_i = \text{number of precursors (ith group) born per neutron lost}$$

$$k_{\text{eff}}\beta_i n/l_o = \text{number of precursors (ith group) born per second, Thus,}$$

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$$\text{Production} = k_{\text{eff}} \beta_i n / l_o$$

$$\text{Loss} = \lambda_i C_i = \text{number of precursors (ith group) lost per second}$$

$$\text{Rate of Change of precursors (ith group)} = C_i$$

Upon substitution of the terms into the proper equation above

$$C_i = k_{\text{eff}} \beta_i n / l_o - \lambda_i C_i$$

or Equation 2 results with $l = l_o / k_{\text{eff}}$

$$C_i = \beta_i n / l - \lambda_i C_i$$

As stated, the point reactor model is limited to k_{eff} near 1, and thus the most significant time-dependent parameter in this model will be k_{eff}^{-1} , or the equivalent, total reactivity $\frac{k_{\text{eff}} - 1}{k_{\text{eff}}}$, which in

the CAM models is referred to as δk_t . For example, a reactor is said to be prompt critical where $\rho = \beta$ 0.0071. This implies that k_{eff} 1.0071, which is not far from 1, but causes very large changes in the neutron generation rate.

Total reactivity is a critical function of the reactor control device position, the absolute temperature of core materials, and the production of certain "poisons", such as Xenon, resulting from the fission process. During transient operation this reactivity will change significantly due to temperature feedback effects and coolant density changes. The CAM model postulates that δk_t , as a function of time, is comprised of four terms:

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$$\delta k_t = C_1 \sin^2 (\theta_D/2) + C_2 \phi_c + C_3 \rho_c + C_4$$

where C_1 , C_2 , C_3 , and C_4 are empirical constants

θ_D = the average angle of the control drums from the full inward position, radians

ϕ_c = the average core material temperature, °R and

ρ_c = the average core hydrogen coolant density, lb/ft³.

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TABLE I

NOMENCLATURE

A	Flow Area, ft^2
C	Empirical Fluid Flow Loss Coefficients, ft^{-3}
C_i	Number of Delayed Neutron Precursors (potential neutrons) Same units as n.
D	Effective Flow Diameter, in.
$\frac{D}{Dt} = \frac{\partial}{\partial t} + V_x \frac{\partial}{\partial x}$	Substantial derivative
E	Fluid Enthalpy in Btu/lb
f	Friction factor, dimensionless
g	Acceleration of Gravity, 32.2 ft/sec^2
G	Heat Generation Rate, Btu/lb-sec
h	Overall Heat Transfer Coefficient, $\text{Btu/ft}^2\text{-}^\circ\text{R-sec}$
H	Material Enthalpy, Btu/lb ($\text{Oat} = 0$)
HD	Temperature Dependent Part of Heat Transfer Coefficient
J	In.-lb/Btu
J'	Mechanical Equivalent of Heat ft-lb/Btu
k	Thermal Conductivity, Btu/ft $^\circ\text{R-sec}$
K	Empirical Coefficient of Heating in Materials, $\frac{\text{Btu}}{\%\text{-sec}}$
k_{eff}^{n/l_o}	Number of Neutrons Lost per Second
$k_{\text{eff}}^{(1-\beta)}$	Number of Prompt Neutrons Born per Neutron Lost
$k_{\text{eff}}^{(1-\beta)n/l_o}$	Number of Prompt Neutrons Born per Second
l_o	Prompt Neutron Life Time, Sec
l^* (or l)	Prompt Neutron Generation Time, Sec
L	Length of Flow Channel, ft
M	Mass of Fluid ($=\bar{\rho}V$) or Material, lb
P	Pressure, psia
Q	Heat Lost or Gained from Surroundings, Btu/lb
Q_i	Heat Produced by <u>ith</u> Group of Fission Products, %

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TABLE I (cont.)

