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ABSTRACT

Neutrino scattering processes are studied in the framework of general local current-current interaction. The scattering amplitudes for such processes are expressed in a simple factorized form in terms of helicity form factors for the current vertex functions. These expressions explicitly display the full kinematic content of the local current-current interaction and are used as the basis for a systematic study of possible ways to test the V-A structure of the weak current in high energy neutrino scattering processes. A direct test in an "inclusive experiment" consists of measuring the outgoing lepton polarization. Alternative tests from angular and spin correlations are possible only in "exclusive experiments." Several examples of this latter type of experiments are given: (i) pure lepton scattering processes, (ii) neutrino scattering off spin zero (nuclei) targets, (iii) quasi-elastic scattering off polarized neucleon, (iv) quasi-elastic hyperon production (decay asymmetry) and (v) single pion production in the N^* region.

I. Introduction

All present experimental evidence in weak decay processes is consistent with the V-A theory of weak interactions.[1] The weak vector (V) and axial-vector (A) currents played an essential role in the remarkable theoretical developments beginning with the Conserved-Vector-Current hypothesis and culminating in the successes of "current algebra".[1,2]

The weak decay processes all involve rather limited ranges in the energy and momentum-transfer variables. The neutrino scattering processes, which are just becoming experimentally feasible (both in existing laboratories and with the projected new generation of accelerators) promise to extend the range of these variables to entirely new territories. This will open up vast domains of the weak interaction hitherto unavailable for our scrutiny.[3] The first obvious questions are: at these high energies and large momentum transfers, are these processes still describable by an effective current-current local interaction Hamiltonian and, if so, is this interaction still a V and A combination? These questions must, in principle, be answered by experiments in the affirmative before further comparison with more detailed theoretical predictions based on this basic structure together with other dynamical assumptions can be made truly meaningful.

Consequences of the local V-A current-current interaction in neutrino scattering processes have been studied before.[4,5] Pais and Treiman, in particular, have exhibited the "full content" of the locality assumption for such processes by giving the energy and angular distributions for the outgoing lepton.[5] Assuming the most general form of local (non-derivative) interaction, we study in this paper in considerable detail the resulting energy and angle spectrums as well as angular and spin correlations in an arbitrary neutrino-scattering process with the specific aim of seeking particular cases in which the presence of any local scalar (s), pseudo-scalar (P) and tensor (T) admixture to the V-A interaction can be experimentally detected.

The systematic investigation of all possible processes of this type for our particular purpose is facilitated by a helicity-like formalism[6,7] which (a) suggests the most natural variables to use in analysing such processes, (b) defines helicity form factors for arbitrary current vertices which are natural generalizations of the familiar G_E and G_M form factors for nucleon electromagnetic current vertex and (c) allows us to write down a general expression for the scattering amplitude which compactly displays all the kinematical contents of current-current interaction in a factorized form (in terms of the helicity form factors) and leads to expressions for angular and spin correlation functions in the form of simple matrix products.

In Section II we spell out our basic assumptions. In Section III we introduce the helicity expansion for the current vertices functions. In Section IV we give the general expressions for the transition amplitude and intensity distribution of an arbitrary neutrino scattering process with general local interactions. It is pointed out, as is undoubtedly known to others, that if the lepton mass can be neglected, the most unambiguous way to distinguish the V-A interaction from the other types (S, T or P)is to look at the polarization of the outgoing lepton. For neutrino initiated reactions, this should be purely left-handed if V-A is the only interaction involved and purely right-handed if any combination of S, T and P interaction is responsible. For anti-neutrino initiated reactions, the result is the converse. Since the measurement of lepton (mainly muon) polarization at very high energies is an extremely difficult task, we examine possibilities for testing the V-A theory through angular and (target) spin correlation experiments. The method used is explained at the end of Section IV and specific examples are given in Section V. These neutrino processes include: (i) Pure leptonic scatterings; (ii) Quasi-elastic scattering off spin zero (nuclear) targets; (iii) Quasi-elastic scattering off polarized nucleon targets; (iv) Decay angular correlations in hyperon production, and (v) Angular correlations in single pion-production in the N* region. These tests involve the measurement of the outgoing lepton angular distribution (one variable only) in either the differential cross-section [(i) and (ii)] or in certain asymmetry functions [(iii), (iv) and (v)]. For maximum statistics, data obtained at different incident energies can be combined and integrated over to obtain the needed spectrum.

For high energy scattering processes, the lepton mass can, for all practical purposes, be neglected. In Appendix A we briefly indicate the lepton mass correction effects. In Appendix B we describe some properties of the helicity form factors and give the explicit relations to the conventional invariant form factors for the case of spin 1/2 particles. In Appendix C we give some detailed formulae on the single-pion-production process discussed in the text.

II. BASIC ASSUMPTIONS

We are interested in the neutrino scattering processes, (Fig. 1):

$$\left\{ {\nu \atop \bar{\nu}} \right\} + A \to \left\{ {\ell \atop \bar{\ell}} \right\} + B \tag{1}$$

where ℓ stands for either the electron or muon and A and B can be leptons (pure leptonic processes) or hadronic systems (semi-leptonic processes). In the latter case, A is usually a single particle state while B may be a single particle or multi-particle hadronic system with or without additional lepton pairs. For definiteness, we shall consider explicitly the neutrino initiated processes (first line in (1)) and refer to the A, B systems as hadrons, all considerations obviously remain valid for the anti-neutrino initiated reactions and for the case where A, B are leptons with little change in the resulting formulas. We shall remark on the necessary changes at the appropriate places. We denote by k, k', p, p' the 4-momenta and $\lambda, \lambda', \sigma, \sigma'$ the polarization indices of the states ν, ℓ, A and B respectively. In general, charge labels will be omitted for conservation of indices.

We assume a general local current-current interaction. The transition amplitude for the process (1) (first line) is of the form

$$f = (G/\sqrt{2})\ \langle k'\lambda'|j^{+}(0)|k\lambda\rangle\langle p'\sigma'|J(0)|p\sigma\rangle \tag{2}$$

where j(x) and J(x) are the weak currents associated with the ν–ℓ (leptonic) and A-B ("hadronic") vertices respectively. These currents can be an arbitrary combination of the five types of possible currents S, V, T, A and P.

Since the incoming neutrinos usually come from pion (or K-meson) decay in flight and are known to be purely left-handed, we can assume they are described by the two component theory with

$$\langle k'\lambda'|j^{+}(0)|k\lambda\rangle = \bar{u}_{\lambda}'(k')\Gamma(1+\gamma_5)u_{\lambda}(k) \tag{3}$$

where Γ stands for some combination of 1, γ^{μ} and $\sigma^{\mu\nu}$ and $\lambda \equiv -1/2$ because of the factor $(1+\gamma_5)$. Since Eq. (3) automatically implies parity non-conservation in these processes, the distinction between the scalar and pseudo-scalar currents as well as the vector and axial vector currents at the hadronic vertex (Eq. (2)) becomes unnecessary. From now on, $\mathcal{J}$ stands for some combination of S (scalar as well as pseudo-scalar), V (vector as well as axial vector) and T (tensor) currents and Eq. (2) becomes

$$\begin{aligned} f_{\lambda'\sigma',\sigma} = (G/\sqrt{2})\ [&\langle k'\lambda'|s^{+}|k,-\tfrac{1}{2}\rangle\ \langle p'\sigma'|S|p\sigma\rangle \\ &+\langle k'\lambda'|v^{+}_{\mu}|k,-\tfrac{1}{2}\rangle\ \langle p'\sigma'|V^{\mu}|p\sigma\rangle \\ &+\langle k'\lambda'|t^{+}_{\mu\nu}|\ k,-\tfrac{1}{2}\rangle\ \langle p'\sigma'|T^{\mu\nu}|p\sigma\rangle] \end{aligned} \tag{4}$$

The first factor of each term is given by an expression of the form (3).

We shall neglect the lepton mass for most of our considerations since we are mainly interested in high energy regions where the existing theory has not been tested before. The lepton mass can be easily incorporated, however, in the ensuring considerations. Appendix A indicates how this can be done should it become necessary. We shall not consider radiative corrections which modify strict locality, nor shall we consider second order weak interaction effects for which we know of no reliable estimates.

To begin with, we study the kinematical structure of each of the current vertex functions appearing in Eq. (4).

III. THE CURRENT VERTEX FUNCTIONS[8]

The current vertex function $<p'\lambda'|\ J(0)|\ p\lambda>$ is related to the decay current matrix element $<p'\lambda',\ \bar{p}\bar{\lambda}\ |J(0)|0>$ by crossing. It is a familiar fact that in such decay processes, the kinematics of the angular and spin correlations are much simplified if the transition amplitudes are expressed in terms of the center of mass (CM) variables of the $(p',\ \bar{p})$ pair and if λ', $\bar{\lambda}$ are chosen to be the helicity indices.[9] This suggests that for the matrix element $<p'\lambda'|J(0)|p\lambda>$, it is most natural to choose our variables in the Brick-Wall (BW) frame in which the space like 4-momentum transfer $q = p-p'$ is of the standard form $(0,0,0\ \sqrt{q^2})$. Refering back to Eqs. (2) or (4), because of the relation $q = p-p' = k'-k$ we see that both current vertices can be simplified at the same time by this choice. Furthermore, since the transiton amplitude f is a Lorentz invariant (j and J are always contracted), there is no loss of generality in choosing a particular frame to evaluate the vertex functions.

