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C00 264-553

η - 3π DECAY WITH SUSSKIND'S MODEL

Yoshio Miyatake

The Enrico Fermi Institute
The University of Chicago, Chicago, Illinois 60637

JULY, 1970

Contract No. AT (11-1)-264

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ABSTRACT

We calculate $\eta \rightarrow \pi^+ + \pi^- + \pi^0$ decay amplitude with Susskind's formula. The result is not good if we take all multiperipheral tree-diagrams. although some of them show very good agreement with experiment. Charge symmetry is satisfied. We give the interpretation to Susskind's formula by analogy with the Two-Fluid Model in superconductivity.

⁺Supported in part by the U. S. Atomic Energy Commission.

^{*}Address after summer; Department of Liberal Arts, Kyoto University, Kyoto, Japan.

Recently Susskind¹⁾ has proposed an interesting model. He pictures that realistic weak and electromagnetic currents are associated with the "localized" states in the string model^{2),3)} of the Veneziano amplitude⁴⁾. It seems to suggest some possibilities of treating the weak and the electromagnetic interactions with the same formula as Veneziano's in the strong interactions, if we use "localized" state.

Following the above circumstances, we calculate, for instance, the $\eta \rightarrow \pi^+ + \pi^- + \pi^0$ decay amplitude with Susskind's formula.

First, we briefly describe Susskind's formulation. The process through which a scalar field quantum is absorbed at a space-time point x by a hadronic current $\rho(x)$ is described by the matrix elements.

$$\langle V | \rho(x) | k_1, \dots, k_n \rangle, \quad (1)$$

where $\langle V |$ is a vacuum state and $| k_1, \dots, k_n \rangle$ is a ground state of n hadrons. If these matrix elements are dominated by the resonant hadronic states in the channel $\sum_i k_i$, Eq. (1) can be written as

$$\sum_i \langle V | \rho(x) | i \rangle \frac{1}{g^2 + M_i^2} \langle i | T | k_1, \dots, k_n \rangle, \quad (2)$$

where $g = \sum_i k_i$. The vectors $| i \rangle$ are the 4-dimensional oscillator system in Ref. 3) and the vector $\langle V | \rho(x)$ is assumed to describe the hadronic string, all elements of which are localized at a point x . This means that when a local current acts it creates a hadron including its quarks and connecting string all at a point. This is of course only an assumption

and may be wrong but we think the suggestion is worth an investigation. Note that the localization of the string is always at its center of mass point and the Fourier transformation of its state vector $\langle \psi(q) |$ is described.

$$\langle \psi(q) | = \langle 0 | \prod_{l=1}^{\infty} \prod_{\mu=1}^4 e^{a_{\mu}^{-}(l)^2/2} e^{-i q X_{c.m.}}, \quad (3)$$

where $a_{\mu}^{-}(l)$ are annihilation operators of harmonic oscillator of component μ and mode l , and $X_{c.m.}$ is a center of mass value of coordinate point x on the string. The strong amplitudes $\langle i | T | k_1, \dots, k_n \rangle$ are the Veneziano amplitudes and we can write Eq. (2)

$$\langle \psi(q) | \frac{1}{q^2 + M^2} T(k_1) \frac{1}{s_1 - M^2} T(k_2) \dots e^{i k_n X_{c.m.}} | 0 \rangle, \quad (4)$$

(Fig. 1) where $T(k_i)$ is a vertex function at a point where the particle with the 4-momentum k_i enters and $s_1 = -(q + k_1 + \dots + k_i)^2$.

Now, when we apply Eq. (4) to $\eta \rightarrow \pi^+ + \pi^- + \pi^0$, we have only to put $q = 0$ and regard k_1, k_2, k_3 and k_4 as the 4-momentum of η, π^+, π^- and π^0 respectively but we must assume the quantum numbers of the state $\langle \psi(q=0) |$ as $I^G J^P = 1^- 0^+$ to have the resonant trajectories on the propagator $\frac{1}{q^2 + M^2}$ ($q=0$) and to make the process possible.