R	Universal Gas Constant, $\frac{\text{ft}^3}{\text{in.}^2 \cdot ^\circ\text{R}}$
S _o	Source of Neutrons (same units as n)
S _n	Number of Neutrons (or neutron density, neutronic power, etc)
S	Total Thermal Power, %
SQN	Power Term for Nonfueled Elements, %
t	Time, seconds
T	Fluid Temperature, °R
U	Internal Energy, Btu/lb
v _x	Fluid Velocity in x Direction, ft/sec
v	Specific Gravity in ft ³ /lb
V	Volume, ft ³
W	Work, ft or Mass Flow, lb/sec
y	Direction Normal to Fluid Flow, ft
α	Heating surface, ft ²
β _i	Fraction of neutrons in <u>i</u> th delay group
β _T	Total fraction of delayed neutrons
θ _D	The average angle of the control drums from the full inward position, radians
λ _i	Decay constant for delayed neutron precursors of the <u>i</u> th group
λ _i C _i	Number of delayed neutrons of the <u>i</u> th group born per second
ρ	Density, lb/ft ³
φ	Material temperature, °R
φ _c	Average core material temperature, °R
δk _t	Total reactivity = $\frac{k_{\text{eff}} - 1}{k_{\text{eff}}}$
δk _D	Reactivity contribution of drums (=0 at 89°)
δk _f	Feedback reactivity (with respect to cold critical)
i	Decay constant for <u>i</u> th group of fission products

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2. CAM X E-1 Nuclear Subsystem Equations

This section contains a detailed list of equations and constants that constitute the model.

a. Reflector

$$\frac{d}{dt} E_r = \frac{1}{\bar{\rho}_r V_r} \left[2W_r (E_5 - E_r) + 144.0 h \alpha_r (\phi_r - T_r) \right] \quad (1)$$

$$\bar{\rho}_r V_r = 1 \text{ lb}$$

$$D_r = 7.06 \text{ in.}$$

$$h = 3.31 \frac{HD_r W_r^{18}}{D_r^{1.8}} \frac{\text{Btu}}{\text{sec-}^\circ\text{R-ft}^2}$$

$$\alpha_r = 609 \text{ ft}^2$$

$$T_r = 0.2725 E_r + 0.008875 (P_5 + P_6) + 44.0 \quad (2)$$

$$\frac{d}{dt} H_r = \frac{1}{M_r} h \alpha_r (T_r - \phi_r) + \frac{K_r}{M_r} \text{SQN} \quad (3)$$

$$h = 0.0982 HD_r W_r^{.8} \frac{\text{Btu}}{\text{sec-}^\circ\text{R-ft}^2}$$

$$M_r = 4246 \text{ lb}$$

$$\alpha_r = 609 \text{ ft}^2$$

$$K_r = 234.4 \text{ Btu/\%-sec}$$

$$HD_r = 0.165 + 0.000833 T_r, HD_r \leq 0.29 \quad (4)$$

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$$\phi_r = f_1(H_r) \quad (5)$$

$$\frac{d}{dt} W_r = \frac{144 A_r g}{L_r} (P_5 - P_6) - C_r \frac{W_r^2}{\rho_6} \quad (6)$$

$$A_r = .595 \text{ ft}^2$$

$$g = 32.2 \text{ ft/sec}^2$$

$$L_r = 4.46 \text{ ft}$$

$$C_r = 2.531 \text{ ft}^{-3}$$

b. Dome Plenum

$$E_6 = 2E_r - E_5 \quad (7)$$

$$T_6 = 0.2651 E_6 + 0.03218 P_6 + 45.0 \quad (8)$$

$$P_6 = R \rho_6 T_6 \quad (9)$$

$$R = 5.32 \frac{\text{ft}^3}{\text{in.}^2 \cdot ^\circ\text{R}} \quad (10)$$

$$\frac{d}{dt} \rho_6 = \frac{1}{(V_r + V_6)} (W_r - W_s - W_{db}) \quad (11)$$

$$V_r + V_6 = 8.71 \text{ ft}^3 \quad (12)$$

c. Shield

$$\frac{d}{dt} E_s = \frac{1}{\rho_s V_s} \left[2W_s (E_6 - E_s) + h\alpha_s (\phi_s - T_s) \right] \quad (13)$$

$$\bar{\rho}_s V_s = 0.5 \text{ lb} \quad (14)$$

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$$D_s = 6.72 \text{ in.}$$

$$h = 3.31 \frac{HD_s W_s^{.8}}{D_s^{1.8}} \frac{\text{Btu}}{^\circ\text{R-ft}^2\text{-sec}}$$