(i) Definition of States[10]

In analogy to the well known Jacob and Wick[9] CM system helicity states, we proceed to define the particle states $|p\lambda>$ and $|p'\lambda'>$in the BW Frame as follows: the standard BW frame state for the "incoming" (or first) particle is defined to be

$$|p_s\lambda> \;=\; B_3(u)\ |0\lambda> \qquad (5)$$

while that for the "outgoing" (or second) particle is

$$|p_s'\lambda'> = B_3(-u')|0,\lambda'> \; . \qquad (6)$$

Here $B_3(u)$ denotes a boost along the positive 3-direction characterized by the hyperbolic angle u and $|0,\lambda>$ are the usual rest frame angular momentum states. The standard BW Frame vectors (p_s, p_s') are of the form

$$\begin{aligned} p_s &= M(\text{ch } u, 0, 0, \text{sh } u) \\ p_s' &= W(\text{ch } u', 0, 0, -\text{sh } u') \end{aligned} \tag{7}$$

with $M^2 = -p^2$, $W^2 = -p'^2$ and

$$\begin{aligned} M \text{ sh } u &= (q^2 - M^2 + W^2)/2\sqrt{q^2} \\ W \text{ sh } u' &= (q^2 - W^2 + M^2)/2\sqrt{q^2} \end{aligned} \tag{8}$$

so that $q = p - p'$ is of the standard form $(0,0,0,\sqrt{q^2})$. A <u>general</u> configuration of the vectors (p,p') in the BW frame can be obtained from the standard vectors, Eq. (7), by a <u>common</u> SO(2,1) transformation which leaves the vector q invariant (i.e. Lorentz transformations involving the 0, 1, 2 axes only). Denoting this tranformation by $O(\psi, \phi)$ we have:

$$\begin{aligned} |p\lambda> &= O(\psi,\phi)|p_s\lambda> \\ |p'\lambda'> &= O(\psi,\phi)|p_s'\lambda'> \end{aligned} \tag{9}$$

where $O(\psi,\phi)$ is chosen to be a boost along the 1-axis by the (hyperbolic) angle ψ followed by a rotation around the 3-axis by the angle ϕ, i.e.

$$O(\psi,\phi) = R_3(\phi)B_1(\psi). \tag{10}$$

the general BW frame momenta (p,p') are therefore parametrized as:[10]

$$\begin{aligned} p &= M(\text{ch } u \text{ ch } \psi, \text{ ch } u \text{ sh } \psi \cos\phi, \text{ ch } u \text{ sh } \psi \sin\phi, \text{ sh } u) \\ p' &= W(\text{ch } u' \text{ch } \psi, \text{ ch } u' \text{sh } \psi \cos\phi, \text{ ch } u' \text{sh } \psi \sin\phi, -\text{sh } u'). \end{aligned} \tag{11}$$

With these definitions, let us turn to the various vertex functions that enter Eqs. (2) and (4).

(ii) Scalar current

Using the definition (9), the scalar vertex function in the brick wall frame can be written

$$\begin{aligned} \langle p'\lambda' \,|S|\, p\lambda\rangle &= \langle p_s'\lambda' | O^{-1}(\psi,\phi) S \, O(\psi,\phi) | p_s\lambda\rangle \\ &= \langle p_s'\lambda' \,|S|\, p_s\lambda\rangle = S_{\lambda'\lambda}(q^2) \end{aligned} \tag{12}$$

where we used the fact that $O^{-1}SO = S$ (S is a scalar) and that p_s and p_s' depend only on the invariant variable q^2, Eq. (7). The second line in Eq. (12) defines the "scalar form factor" or "reduced matrix element" $S_{\lambda'\lambda}(q^2)$. This equation shows that the vertex function $\langle p'\lambda'|S|p\lambda\rangle$ is independent of the variables ψ and ϕ . It is shown in Appendix B that

$$S_{\lambda'\lambda}(q^2) = \delta_{0,\lambda+\lambda'} S_\lambda(q^2) \tag{13}$$

which expresses angular momentum conservation.

(iii) Vector currents

Let the unit vectors in the BW frame be $\{e^\mu_{(0)}, e^\mu_{(1)}, e^\mu_{(2)}, e^\mu_{(3)}\}$. We define

$$e^\mu_{(\pm 1)} = (e^\mu_{(1)} - i\, e^\mu_{(2)})/\sqrt{2} \tag{14}$$

and use the set $\{e^\mu_{(\alpha)};\ \alpha=+1,0,-1,3\}$ as our basis vectors. They satisfy the orthonormality and completeness conditions:

$$e_{(\alpha)}{}^{\mu *} e^{(\beta)}{}_{\mu} = \delta_{\alpha}{}^{\beta}$$

$$e^{(\alpha)\mu} e_{(\alpha)}{}^{\nu *} = g^{\mu\nu} \tag{15}$$

where $e^{(0)} = -e_{(0)}$ and $e^{(i)} = e_{(i)}$, $i = \pm 1,3$. The vector current can be written as

$$V^{\mu} = e_{(\alpha)}{}^{\mu} V^{(\alpha)} = e_{(3)}{}^{\mu} V^{(3)} + e_{(m)}{}^{\mu} V^{(m)} \tag{16}$$

where $m = +1, 0, -1$ and $V^{(\alpha)} = e^{(\alpha)\mu *} V_{\mu}$. In analogy with the scalar case, we can now write down the general expression for the vector vertex function in the BW frame,

$$\langle p'\lambda'|V^{\mu}(0)|p\lambda\rangle = \langle p_s'\lambda'|O^{-1}(\psi,\phi)V^{\mu}(0)O(\psi,\phi)|p_s\lambda\rangle$$

$$= e_{(\alpha)}{}^{\mu} D(\psi,\phi)^{\alpha}{}_{\beta} V^{(\beta)}_{\lambda'\lambda}(q^2) \tag{17}$$

where

$$V^{(\alpha)}_{\lambda'\lambda}(q^2) = \langle p_s'\lambda'|V^{(\alpha)}(0)|p_s\lambda\rangle \tag{18}$$

$$D(\psi,\phi)^{3}{}_{3} = 1,\ D(\psi,\phi)^{3}{}_{m} = D(\psi,\phi)^{m}{}_{3} = 0$$

$$D(\psi,\phi)^{m}{}_{n} = e^{-im\phi} \bar{d}(\psi)^{m}{}_{n} \tag{19}$$

and

$$\bar{d}(\psi)^{m}{}_{n} = \begin{pmatrix} (1/2(1+\mathrm{ch}\,\psi) & -\mathrm{sh}\psi/\sqrt{2} & (1/2)(1-\mathrm{ch}\,\psi) \\ -\mathrm{sh}\psi/\sqrt{2} & \mathrm{ch}\,\psi & \mathrm{sh}\psi/\sqrt{2} \\ (1/2)(1-\mathrm{ch}\,\psi) & \mathrm{sh}\,\psi/\sqrt{2} & (1/2)(1+\mathrm{ch}\psi) \end{pmatrix}$$

Eq. (17) is an expansion of the vertex functions in terms of the form factors $V^{(\beta)}_{\lambda'\lambda}$ which depend only on the invariant variable q^2 (together with possible "internal variables" if the final state is a complex system). The (ψ,ϕ)-dependence of the vector vertex function is explicitly exhibited in the D-functions. Angular momentum conservation conditions again impose the constraints:

$$V^{(3)}_{\lambda'\lambda}(q^2) = \delta_{0,\lambda+\lambda'}V^{(3)}_{\lambda}(q^2)$$
$$V^{(m)}_{\lambda'\lambda}(q^2) = \delta_{m,\lambda+\lambda'}V^{(m)}_{\lambda}(q^2), \ m=+1,0,-1. \tag{20}$$

The last equation indicates that the index m has the physical interpretation of being the "helicity" of the current (with 4-momentum q).

(iv) Tensor current

We define basis tensors $\{e^{\mu\nu}_{(m)}, \tilde{e}^{\mu\nu}_{(m)}; m=+1,0,-1\}$,

$$e_{(m)}{}^{\mu\nu} = i[e_{(3)}{}^{\mu}e_{(m)}{}^{\nu}-e_{(m)}{}^{\mu}e_{(3)}{}^{\nu}]/\sqrt{2} \tag{21}$$
$$\tilde{e}_{(m)}{}^{\mu\nu} = -(1/2)i\varepsilon^{\mu\nu\lambda\sigma}e_{(m)\lambda\sigma}.$$

They satisfy the orthonormality and completeness conditions:

$$e_{(m)}{}^{*}.e^{(n)} = -\tilde{e}_{(m)}{}^{*}.\tilde{e}^{(n)} = \delta_m{}^n$$

$$e_{(m)}{}^{*}.\tilde{e}^{(n)} = \tilde{e}_{(m)}{}^{*}.e^{(n)} = 0 \tag{22}$$

$$e_{(m)\mu\nu}e^{(m)}{}_{\lambda\sigma}{}^{*} - \tilde{e}_{(m)\mu\nu}\tilde{e}^{(m)*}{}_{\lambda\sigma} = \frac{1}{2}(g_{\mu\lambda}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\lambda})$$

It is easy to see that under the SO(2,1) transformations (10), the two sets $\{e_{(m)}{}^{\mu\nu}\}$ and $\{\tilde{e}_{(m)}{}^{\mu\nu}\}$ transform <u>separately</u> as vectors in the (0,1,2) subspace. Thus, for example,

$$O(\psi,\phi)^{\mu}{}_{\lambda}\, O(\psi,\phi)^{\nu}{}_{\sigma}e_{(m)}{}^{\lambda\sigma} = e_{(n)}{}^{\mu\nu}D(\psi,\phi)^{n}{}_{m} \tag{24}$$

where $D(\psi,\phi)$ is given by Eq. (19).

