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ANALYSIS OF THE PHOENIX
FUEL EXPERIMENTS

C. M. Heeb

October 1970

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C. M. Heeb

Systems Engineering Department
Physics and Engineering Division

October 1970

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ABSTRACT

A depletion experiment on a plutonium-aluminum-alloy fueled core in the Materials Testing Reactor (MTR) was completed in the first half of 1970. The supporting integral experiments and the installed neutronic behavior of this "Phoenix core" are described briefly.

The analysis of these experiments is described in detail. It was found that increasing the absorption of the 1 eV ^{240}Pu resonance to agree with measured k_{∞} values results in eigenvalues which are within 1% of the experimental values over a range in ^{240}Pu concentrations from 8% to 23%.

The observed reactivity time behavior of the core shows a more abrupt loss of reactivity with time than had been calculated. A method of partitioning the computed reactivity time slope is presented which defines the contribution each burnable isotope makes to the total slope. A comparative analysis of two methods plus nuclear data is carried out using this technique to illustrate the complexities involved in burnup analysis.

The combination of integral experiments and the experiment conducted in the MTR with the Phoenix core is a unique fund of observational knowledge on the neutronic behavior of cores containing only plutonium which can be used to test analysis methods in the absence of interference from uranium isotopes.

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I. INTRODUCTION

The Phoenix Fuel Program at the Pacific Northwest Laboratory (PNL) has as its primary objective the exploration and evaluation of the longer lifetime potential inherent in the use of the isotope ^{240}Pu as a burnable poison in nuclear reactor cores. The unusual property of ^{240}Pu as a burnable poison is that it converts to the fissile isotope ^{241}Pu upon absorbing a neutron. This is known as the Phoenix effect. Unlike ^{238}U , however, ^{240}Pu is formed directly by non-fissioning neutron capture by the fissile isotope ^{239}Pu . The potential is therefore present for a reactor core design exhibiting a long endurance with relatively small reactivity change over the lifetime of the core.

Early survey work⁽¹⁾ singled out a small, high power density water core as the most likely vehicle for the Phoenix effect. Since the ultimate demonstration of the Phoenix effect would require a full scale core depletion experiment, and since the desired high power density, low core volume criteria were satisfied by the Materials Testing Reactor (MTR) type core, a burnup experiment was designed for the MTR itself.

A series of experiments was undertaken at PNL to provide support for the eventual burnup experiment in the MTR and more general integral information for pure plutonium systems. This campaign was conducted in four stages with decreasing generality as the MTR experiment became more firmly established:

- 1) Measurements of the infinite multiplication factor, k_{∞} , of arrays of borated polyethylene and Pu-Al alloy discs

- in the Physical Constants Testing Reactor (PCTR).⁽²⁾
- 2) Critical approach experiments with small cylindrical arrays of cans containing polyethylene and Pu-Al fuel discs in the Critical Approach Facility (CAF).⁽³⁾
 - 3) Critical experiments in a mockup assembly designed to resemble the MTR reflector and the projected Phoenix core in the Plutonium Recycle Critical Facility (PRCF).⁽⁴⁾
 - 4) Startup critical experiments in the MTR with the Phoenix core prior to beginning the burnup experiment.⁽⁵⁾

This report has two main objectives. The first is to describe the experiments in sufficient detail to allow the reader to perform calculations to compare his data and methods with the experimental results. The second is to describe the analytic techniques and theoretical methods used in the analysis of all the experiments at PNL and at the MTR. Collectively, these techniques and methods have been successful in describing the critical behavior of these plutonium systems and should be useful for Phoenix design calculations.

II. SUMMARY AND CONCLUSIONS

The MTR-Phoenix experiment demonstrated that a longer life with a lower control requirement can be obtained from a plutonium-fueled core as compared to the highly enriched ^{235}U standard MTR core.

The analysis of the MTR-Phoenix critical experiments and the supporting critical experiments done at PNL yield predicted critical

results within 1% over the entire range of geometries and isotopic concentrations. This agreement is obtained through the use of a special prescription called the Wings-240 treatment which increases the calculated absorption in the ^{240}Pu isotope beyond that justified by the current nuclear data and resonance absorption calculational techniques.

The calculated reactivity-exposure slope is much less than that observed. The reasons for this are currently under investigation. The isotopic analysis of the spent fuel, which will be completed in the near future, will help to determine the direction that further analysis must take to provide an understanding of observed depletion characteristics of the Phoenix core.

III. DESCRIPTION AND ANALYSIS OF POISONED PHOENIX LATTICES

A. PCTR Experiments

The Physical Constants Testing Reactor (PCTR) was used to measure k_{∞} for plutonium of three isotopic compositions in three discrete geometries. The technique⁽⁶⁾ is limited to the measurement of assemblies with $k_{\infty} = 1.00$. It was necessary to add boron to the cell in order to meet this requirement while still maintaining the heavy plutonium loading and the overall H/Pu ratio that would exist in a Phoenix core. The boron addition was accomplished by using a mixture of pure polyethylene and borated polyethylene to simulate the coolant passages between fuel plates.

The components of the experiments were:

- 1) Pu-Al fuel discs, 20 wt% Pu, 1.950 inches in diameter, 20 mils thick.
- 2) Polyethylene discs, 1.960 inches in diameter, 20 mils thick.
- 3) Borated polyethylene discs, 0.90 wt% natural boron, 1.960 inches in diameter, 20 mils thick.

The discs were stacked into aluminum cans 12 inches inside length, 2.0 inches I.D., with a 1/16 inch wall thickness. A typical unit cell is shown in Figure 1. The cans were loaded into the central test region of the PCTR as shown schematically in Figure 2. The fuel, moderator and poison disks were stacked in layers within the cans so that continuous slabs of each material were formed. For each system the boron concentration was varied until $k_{\infty} = 1.00$. This condition was detected by determining that the removal of a single can in the center of the test lattice had no effect on the PCTR reactivity, and by using foil measurements to assure that a spectrum match existed within the central test region.

The plutonium isotopic mixtures used are shown in Table I.

FIGURE 1. PCTR UNIT CELL CONFIGURATION

- FUEL CAN - PuAl, POLYETHYLENE, BORATED POLYETHYLENE
- BUFFER CAN* - UA1 (33 W/O U, ENRICHED TO 93 W/O ^{235}U), BORATED POLYETHYLENE
- EMPTY CAN - AIR

* THESE CANS FORM A REGION WHERE THE THERMAL SPECTRUM OF THE PCTR IS HARDENED TO MORE CLOSELY MATCH THE CHARACTERISTIC SPECTRUM OF THE PuAl LATTICE

FIGURE 2. PCTR EXPERIMENTAL CONFIGURATION

Table I

PCTR Plutonium Isotopic Compositions

<u>Type</u>	<u>At % ^{239}Pu</u>	<u>At % ^{240}Pu</u>	<u>At % ^{241}Pu</u>	<u>At % ^{242}Pu</u>
I	90.48	8.19	1.25	0.08
II	77.80	18.89	3.01	0.30
III	67.42	26.86	4.76	0.96

The experimental results are shown in Table II.

Table II

$k_{\infty} = 1.00$ Combinations in PCTR

<u>Plutonium Type</u>	<u>H/Pu Atom Ratio</u>	<u>Pu-Al Plate Thickness (mil)</u>	<u>$^{10}\text{B/Pu}$ Atom Ratio</u>	<u>Cd. Ratio 0.02 in. Dia. Gold</u>
I (8% ^{240}Pu)	208	100	0.179 $\pm$.002	1.61
II (19% ^{240}Pu)	214	20	0.198 $\pm$.004	
II (19% ^{240}Pu)	200	100	0.141 $\pm$.002	
III (27% ^{240}Pu)	203	20	0.152 $\pm$.003	1.61
III (27% ^{240}Pu)	202	60	0.135 $\pm$.003	
III (27% ^{240}Pu)	199	100	0.114 $\pm$.002	1.88
III (27% ^{240}Pu)	960	20	0.089 $\pm$.002	5.17

Cadmium ratios were measured in some of the assemblies and these are included as measured indicators of the cell neutron spectrum relationship.

B. PCTR Experiment Analysis

Since the quantity of interest is the system neutron multiplication and not the critical boron concentration, the experimental boron concentration at measured $k_{\infty} = 1.00$ was used in each calculation, and the calculated results are given in terms of the computed values of k_{∞} .