Now, from the form of Susskind's formula (4) we have only to consider the tree-diagrams such as Figs. 2-5 and those obtained by all permutations of π^+, π^- and π^0 mesons in each diagram as a "Born" approximation. Figs. 4, 5 and those obtained from them by permutations of π mesons have the same contributions as those from Figs. 2 and 3 because of duality if we calculate them

taking all possible trajectories. But from the field theoretical view point, it is still unresolved whether we should add them or not, so here, we add them temporarily.*

We have many trajectories on the internal lines as in Figs. 2-5 and therefore, we must calculate the amplitudes for each trajectory on the internal lines and add them together. We, however, do not expect the model to give detailed predictions but only the general features of the decay process. In this spirit, we will take into account only the lightest particles on the trajectories; for examples, the δ meson on the first internal line in all figures and the δ meson on the third internal line in Figs. 4 and 5. (The A_1 meson on the second internal line in Fig. 2 has the π meson as the ground state of the same trajectory and there is no problem in this case.) In fact we notice from the calculations that these lowest resonances dominate the contributions to the amplitudes.

We calculate the amplitude for Fig. 2, following the methods of Ref. 3) in Appendix, and obtain the result

$$A(\eta, +, -, 0)$$

$$= -31 - 3.23 T_2 - 2.2 T_3 + 1.2 T_4 - 3532 T_2^2 - 4.2 T_3^2$$

$$+ 3.9 T_4^2 - 9.7 T_2 T_3 + 0.9 T_2 T_4 + 0.3 T_3 T_4, \quad (5)$$

*This point is due to Professor Nambu.

where T_2 , T_3 and T_4 are the kinetic energies of the π^+ , π^- and π^0 mesons respectively. We can calculate $A(+, \eta, -, 0)$ (Fig. 3) in the same way as $A(\eta, +, -, 0)$ and obtain for the sum of them

$$\begin{aligned} & A(\eta, +, -, 0) + A(+, \eta, -, 0) \\ &= -23 - 229 T_2 - 1.9 T_3 + 1.1 T_4 - 2495 T_2^2 - 3.6 T_3^2 \\ &+ 3.5 T_4^2 - 8.2 T_2 T_3 + 0.8 T_2 T_4 + 0.3 T_3 T_4. \end{aligned} \quad (6)$$

For the diagrams obtained by the permutations of π mesons we have only to permute the suffices in Eq. (5) or Eq. (6); for instance,

$$A(\eta, +, 0, -) + A(+, \eta, 0, -) = \{3 \leftrightarrow 4 \text{ in (6)}\}.$$

Thus we easily have $A(\eta, -, +, 0) + A(-, \eta, +, 0)$, $A(\eta, -, 0, +) + A(-, \eta, 0, +)$, $A(\eta, 0, +, -) + A(0, \eta, +, -)$ and $A(\eta, 0, -, +) + A(0, \eta, -, +)$.

In order to obtain total amplitude we must add these amplitudes multiplying them by the following isotopic spin projection operators respectively,

$$\begin{aligned} & \text{Tr}(\tau_\psi \tau_+ \tau_- \tau_0) = -\text{Tr}(\tau_\psi \tau_+ \tau_0 \tau_-) \\ &= \text{Tr}(\tau_\psi \tau_- \tau_+ \tau_0) = -\text{Tr}(\tau_\psi \tau_- \tau_0 \tau_+) \\ &= \text{Tr}(\tau_\psi \tau_0 \tau_+ \tau_-) = \text{Tr}(\tau_\psi \tau_0 \tau_- \tau_+) \\ &= 1, \end{aligned} \quad (7)$$

where the τ spin matrix τ_ψ corresponds to the charge state of $\langle \psi(0) |$ and equal to τ_0 . Then we obtain the decay amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ from Figs. 2, 3 and those obtained from them by the permutations of π mesons proportional to (apart from a suitable normalization constant)

$$F' = -46 - 4(T_2 + T_3) - 452T_4 - 7(T_2^2 + T_3^2) - 4976T_4^2 - 17T_2T_3 + 2(T_2 + T_3)T_4 \quad (8)$$

$$\doteq -45 - 446T_4 - 4985T_4^2 - 3T_2T_3$$

$$\sim 1 + 10T_4 + 111T_4^2 \quad (9)$$

Eq. (8) satisfies the charge symmetry between π^+ and π^- but even if we take into account the fact that the maximum value of each pion kinetic energy is about 0.07 Bev, Eq. (8) or (9) does not express a good linear approximation. We thus propose extrapolation, in which higher order terms contribute considerably. Comparing with the experimental formula