The anti-symmetric tensor current operator may be written as

$$T^{\mu\nu} = e_{(m)}{}^{\mu\nu} T^{(m)} + \tilde{e}_{(m)}{}^{\mu\nu} \tilde{T}^{(m)} \tag{25}$$

where
$$T_{(m)} = e_{(m)}{}^{\mu\nu *} T_{\mu\nu}$$

$$\tilde{T}_{(m)} = -\tilde{e}_{(m)}{}^{\mu\nu *} T_{\mu\nu}. \tag{26}$$

Using (24), (25) we obtain:

$$\begin{aligned} \langle p'\lambda'|T^{\mu\nu}(0)|p\lambda\rangle &= e_{(m)}{}^{\mu\nu} D(\psi,\phi)^m{}_n\, T^{(n)}_{\lambda'\lambda}(q^2) \\ &+ \tilde{e}_{(m)}{}^{\mu\nu} D(\psi,\phi)^m{}_n\, \tilde{T}^{(n)}_{\lambda'\lambda}(q^2) \end{aligned} \tag{27}$$

where the tensor form factors are defined by:

$$\begin{aligned} T^{(m)}_{\lambda'\lambda}(q^2) &= \langle p_s'\lambda'|T^{(m)}(0)|p_s\lambda\rangle \\ \tilde{T}_{\lambda'\lambda}{}^{(m)}(q^2) &= \langle p_s'\lambda'|\tilde{T}^{(m)}(0)|p_s\lambda\rangle. \end{aligned} \tag{28}$$

Again we have the constraints,

$$\begin{aligned} T_{\lambda'\lambda}{}^{(m)}(q^2) &= \delta_{m,\lambda+\lambda'}\, T^{(m)}_{\lambda}(q^2) \\ \tilde{T}_{\lambda'\lambda}{}^{(m)}(q^2) &= \delta_{m,\lambda+\lambda'}\, \tilde{T}^{(m)}_{\lambda}(q^2) \end{aligned} \tag{29}$$

(v) Remarks

a) The expansions (12), (17) and (27) separate out the dependence of the vertex function on the (ψ,ϕ) variables in the form of D-functions (reflecting the Lorentz transformation properties of the specific current involved) from the dependences on q^2 and other internal variables which are determined by dynamics.

b) What we have called $S_{\lambda'\lambda}$, $V^{(3)}_{\lambda'\lambda}$, $V^{(m)}_{\lambda'\lambda}$, $T_{\lambda'\lambda}^{(m)}$ and $\tilde{T}_{\lambda'\lambda}^{(m)}$ can in a certain sense be regarded as the "helicity amplitudes" for the process[6], Fig. (2),

$$J + A \to B \tag{30}$$

on account of the physical meaning given to the index (m). For convenience, let us call them helicity form factors. These form factors are the natural generalizations of the familiar G_E and G_M form factors to general currents and arbitrary states A and B. They diagonalize the (unpolarized) intensity distribution function for the general process (1) (as will be shown in the next section) and, when q^2 is small and a non-relativistic reduction procedure makes sense, are simply related to the familiar multipole moments with the associated physical interpretations.[12] Some further properties of these form factors are briefly enumerated in Appendix B.

We should emphasize, however, the polarization indices λ,λ' are helicities in the BW frame and should not be identified with the Jacob and Wick helicities[9] defined for two particle states in the CM frame for scattering processes.

IV. TRANSITION AMPLITUDE AND INTENSITY DISTRUCTIONS

We can now use the results of the last section to evaluate the transition amplitude, Eq. (4) for the general local interaction. In the Brick Wall frame we can choose the coordinate axies such that the momenta (p,p') associated with the A-B vertex are of the standard form (p_s,p_s'), Eq. (7). Then, using (12), (17) and (27) for the current vertices and the orthonormality conditions for the basis vectors, we obtain

$$f_{\lambda'\sigma',\sigma}(\psi,\phi,q^2\ldots) = (G/\sqrt{2})j_{\lambda'}{}^{(\beta)*}(q^2)D^*(\psi,\phi)^{\alpha}{}_{\beta}J_{(\alpha)\sigma'\sigma}(q^2\ldots)$$
$$=(G/\sqrt{2})\{D^*(\psi,\phi)^m{}_n[v_{\lambda'}{}^{(n)*}V_{(m)\sigma'\sigma}+t_{\lambda'}{}^{(n)*}T_{(m)\sigma'\sigma}+\tilde{t}_{\lambda'}{}^{(n)*}\tilde{T}_{(m)\sigma'\sigma}] \quad (31)$$
$$+s^*_{\lambda'}S_{\sigma'\sigma} + v_{\lambda'}{}^{(3)*}V_{(3)\sigma'\sigma}\}$$

Note that Eq. (31) (first line) appears in a "factorized" form consisting of two vertex functions each depending only on q^2 (and its own internal variables, if any) and they are connected by a D-function depending on the variables (ψ,ϕ) which specify the relative coordinates of the two vertices (cf. Fig. 1).

To make the choice of variables clearer, we recall that our BW frame is chosen such that

$$\begin{aligned} q &= p-p' = k'-k = (0,0,0,\sqrt{q^2}) \\ p &= (p^0,0,0,p) \\ k &= k(\mathrm{ch}\,\psi,\ \mathrm{sh}\,\psi\cos\phi,\ \mathrm{sh}\,\psi\sin\phi,-1) \end{aligned} \qquad (32)$$

where $p = (q^2-M^2+W^2)/2\sqrt{q^2}$, $p^0 = (p^2+M^2)^{1/2}$

$k = (q^2+m_\ell^2)/2\sqrt{q^2}$,

M being the mass of the target particle A, m_1 that of the lepton ℓ and W the effective mass of the system B (See Fig. 3). These variables are related to the Laboratory frame incoming neutrino energy ε, the outgoing lepton energy ε' and scattering angle θ_L and the magnitude of the 3-momentum transfer $|\underset{\sim}{q}_L|$ by

$$\mathrm{ch}\,\psi = (\varepsilon + \varepsilon')/|\underset{\sim}{q}_L|$$

$$\mathrm{sh}\,\psi = (\sqrt{q^2}/|\underset{\sim}{q}_L|)\ \mathrm{ctg}\ \tfrac{1}{2}\theta_L \tag{33}$$

in the approximation of $m_l = 0$.

The leptonic form factors are, of course, explicitly known from (3). Straightforward calculations yield

$$v^{(-1)}_{-\frac{1}{2}} = t^{(0)}_{\frac{1}{2}} = \tilde{t}^{(0)}_{\frac{1}{2}} = -\sqrt{2}\ s_{\frac{1}{2}} = -\ 2\sqrt{2q^2}\ (1 + m_\ell^2/q^2)^{\frac{1}{2}} \tag{34a}$$

and

$$v^{(0)}_{\frac{1}{2}} = v^{(3)}_{\frac{1}{2}} = -\ \frac{1}{2}\ t^{(-1)}_{-\frac{1}{2}} = -\frac{1}{2}\ \tilde{t}^{(-1)}_{-\frac{1}{2}} = 2m_\ell(1 + m_\ell^2/q^2)^{\frac{1}{2}} \tag{34b}$$

and all other form factors vanish due to the conditions(13), (20) and (29). We note also that the form factors given in Eq. (34b) are proportional to the lepton mass. In the limit $m_l = 0$, which is a good approximation for high energy processes, the only surviving (leptonic) form factors are those in Eq. (34a) and they are all proportional to $\sqrt{q^2}$. The transition amplitude (31) can, therefore, be written

$$f_{\frac{1}{2}\sigma',\sigma} = -\ 2G\sqrt{q^2}[S_{\sigma'\sigma} + e^{im\phi}\bar{d}(\psi)^m{}_0\ T_{(m)\sigma'\sigma}] \tag{35a}$$

$$f_{-\frac{1}{2}\ \sigma',\sigma} = -\ 2G\sqrt{q^2}\ e^{im\Phi}\bar{d}(\Psi)^m{}_{-1}V_{(m)\sigma'\sigma} \tag{35b}$$

where, for simplicity, we have written $T_{(m)}$ for the combination $\tilde{T}_{(m)}-T_{(m)}$ and absorbed a factor $(-1/\sqrt{2})$ into the definition of $S_{\sigma'\sigma}$.

Eq. (35) exhibits very compactly the full kinematic content of the general local current-current interaction for all neutrino scattering processes. The most obvious feature of this equation is the separation of the S-T and V interactions according to the helicity of the outgoing lepton. This is expected (as can be seen by a close examination of (3)) and holds true for all possible targets A and final states B. This points to the most unambiguous way of testing the structure of the local current-current interaction at high energies and large momentum transfers--to the extent that the lepton mass can be neglected, a purely left-handed outgoing lepton indicates V interaction, a purely right-handed one indicates S-T interaction and the coexistence of both helicities indicates a mixture of the two. Although this sounds very simple, the practical difficulties of measuring the polarization of the very high energy outgoing lepton (muon in almost all planned experiments) are quite formidable though perhaps not entirely insurmountable.