The epithermal neutron spectrum used in cross section averaging was computed using the P1 approximation to the transport equation as contained in HRG.⁽⁷⁾ The spectra were computed for infinite media with spatial dependence present only in the heterogeneous resonance treatment of the fuel, where the slab geometry was treated by either the NR (Narrow resonance) or the NRIA (narrow resonance infinite absorber) heterogeneous approximations. Slab to slab interactions which are very important in these closely-spaced geometries were computed using the Bell Correction.⁽⁸⁾

The treatment of the major ^{240}Pu resonance at 1 eV was studied in considerable detail prior to undertaking the analysis. The experimental determinations of the ^{240}Pu resonance integral by P. F. Nichols⁽⁹⁾ were reanalyzed⁽¹⁰⁾ using the Nordheim integral evaluation of the ^{240}Pu resonance integral contained in ZUT.⁽¹¹⁾ Good agreement was found between the experimental results and those from the integral method. The NRIA approximation for ^{240}Pu in turn closely approximated the integral result. Thus, it is believed that this most important resonance is treated correctly by the NRIA approximation. However, the entire resonance contribution

was placed into only one quarter lethargy fine group in the version of HRG in use at the time the analysis was made.*

The effect of placing all of the huge resonance integral in one fine group was to grossly underestimate the absorption in ^{240}Pu . To overcome this difficulty within the existing framework of the HRG code a change was made in the neutron width (Γ_n) of the resonance from 2.468×10^{-3} eV to 2.20×10^{-3} eV. The resulting decrease in the resonance integral was compensated for by the addition of a fixed absorption to four of the off-resonance fine groups. The allocation used is shown in Table III. The infinitely dilute resonance integral is reduced from 8559 barns to 8379 barns by this cross section allocation. Both values agree with the experimental resonance integral within the uncertainties as quoted in the standard references. In the case of 40 mil fuel plates, the effective resonance integral for the ^{240}Pu 1 eV resonance is 1570 barns using the unmodified resonance parameters. With the allocation of the fixed absorption to the fine groups immediately surrounding the group containing the resonance, the value of the effective resonance integral becomes 2208 barns, of which 661 barns is the fixed absorption contribution and 1547 barns is contained within the resonance group. Thus this "wings prescription" adds 661 barns to the effective cross section in addition to increasing the

* Subsequent modifications to HRG made allocations to the quarter lethargy fine groups surrounding the fine group containing the 1 eV resonance. (12)

Table III

Wings-240 Resonance Allocation

<u>Fine Group #</u>	<u>E Upper (eV)</u>	<u>E Lower (eV)</u>	<u>Fixed Absorption Cross Section (barns)</u>
62	2.38	1.86	8
63	1.86	1.44	31.4
64	1.44	1.125	430.0
65	1.125	.876	reduced resonance
66	.876	.683	192.2

absorption computed by allocation to more than one fine group. The latter effect is physically reasonable; however, the addition of the 661 barns must be regarded as an engineering adjustment to the nuclear data whose entire justification is that it allows the eigenvalues to be calculated quite well over the range of plutonium concentrations represented by the set of Phoenix experiments. Further data are necessary to determine whether or not this adjustment to the ^{240}Pu cross section is valid.

The results of calculations of k_{∞} for the PCTR experiment using HRG for the epithermal group constants and the BATTELLE-REVISED-THERMOS code⁽¹³⁾ for the thermal group constants are shown in Table IV for cases which do and do not include the wings prescription for ^{240}Pu .

Table IV

Wings Prescription Comparison of Calculated k_{∞}
for Lattice Configurations with Measured $k_{\infty} = 1.00$

The lattice configuration chosen is the PCTR experiment - H/Pu ~ 200 .
Wings prescription values are shown in parenthesis.

Plate Thickness (mils)	% ^{240}Pu		
	<u>8</u>	<u>19</u>	<u>27</u>
20		1.034 (1.014)	1.056 (1.025)
60			1.025 (0.995)
100	0.980 (0.972)	0.989 (0.969)	1.004 (0.973)

The use of the wings prescription reduces the chronically high calculated values of k_{∞} at low plate thickness, but also moves the computed values farther from the experimental result at large plate thickness. Using a linear interpolation for the 19 wt% ^{240}Pu case, k_{eff} is only 0.3% high for a plate thickness of 40 mils. Since this condition most nearly resembles the MTR core geometry, the wings method of treating the 1 eV resonance appears to be very appropriate for that geometry. Indeed, the wings treatment is determined by this experimental value.

An additional fact is evident from an examination of Table IV: there is a systematic variation with the plate thickness in the discrepancy between the calculated and experimental values of k_{∞} . The variation is

larger than that caused by the % ^{240}Pu and the use of the wings prescription. In an attempt to identify the source of this discrepancy a comparison was made between the regional thermal flux depression as calculated by the discrete ordinate approximation (S_N method)^(14, 15) and the integral transport method used in THERMOS-type codes. The thinly-spaced slab geometry made it necessary to use up to S_{36} in order to get a satisfactory convergence from the S_N code. The results are summarized in Table V.

Table V

S_N Comparison of Calculated k_∞ for Lattice Configurations

with Measured $k_\infty = 1.00$

The lattice configuration chosen is the PCTR Experiment - H/Pu 200 and values shown are for the wings prescription in HRG and using THERMOS and using S_N (in parenthesis).

Plate Thickness (mils)	% ^{240}Pu	
	<u>19</u>	<u>27</u>
20	1.014 (1.015)	1.025 (1.027)
60		0.995 (1.005)
100	0.969 (0.993)	0.973 (0.997)

The use of the S_N method substantially reduces the systematic variation with plate thickness and brings the total range of agreement to 3% or less over the range of plate thicknesses. The hydrogen scattering is computed in BATTELLE-REVISED-THERMOS using the Goldman⁽¹⁶⁾ kernel for polyethylene.

IV. THE CRITICAL APPROACH EXPERIMENT

A. CAF Experiment Description

The critical approach experiments conducted in the Critical Approach Facility (CAF) at Pacific Northwest Laboratory were designed to be the next step toward a real reactor system after the measurements of infinite medium neutron multiplication were completed in the PCTR.

Cooling water channels between fuel plates were simulated with polyethylene discs. A fixed Pu-Al fuel plate thickness of 40 mils was used with 20 mils of aluminum on each side to simulate cladding. The unit cell arrangement is shown in Figure 3. The discs were loaded into 2.00 inch I.D., 2.25 inch O.D. aluminum cans, 30 inches long. It was found that a maximum of 145 unit cells consisting of 120 mils polyethylene, 20 mils aluminum, 40 mils Pu-Al alloy, and 20 mils aluminum could be loaded into the 30 inch aluminum cans. The fuel discs were Al-20 wt% Pu alloy of 3.20 g/cm^3 density. The plutonium isotopic composition was 90.5 wt% ^{239}Pu , 8.2 wt% ^{240}Pu , 1.2 wt% ^{241}Pu and 0.1 wt% ^{242}Pu .

An approach to critical was made by varying the number of unit cells loaded into a 19-can array. The entire assembly was immersed in water inside the critical approach tank. Thus there were water reflectors at the top and bottom except for the unfueled ends of the cans which were loaded with polyethylene discs above and below the core discs such that the core area was centered in the cans.

Of the number of approaches made, two geometries are of most interest to the calculational effort. These were:

- 1) A nineteen-can array on a core pitch of 2.25 in surrounded by a ring of beryllium cans to provide a partial beryllium reflector.

FIGURE 3. CAF DISC ARRANGEMENT

- 2) The same configuration as above, but with the center can replaced by a simulated control rod. The pseudo control rod was made from a fuel can by first placing 40 mils of cadmium around the inside wall and then filling the can with polyethylene.

The extrapolated critical heights of both of these arrays were determined by increasing the number of unit cells loaded into the fuel cans until a reliable extrapolation could be made.

The approach with the rod-free core was extrapolated to predict a critical loading of 103 ± 1 cells, or a critical core height of 21.31 inches.

The rodded core was completely loaded with 145 cells or 30.00 inches core height; however, a reliable extrapolation to critical could not be made from the data.