$$\alpha_s = 124 \text{ ft}^2$$

$$T_s = 0.2651 E_s + 0.01609 (P_6 + P_7) + 45.0 \quad (15)$$

$$HD_s = 0.165 + 0.000833T_s, \quad HD_s \leq 0.29 \quad (16)$$

$$\frac{d}{dt} H_s = \frac{1}{M_s} h \alpha_s (T_s - \phi_s) + \frac{K_s}{M_s} SQ \quad (17)$$

$$M_s = 2813 \text{ lb}$$

$$h = .1072 HD_s W_s^{.8} \frac{\text{Btu}}{\text{sec-ft}^2\text{-}^\circ\text{R}}$$

$$\alpha_s = 124 \text{ ft}^2$$

$$K_s = 27.8 \frac{\text{Btu}}{\%\text{-sec}}$$

$$\phi_s = f_2 (H_s) \quad (18)$$

$$\frac{d}{dt} W_s = \frac{144 A_s g}{L_s} (P_6 - P_7) - C_s \frac{W_s^2}{\rho_7} \quad (19)$$

$$A_s = 0.477 \text{ ft}^2$$

$$g = 32.2 \text{ ft/sec}^2$$

$$L_s = 2.71 \text{ ft}$$

$$C_s = 1.046 \text{ ft}^{-3}$$

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d. Core Inlet Plenum

(U) $E_7 = 2.0 E_s - E_6$ (20)

$T_7 = 0.2651 E_7 + 0.03218 P_7 + 45.0$ (21)

$P_7 = R \rho_7 T_7$ (22)

$R = 5.32 \frac{\text{ft}^{-3}}{\text{in.}^2 \text{-}^\circ\text{R}}$

$\frac{d}{dt} \rho_7 = \frac{1}{(1/2 V_c + V_s + V_7)} (W_s - W_c - W_{tr})$ (23)

$1/2 V_c + V_s + V_7 = 12.65 \text{ ft}^3$ (24)

(CRD)

e. Core

$\frac{d}{dt} E_{c.1} = \frac{1}{\bar{\rho}_{c.1} V_{c.1}} \left[2W_c (E_7 - E_{c.1}) + h\alpha_{c.1} (\phi_{c.1} - T_{c.1}) \right]$ (25)

$\bar{\rho}_{c.1} V_{c.1} = 1/6 \text{ lb}$

$D_c = 9.47 \text{ in.}$

$h = 3.31 \text{ HD}_{c.1} \frac{W_c^{.8}}{D_c^{1.8}} \frac{\text{Btu}}{\text{sec-ft}^2 \text{-}^\circ\text{R}}$

$\alpha_{c.1} = \frac{\alpha_c}{3} = 1123 \text{ ft}^2$

$\frac{d}{dt} E_{c.2} = \frac{1}{\bar{\rho}_{c.2} V_{c.2}} \left[2W_c (E_{c.1e} - E_{c.2}) + h\alpha_{c.2} (\phi_{c.2} - T_{c.2}) \right]$ (26)

$\bar{\rho}_{c.2} V_{c.2} = 1/6 \text{ lb}$

(CRD)

$$h = .0579 \text{ HD}_{c.2} W_c^{.8}$$

$$\alpha_{c.2} = \frac{\alpha_c}{3} = 1123 \text{ ft}^2$$

$$\frac{d}{dt} E_{c.3} = \frac{1}{\bar{\rho}_{c.3} V_{c.3}} 2W_c (E_{c.2e} - E_{c.3}) + h \alpha_{c.3} (\phi_{c.3} - T_{c.3}) \quad (27)$$

$$\bar{\rho}_{c.3} V_{c.3} = 1/6 \text{ lb}$$

$$h = .0579 \text{ HD}_{c.3} W_c^{.8} \frac{\text{Btu}}{\text{sec-ft}^2-\text{°R}}$$