We are thus led to explore the more detailed structure of Eq. (35) and to seek to distinguishing the two types of interaction through angular or (target) spin correlation experiments. To this end, we write down the transiton probability for the general reaction (1) when the lepton helicity is not observed:

$$I = \rho^{A}_{\sigma\tau}\rho^{B}_{\sigma'\tau'}[f_{-\frac{1}{2}\sigma';\sigma}f^{*}_{-\frac{1}{2}\tau';\tau} + f_{\frac{1}{2}\sigma';\sigma}f^{*}_{\frac{1}{2}\tau';\tau}]. \qquad (36)$$

Here ρ^{A} and ρ^{B} are the density matrices for the states A and B respectively. Substituting (35) into (36) and using the explicit expressions for $\bar{d}(\psi)$, we obtain the following general distribution in the variables (ψ,ϕ),[6]

$$I = 2G^2q^2\{sh^2\psi\ I_1 + (1\text{-ch}\ \psi)^2 I_2 + (1\text{+ch}\psi)^2 I_3$$
$$+ sh\psi(1\text{-ch}\ \psi)(\cos\phi I_4 + \sin\phi I_5) + sh\psi(1\text{+ch}\ \psi)(\cos\phi I_6 + \sin\phi I_7)$$
$$+sh^2\psi(\cos 2\phi I_8 + \sin 2\phi I_9) \tag{37}$$

where the coefficients I_i, are given by

$$\begin{aligned}
I_1 &= V^{(0)}V^{(0)*} + T^{(+)}T^{(+)*} + T^{(-)}T^{(-)*} + T^{(0)}T^{(0)*} - SS^* \\
I_2 &= \tfrac{1}{2}[V^{(+)}V^{(+)*} + (T^{(0)} + S)(T^{(0)} + S)^*] \\
I_3 &= \tfrac{1}{2}[V^{(-)}V^{(-)*} + (T^{(0)} - S)(T^{(0)} - S)^*] \\
I_4 &= -\sqrt{2}\ \mathrm{Re}[V^{(+)}V^{(0)*} + (T^{(+)} - T^{(-)})(T^{(0)} + S)^*] \\
I_5 &= \sqrt{2}\ \mathrm{Im}[V^{(+)}V^{(0)*} + (T^{(+)} + T^{(-)})(T^{(0)} + S)^*] \\
I_6 &= -\sqrt{2}\ \mathrm{Re}[V^{(0)}V^{(-)*} + (T^{(+)} - T^{(-)})(-T^{(0)} + S)^*] \\
I_7 &= \sqrt{2}\ \mathrm{Im}[V^{(0)}V^{(-)*} + (T^{(+)} + T^{(-)})(-T^{(0)} + S)^*] \\
I_8 &= -\tfrac{1}{2}\ \mathrm{Re}[V^{(+)}V^{(-)*} + 2T^{(+)}T^{(-)*}] \\
I_9 &= \tfrac{1}{2}\ \mathrm{Im}[V^{(+)}V^{(-)*} + 2T^{(+)}T^{(-)*}]
\end{aligned} \tag{38}$$

For simplicity, in Eq. (38) we have: (a) used $(\pm)$ in place of (± 1) for the superscript (m) and (b) omitted a common factor consisting of the density matrices $\rho^A\rho^B$. In other words, each term in Eq. (39) stands for:

$$J^{(\alpha)}J^{(\beta)*} \equiv \rho^A_{\sigma\tau}\rho^B_{\sigma'\tau'}J^{(\alpha)}_{\sigma'\sigma}J^{(\beta)*}_{\tau'\tau} \tag{39}$$

We note a few features of Eqs. (37), (38) and (39): (a) The form of the 9-term distribution function is the same for V and T-S interactions in general.[3,5] (b) When the state B consists only of a single particle or includes all possible final states, the form factors can be taken to be real provided time reversal invariance holds. In that case, the sin ϕ terms (I_5, I_7 and I_9) vanish and we have a six term distribution function. In all of these processes, time reversal invariance can be tested by measuring the asymmetry in ϕ with all other variables integrated over.[13)] (c) The first three terms in (37) (I_1, I_2 and I_3) which are the only surviving terms in the unpolarized quasi-two-body cross-sections, are diagonal in the helicity form factors. The other terms involve simple interference terms. The same distribution functions when written in terms of the Lab. or CM frame variables and the conventional invariant form factors are invariably so complicated as to be almost untractable except in the simplest cases.

We remark at this point some necessary sign changes in the formulas presented so far if they are to be applied to anti-neutrino scattering. The first change comes in Eqs. (34a, b) where all the signs of the polarization indices (λ' and m) should be reversed. In addition, all form factors appearing in Eq. (34a) change sign. These changes imply that the right hand sides of Eqs. (35a,b) change sign and $\partial(\psi)^m_{-1}$ is replaced by $\partial(\psi)^m_1$. These, in turn, imply that in Eq. (37) we should replace sh ψ by (-sh ψ) and interchange (1-ch ψ) and (1+ch ψ).

Although the general distribution function (37) is the same for all types of local interactions, it is possible to test the structure of the interaction in special cases when the contribution of a specific interaction (S, V or T) to some I_i's (or combinations thereof) vanish due to the angular momentum constraints (13), (20) and (29). A close examination of these constraints together with the general formulae (37), (38) and (39) should convince oneself that no "inclusive experiments"[14] (in which the final state B is only partially detected) can lend itself to such tests. In the next section we explicitly work out examples of "exclusive experiments"[14] in which the V-A interaction can be distinguished from the other types of interactions. These examples also serve to illustrate how the general formalism developed in the preceding sections can be applied to individual cases.

V. EXAMPLES

(i) LEPTON-LEPTON "ELASTIC" SCATTERING

We first consider the reaction,

$$\nu_{\ell} + \ell' \rightarrow \ell + \nu_{\ell'} \tag{40}$$

which is among the experiments being comtemplated at NAL.[15] The intensity distribution for this reaction can be easily obtained from the general formulas of the previous section by setting $V^{(-)}=v^{(-1)}$, $S = \frac{1}{2}$ s and all other form factors zero (in particular, note $T^{(m)} = t^{(m)}-\tilde{t}^{(m)}=0$).

We get

$$I(q^2,\psi) = 8\ G^2 q^4 (1+\mathrm{ch}\ \psi)^2 \tag{41}$$

if V-A interaction still holds at high energies. The S-P interaction, if present, would contribute

$$I(q^2,\psi) = 8\ G'^2 q^4 \tag{42}$$

where we wrote G' in place of G to allow for a different coupling constant. The tensor interaction does not contribute to this process.[16]

We also mention another lepton-lepton interaction which will be investigated experimentally for its own right as well as in connection with the search for the intermediate boson.[15] This is neutrino lepton-pair creation in the Columb field of nuclei, e.g.

$$\nu_{\ell} + Z \rightarrow \ell + \overline{\ell}' + \nu_{\ell'} + Z \tag{43}$$

Here, because of the photon interaction, the strict locality of the 4-fermion interaction is modified. However, it has been shown[17] that the μ^- and μ^+ energy spectra are markedly different for V-A and S-P interactions. (Again the tensor current, even if it is present, does not contribute to this process.[16])

(ii) Quasi-elastic Scattering off Spinless Targets

The cases where both A and B in reaction (1) consist of spinless particles (nuclei) offer the only possibilities for testing the V-A interaction in semi-leptonic processes from the lepton intensity distribution alone.

Consider the quasi-elastic process where both A and B are single particles of spin 0. There is only one form factor for each type of current and we get, from Eqs. (37) and (38),

$$I = 2G^2q^2[(|V|^2 + |T|^2 - |S|^2)\mathrm{sh}^2\psi + (1/2)|T + S|^2(1-\mathrm{ch}\ \psi)^2 + (1/2)|T-S|^2(1+\mathrm{ch}\ \psi)^2] \tag{44}$$

We should therefore observe a pure $\mathrm{sh}^2\ \psi$ distribution if the vector (V-A in usual language) action is the only one present. Deviation from such a distribution indicates the presence of tensor or scalar admixtures. This case is completely analogous to that of the K_{e3} decay where the hyperbolic angle ψ should be replaced by the decay angle θ in the CM system of the leptons.[18]

The case where the final state B consists of two spinless particles is also, in principle, capable of distinguishing the V-A interaction from the other possibilities. There are only three independent vector-axial vector form factors, $V_{om}^{(m)}$, entering the 9-term intensity distribution (37). We shall not enter into the details here because of the lack of practical applications. It is perhaps worth noting, however, that this type of process is again very closely related to the K_{e4} decay[19] in its kinematics.

(iii) Quasi-Elastic Scattering off Polarized Target

We now consider the process

$$\nu_\ell + N \to \ell + N' \tag{45}$$

where the initial nucleon is polarized. The final particle N' can be any spin $\frac{1}{2}$ baryon. Our standard BW frame is related to the laboratory frame by a boost along the recoil N' direction which can be chosen conveniently to be the negative 3-axis. Let us further choose the polarization vector of the target N to lie in the 1-3 plane and form an angle θ with the positive 3-axis in the laboratory system. The scattering amplitudes depend on four variables which we choose to be q^2, ψ,ϕ and θ.