B. Critical Approach Experiment Analysis

The number of neutron energy groups required to represent the neutron energy changes in the core and reflector was computed directly for both diffusion theory and transport theory. Using a one dimensional diffusion theory⁽¹⁷⁾ representation of the core and reflector in cylindrical geometry, the eigenvalue was computed for 4, 7, 10, 13 and 17 energy groups. The results obtained are shown in Figure 4. The energy boundaries for the multigroups used are given in Table VI. The change in eigenvalue in going from 4 to 17 groups was about 1.6% in Δk . A water-reflected assembly was chosen for this calculation because the effect of the water reflector is to

FIGURE 4. CALCULATED k_{eff} VERSUS NUMBER OF ENERGY GROUPS IN DIFFUSION THEORY FOR THE WATER REFLECTED CAF ASSEMBLY

Table VI

Multigroup Energies in CAF Analysis

Group Number	17 Lower Boundary (eV)	13 Lower Boundary (eV)	10 Lower Boundary (eV)	7 Lower Boundary (eV)	4 Lower Boundary (eV)
1	2.23 + 6	2.23 + 6	2.23 + 6	2.23 + 6	4.98 + 5
2	1.05 + 6	1.05 + 6	1.05 + 6	4.98 + 5	3.06
3	4.98 + 5	4.98 + 5	4.98 + 5	2.48 + 4	0.683
4	2.35 + 5	2.35 + 5	1.11 + 5	1.23 + 3	0.0
5	1.11 + 5	1.11 + 5	2.48 + 4	3.06	
6	5.25 + 4	5.25 + 4	1.23 + 3	0.683	
7	2.48 + 4	2.48 + 4	6.14 + 1	0.0	
8	1.17 + 4	5.53 + 3	3.06		
9	5.53 + 3	1.23 + 3	0.683		
10	2.61 + 3	6.14 + 1	0.0		
11	1.23 + 3	3.06			
12	2.75 + 2	0.683			
13	6.14 + 1	0.0			
14	1.37 + 1				
15	3.06				
16	0.683				
17	0.0				

increase the group sensitivity of the eigenvalue above that of the partial beryllium reflector. Seventeen groups in a diffusion theory calculation should then be more than sufficient for the beryllium-reflected assemblies.

The group sensitivity of the discrete ordinate approximation to the transport equation was determined by a direct calculation with the same group structure as in the diffusion theory group sensitivity calculation. The S_N code DTF-IV⁽¹⁸⁾ was used for this purpose. The group sensitivity was much larger in the case of transport theory than with diffusion theory, so 17 groups were used. The difference between diffusion theory and transport theory at seventeen energy groups is 2.6% in k_{eff} with diffusion theory giving the lower values.

This effect is somewhat dependent on the approximation used in the spectrum averaged cross section set. A two-table approximation was used with the total cross section including zeroth moment scattering in the first set, and only first moment scattering for the total cross section in the second table position. Various transport approximations⁽¹⁹⁾ can reduce the group sensitivity somewhat for both single and multitable formalisms, so the result quoted here may not be taken as a general conclusion.

The classic assumption under which the rough array of fuel cans is replaced by a smooth cylinder of equal area was also checked. Using two dimensional diffusion theory in four energy groups and in XY geometry,⁽²⁰⁾ the rough boundary of the core and reflector interface was approximated in great detail. This calculation was compared to a cylindrical calculation in one dimension with core and reflector area the same as in the XY calculation. Eigenvalues for the comparable cases were well within 1% in Δk_{eff} .

The separability of the neutron flux into independent axial and radial functions was determined by comparing a two dimensional diffusion theory calculation in four energy groups in RZ geometry to a one dimensional calculation in cylindrical geometry with the vertical dimension represented by an axial buckling. Here the eigenvalue agreement was within 0.3% in k_{eff} .

The reflector savings used in the axial buckling calculation was based on a four group diffusion theory calculation in slab geometry. The eigenvalue was computed for a slab region representing the core with thickness equal to the core height and surrounded by a reflector region of thickness equal to the water reflector thickness. Next, the reflectors were removed and the bare slab core thickness was increased until the reflected core eigenvalue was again attained. The reflector savings is then the difference between the unreflected and reflected slab thickness at the same eigenvalue.

Extensive sampling records were kept of fuel disc thickness and weight percent plutonium. The uncertainties in these numbers were of the order of 3 to 5 percent. This problem of determining what effect composition uncertainties might have on an integral experiment outcome is encountered frequently.

To answer the question of the effect of uncertainties on k_{eff} , use was made of results from elementary statistics on the variance of linear functions: if $Z = aX + bY + c$, then $\text{Var}[Z] = a^2\text{Var}[X] + b^2\text{Var}[Y]$. Clearly if a linear regression can be obtained for k_{eff} as a function of disc thickness (X) and plutonium weight percent (Y), then the expected variance of k_{eff} can be computed using the regression coefficients. A grid of cases

at various wall thicknesses and plutonium composition was laid out. Spectrum-averaged cross sections were computed with HRG and BATTELLE-REVISED-THERMOS for each point selected on the grid. These were used in a four group diffusion theory calculation of k_{eff} for each point on the grid. From these results a very good linear fit was obtained for k_{eff} :

$$k_{eff} = 0.0086853X + 0.00356096Y + 0.9446643.$$

To determine the variances in wall thickness and plutonium composition, samples of 240 discs were selected from the total population of discs. The mean wall thickness and the mean plutonium composition for each 240 disc sample was computed, and the variance of these quantities determined. The computed variation in k_{eff} at the 95% confidence level was ± 1.6 milli k. Thus variances of ± 0.0005 inch and ± 0.0042 weight fraction combine to produce a variance of 0.0016 in k_{eff} . The possible error due to composition uncertainty is therefore acceptably small.

On the basis of the above results a 17 energy group transport calculation with first order anisotropic scattering was performed on each of the two approach configurations. For the rod-free core a k_{eff} of 0.9915 was obtained which compares with the experimental $k_{eff} = 1.0000$. The extrapolated data for the array with the pseudo control rod at the center indicated that the critical height would be greater than 39.7 inches. The calculated k_{eff} for this height was 0.9403 which confirms the experimental conclusion that a reliable extrapolation to critical was not possible.

V. THE ENGINEERING MOCKUP CRITICAL EXPERIMENT (PRCF)

A. Experimental Description

A full-scale mockup of the Phoenix core and MTR reflector was de-

signed and assembled at PNL. The entire assembly was placed in the Plutonium Recycle Critical Facility (PRCF). Critical measurements were performed within the envelope of PRCF operating procedures and safeguards, which provide complete neutron multiplication and attenuation instrumentation and shutdown control.

The experimental mockup of the MTR Phoenix core was as close an approximation to the actual MTR with the Phoenix core as could be achieved with the materials available. As assembled, the main differences between the mockup and the real core were as follows:

- Plutonium composition; 76.8/19.3/3.2/0.7 versus 65.5/23.3/7.7/3.5 for the MTR itself in units of wt% ^{239}Pu , ^{240}Pu , ^{241}Pu , ^{242}Pu respectively.
- Reflector composition; the beryllium reflector in the mockup contained regions of two beryllium densities. Immediately adjacent to the core the beryllium pieces were more densely packed so that there was about 5 volume% water in the region between the core and the control slabs (See Figures 5 and 9) and 10 to 20% outside this area. The entire MTR beryllium reflector was less than 3 volume% water.
- Core boundary; the mockup required a 1/4 inch aluminum wall between the core and the reflector which had no counterpart in the MTR itself.
- Control slots; the mockup required control sheets for shutdown and approach control reasons. These are shown schematically in Figure 5. There is no counterpart in the MTR.

- 19 FUEL BOX
- 8 SIMULATED MTR CONTROL ROD
WITH FOLLOWER FUEL
- EXPERIMENTAL SHUTDOWN AND
APPROACH CONTROL SHEETS -
CADMIUM WITH LEXAN FOLLOWERS

FIGURE 5. GENERAL CONFIGURATION OF MTR MOCK-UP IN PRCF

- ° Beam Tubes; the MTR beam holes were represented entirely by six solid aluminum structures in the PRCF mockup reflector structure.

The salient similarities include the 3x9 core plus shim rod-follower fuel array surrounded by a beryllium water reflector. A schematic drawing of the fuel element unit cell is shown in Figure 6. Overall core dimensions are shown in Figure 7.