$$\alpha_{c.3} = \frac{\alpha_c}{3} = 1123 \text{ ft}^2$$

$$T_{c.1} = 0.2791 E_{c.1} + 56.0 \quad (28)$$

$$E_{c.1e} = 2.0 E_{c.1} - E_7 \quad (29)$$

$$T_{c.2} = 0.2791 E_{c.2} + 56.0 \quad (30)$$

$$E_{c.2e} = 2.0 E_{c.2} - E_{c.1e} \quad (31)$$

$$T_{c.3} = f_3 (E_{c.3}) \quad (32)$$

$$E_{c.3e} = 2.0 E_{c.3} - E_{c.2e} \quad (33)$$

$$T_{c.3e} = f_3 (E_{c.3e}) \quad (34)$$

$$\text{HD}_{c.1} = 0.275 + 0.0000475 T_{c.1} \quad (35)$$

$$\text{HD}_{c.2} = 0.275 + 0.0000475 T_{c.2} \quad (36)$$

$$\text{HD}_{c.3} = 0.275 + 0.0000475 T_{c.3} \quad (37)$$

(CRD)

$$\frac{d}{dt} H_{c.1} = \frac{1}{M_{c.1}} h \alpha_{c.1} (T_{c.1} - \phi_{c.1}) + \frac{K_{c.1}}{M_{c.1}} S \quad (38)$$

$$M_{c.1} = \frac{M_c}{3} = 768.16$$

$$h = 0.579 HD_{c.1} W_c^{.8} \frac{\text{Btu}}{\text{sec-ft}^2-\text{°R}}$$

$$K_{c.1} = 3430 \frac{\text{Btu}}{\text{°-sec}}$$

$$\alpha_{c.1} = 1123 \text{ ft}^2$$

$$\frac{d}{dt} H_{c.2} = \frac{1}{M_{c.2}} h \alpha_{c.2} (T_{c.2} - \phi_{c.2}) + \frac{K_{c.2}}{M_{c.2}} S \quad (39)$$

$$M_{c.2} = \frac{M_c}{3} = 768.16$$

$$h = .0579 HD_{c.2} W_v^{.8} \frac{\text{Btu}}{\text{sec-ft}^2-\text{°R}}$$

$$K_{c.2} = 4970 \frac{\text{Btu}}{\text{°-sec}}$$

$$\alpha_{c.2} = 1123 \text{ ft}^2$$

$$\frac{d}{dt} H_{c.3} = \frac{1}{M_{c.3}} h \alpha_{c.3} (T_{c.3} - \phi_{c.3}) + \frac{K_{c.3}}{M_{c.3}} S \quad (40)$$

$$M_{c.3} = \frac{M_c}{3} = 768 \text{ lb}$$

$$h = .0579 HD_{c.3} W_c^{.8} \frac{\text{Btu}}{\text{sec-ft}^2-\text{°R}}$$

$$K_{c.3} = 2170 \frac{\text{Btu}}{\text{°-sec}}$$

$$\alpha_{c.3} = 1123 \text{ ft}^2$$

$$(CRD) \quad \phi_{c.1} = f_4 (H_{c.1}) \quad (41)$$

$$\phi_{c.2} = f_4 (H_{c.2}) \quad (42)$$

$$\phi_{c.3} = f_4 (H_{c.3}) \quad (43)$$

$$\frac{d}{dt} W_c = 144.0 \frac{A_c g}{L_c} (P_7 - P_8) - C_{c1} \frac{W_c^2}{\rho_7} - C_{c2} \frac{W_c^2}{\rho_8} \quad (44)$$

$$A_c = 1.485 \text{ ft}^2$$

$$g = 32.3 \text{ ft/sec}^2$$

$$L_c = 4.46 \text{ ft}$$

$$C_{c1} = 4.59 \text{ ft}^{-3}$$

$$C_{c2} = .993 \text{ ft}^{-3}$$

(45)

$$\phi_c = 1/8 (3.0 \phi_{c.1} + 2.0 \phi_{c.2} + 3.0 \phi_{c.3}) \quad (46)$$

(U) f. Three-Group Log Kinetics

$$\frac{d}{dt} \ln S_n = \frac{(Sk_t - \beta_T)}{1^*} + \lambda_1 \frac{C_1}{S_n} + \lambda_n \frac{C_2}{S_n} + \lambda_3 \frac{C_3}{S_n} \quad (47)$$