The general formulas (37) - (39) can be applied to this case with the following substitutitons for the initial and final density matrices:

$$\rho^A_{\sigma\tau} = \frac{1}{2}(1+p\underset{\sim}{\sigma}\cdot\underset{\sim}{n})_{\sigma\tau} = \sum_\kappa d^{\frac{1}{2}}_{\sigma\kappa}(\theta)p_\kappa\, d^{\frac{1}{2}}_{\kappa\tau}(-\theta) \tag{46}$$

$$\rho^B_{\sigma'\tau'} = \delta_{\tau'\sigma'}$$

where p is the polarization of the target, $p_{\pm\frac{1}{2}} = \frac{1}{2}(1 \pm p)$ and $d^{1/2}(\theta)$

is the usual spin $\frac{1}{2}$ rotational matrix. Substituting into (39), one obtains

$$J^{(0)}J^{(0)*} = \frac{1}{2}[|J_{-+}|^2 + |J_{+-}|^2] + p\cos\theta\frac{1}{2}[|J_{-+}|^2 - |J_{+-}|^2]$$

$$J^{(\pm)}J^{(\pm)*} = \frac{1}{2}(1\pm p\cos\theta)|J_{\pm,\pm}|^2$$

$$J^{(+)}J^{(0)*} = \frac{1}{2}p\sin\theta\ J_{++}\ J_{+-}^{*}$$

$$J^{(0)}J^{(-)*} = \frac{1}{2}p\sin\theta\ J_{-+}J_{--}^{*}$$

$$J^{(+)}J^{(-)*} = 0 \tag{47}$$

where we have omitted the superscript (m) on the right since $m = \sigma'+\sigma$. These results can be applied to each term in Eq. (38) yielding,

$$\begin{aligned} I_i &= a_i + p\cos\theta\ \ b_i & & i = 1,2,3 \\ &= p\sin\theta\ a_i & \text{for}\quad & i = 4,5,6,7 \\ &= 0. & & i = 8,9 \end{aligned} \tag{48}$$

Eq. (48), when substituted into Eq. (37), gives rise to a ten-term combined correlation function in the variables ψ,ϕ and θ. The contribution of the vector form factors to the coefficient functions are:

$$a_1 = \frac{1}{2}[|V_{-+}|^2 + |V_{+-}|^2]$$

$$b_1 = \frac{1}{2}[|V_{-+}|^2 - |V_{+-}|^2]$$

$$a_2 = b_2 = \frac{1}{4}|J_{++}|^2$$

$$a_3 = -\ b_3 = \frac{1}{4}|J_{--}|^2$$

$$a_4 = -\ (1/\sqrt{2})\ \mathrm{Re}\ V_{++}V_{+-}^{*}$$

$$a_5 = (1/\sqrt{2})\ \mathrm{Im}\ V_{++}V_{+-}^{*}$$

$$a_6 = -(1/\sqrt{2})\ \mathrm{Re}\ V_{-+}V^*_{--}$$
$$a_7 = (1/\sqrt{2})\ \mathrm{Im}\ V_{-+}\ V^*_{--} \qquad (49)$$

The four vector form factors appearing above are essentially the usual G_E, G_M and their axial-vector counterparts. The exact relations between the two sets are given in Appendix B. It is clear from (49) that if this process is mediated only by the usual vector current then there exist four relations among the ten coefficient functions a_i, b_i. The two more useful ones among these relations are already explicitly displayed in the third and fourth equations in (49). From Eqs. (37), (38), (39), (47) and (48), it is straightforward to verify that the presence of any scalar or tensor currents will spoil these relations. This suggests the possibility of testing the structure of the basic interaction by measuring the correlation functions and checking these relations among the coefficients. In practice, it is very unlikely that the complete correlation distributions (37) (38) and (48) can be obtained experimentally. It is therefore desireable to see how much of the phase space volume can be integrated over without distroying the relevent information to be extracted.

First of all, the ϕ-dependence is not of particular interest for our purpose, thus can be integrated over. The resulting differential cross-section (for fixed incident energy) is:

$$\frac{d\sigma}{dq^2} = \frac{G^2q^2}{32\pi\varepsilon^2M^2} \cdot$$

$$\{sh^2\psi\ a_1(q^2)+(1-ch\ \psi)^2a_2(q^2)+(1+ch\ \psi)^2a_3(q^2)$$

$$+p\cos\theta[sh^2\psi b_1(q^2)+(1-ch\ \psi)^2b_2(q^2)+(1+ch\ \psi)^2b_3(q^2)]\} \qquad (50)$$

where ε, the Lab. neutrino energy, is given in turns of the BW frame variables by

$$\varepsilon = (1/4M)\{(q^2-M^2+M'^2)+[q^2+(M'-M)^2]^{\frac{1}{2}}[q^2+(M'-M)^2]^{\frac{1}{2}} \text{ ch } \psi\}.$$

The first three terms (a_i) give the spectrum function for <u>unpolarized target</u>

$$A_+ (q^2;\psi) = \frac{d\sigma}{dq^2}\Big|_{\theta=0} + \frac{d\sigma}{dq^2}\Big|_{\theta=\pi} \tag{51}$$

The last three terms (b_i) can be isolated by forming the <u>asymmetry function</u>

$$A_-(q^2;\psi) = \frac{d\sigma}{dq^2}\Big|_{\theta=0} - \frac{d\sigma}{dq^2}\Big|_{\theta=\pi} . \tag{52}$$

The q^2- and ψ -dependences can be separated only by measuring $A_\pm$ at many different incident energies. (At fixed energy, the two variables are related.) With these available, one can divide $A_\pm$ by the common factor $(G^2q^2/32\,\pi\,\varepsilon^2M^2)$ and integrate over the experimentally available range of the variable q^2 <u>at fixed ψ .</u> The resulting ψ-spectra for the two cases are

$$sh^2\psi \left\{ {\langle a_1\rangle \atop \langle b_1\rangle} \right\} + (1-ch\psi)^2\left\{ {\langle a_2\rangle \atop \langle b_2\rangle} \right\} + (1+ch\psi)^2\left\{ {\langle a_3\rangle \atop \langle b_3\rangle} \right\} \tag{53}$$

where $\langle a_i\rangle = \int dq^2\, a_i(q^2)$ and $\langle b_i\rangle = \int dq^2\, b_i(q^2)$. From (49) we infer that if the weak current remain to be a vector at high energies, we should expect

$$\begin{aligned} \langle a_2\rangle &= \langle b_2\rangle \\ \langle a_3\rangle &= -\langle b_3\rangle \end{aligned} \tag{54}$$

We also point out that if second class currents are absent, $V_{+-} = V_{-+}$ (cf. Appendix B) and consequently (cf. Eq. (49)),

$$b_1 = \langle b_1 \rangle = 0. \tag{55}$$

It is worth noting that although we integrate over q^2 in order to gain maximum statistics, <u>the tests (54), (55) are free from any assumption on the q^2-dependence of the form factors</u>.

For anti-neutrino scattering the coefficients of the $(1 \pm \mathrm{ch}\,\psi)$ terms are interchanged. Consequently, the relevent relations are,

$$\begin{aligned} \langle a_2 \rangle &= - \langle b_2 \rangle \\ \langle a_3 \rangle &= \langle b_3 \rangle \end{aligned} \tag{56}$$

and

$$\langle b_1 \rangle = 0 .$$

(iv) Quasi-Elastic Hyperon Production

Here we consider processes of the type

$$\begin{aligned} \nu + N \to \ell + {} & Y \\ & \hookrightarrow N' + \pi \end{aligned} \tag{57}$$

where N is a target nucleon (unpolarized) and Y a hyperon. We choose the hyperon decay plane to be the 1-3 plane and denote by θ the decay angle of N' in the hyperon CM frame. The overall process (57) is specified by the four independent variables q^2, ψ, ϕ and θ .

The density matrixes for the initial and final states are

$$\rho^A_{\sigma\tau} = \frac{1}{2}\delta_{\sigma\tau}$$

$$\rho^B_{\sigma'\tau'} = \sum_\kappa \underset{\sim}{d}^{\frac{1}{2}}_{\sigma'\kappa}(\theta)|a_\kappa|^2 d^{\frac{1}{2}}_{\kappa,-\tau'}(\theta)$$

$$= a^2 \frac{1}{2}[1-\alpha\underset{\sim}{\sigma}\cdot\underset{\sim}{n}]_{\sigma'\tau'} \qquad (58)$$

where $a_{\pm\frac{1}{2}}$ are the Y-decay amplitudes for $\pm$ helicity outgoing N' (they are simply the sum and difference of the conventional a_s, a_p) respectively; $|a|^2 = |a_+|^2 + |a_-|^2 = |a_s|^2 + |a_p|^2$;$\alpha$ is the asymmetry parameter $[|a_+|^2 - |a_-|^2]|a|^{-2} = 2(\mathrm{Re}\, a_s a_p^*)|a|^{-2}$ and n the polarization vector with components ($\sin\theta$, 0, $\cos\theta$).

The similarity between this process and the polarized target case is obvious (compare Eq. (58) with (46)). All considerations of the last subsection can be carried over with very little change. In particular, the combined correlation distribution in the variables ψ, ϕ and θ is given by Eqs. (37), (48). The coefficients $\{a_i, b_i\}$ are expressed in terms of the vector form factors by formulas similar to Eqs. (49) with the following changes: (a) an overall factor of $\frac{1}{2} a^2 = \frac{1}{2}(|a_s|^2+|a_p|^2)$ is inserted, (b) the form factors V_{+-} and V_{-+} are interchanged and (c) b_1, b_2, b_3, a_4, a_5, a_6 and a_7 acquire an additional minus sign. The phase space for the present process involves one more integration in the θ-variable. The two spectrum-functions $A\pm(q^2, \psi)$ are obtained in this case by integrating over the entire θ-range for A_+ and taking the difference of a two-fold division of events with $0 < \theta < \frac{\pi}{2}$ and $\frac{\pi}{2} < \theta < \pi$ for A- (asymmetry of N' with respect to N in the Y CM frame).

Again the q^2-variable may be integrated over in the manner described before. The relations which serve as tests of the V-A structure of the weak current are now (56) for the neutrino initiated reactions and (54) for the anti-neutrino initiated reactions.