Many experiments which were performed in the mockup were done to provide information to ensure safe operation and were not designed to be readily amenable to analysis. The complete set is described in Reference 4. The following experiments were selected for detailed analysis:

1. Clean critical loading: with the core cavity in the reflector completely filled with water, a minimum critical loading was obtained by loading 14-1/4 fuel boxes. The critical array was approximately centered in the core cavity with a water reflector at each end.
2. Rodded critical loading: an array loaded from one end of the central cavity and containing a simulated cadmium control rod bordering on the short side beryllium reflector and centered on the long axis of the core cavity. This configuration required 15-5/8 fuel boxes to be critical.

FIGURE 6. PRCF-PHOENIX CORE UNIT CELL SCHEMATICS WITH DIMENSIONS

FIGURE 7. PRCF MOCK-UP DIMENSIONAL SCHEMATIC

3. Full Core Loading: when the core was completely loaded, the critical bank height was determined for all eight simulated control rods. The core went critical with the tips of all eight rods essentially at the core midplane.

B. Mockup Critical Experiment Analysis

Preliminary calculations indicated that the effect on computed k_{eff} of the number of energy groups is much smaller than that seen in the CAF experiments. Values of k_{eff} were calculated using spectrum averaged group cross sections ranging from 17 to 2 energy groups in a one dimensional diffusion theory representation of the core and adjacent reflector. With four groups k_{eff} is only +0.4% above the value for 17 groups; with two energy groups the difference is + 3.9%. The conclusion was that four energy groups would be sufficient and that two energy group results should be valid only for comparison of relative effects, or perhaps not valid at all.

The reasons for the reduced group number sensitivity are complex. The greatest single contributing effect is the $n, 2n$ reaction in the beryllium. This effect is + 2% in Δk_{eff} above seven energy groups and is reduced to + 1.5% at four energy groups. This reduction in the computed worth of the beryllium $n, 2n$ reaction partly compensates for the normal increase in k_{eff} as the number of groups is reduced. The net result of these oppositely directed effects is an overestimation in k_{eff} of only 0.4%.

The four group structure chosen for most of the analysis was:

- 1) Group 1: 10 MeV to 1.17×10^4 eV; a single fast group to contain all fission source neutrons.

- 2) Group 2: 1.17×10^4 eV to 2.38 eV, a resonance reaction group to contain all unresolved and all upper level resolved resonances.
- 3) Group 3: 2.38 eV to 0.683 eV; a group designed to contain the bulk of the absorption from the giant 1 eV resonance in ^{240}Pu .
- 4) Group 4: 0.683 eV to 0 eV; a thermal group.

Epithermal spectra and group averaged cross sections were all computed with HRG. For the fuel region the unit cell was taken to be the entire fuel box with sixteen plates and small amounts of surrounding water as shown in Figure 6. Homogeneous nuclei densities include the volume fractions appropriate to that assumption. Thus the aluminum density includes the aluminum side walls and all other aluminum structure along with the clad aluminum and the alloyed fuel region aluminum. No leakage was included in the core spectrum calculation.

The resonance treatment in slab geometry included the aluminum as an admixed light mass scatterer. The Bell correction was used, as in the CAF analysis, to take into account plate-to-plate interactions. The wings-240 prescription was used in the ^{240}Pu resonance calculation.

Reflector spectra were all infinite media, homogeneous calculations. When spectrum-averaged cross sections were desired for a two spatial dimension calculation, vertical homogenization was employed to represent the reflector pseudo beam tube structures in the axial direction.

Thermal spectra for the core were calculated by a two step process which is the direct analog of that used in the CAF analysis. (See Figure 8.) Using BATTELLE-REVISED-THERMOS, an initial calculation was done in slab geometry for a plate unit cell in the spatial sequence:

- 1) reflecting boundary
- 2) half channel thickness of water-59.3 mils
- 3) aluminum clad-20 mils
- 4) half thickness of fuel plate-20 mils
- 5) reflecting boundary

The ratio of the region average flux to cell average flux from this calculation was used to adjust the cell nuclear densities for the i th isotope in the j th region (N_{ij}) according to the overall thermal rate preserving relationship. $N_i = \sum_j v_j f_j N_{ij}$, where v_j and f_j are the volume fraction and the region to cell average flux ratio for the j th region respectively. The "smeared" nuclear densities (N_i) were then used in a second thermalization calculation which included the whole fuel box unit cell. The spatial sequence for this second calculation, also in slab geometry, was:

1. reflecting boundary
2. box external water-2.25 mils
3. side plate-187 mils
4. aluminum and water region between side plate and fuel
active section - 52 mils
5. fuel region containing flux adjusted densities from the
first calculation - 1.26 inches
6. reflecting boundary

FIGURE 8. SCHEMATIC OF CORE THERMAL SPECTRUM CALCULATION METHOD

The result of this two step process is to produce spectrum averaged thermal cross sections which take account of all material in the core.

Thermal spectra for the reflector were calculated over the same homogeneous region as used in the epithermal HRG calculation.

The analyses of the minimum critical loading core and the single rod critical experiment were very similar. Both configurations were chosen to be amenable to two dimensional analysis. The minimum critical core was approximated by a quarter core diffusion theory model. The actual core was off-center by two plates, but the two different quarters (the water reflector thickness off the core end was varied at constant core volume) gave the eigenvalue to within 0.1%.

The single rod critical core had exact half core symmetry, hence the diffusion theory model could be made half core symmetric. This is shown in Figure 9.

The active section length of the fuel plates was 23.52 inches. The top reflector savings obtained from one dimensional diffusion theory as explained previously was 6.51 cm, while the bottom reflector savings was 6.38 cm. The difference comes about from the slightly different aluminum to water ratio for the material above and below the core. The vertical buckling based on these quantities is $1.79 \times 10^{-3} \text{ cm}^{-2}$, and is the same for both the minimum and single rod critical experiments.

Models of both critical assemblies were constructed using two dimensional diffusion theory in X-Y geometry. The code 2DB⁽²¹⁾ was used for this purpose. Details of the diffusion theory model for each configuration are given in

FUEL BOXES, 16 PLATES

Cd-A1 CONTROL ROD

14 FUEL BOXES WITH 16 PLATES

2 FUEL BOXES WITH 9 PLATES

1 FUEL BOX WITH 8 PLATES

15-5/8 FULL 16 PLATE BOXES CRITICAL

FIGURE 9. PRCF-PHOENIX SINGLE ROD CRITICAL

Figures 10 and 11. The calculated eigenvalue for the rod-free assembly was 1.0038 and the calculation for the single rod critical gave an eigenvalue of 1.0059.

The fully-loaded core with rods inserted to the critical position required a calculation in three dimensions. The 3D diffusion theory code WHIRLAWAY⁽²²⁾ was available for this purpose; however it is limited to only two neutron energy groups. This is clearly not enough to provide eigenvalues which correspond to k_{eff} exactly, based on the group sensitivity studies quoted previously. Additional disadvantages are the rather large amounts of computer time required for convergence and the coarse mesh spacing due to computer storage limitations (65K in the case of the UNIVAC 1108 version). In spite of these disadvantages in the utility of the eigenvalue, the flux and resultant power distribution calculations are the only ones using analytic methods which reflect the complexities of the MTR Phoenix core.

A calculation of the eigenvalue from the model shown in part in Figure 12 with the rods at the critical position gave an eigenvalue of 1.017. Even this rather poor agreement is most certainly fortuitous. This can be seen by comparing three dimensional calculations of the PRCF core with rods completely withdrawn and inserted with two dimensional calculations of the same conditions. In the two dimensional calculations, with the rods at the two extreme positions, the core regions are invariant in the axial direction and a single two dimensional plane with the appropriate axial buckling can be used to represent the entire core. Two dimensional calculations with four energy groups do not compare well with the corresponding three dimensional, two group calculations, as can be seen from Table VII. Even the rod

* THE NUMBER OF MESH POINTS SHOWN IN EACH INTERVAL ARE EQUALLY-SPACED

AXIAL BUCKLING $1.791 \times 10^{-3} \text{ cm}^{-2}$ INCLUDING REFLECTOR SAVINGS AND
EXTRAPOLATION DISTANCE

FIGURE 10. X-Y DIFFUSION THEORY QUARTER CORE MODEL OF PRCF
CLEAN CRITICAL

FIGURE 11. X-Y DIFFUSION THEORY QUARTER CORE MODEL OF PRCF SINGLE ROD CRITICAL

Table VII

Comparison of Two Dimensional, Four Group Models
with Three Dimensional, Two Group Models in PRCF Analysis

	<u>2D, 4 Group</u>	<u>3D, 2 Group</u>	<u>Δ</u>
Rods In	0.737	0.58	-0.157
Rods Out	1.066	1.12	+0.05
Rod Worth $\Delta k/k$	-0.309	-0.48	

worths are very different, as shown in Table VII. The 1.7% error obtained for the critical rod position came about through a cancellation of the larger rods-out error (+.05) with the still larger rods-in error (-0.157) for banked rod positions near the midplane.