$$\beta_T = 0.0071$$

$$1^* = 2.5 \times 10^{-5} \text{ sec}$$

$$\lambda_1 = .0276 \text{ sec}^{-1}$$

$$\lambda_2 = 0.235 \text{ sec}^{-1}$$

$$\lambda_3 = 1.65 \text{ sec}^{-1}$$

$$\frac{d}{dt} \frac{C_1}{S_n} = \frac{\beta_1}{1^*} - \lambda_1 \frac{C_1}{S_n} - \frac{C_1}{S_n} \frac{d}{dt} \ln S_n \quad (48)$$

$$\beta_1 = .001945$$

$$1^* = 2.5 \times 10^{-5} \text{ sec}$$

$$\lambda_1 = .0276 \text{ sec}^{-1}$$

$$\frac{d}{dt} \frac{C_2}{S_n} = \frac{\beta_2}{1^*} - \lambda_2 \frac{C_2}{S_n} - \frac{C_2}{S_n} \frac{d}{dt} \ln S_n \quad (49)$$

$$\beta_2 = 0.003975$$

$$1^* = 2.5 \times 10^{-5} \text{ sec}$$

$$\lambda_2 = 0.235 \text{ sec}$$

$$\frac{d}{dt} \frac{C_3}{S_n} = \frac{\beta_3}{1^*} - \lambda_3 \frac{C_3}{S_n} \frac{C_3}{S_n} \frac{d}{dt} \ln S_n \quad (50)$$

$$\beta_3 = 0.001180$$

$$1^* = 2.5 \times 10^{-5} \text{ sec}$$

$$\lambda_3 = 1.65 \text{ sec}^{-1}$$

g. Six-Group Kinetics

$$\begin{aligned} \frac{d}{dt} S_n = & \frac{(\delta_{kt} - \beta_T)}{1^*} S_n + \lambda_1 C_1 + \lambda_2 C_2 + \lambda_3 C_3 + \lambda_4 C_4 \\ & + \lambda_5 C_5 + \lambda_6 C_6 \end{aligned} \quad (51)$$

$$\beta_T = .0071$$

$$1^* = 2.5 \times 10^{-5} \text{ sec}$$

$$\lambda_1 = .0127 \text{ sec}^{-1}$$

$$\lambda_2 = .0317 \text{ sec}^{-1}$$

$$\lambda_3 = .115 \text{ sec}^{-1}$$

$$\lambda_4 = .311 \text{ sec}^{-1}$$

$$\lambda_5 = 1.140 \text{ sec}^{-1}$$

$$\lambda_6 = 3.87 \text{ sec}^{-1}$$

$$\frac{d}{dt} C_1 = \frac{\beta_1}{1^*} S_n - \lambda_1 C_1 \quad (52)$$

$$\beta_1 = .00027$$

$$\frac{d}{dt} C_2 = \frac{\beta_2}{1^*} S_n - \lambda_2 C_2 \quad (53)$$

$$\beta_2 = .001512$$

$$\frac{d}{dt} C_3 = \frac{\beta_3}{1*} S_n - \lambda_3 C_3 \quad (54)$$

$$\beta_3 = .001335$$

$$\frac{d}{dt} C_4 = \frac{\beta_4}{1*} S_n - \lambda_4 C_4 \quad (55)$$

$$\beta_4 = .003890$$

$$\frac{d}{dt} C_5 = \frac{\beta_5}{1*} - \lambda_5 C_5 \quad (56)$$

$$\beta_5 = .000906$$

$$\frac{d}{dt} C_6 = \frac{\beta_6}{1*} - \lambda_6 C_6 \quad (57)$$

$$\beta_6 = .000186$$

h. Feedback Equations

$$\delta_{kt} = \delta_{kf} + \delta_{kD} \quad (58)$$

$$\delta_{kf} = 0.00204 + 0.3694 \quad 8 - 5.725 \times 10^{-6} \phi_c \quad (59)$$

$$\delta_{kD} = 0.0582 \sin^2 \frac{\theta_D}{2} - 0.0285 \quad (60)$$

i. Decay Heat

$$S = S_n + Q_1 + Q_2 + Q_3 \quad (61)$$

$$SQ = 0.6547 S_n + 6.409 (Q_1 + Q_2 + Q_3) \quad (62)$$

(U) $\frac{d}{dt} Q_1 = \alpha_1 S_n - \zeta_1 Q_1$ (63)