It may be interesting to note that similar tests of the V-A interaction can be carried out in polarized hyperon β-decay. Essentially all the above analysis go through if the variables (ψ,ϕ) are replaced by the decay angles of the lepton in the $(\ell\ \nu)$ CM frame. Data on these processes are being accumulated at such a rate that it appears such an analysis may be feasible in the very near future.[20]

(v) Single Pion Production in the N* Region

Finally, we consider the single pion production process

$$\nu + N \rightarrow \ell + N' + \pi \tag{59}$$

with the final $(N'\ \pi)$ invariant mass in the low energy region where N* dominates. As a first approximation, we assume that the final $(N'\pi)$ system is in a pure $J^P = \frac{3}{2}^+$ state (N*) in its CM frame. Then the initial and final density matrices can be written,

$$\begin{aligned} \rho^A_{\sigma\tau} &= \frac{1}{2}\delta_{\sigma\tau} \\ \rho^B_{\sigma'\tau'} &= |a|^2 \sum_{\kappa=\pm\frac{1}{2}} d_{-\tau'\kappa}(\theta)\ d_{-\sigma'\kappa}(\theta)\ . \end{aligned} \tag{60}$$

where $|a|$ stands for the magnitude of the N* decay matrix element $\langle N^*|\ N'\pi\rangle$. Substituting (60) into (39), one can obtain the expected distribution in the variables ψ,ϕ and θ from (37) and (38).

As before, we integrate over the ϕ-variable obtaining a six-term distribution expressed by

$$I_i = a_i + b_i \cos^2\theta \qquad \text{for} \qquad i = 1,2,3$$
$$= 0 \qquad\qquad\qquad\qquad i = 4,5,6,7,8,9 \tag{61}$$

The vector current contribution to a_i, b_i can be easily worked out and they are

$$3a_1 = b_1 = \frac{3}{4}\left[|V^{(0)}{}_{+}|^2 + |V^{(0)}{}_{-}|^2\right]$$
$$a_2 = \frac{1}{8}\left[|V^{(+)}{}_{+}|^2 + |V^{(+)}{}_{-}|^2\right]$$
$$b_2 = \frac{3}{8}\left[|V^{(+)}{}_{+}|^2 - |V^{(+)}{}_{-}|^2\right]$$
$$a_3 = \frac{1}{8}\left[|V^{(-)}{}_{-}|^2 + |V^{(-)}{}_{+}|^2\right] \tag{62}$$
$$b_3 = \frac{3}{8}\left[|V^{(-)}{}_{-}|^2 - |V^{(-)}{}_{+}|^2\right].$$

Here the form factors are labelled by the current helicity (m) and initial N polarization. with $\sigma' = m-\sigma$ omitted. The first equation in (62) can be used as a test of the V-A interaction in the same manner as described in the previous two sections.

In order to improve on the (rather drastic) approximation made above, we may include into our consideration correction terms brought about by other partial waves. In the N* region, the $J^P = \frac{1}{2}^{\pm}$ states are expected to have some influence. An analysis may be carried out retaining the three partial waves $\frac{1}{2}^{\pm}$ and $\frac{3}{2}^{+}$. The procedure used is the same as before, the algebra becomes slightly more involved. We give the detailed results in Appendix C. The combined ψ, ϕ and θ correlation distribution consists of 19 independent terms. Since the number of form factors proliferates with the inclusion of more states, we found it convenient to assume time reversal invariance. This enables us to invoke the Watson's theorem to fix the phases of the form factors in terms of the measured (πN) phase shifts, thus reducing the number of unknowns. This way, the 19 coefficients in the correlation function can be expressed in terms of three known phase shifts and 14 (modulus of) vector form factors if V-A interaction alone contributes. Thus, five relations among the 19 coefficients exist and again serve as the basis for a test of the V-A interaction at high energies. For the details of this calculation we again refer the reader to Appendix C. Here we only note that the procedure used is, in a sense, the reverse of that used by Pais and Treiman in K_{e4} and $\Xi_{\ell 4}$ decays.[19] There, the V-A interaction is assumed and the phase shifts treated as unknowns to be solved from the coefficients of the correlation function.

(vi) Conclusions

We have given a formula for the general intensity distribution function for an arbitrary neutrino scattering process in the local current-current picture in terms of helicity form factors. We have discussed various ways to test whether the basic V-A interaction structure of the weak interaction Hamiltonian deduced from low energy decay experiments still remains valid for high energy neutrino scattering processes. All considerations are independent of dynamical assumptions.[21] It is quite evident that none of these proposed tests are easy to carry out in the laboratory. However, given the importance of the issue involved, the clear lack of other alternatives as revealed by this analysis and the rapid advancement in experimental techniques we hope, in time, most of these tests can be carried out.[22]

Aside from the proposed tests, we hope the analyses in this paper also succeed in demonstrating the usefulness of the BW frame variables[6] (ψ,ϕ) and the helicity form factors, in describing all types of weak scattering processes. The same halicity form factors, in fact, can be used in other types of weak and electromagnetic processes to great advantage.[6]

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1) For a review and references, see for instance, T. D. Lee and C. S. Wu, Annual Review of Nuclear Sciences, Vol. 15 (1965) and Vol. 16 (1966), Annual Reviews, Inc. (U.S.A.); or R. Marshark, Riazuddin and C. P. Ryan, Weak Interactions in Particle Physics, Wiley-Interscience (1969).

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7) The essense of the method used in the present paper is the same as that of reference 6 although the role played by the form factors is largely by-passed in the latter.

8) For earlier studies of the current vertex functions see, for example, L. Durand, III, P. C. DeCelles and R. B. Marr, Phys. Rev. 126, 1882 (1962); M. Scadron, Phys. Rev. 165, 1640 (1968); M. Bander, Phys. Rev. 173, 1568 (1968); T. L. Trueman, Phys. Rev. 182, 1469 (1969).

9) M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).

10) The choice of definition of states used here is one of the possibilities enumerated in J. Strathdee, J. F. Boyes, R. Delbourgo and A. Salam,

Refs. (continued)

Trieste preprint IC/67/9 (unpublished).

11) The factors of i adopted in the following definiitons are to render the resulting form factors real whenever time reversal invariance can be applied.

12) For a discussion of G_E and G_M see F. J. Ernst, R. G. Sachs and K. C. Wali, Phys. Rev. 119, 1105 (1960). Generalization of G_E and G_M to the arbitrary spin case and the relation to multipole moments are studied by Durand et. al. reference 8. However, this paper used the Jacob and Wick definition of the states and chose a "BW" frame in which $p_3 = -p_3'$ (instead of $p_3 - p_3' = \sqrt{q^2}$ and $p_0 = p_0'$). As a result many of the appealing features of the helicity form factors are lost.

13) To observe the asymmetry, one needs a prefered direction with respect to which the asymmetry is defined. This can only be supplied by the polarization vector of either A (polarized target) or B (final polarization measured).

14) R. Feynman, Phys. Rev. Letters, 23, 1415 (1969) and contribution to High Energy Collisions, Third International Conference, Gorden and Beach, New York (1969).

15) National Accelerator Laboratory, Summer Study, 1968 and 1969.

16) The vanishing of tensor contribution to four-point lepton processes (of which this is one example) can be proved in another way. By using the Fierz transformation one obtains

$$\bar{\psi}_\ell \sigma_{\mu\nu}(1+\gamma_5)\, \psi_\nu \bar{\psi}_{\nu'}(1-\gamma_5)\sigma^{\mu\nu}\psi_{\ell'}$$

Refs. (continued)

$$= -\frac{1}{2}\,\bar{\psi}_{\nu'}(1-\gamma_5)\sigma_{\mu\nu}(1+\gamma_5)\psi_\nu\;\bar{\psi}_\ell\sigma^{\mu\nu}\psi_{\ell'}$$

$$+\frac{1}{4}\,\bar{\psi}_{\nu'}(1-\gamma_5)(1+\gamma_5)\psi_\nu\;\bar{\psi}_\ell\psi_{\ell'}$$

$$+\frac{1}{4}\,\bar{\psi}_{\nu'}(1-\gamma_5)\gamma_5(1+\gamma_5)\psi_\nu\;\bar{\psi}_\ell\psi_5\psi_{\ell'}$$

$$= 0$$

We thank Dr. K. Fujikawa (see next reference) for pointing out this proof to us.

17) K. Fujikawa, Princeton thesis (1970) and to be published.

18) The latest experimental upper limit for $|S/V|$ and $|T/V|$ are 0.23 and 0.58 respectively in K_{e3} decay. See Botterill et. al., Phys. Rev. 174, 1661 (1968).

19) A. Pais and S. Treiman, Phys. Rev. 168, 1858 (1968) and 178, 2365 (1969).

20) For example, J. Lindquist et. al. (Chicago-Washington-Ohio State-Argonne collaboration), contribution to the XVth International Conference on High Energy Physics, Kiev (1970).

21) There is one exception. That is the omission of other partial waves in the ($N''\pi$) system in subsection (v) of Section (V). This assumption can be independently checked experimentally.

22) A quick look at the list of proposed experiments at the National Accelerator Laboratory should offer much encouragement to one's optimism on this point.

Appendix A

We briefly indicate the lepton mass corrections to the various formulae in the text. This is rather easy to do in our formalism. Thus, substituting Eqs. (34a) and (34b) into Eq. (31), the general scattering amplitudes are, (compare with Eq. (35)),

$$f_{\frac{1}{2}\sigma',\sigma} = -\ 2G(q^2+m_\ell^2)^{\frac{1}{2}}[S_{\sigma'\sigma} + e^{im\phi}\bar{d}(\psi)^m_o\ T_{(m)\sigma'\sigma}]$$

$$+\sqrt{2}Gm_\ell(1+m_\ell^2/q^2)^{\frac{1}{2}}\ [V^{(3)}_{\sigma'\sigma} + e^{im\phi}\bar{d}(\psi)^m_o\ V_{(m)\sigma'\sigma}]$$

$$f_{-\frac{1}{2}\ \sigma',\sigma} = -\ 2G[(q^2+m_\ell^2)^{\frac{1}{2}}\ V_{(m)\sigma'\sigma} - \sqrt{2}m_\ell(1+m_\ell^2/q^2)^{\frac{1}{2}}]e^{im\phi}\bar{d}(\psi)^m_{-1} \qquad \text{(A-1)}$$

The intensity distribution can then be obtained from (A-1) and (36). The distribution function is again of the form Eq. (37) but with I_i containing more terms than those given in Eq. (38) due to contributions from the second terms in Eqs. (A-1). They can be easily worked out when necessary, we do not give the complete expressions here.