On the basis of this set of calculations, a rod model was developed which attempts to correct the two group eigenvalue to the four group value:

$$k_{\text{eff}} = (k_{4G0} - k_{2G0}) + \left(\frac{X}{L}\right)^{2.5} [k_{2G0} - k_{2GI} - k_{4G0} + k_{4GI}] + k_{2G}$$

where

k_{4G0} = the eigenvalue for four groups, rods out;

k_{2G0} = the eigenvalue for two groups, rods out;

k_{2GI} = the eigenvalue for two groups, rods in;

k_{4GI} = the eigenvalue for four groups, rods in;

k_{2G} = the eigenvalue for two groups, rods at X;

X = distance from the top of the core that the rods are inserted;

L = total rod travel distance through the active section;

k_{eff} = the eigenvalue corrected to four groups;

The model has the following properties:

- 1) at $X=0$, $k_{\text{eff}} = k_{4G0}$
- 2) at $X=L$, $k_{\text{eff}} = k_{4GI}$
- 3) at $X=L/2$, $k_{\text{eff}} = 1.000$

Thus it goes through the three known points exactly, but there are no degrees of freedom left to test the goodness of fit. A three dimensional calculation in four groups would be required to do this, but the lack of such a calculation is the reason for the model being developed in the first place. Nor is it possible to derive such a relation directly since it merely relates the relative error in two different numerical approximations to the energy dependent diffusion equation. The first two properties must be satisfied *a priori*, the third property is satisfied by the choice of 2.5 for the exponent of the (X/L) coefficient.

The comparison of measured and calculated power distributions are necessarily complex because of the wealth of comparison points. Simple generalizations are difficult to formulate. The best approach is to display the results graphically and discuss each in its turn.

Starting with the horizontal power shape down the long axis of the core and as far removed from rods and reflectors as possible, a line six inches below the core horizontal midplane from the center of the core to the short side reflector was chosen to compare the measured and calculated power distribution. This is displayed in Figure 13. The agreement everywhere is within 12%.

FIGURE 13. COMPARISON OF THREE DIMENSIONAL DIFFUSION THEORY
 CALCULATED POWER DISTRIBUTION AND MEASURED POWER DISTRIBUTION IN
 PRCF MOCK-UP CRITICAL

The next comparison, shown in Figure 14, is a corner box traverse six inches below the core horizontal midplane and into the short side beryllium reflector. Here the agreement is fair, the maximum error being approximately 10%.

An axial power shape at the center of the center fuel element is shown in Figure 15. The agreement here is good except at the bottom of the core. There the large power spike in the calculated power distribution illustrates what sort of distribution would have resulted if the fuel plates had not been tapered at the bottom. The experimental measurement includes the effect of the taper. The calculated bottom peak of nearly 3.6 has been reduced to an innocuous 1.1 by tapering the fuel alloy over a distance of approximately one inch at the bottom of the core. The good agreement in all but the bottom portion leads one to believe that if enough mesh points had been available to include the tapered plates in the calculation the overall agreement in axial shape at the center would have been well within 10% too.

The next axial shape in Figure 16 illustrates the rather poor agreement achieved at the centers of the fuel elements along the long side beryllium reflector. Here, as in Figure 15, the calculated shape excludes the effect of the taper.

The power distribution in the follower fuel at the center of the follower fuel is shown in Figure 17. The agreement here is within 13% in the worst case.

Figure 18 shows the follower fuel power distribution where the point chosen is next to the aluminum side wall structure. The agreement is within 10% in this case.

The greatest difficulty was experienced in predicting the single element power distribution from the center to the edge of the long side beryllium

FIGURE 14. BOX POWER SHAPE INTO SHORT SIDE REFLECTOR

FIGURE 15. AXIAL POWER SHAPE AT CENTER OF CORE

FIGURE 16. AXIAL POWER SHAPE AT CENTER OF EDGE FUEL BOX

FIGURE 17. COMPARISON OF THREE DIMENSIONAL DIFFUSION THEORY CALCULATED POWER DISTRIBUTION AND MEASURED POWER DISTRIBUTION AT CENTER OF SHIM FOLLOWER FUEL ELEMENT

FIGURE 18. COMPARISON OF THREE DIMENSIONAL DIFFUSION THEORY CALCULATED POWER DISTRIBUTION AND MEASURED POWER DISTRIBUTION AT EDGE OF SHIM FOLLOWER FUEL ELEMENT

reflector. The three dimensional, two group WHIRLAWAY calculations indicated an extremely high power peak at the edge of the power zone next to the reflector. This is illustrated in Figure 19. The point power-to-core average power ratio at the edge is calculated to be 2.76 as compared to the measured value of 2.14.

The reasons for this major discrepancy were studied in detail. The use of diffusion theory was initially suspect in this application, so one dimensional transport methods were used as a test. Integral transport theory using BATTELLE-REVISED-THERMOS in two different ways was employed. First a slab calculation with a spatially independent thermal neutron source was used. Then a spatially dependent source obtained by collapsing the three epithermal groups of a four group S_N transport calculation was used in BATTELLE-REVISED-THERMOS. These two results are shown in Figure 20 along with the experimental numbers. The use of the spatially-dependent thermal source results in only a slight improvement in the poor agreement with experiment. Even the four group S_N calculation (DTF-IV) is equally poor as is shown in Figure 20. In fact the agreement between the S_N and integral methods where the spatial source is used is very good and both are in very poor agreement with the experiment.

The next avenue of agreement was sought in the direction of the problem spatial detail. In all previous calculations, the fueled region was represented as a homogeneous power zone containing fuel, clad and channel coolant. This homogenized treatment works well at the core reflector interface when the fuel plates are parallel to the reflector, as seen in Figure 14. In the present case the fuel plates are at right angles to the reflector. One can imagine that the space-spectrum interactions are improperly treated by introducing a homogeneous mix of fuel and water right

FIGURE 19. COMPARISON OF THREE DIMENSIONAL DIFFUSION THEORY CALCULATED POWER DISTRIBUTION AND MEASURED POWER DISTRIBUTION INTO THE BERYLLIUM ALONG THE AXIS OF A FUEL PLATE

FIGURE 20. ONE DIMENSIONAL POWER DISTRIBUTION INTO THE BERYLLIUM ALONG THE AXIS OF A FUEL PLATE

up to the reflector boundary when, in fact, the water and metal regions alternate along that boundary. To check the assumption, a long thin two dimensional model was developed. Half of a coolant channel, the clad and half of the fuel zone thickness were represented in one direction while the other direction ran from the center of the core to the outside of the beryllium reflector. Reflecting boundary conditions were applied to all boundaries except for the outer reflector boundary which was a leakage boundary. The general arrangement is shown in Figure 21. The fuel plate power distribution as determined by a diffusion theory calculation in four energy groups is shown in Figure 20. This represents the best agreement of any of the methods investigated, so it is at least a fair inference that the major cause of the power calculation error is the spatial detail that is sacrificed in the calculational model. This necessary approximation is apparently entirely acceptable when the fuel plates are parallel to the reflector, but introduces a serious error when the plates are at right angles to the reflector. The magnitude of this error can be appreciated by comparing the two dimensional result in Figure 20 with the one dimensional result using the homogenized representation.

An attempt to use S_N transport theory on the two dimensional model led to convergence difficulties so no transport verification was possible.

The residual discrepancy is small enough to be explained by the switch to gross one dimensional slab calculations from the more exact gross three dimensional treatment. That is, a full three dimensional treatment of the core in which the discrete fuel plates were included would produce entirely acceptable power distributions. Unfortunately such a detailed model is beyond current computer capabilities.