$$\sim 1 - 7.4 T_4^{5)} \quad (10)$$

or

$$\sim 1 - 0.37 T_4 - 25.3 T_4^2 \quad 6) \quad (11)$$

we find Eq. (9) is not satisfactory. But if we add only some parts of all amplitudes, we find the results are in good agreement with experiment. For examples,

$$\begin{aligned} & \text{Tr}(\tau_\psi \tau_+ \tau_- \tau_0) \{ A(\eta, +, -, 0) + A(+, \eta, -, 0) \} \\ & + \text{Tr}(\tau_\psi \tau_- \tau_+ \tau_0) \{ A(\eta, -, +, 0) + A(-, \eta, +, 0) \} \end{aligned}$$

$$\doteq -126 + 983 T_4 - 2500 T_4^2 + 4982 T_2 T_3$$

$$\propto 1 - 7.8 T_4 + 19.8 T_4^2 - 39.5 T_2 T_3$$

$$\sim 1 - 7.8 T_4 - 39.5 T_2 T_3$$

and

$$\begin{aligned} & \text{Tr}(\tau_\psi \tau_+ \tau_- \tau_0) \{ A(\eta, +, -, 0) + A(+, \eta, -, 0) \} + \text{Tr}(\tau_\psi \tau_+ \tau_0 \tau_-) \\ & \times \{ A(\eta, +, 0, -) + A(+, \eta, 0, -) \} + \text{Tr}(\tau_\psi \tau_- \tau_+ \tau_0) \{ A(\eta, -, +, 0) \\ & + A(-, \eta, +, 0) \} + \text{Tr}(\tau_\psi \tau_- \tau_0 \tau_+) \{ A(\eta, -, 0, +) + A(-, \eta, 0, +) \} \end{aligned}$$

$$\doteq -0.5 + 12.0 T_4 - 2.0 T_4^2 - 4.0 T_2 T_3$$

$$\propto 1 - 22 T_4 + 7.4 T_2 T_3$$

These results are in rather good agreement with Eq. (10) except the cases in which T_4 is very small because of the smallness of each T_i (max. $T_i \simeq 0.07$ BeV). From these results the disagreement of Eq. (9) comes from the sum of the contributions of $\{A(\eta, 0, +, -) + A(0, \eta, +, -)\}$ and $\{A(\eta, 0, -, +) + A(0, \eta, -, +)\}$, corresponding to the diagrams in which π^0 meson is created first and their bad contributions are due to the appearance of ρ meson which has a relatively small mass on the third internal line. Next, we calculate the amplitudes from Figs. 4, 5 and those obtained from them by permutations of π mesons with the lowest trajectory on each internal line. With the same method as Figs. 2 and 3 we have the decay amplitude proportional to

$$\begin{aligned}
 F'' &= 5.4 + 9.2(T_2 + T_3) - 3.8T_4 + 19.4(T_2^2 + T_3^2) \\
 &\quad - 13.2T_4^2 + 38.0T_2T_3 - 5.2(T_2 + T_3)T_4 \\
 &= 7.0 - 19T_4 + 11T_4^2 - 0.8T_2T_3
 \end{aligned} \tag{12}$$

$$\propto 1 - 2.7T_4 \tag{13}$$

This is to be said to agree with Eq. (10) rather well. But when we add (8) and (12) we have

$$\begin{aligned}
 F &= F' + F'' \\
 &= -38 - 465T_4 - 4974T_4^2 - 4T_2T_3 \\
 &\propto 1 + 12T_4 + 131T_4^2
 \end{aligned} \tag{14}$$

This is still bad, so we have found that the good contributions from Figs. 4, 5 and more parts of Figs. 2, 3 and those obtained from them by permutation of π mesons are dominated by the bad contributions from the rest fewer parts of Figs. 2 and 3. But there are some not clear and inaccurate points in this result: We had Eq. (6) by adding temporarily the contributions from Figs. 2 and 3 with equal weight (it is the same circumstance as Eq. (14)) but we should add them with suitable weight factors coming from the strengths of coupling constants in both diagrams. They are however, not clear. Furthermore, if we want to have a better approximation, we must take more internal trajectories and, when we have much more possible trajectories in Figs. 4 and 5, (the trajectories shown in Figs. 4 and 5 are not enough) there remains a possibility that the good contributions from Figs. 4 and 5 dominate. On the other hand, if we have other trajectories heavier than ρ meson on the third internal lines in Figs. 2 and 3, their contributions have a tendency to cancel the bad contribution from the ρ meson trajectory.