$$\alpha_1 = .000117$$

$$\zeta_1 = .00322$$

$$\frac{d}{dt} Q_2 = \alpha_2 S_n - \zeta_2 Q_2$$
 (64)

$$\alpha_2 = .000755$$

$$\zeta_2 = .05525$$

$$\frac{d}{dt} Q_3 = \alpha_3 S_b - \zeta_3 Q_3$$
 (65)

$$\alpha_3 = .00372$$

$$\zeta_3 = .2707$$

(CRD) j. Support Element

$$\frac{d}{dt} H_{sg} = F_1 (\phi_{c.2} - \phi_{sg}) + F_2 (\phi_{pg} - \phi_{sg}) + \frac{K_{sg}}{M_{sg}} SQN$$
 (66)

$$F_1 (\phi_{sg}) = \frac{2d_{sgo}}{M_{sg}} \frac{K_{sg} \cdot K_{H2}}{(\Delta_{sg} k_{H2} + 2\Delta H_2 k_{sg})}$$
 (67)

$$F_2 \left(\frac{(\phi_{sg} + \phi_{pg})}{2} \right) = \frac{2\alpha_{sgj}}{M_{sg}} \frac{k_{sg} \cdot k_{pg}}{(\Delta_{pg} k_{sg} + k_{pg} \Delta_{sg})}$$
 (68)

$$H_{sg} = f(\phi_{sg})$$
 (69)

(CRD)

$$\frac{d}{dt} H_{pg} = F_3 (\phi_{sg} - \phi_{pg}) + F_4 (T_{H_2} - \phi_{pg}) + \frac{K_{pg}}{M_{pg}} SQN \quad (70)$$

$$F_3 = \frac{F_2 \cdot M_{sg}}{M_{pg}} = 5.98F_2 \quad (71)$$

$$F_4 = \frac{(\phi_{pg} + T_{H_2})}{2} = \frac{2}{M_{pgj}} \frac{k_{H_2} \cdot k_{pg}}{(\Delta P_g k_{H_2} + \Delta H_2 k_{pg})} \quad (72)$$

$$H_{pg} = f(\phi_{pg}) \quad (73)$$

$$T_{H_2} = \phi_{pg} + \frac{F_6}{F_5} (\phi_{ss} - T_{H_2}) \quad (74)$$

$$F_5 = F_4 \cdot M_{pg} = 0.221F_4 \quad (75)$$

$$F_6 = 2_{ss_o} \frac{k_{H_2} k_{ss}}{(\Delta SS k_{H_2} + \Delta H_2 k_{ss})} \quad (76)$$

$$\frac{d}{dt} H_{ss} = F_7 (T_H - \phi_{ss}) + \alpha_{ss_j} \frac{h_o}{M_{ss}} (T_g - \phi_{ss}) + \frac{K_{ss}}{M_{ss}} SQN \quad (77)$$

$$\phi_{ss} = f(H_{ss}) \quad (78)$$

$$F_7 = \frac{F_6}{M_{ss}} = 17F_6 \quad (79)$$

$$E_g = E_7 + 289 \frac{\alpha_{ss_j} h_o}{2W} (\phi_{ss} - T_g) + 289 \frac{\alpha_{tr_o} h_i}{2W} (\phi_{tr_o} - T_g) \quad (80)$$

$$\phi_{tr_o} = \phi_{tr} + \frac{\Delta tr h_i}{2k_{tr}} \quad (81)$$

$$k_{tr} = f(\phi_{tr}) \quad (82)$$

(CRD)

$$M_{tr} \frac{d}{dt} H_{tr} = 2\alpha_{tr_o} \frac{k_{tr}}{\Delta tr} (\phi_{tr_o} - \phi_{tr}) + k_{tr} S \quad (83)$$

$$\phi_{tr} = f(H_{tr}) \quad (84)$$

$$T_g = f(E_g) \quad (85)$$

$$h_i = h_o = h \quad (86)$$

$$h = 0.00177W^{0.8} \quad (87)$$

$$T_{tre} = 2T_g - T_7 \quad (88)$$