FIGURE 21. TWO DIMENSIONAL POWER CUT MODEL

The three dimensional model used gives the bulk power shapes to within 15% in the worst cases and averages within 10%, except for the core-reflector interface along the long side of the rectangular core. Here a localized two dimensional calculation is necessary to provide a local peaking factor to correct the three dimensional calculation.

The sensitivity of the eigenvalue to changes in the reflector structure was investigated for two reasons: there were large differences between the actual reflector structure and the mockup experiment reflector; and the relative amounts of water and beryllium in the outer mockup reflector were not known accurately. Variations in the beryllium-to-water ratio and in the beryllium cross sections used had little effect on the eigenvalue. It was therefore concluded that the eigenvalue was relatively insensitive to the reflector details.

V. THE MTR CRITICAL EXPERIMENTS

A. Description of the Experiment

The initial loading of the MTR Phoenix core was accomplished by substituting the plutonium fuel elements for beryllium pieces in a stepwise fashion.⁽²³⁾ The pertinent steps are shown in Figure 22. After reaching the minimum critical loading, the control rods were inserted as required to maintain criticality as additional fuel elements were added. The depth of insertion was the same for all rods.

After the core was fully loaded a series of four low power irradiations was undertaken to provide fission product activities for power shape determinations by gamma scanning techniques. Other measurements of interest to the core analyst were the rod calibrations and rod drop measurements.

Be	SF	FF	SF	Be
	FF	FF	FF	
	SF	FF	SF	

CALCULATED $k_{eff} = 0.9743$, OBSERVED $k_{eff} < 1.00$

Be	SF	FF	SF	Be
	FF	FF	FF	
	SF	FF	SF	

CALCULATED $k_{eff} = 0.9977$, OBSERVED $k_{eff} < 1.00$

Be	SF	FF	SF	Be
	FF	FF	FF	
	SF	FF	SF	

CALCULATED k_{eff} WITH $Pu^{241} \rightarrow Am^{241}$ DECAY CORRECTION = 1.010,
OBSERVED $k_{eff} = 1.000 \dagger$

FF	SF	FF	SF	FF	SF	FF	SF	FF
FF	FF	FF	FF	FF	FF	FF	FF	FF
FF	SF	FF	SF	FF	SF	FF	SF	FF

CALCULATED $k_{eff} = 1.107$, OBSERVED $k_{eff} = 1.103 *$

FF FIXED FUEL ELEMENTS

SF SHIM FOLLOWER FUEL

$\dagger$ NEGLECTS THE LESS THAN 1 MK WORTH OF RODS LEFT IN THE CORE AT CRITICAL

* ESTIMATED FROM EXPERIMENTAL ROD WORTHS

FIGURE 22. MTR PHOENIX CRITICAL LOADING SEQUENCE

B. Critical Predictions and Comparison with Experimental Results

The initial loading configurations which are free of control rods can be represented accurately in two dimensions. The configurations with partly inserted shim rods cannot be represented in two dimensions. The experience with the three dimensional, two group method discussed under the PRCF analysis indicates that only poor k_{eff} predictions can be expected from direct three dimensional WHIRLAWAY calculations.

A simplified three dimensional model designed for depletion calculations was utilized in predicting the critical position for partially inserted rods. This 12x12x11 mesh point, quarter core model supplied a relationship between the fraction of the total control span and the fraction inserted of the banked shim rods, X/L :

$$\frac{k_{2G} - k_{2GI}}{k_{2G0} - k_{2GI}} = F(X/L), \text{ where the subscripted k's are as defined in the}$$

rod model section under the PRCF. Now the assumption was made that this same functional form would apply for the four group control swing fraction at the critical rod position:

$$\frac{1.0 - k_{4GI}}{k_{4G0} - k_{4GI}} = F(X/L). \text{ One then calculated } k_{4GI}, \text{ and } k_{4G0} \text{ exactly from}$$

the two dimensional four group models. This assigned a value to $F(X/L)$, and from this form as determined by the three dimensional simple model a value of the critical rod position X/L was obtained. One would expect that the cardinal assumption that $F(X/L)$ is the same for the four group and two group treatment is best satisfied only for the fully loaded core where the two and three dimensional models are the same. In applying this method to some

of the partial loadings one is really comparing the fully loaded core in two energy groups with some incomplete core configuration in four energy groups. Therefore intermediate rod position predictions will probably be less accurate than the final rod position prediction for the completely loaded core.

The MTR reflector beam tube system was represented in the two dimensions by reducing the beryllium nuclei in a vertical region homogenization. The horizontal through tube (HT-1) facility and the HG-9 thermal column opening were the only neutron egress facilities abutting directly on the core. All other beam tube openings had at least one row of beryllium blocks between the core and the beam opening. The basic two dimensional model used is shown in Figure 23, illustrating the reflector details. The HG-9 was to have been provided with a beryllium plug and so it was not included in the initial calculations. The plug was not present during the actual critical loading experiment. However, an after-the-fact calculation showed this to have only a small effect on k_{eff} even for the fully loaded core. For the partially loaded core where there are beryllium pieces between the HG-9 and the fuel, the effect will be even smaller.

The critical predictions were documented and distributed to INC and the AEC prior to the critical experiment. These predictions and the observed results are summarized in Table VIII.

The critical loading was reached with 7 fixed and 4 follower fuel elements as predicted. The 1.45 inches of banked shim rod inserted is worth almost nothing as judged by both the experimental and calculational rod worth curves. The minimum critical loading is therefore essentially rod free.

FIGURE 23. TWO DIMENSIONAL DIFFUSION THEORY MODEL USED IN PREDICTING THE MTR-PHOENIX CRITICAL LOADING

TABLE VIII

MTR-Phoenix Core Critical Configurations:

Predictions and Experimental Results

<u>Assembly</u> ⁽¹⁾	<u>Number of Fixed Fuels</u>	<u>Number of Shim Rods</u>	<u>Shims Out k_{eff}</u>	<u>Shims In k_{eff}</u>	<u>Predicted⁽²⁾ Rod Position</u>	<u>Observed Rod Position</u>	<u>Estimated⁽³⁾ error in ΔK</u>
1	5	4	0.9743				--
2	6	4	0.9977				--
3	7	4	1.0163	0.6968	+5.100"	+10.30"	0.016
4	11	4	1.0696	0.7147	+1.034"	+ 2.90"	0.0269
5	13	4	1.0848	0.8679	-0.864"	+ 1.80"	0.0505
6	13	8	1.0859	0.7446	+0.423"	+ 0.50"	0.0041
7	15	8	1.0964	0.7640	+0.117"	+ 0.25"	0.0061
8	19	8	1.1126	0.8078	-0.587"	- 0.45"	0.0055

(1) The assemblies are shown schematically in Figure 17

(2) Rod positions are in inches above (+) or inches below (-) the core horizontal midplane. The rods out position is +11.75" on this scale.

(3) The estimated errors in $\Delta K = k_{calculated} - k_{observed}$ for the rodded cores are estimated by inferring an observed rods-out k_{eff} for the observed rod position and the $F(X/L)$ function used in making the critical rod position. The smaller the difference between the observed and predicted rod position is, the less important this particular choice of $F(X/L)$ becomes.

This means that the error at critical is 1.6%. The source of part of this error was identified as the decay of ^{241}Pu to ^{241}Am during the three year period between the date of the isotopic analysis and the critical measurements. If this is taken into account the error is 1.0% in Δk_{eff} . The only other known difference in the configuration as calculated and the experimental configuration which went critical are the neutron source block in the reflector at one end of the configuration and the axial flux detectors in the reflector on the long side of the core. The combined effect of these is 0.2% in Δk_{eff} for a total *a posteriori* error of 0.8%.

The source of this residual error is unknown; however one might point out that the known core reflector interactions which make possible the use of four energy groups were tested on the rectangular PRCF critical arrays as discussed previously. The cruciform shape of the critical array may introduce a different set of compensations which are large enough to cause a 0.8% error.

The estimated errors in assemblies 4 through 7 in Table VIII are subject to the problem mentioned previously of comparing the completely loaded two group models with the partly-loaded four group calculational models. The final fully-loaded configuration has a 0.55% Δk_{eff} error which may be entirely explained by the ^{241}Pu decay to ^{241}Am .

One can therefore conclude that when the known neutronic and structural effects are taken into account, the error in k_{eff} was less than 1% in the two cases where the calculational models mirror the critical configuration exactly, i.e., assemblies 3 and 8 in Table VIII. Furthermore the initial critical loading was predicted and the final critical rod positions specified to slightly greater than 1/8" in error on a completely *a priori* basis.