As a consequence of the lepton mass terms, the outgoing lepton polarization is not 100% for pure V-A or S-T interactions. The correction is roughly of the order m_ℓ^2/q^2 (see Eq. (A-1)). Similarly, the distribution functions (41), (44), (49), (62) and the constraints (54), (56), (62) all are subject to additional corrections of this order. For electron neutrino interactions m_ℓ^2/q^2 is for all practical purposes zero. For muon neutrino interactions $m_\ell^2/q^2 = 0.01$ for $q^2 = 1(\text{Gev/c})^2$.

Appendix B

We briefly enumerate a few relevent properties of the helicity form factors defined in the text. We recall the definition of such form factors,

$$J^{(m)}_{\lambda'\lambda} = \langle p'_s\lambda'|J^{(m)}(0)|p_s\lambda\rangle = \langle 0-\lambda'|B_3(-u')J^{(m)}(0)B_3(u)|0\lambda\rangle \tag{B-1}$$

Here, as in the text, $J^{(m)}$ denotes the m-component of some general current operator. The m-index is defined such that

$$[J^{(m)}(0),J_3] = mJ^{(m)}(0) \tag{B-2}$$

where J_3 denotes the angular momentum operator along the 3-axis. (An exception to this statement is $V^{(3)}$ which corresponds to m = 0 but, for obvious reasons, we cannot, and did not, label it $V^{(0)}$). Sandwich Eq. (B-2) in between the states $\langle p'_s\lambda'|$ and $|p_s\lambda\rangle$ we get $(\lambda + \lambda')J^{(m)}_{\lambda'\lambda} = mJ_{\lambda'\lambda}$, which implies

$$J^{(m)}_{\lambda'\lambda} = \delta_{m,\lambda+\lambda'}J^{(m)}_{\lambda}. \tag{B-3}$$

The hadronic states $|p_s\lambda\rangle$ and $|p'_s\lambda'\rangle$ have definite transformation properties under the space-and time-inversion operators. It is therefore meaningful to spearate the S, V, T currents in the text into the usual S, P, V, A and T currents which also have definite transformation properties under these discrete transformations. One can then derive consequences due to covariance under these transformations. This is easy to do with the explicity definition (B-1), we shall not go into it explicitly here but rather confine ourselves to a few remarks. Constrainsts on the form factors due to these symmetries arise only if all the momenta (both the initial and final states may be composite systems with internal momenta) lie in a plane. (So that the operators $U_pR_2(\pi)$ and $U_TR_2(\pi)$ will leave the momenta unchanged.) Otherwise one only relates form factors at different momentum values. As is well known parity relates $J^{(m)}_{\lambda'\lambda}$ to $J^{(-m)}_{-\lambda'-\lambda}$ and time reversal invariance yields information on the phase of the form factors and, in the special case where the initial and final states are identical, symmetry relations in the initial and final variables.

As shown in the text the helicity form factors diagonalize the unpolarized cross-section formula and yield simple expressions for spin- and angular-correlation functions. The relation between these form factors and the conventional invariant form factors can be worked out easily for any particular case from Eq. (B-1) according to the specific way the latter are defined. They can be also related to the non-relativistic multipole moments familiar in atomic and nuclear physics. To do this, simply observe that the operator $B_3(-u')J^{(m)}(0)B_3(u)$ in Eq. (B-1) can be decomposed into a sum of terms irreducible under the rotation group O(3) (with parity). When sandwiched between the rest frame states in (B-1), one obtains reduced matrix elements which are generalizations of the multipole moments.[8]

Finally we give the explicit relations between the vector and axial vector helicity form factors and the conventional invariant form factors for spin 1/2 particles. From the definition

$$V^{\mu}_{\lambda'\lambda} = \bar{u}_{\lambda'}(p_s')[\gamma^{\mu}f_1 + i\sigma^{\mu\nu}q_{\nu}f_2 + q^{\mu}f_3$$

$$+ \gamma^{\mu}\gamma_5 g_1 + i\sigma^{\mu\nu}q_{\nu}\gamma_5 g_2 + q^{\mu}\gamma_5 g_3]u_{\lambda}(p_s)$$

$$q^{\mu} = p_s^{\mu} - p_s'^{\mu}, \tag{B-4}$$

straightforward calculations yield

$$V^{(3)}_{-\lambda,\lambda} = (\Delta M f_1 + q^2 f_3)(1+4\bar{M}^2/q^2)^{\frac{1}{2}} - 2\lambda(2\bar{M}g_1 + q^2 g_3)(1+\Delta M^2/q^2) \qquad \lambda = \pm\frac{1}{2}$$

$$V^{(0)}_{-\lambda,\lambda} = (2\bar{M}f_1 - q^2 f_2)(1+\Delta M^2/q^2)^{\frac{1}{2}} - 2\lambda(\Delta M g_1 - q^2 g_2)(1+4\bar{M}^2/q^2)$$

$$V^{(+1)}_{\frac{1}{2},\frac{1}{2}} = \sqrt{2q^2}[-(f_1+2\bar{M}f_2)(1+\Delta M^2/q^2)^{\frac{1}{2}} + (g_1+\Delta M g_2)(1+4\bar{M}^2/q^2)$$

$$V^{(-1)}_{-\frac{1}{2},-\frac{1}{2}} = \sqrt{2q^2}[-(f_1+2\bar{M}f_2)(1+\Delta M^2/q^2)^{\frac{1}{2}} - (g_1+\Delta M g_2)(1+4\bar{M}^2/q^2)$$

(B-5)

where $\Delta M = M' - M$ and $\bar{M} = (1/2)(M+M')$.

The vector parts of the helicity form factors $V^{(0)}$ and $V^{(\pm 1)}$ can be readily recognized as multiples of the conventional G_E and G_M form factors respectively.

Appendix C

We give some detailed results on single pion production in the N* region. The process under considerations is

$$\nu + N \to \mu + N' + \pi \tag{C-1}$$

In our convention, the toal momentum of the $(N'\pi)$ system is p'. Let us denote the relative momentum of this two particle state by r and the N' polarization by κ. This process is described by five independent variables $(\psi,\phi,q^2,\theta$ and $W)$ where W is the invariant mass of the $N'\pi$ system. In the region where W is near or below the two pion production threshold, we assume the matrix elements of the current operators between the states $(N'\pi)$ and N are dominated by the $(N'\pi)$ system in the $J^P = \frac{1}{2}^\pm, \frac{3}{2}^+$ states. The helicity form factors which enter into Eqs. (31), (35) and (38) can be written:

$$\langle p_s'; r\kappa|J^{(m)}|p_s\sigma\rangle = \langle 0;r,\kappa|B_3(-u')J^{(m)}B(u)|0\sigma\rangle$$

$$=\Sigma_{\sigma'} d^{\frac{1}{2}}_{-\sigma',\kappa}(\theta)[J^{(m)(\frac{1}{2}+)}_{\sigma'\sigma}+2\kappa J^{(m)(\frac{1}{2}-)}_{\sigma'\sigma}]+\Sigma_{\sigma'} d^{\frac{3}{2}}_{-\sigma',\kappa}(\theta)J^{(m)(\frac{3}{2}+)}_{\sigma'\sigma} \tag{C-2}$$

where the second superscript indicates the J^P state of the $(N'\pi)$ system.

In the region of W-variable of interest to us, elastic unitarity in the $(N'\pi)$ channel holds. If one assumes time reversal invariance, as is consistent with present experimental evidences in high energy scattering, then the phases of the form factors in (C-2) are identical to the elastic scattering phase shifts in the appropriate $(N'\pi)$ states. For simplicity in notation, let us denote the phase shifts in the $\frac{1}{2}^+$, $\frac{1}{2}^-$ and $\frac{3}{2}^+$ states by α, β, γ and the form factors $J^{(m)(J^P)}_{\sigma'\sigma}$ in these states by $\alpha^{(m)}_\sigma$ m $\beta^{(m)}_\sigma$, $\gamma^{(m)}_\sigma$ respectively. Then, substituting (C-2) into Eqs. (37) and (38) yields the combined correlation distribution in ψ, ϕ, θ with the result

$$I_i = a_i + b_i \cos^2\theta + c_i \cos\theta \qquad\qquad i = 1,2,3$$

$$= (a_i + c_i \cos\theta)\sin\theta \qquad \text{for} \qquad i = 4,5,6,7$$

$$= a_i \sin^2\theta. \qquad\qquad i = 8,9 \qquad\qquad \text{(C-3)}$$

We give the contribution of the vector current form factors to the coefficients $\{a_i, b_i, c_i\}$:

$$a_1 = (\alpha_+^0)^2 + (\alpha_-^0)^2 + (\beta_-^0)^2 + \frac{1}{4}[(\gamma_+^0)^2 + (\gamma_-^0)^2]$$

$$-(\beta_+^0\gamma_+^0 - \beta_-^0\gamma_-^0)\cos(\beta-\gamma)$$

$$b_1 = 3\{\frac{1}{4}[(\gamma_+^0)^2 + (\gamma_-^0)^2] + (\beta_+^0\gamma_+^0 - \beta_-^0\gamma_-^0)\cos(\beta-\gamma)\}$$