The role of the expensive three dimensional calculations was limited to predicting the three dimensional power distribution. A detailed comparison of these distributions with the experimental gamma scans will be given in another report. The agreement achieved is comparable to that for the PRCF three dimensional power distribution discussed previously.

VII. GENERAL ANALYTIC CONSIDERATIONS

A. The Wings-240 Prescription

The use of the wings-240 prescription makes possible the prediction of the neutron multiplication for the plutonium systems under study here to within 1% Δk_{eff} . The errors introduced if one uses unadjusted nuclear data range from +2% for the 8% ^{240}Pu CAF experiment to +5% for the ~20% ^{240}Pu PRCF and MTR-Phoenix cores. It is therefore natural to conclude that the ^{240}Pu absorption itself must be increased. A single large change to any of the other isotopes of plutonium would not suffice to bring the entire range of experimental isotopics into relatively close agreement with the Phoenix experimental results. Of course a series of smaller errors in more than one of the plutonium isotopes might result in a trade off in absorption and/or fission rates which would mirror the experimental results.

After the development of the wings prescription, HRG was revised to provide an allocation of the resonance integral to more than one fine group.⁽¹²⁾ Two other improvements were made which affect the ^{240}Pu absorption. First, the intermediate resonance treatment for the admixed scatterer was added. This is a method of accounting for different rates of removal from the resonance energy of neutrons colliding with admixed scatterers of various masses. It is an improvement over the usual NR1A and NR approximations in which the assumption is that any single collision with a "light" admixed scatterer suffices to remove the neutron from the region of the resonance. Instead of rather arbitrarily classifying the

admixed moderator as "light" or "heavy" there is a smooth transition from "complete removal" to "no effect" depending on the scattering mass of the admixed moderator isotope. The other improvement consisted of an addition to the library of a special water isotope wherein the absorption cross section of the lowest fine groups has been increased to take into account the upscattering of neutrons from the thermal group.

The effect of these coding changes has been studied in detail by Jenquin and Gelhaus;⁽²⁴⁾ their results are summarized in Table IX.

TABLE IX
Results of HRG Calculations for the PRCF System

	<u>Data</u>	<u>Program</u>	σ_a^{240} (2.38 to 0.683) barns	k_∞
1. Wings 240	1967	1967	1617	1.514*
2. No wings 240	1967	1967	1114	1.564
3. No wings 240	1967	1969+	1275	1.549
4. No wings 240	1969	1969	1311	1.545
5. H ₂ O Upscatter	1969	1969	1321	1.531
6. Intermediate res.	1969	1969	1170	1.547

* Agrees with experiment to within 1%.

+ 1969 program includes the effect of the allocation of resonances.

The comparisons are made across a change in nuclear data: the 1967 data used in this work, and the 1969 revised data. Lines 3 and 4 in the table show that this effect is a 0.4% decrease in k_∞ . This is not nearly large

enough to produce experimental agreement. Line 6 is the most all-inclusive treatment and it is still 3.3% above the line 1 wings prescription value. The nearest agreement is obtained in line 5 which excludes only the intermediate resonance treatment. To include the admixed aluminum as a "light" scatterer as was done causes an over-estimation of ^{240}Pu absorption which is as arbitrary as the wings prescription itself, so this "best agreement" is of little direct interest.

A possible method of generalizing the wings prescription would depend on the coding assumptions with which it was intended to be used. Clearly, a code which does not allocate the ^{240}Pu to cover the range span of the 1 eV resonance would require the largest adjustment. In the case of the 1967 version of HRG this amounts to an increase in the ^{240}Pu effective resonance integral from 1547 barns to 2208 barns, or a 43% increase. The effect of the allocation alone as judged from a comparison of line 1 with line 3 in σ_a^{240} would cause the prescription to be modified by the ratios of the appropriate σ_a^{240} values: $(1617-1275)/(1617-1114) = 0.68$. The use of the effective heterogeneous resonance integral should allow one to generalize to geometries other than 40 mil plates.

This bare outline of a more general application of the wings-240 treatment may be used where some experimental information is available to guide the analyst in formulating an engineering adjustment if such is needed.

The wings-240 prescription is coupled somewhat to the problem of an accurate depletion calculation. Increasing the absorption rate in ^{240}Pu increases the formation rate of the fissile isotope ^{241}Pu which tends

to produce a lower reactivity loss rate with increasing exposure. The adjustment which is necessary to predict $k_{\text{effective}}$ thus will tend to increase the computed core endurance. The experimental results in the depletion of the MTR-Phoenix core have a direct bearing on the use of the wings-240 prescription. These are discussed in the next section.

B. Depletion Calculations and Results

A depletion calculation for the MTR-Phoenix core performed as early as 1967 using BATTELLE-REVISED-THERMOS, HRG cross sections and a two dimensional diffusion theory spatial representation⁽²⁵⁾ gave a sustained slope of -0.046 mk/MWD. Applying the wings-240 prescription reduces this slope to -0.039 mk/MWD.

A more sophisticated depletion calculation was done in three dimensions to provide a definite prediction of the actual reactivity exposure relationship. Initially a complete zero dimensional burnup calculation was done on the plate lattice of the power zone using ZODIAC-G.⁽²⁶⁾ This calculation supplied the exposure-dependent behavior of the nine spectrum-averaged nuclear constants (D_1 , D_2 , Σa_1 , Σa_2 , Σf_1 , Σf_2 , $\nu \Sigma f_1$, $\nu \Sigma f_2$ and $\Sigma_{\text{out}1}$) necessary to specify a region and compute a power in the two group diffusion theory regime of WHIRLAWAY. ZODIAC-G is an automated chain of codes consisting of HRG for epithermal spectrum-averaged cross sections, BATTELLE-REVISED-THERMOS for thermal spectrum-averaged cross sections, HFN⁽¹⁷⁾ for computing epithermal and thermal fluxes and ALCHEMY⁽²⁷⁾ for the transmutation calculations. The wings-240 prescription was used in the HRG portion of ZODIAC-G. The calculated burnup slope was -0.042 mk/MWD.

A simplified three dimensional WHIRLAWAY quarter core model of the MTR-Phoenix core was developed to use with this zero dimensional parent calculation. Use was made of the rod model developed in the PRCF analysis to relate the crude model eigenvalues to a physically reasonable k_{eff} based on the four group, two dimensional rods-in and rods-out condition of the reactor. The model was 12x12x11 mesh points with 18 burnable regions including the rod follower fuel. Average criticality was maintained during the burnup simulation to within $\pm 1\% \Delta k_{eff}$ by moving control rod regions in the model. The linear slope of the reactivity exposure relationship was -0.036 mk/MWD. Thus the parent zero dimensional slope of -0.042 mk/MWD is reduced to -0.036 mk/MWD by the three dimensional effects as estimated by the WHIRLAWAY model.

The measured reactivity exposure slope of the MTR-Phoenix core is -0.066 mk/MWD. This slope is much larger than had ever been calculated and points up the inadequacy of the depletion calculations. It is important to note, however, that the Phoenix core ran to 923 MWD as compared to 850 MWD for the maximum exposure on any uranium core in the MTR. This was accomplished with a 4% lower initial excess reactivity, which, when combined with the greater worth of saturable fission products for the Phoenix core as compared to the uranium core, yields a uranium burnup slope of -0.136 mk/MWD - nearly twice that of the Phoenix core.

The problem of attempting to explain the discrepancy is still very much under investigation at this writing.

An initial approach was to choose a completely different combination of method and nuclear data to compute the zero-dimensional burnup slope.

The zero dimensional calculation must contain the main cause of the discrepancy between the analytical and experimental results because it yields -0.042 mk/MWD and it is likely that spatial effects will only algebraically decrease the value to judge from the moderating effect of the three dimensional calculation (parent slope of -0.042 mk/MWD reduced to -0.036 mk/MWD by the 3D model) on the parent slope.

Using ALTHAEA,⁽²⁸⁾ a zero dimensional depletion calculation was undertaken. The parameter settings used were those derived to calculate isotopics for the Yankee core, and were not specifically adjusted for the Phoenix calculation. The results of this calculation are easily summarized: The reactivity-exposure slope was -0.065 mk/MWD, but the plutonium isotopic changes as a function of exposure were substantially the same as in the ZODIAC-G calculation. The isotopics as a function of exposure are shown in figures 24 through 27. To understand how this computed difference in the reactivity-exposure can arise while the isotopics are nearly identical the following expressions for the slope were developed:

$$k_{\infty} = \frac{\text{Product rate}}{\text{Absorption rate}} = P/A, E = \text{exposure}$$

$$\frac{dk_{\infty}}{dE} = \frac{1}{A} \frac{dP}{dE} - \frac{P}{A^2} \frac{dA}{dE}$$

For a depletion at constant power:

$$\frac{dP}{dE} = 0, \text{ to first order.}$$

Therefore:

$$\frac{dk_{\infty}}{dE} \approx -\frac{k_{\infty}}{A} \frac{dA}{dE}$$

FIGURE 24. COMPUTED ISOTOPIC CONCENTRATIONS OF ^{239}Pu

FIGURE 25. COMPUTED ISOTOPIC CONCENTRATIONS OF ^{240}Pu

FIGURE 26. COMPUTED ISOTOPIC CONCENTRATIONS OF ^{241}Pu

FIGURE 27. COMPUTED ISOTOPIC CONCENTRATIONS OF ^{242}Pu

If one assumes linear behavior, the differentials may be replaced by discrete differences and these differences can be partitioned by isotope:

$$\frac{\Delta k_{\infty}}{\Delta E} \sim -\frac{k_I}{A_I} \frac{\Delta A}{\Delta E}, \quad \Delta A = \sum_{i=1}^8 (\phi_{II} \Sigma_{aII}^i - \phi_I \Sigma_{aI}^i),$$

where superscript *i* refers to the isotope, and subscripts *I* and *II* refer to initial and final exposure increments, respectively. The results of this analysis are contained in Table X.

TABLE X

MTR-Phoenix Calculated Reactivity Slope Analysis

$-\Delta k_{\infty} / \Delta E$ (mk/MWD)

<u>ISOTOPE</u>	<u>ALTHAEA</u>	<u>ZODIAC</u>	<u>Δ(ALTHAEA-ZODIAC)</u>
²³⁹ Pu	-0.0819	-0.0846	+0.0027
²⁴⁰ Pu	+0.0313	+0.0240	+0.0073
²⁴¹ Pu	+0.0698	+0.0663	+0.0035
²⁴² Pu	+0.0034	+0.0042	-0.0008
¹³⁵ Xe	+0.0013	+0.0061	-0.0048
¹⁴⁹ Sm	+0.0001	+0.0018	-0.0017
¹⁵¹ Sm	.0	--	
Fission Prod.	<u>+0.0402</u>	<u>+0.0232</u>	<u>+0.0170</u>
Sum	<u><u>+0.0642</u></u>	<u><u>+0.0410</u></u>	<u><u>+0.0232</u></u>

The sum of the isotopic contributions to the burnup slope is very close to the 0.0650 and 0.0410 values of the unpartitioned slopes, which confirms to a large extent the validity of the assumption of linearity. A

comparison of the differences between the various isotopic contributors to the burnup slope shows the largest difference to be between the two fission product models used. The three pseudo isotope fission product model of Walker⁽²⁹⁾ is used in ALTHAEA. The four pseudo fission product model of Nephew⁽³⁰⁾ was combined into a single pseudo product to overcome a limit on the number of fission product isotopes in ZODIAC.

The second most important difference is the Pu-240 contribution. The difference here is largely due to the use of the wings-240 prescription in ZODIAC and not in ALTHAEA. The difference of only 0.0073 mk/MWD is really somewhat smaller than expected.

The other differences are less than 0.0011 mk/MWD in the aggregate with the Xe-135 isotopes the next most important, albeit opposite in direction.

Although this comparison is merely between two calculations and may have no direct bearing on the actual physical interactions which did determine the burnup of the Phoenix core, it does illustrate that major differences can arise during a core burnup, even when the time dependent isotopics are very similar.

The MTR Phoenix experiment will not be complete until the isotopic analysis of the spent fuel is available. These results will most strongly indicate the direction of future efforts in the analysis of the core behavior. Pending these results, a more detailed examination of the plutonium fission product absorption would be very much in order because any increase in absorption which might come from such an examination will

help explain the observed reactivity exposure slope without affecting the agreement on initial reactivity.

C. Reactor Modeling with the Multigroup Method

The Phoenix fuel program has necessarily entailed a very wide use of the multigroup method. From multitable S_N transport approximations to the three dimensional diffusion theory, it was necessary to adapt standard computer codes to the geometrical complexities of a real reactor system. The MTR Phoenix core is a rather severe test of these techniques. The reactor is small; therefore leakage is important and must be calculated correctly. The core reflector interaction is complicated by the very energetic core spectrum impinging upon a reflector of very good moderating properties. The control rods are a large part of the total core volume. The neutron flux contains severe gradients and its shape strongly depends on the neutron energy. In short, most of the "classical limits" on diffusion theory are violated by the MTR-Phoenix core. It is to be expected that any generalization that one can make concerning the use of multigroup methods in the MTR-Phoenix neutronic system would be valid for less stringent systems.

The following procedure is therefore advanced as a general method for applying the multigroup method to any reactor system.

- 1) What is the object of the calculation? Typically the objective will be the neutron multiplication and/or the power distribution.
- 2) Determine the group sensitivity by calculating the quantity of interest over a range of broad groups. A single spatial dimension may be used, but the core and reflector chosen should represent the largest disparity in spectrum energetics between

the equilibrium core and reflector spectra that occurs anywhere in the reactor. A large number of spatial mesh points should be used.

- 3) The sensitivity of the quantity of interest to changes in the spatial mesh should be determined using the group structure selected in the previous step. This calculation must be carried out on the model of the reactor system under investigation. Mesh points should be increased until the change in the quantity of interest with increasing mesh points is acceptably small. If the eigenvalue is the quantity of interest, a very heavy investment in core mesh points at the expense of reflector points will give a convergent eigenvalue with fewer total mesh points. It is particularly wasteful to adopt completely regular mesh grids with equal spacing in core and reflector.

If the power distribution is of particular interest at certain points, the number of mesh points in that area may be increased at the expense of less interesting areas as long as the eigenvalue remains relatively invariant.

The experience with three dimensional diffusion theory was based entirely on the WHIRLAWAY code which is limited to two energy groups and, in this particular application, a crude spatial mesh. The spatial mesh is a necessary consequence of the 65K core storage limitation. At the present stage of computing hardware development this is a good average core size for a large computer. The MTR is a small reactor. The number

of mesh intervals used was $18 \times 28 \times 39$. For a two group calculation this requires 39312 core locations. The approximately 25000 core locations remaining contain the program and the computer software routines. The convergence time from a flat flux start is in the range of three to four hours. With good starting flux guesses convergence takes up to one hour. These observations, when coupled with the observed gross inaccuracy of the eigenvalues obtained indicate that three dimensional diffusion theory in two energy groups is not a very acceptable analysis tool, and that it is very expensive, given the current status of computer hardware development. Its sole justification in the Phoenix core analysis is that it did predict the core gross power distribution to within about 15%; however, the power peaking was underestimated in one location. Therefore for gross power shape estimation the three dimensional, two group method can be used in lieu of experimental information.

Another point which often occurs in multigroup applications is the question of the neutron spectra which are chosen to obtain region cross section averages for use in the multigroup calculation. The procedure followed in this analysis has been to use a single pure core spectrum for all regions within the core for both thermal and epithermal energies. Infinite media spectra were calculated in each case with no attempt made to introduce leakage effects. In no case were the borders of the core and reflector subjected to a mixed core and reflector. These spectrum approximations, when placed in the context of the other multigroup approximations used, yield a close agreement with the experimental multiplication values.

The control rods constitute an exception to this rule. The epithermal spectra were computed for a region containing the volume equivalent of two

fuel elements and one control rod. This ratio of rod-to-fuel volume is true in the rodded portion of the core extending to within one half an element width of the ends of the core. Epithermal cross sections for water, aluminum, and cadmium used in the rod regions were obtained from this spectrum.

Thermal rod cross sections were obtained from a slab section starting at the center of the fuel element and proceeding through the aluminum wall, through the cadmium absorber and through the center water to the center of the rod.

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