$$c_1 = 2(\alpha_+^0\beta_+^0 - \alpha_-^0\beta_-^0)\cos(\alpha-\beta)$$

$$a_2 = \frac{1}{2}[(\alpha_+^+)^2 + (\beta_+^+)^2] + \frac{1}{8}[(\gamma_+^+)^2 + (\gamma_-^+)^2]$$

$$b_2 = \frac{3}{2}\{\frac{1}{4}[(\gamma_+^+)^2 - (\gamma_-^+)^2] - \beta_+^+\gamma_+^+\cos(\beta-\gamma)\}$$

$$c_2 = -\alpha_+^+\beta_+^+\cos(\alpha-\beta) + \gamma_+^+\alpha_+^+\cos(\gamma-\alpha)$$

$$a_3 = \frac{1}{2}[(\alpha_-^-)^2 + (\beta_-^-)^2 + \frac{1}{8}[(\gamma_+^-)^2 + (\gamma_-^-)^2]$$

$$-\frac{1}{2}\beta_-^-\gamma_-^-\cos(\beta-\gamma) \qquad \text{(C-4)}$$

$$b_3 = \frac{3}{2}\{\frac{1}{4}[(\gamma_-^-)^2 - (\gamma_+^-)^2] + \beta_-^-\gamma_-^-\cos(\beta-\gamma)\}$$

$$c_3 = \alpha_-^-\beta_-^-\cos(\alpha-\beta) + \gamma_-^-\alpha_-^-\cos(\gamma-\alpha)$$

$$a_4 = -\sqrt{2}[(\alpha_+^+\beta_+^0 + \beta_+^+\alpha_+^0)\cos(\alpha-\beta)$$

$$+\frac{1}{2}(\gamma_+^+\alpha_+^0 + \alpha_+^+\gamma_+^0 + \sqrt{3}\gamma_-^+\alpha_-^0)\cos(\gamma-\alpha)]$$

$$c_4 = -\sqrt{\frac{3}{2}}\{\gamma_-^+\gamma_-^0 + [\sqrt{3}(\beta_+^+\gamma_+^0 + \gamma_+^+\beta_{+}^{0}) - \gamma_-^+\beta_-^0]\cos(\beta-\gamma)\}$$

$$a_5 = \sqrt{2}[(\alpha_+^+\beta_+^0 - \beta_+^+\alpha_+^0)\sin(\alpha-\beta)$$

$$+\frac{1}{2}(\gamma_+^+\alpha_+^0 + \alpha_+^+\gamma_+^0 + \sqrt{3}\gamma_+^+\alpha_-^0)\sin(\gamma-\alpha)]$$

$$c_5 = \sqrt{\tfrac{3}{2}}\,[\sqrt{3}(\beta_+{}^+\gamma_+{}^0 - \gamma_+{}^+\beta_+{}^0) + \gamma_-{}^+\beta_-{}^0]\ \sin\ (\beta-\gamma)$$

$$a_6 = -\sqrt{2}\ [(\alpha_-{}^0\beta_-{}^- + \beta_-{}^0\alpha_-{}^-)\ \cos\ (\alpha-\beta)$$

$$+ \tfrac{1}{2}(\gamma_-{}^0\alpha_-{}^- - \alpha_-{}^0\gamma_-{}^- - \sqrt{3}\alpha_+{}^0\gamma_+{}^-)\ \cos\ (\gamma-\alpha)]$$

$$c_6 = -\ \sqrt{\tfrac{3}{2}}\ \{-\gamma_+{}^0\gamma_+{}^- + [\sqrt{3}(\beta_-{}^0\gamma_-{}^- + \gamma_-{}^0\beta_-{}^-)$$

$$-\beta_+{}^0\gamma_+{}^-]\ \cos\ (\beta-\gamma)$$

$$a_7 = \sqrt{2}\ [(\alpha_-{}^0\beta_-{}^- - \beta_-{}^0\alpha_-{}^-)\ \sin\ (\alpha-\beta)$$

$$+ \tfrac{1}{2}\ (\gamma_-{}^0\ \alpha_-^- + \alpha_-{}^0\gamma_-{}^- - \sqrt{3}\alpha_+{}^0\gamma_+{}^-)\ \sin\ (\gamma-\alpha)]$$

$$c_7 = \sqrt{\tfrac{3}{2}}\ [\sqrt{3}(\beta_-{}^0\gamma_-{}^- - \gamma_-{}^0\beta_-{}^-) - \beta_+{}^0\gamma_+{}^-]\ \sin\ (\beta-\gamma)$$

$$a_8 = (\sqrt{3}/4)[\tfrac{1}{2}(\gamma_-{}^+\gamma_-{}^- + \gamma_+{}^+\gamma_+{}^-) + (\beta_+{}^+\gamma_+{}^- - \gamma_-{}^+\beta_-{}^-)\ \cos\ (\beta-\gamma)$$

$$a_9 = (\sqrt{3}/4)[-\beta_+{}^+\gamma_+{}^- - \gamma_-{}^+\beta_-{}^-]\ \sin\ (\beta-\gamma)$$

The 19 coefficients are given in terms of 3 known phase shifts and 14 unknown form factors. Thus, in principle there exist five relations among the correlation coefficients which can provide as tests of the V-A interaction. It is not hard to see from Eqs. (C-4) however, that these relations are rather complicated in general. In the text we considered the zeroth order approximation of setting $\alpha_\sigma^{(m)} = \beta_\sigma^{(m)} = 0$ and obtained one simple constraint relation $a_1 = 3b_1$. One can improve on these results by attemptong to solve the equations in (C-4) in the approximation that $(\alpha_\sigma^{(m)} / \gamma_\sigma^{(m)}{}')$ and $(\beta_\sigma^{(m)} / \gamma^{(m')}_\sigma{}')$ are small but non-zero. We shall not do this here explicitly. It can be carried out in a straight-forward manner when the result is called for. Let us simply state that the afor-mentioned relation $a_1 = 3b_1$ still holds to the first order approximation in the small parameters (α/γ) and (β/γ).

Figure Captions

Fig. 1 A general neutrino scattering process

Fig. 2 The effective current-hadron scattering process

Fig. 3 The Brick-Wall frame kinematics

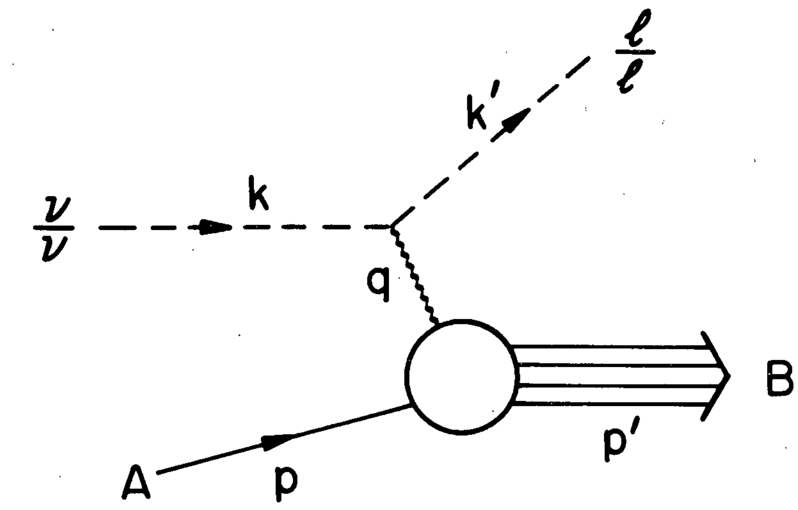

Fig. 1

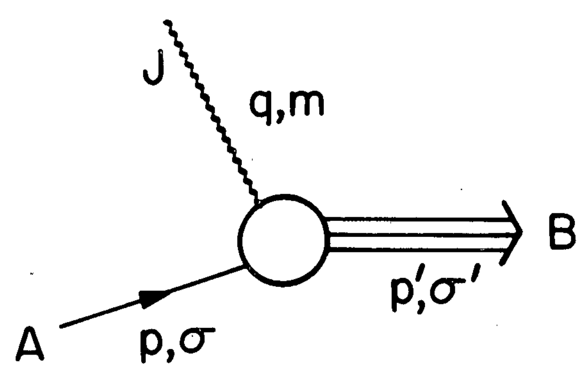

Fig. 2

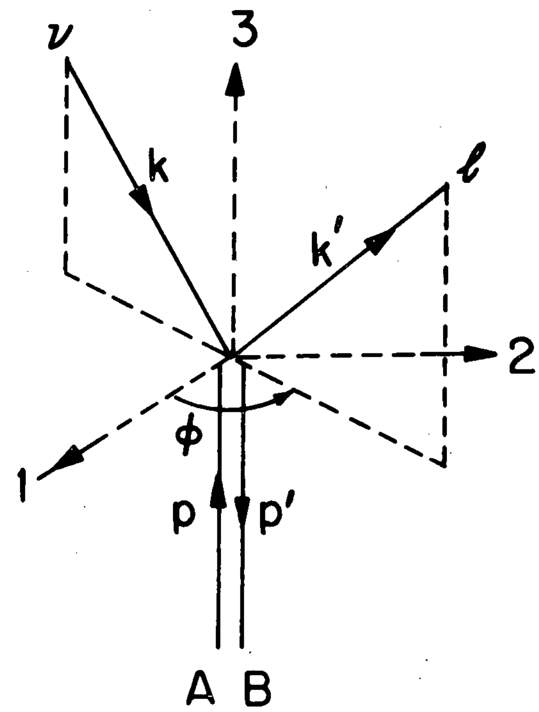

Fig. 3