We obtain Eq. (9) or Eq. (14) by Taylor expansion taking the first few terms only but find that there are no considerable contributions from higher order terms than the second term to T_4 -term, using the relation $T_2 + T_3 + T_4 = m_\eta - 3m_\pi \simeq 0.13$ Bev.

We must bear in mind, however, that Susskind's model is very speculative and has some ambiguities and therefore, it

is no wonder that we have obtained bad results (14) with rough approximation. In spite of these circumstances, we believe that his model has an important suggestion for the treatment of strong and weak interactions: Several years ago⁷⁾ we proposed the idea of the "two-fluid" model to treat the weak and strong interactions by analogy with the same model in superconductivity. We consider the states to be responsible for the strong (weak) interactions when they are in the superconducting (normal) states; electrons can go through the matter in the superconducting state without any resistance and their reaction time is short, so it corresponds to the strong interactions, while in the normal state it conversely corresponds to the weak interactions. The superconducting states are due to the Cooper pairs of electrons and the states with such pairs may be considered as nonlocalized states; the normal states, on the other hand, may be regarded as localized states. Therefore, we may consider the strong interactions to be the phenomena among the particles in the "nonlocalized" states, and while the weak (including the electromagnetic) interactions may be considered as the phenomena among the particles which are related to the "localized" states. The aim of this proposition was to relate both kinds of interactions. We hoped the differences of the strengths of couplings could be explained by those of internal structures of the particles. The fact that the original Veneziano amplitudes, responsible for the strong interactions, can be described very well by

the string model ----- a nonlocal model ----- and now, following Susskind's model, weak and electromagnetic interactions seems to be describable by the same Veneziano amplitudes if "localized" states are used, may give us some powerful hints for the proper formulation of the above problem. Of course his model is very speculative and we are unable to draw a clear cut analogy between the two proposals but as one step to our "two-fluid" model of interactions we have calculated the decay amplitude with his model. Apart from the ambiguity in his model, for instance, the definition of localized state $\langle \psi(q) |$, it is desirable not to treat the "two-fluid" states of the particles separately but to treat them more organically in the future theory. For instance, it is very interesting if we can formulate the mechanism of interactions in the following way; the strong interactions are due to the string states nonlocalized by many quark pairs on the string corresponding to the Cooper pairs of electrons in superconductivity, while the weak interactions are due to the string states localized at a point without any quark pairs.

ACKNOWLEDGEMENTS

I am grateful to Professor Y. Nambu, Drs. L. N. Chang, P. H. Frampton and M. Yoshimura for their patient guidance and helpful suggestions and discussions.

I am also greatly indebted to Professors R. G. Sachs and Y. Nambu for hospitality at the Enrico Fermi Institute.

APPENDIX

$$A(?, +, -, 0)$$

$$= \langle 0 | \prod_{l=1}^{\infty} \prod_{\mu=1}^4 e^{a_{\mu}(l)^2/2} \frac{1}{M^2} T(k_1) \frac{1}{k_1^2 + M^2} T(k_2) \frac{1}{(k_1 + k_2)^2 + M^2} T(k_3) e^{ik_4 X_{c.m.}} | 0 \rangle$$

$$= \int_0^1 dy dx_1 dx_2 y^{-1+m_\delta^2} x_1^{-1+k_1^2+m_\pi^2} x_2^{-1+(k_1+k_2)^2+m_\rho^2}$$

$$\times \prod_{l=1}^{\infty} \prod_{\mu=1}^4 e^{a_{\mu}(l)^2/2} e^{\overleftarrow{\partial}_{\alpha_{\mu}(l)} \overrightarrow{\partial}_{\beta_{\mu}(l)} y^l} e^{i\beta_{\mu}(l) - \gamma_{\mu}(l)} \left\{ \sqrt{\frac{2}{l}} k_{1\mu} + \beta_{\mu}(l) \gamma_{\mu}(l) \right\}$$

$$\times e^{\overleftarrow{\partial}_{\gamma_{\mu}(l)} \overrightarrow{\partial}_{\delta_{\mu}(l)} x_1^l} e^{i\delta_{\mu}(l) - \varepsilon_{\mu}(l)} \left\{ \sqrt{\frac{2}{l}} k_{2\mu} + \delta_{\mu}(l) \varepsilon_{\mu}(l) \right\}$$

$$\times e^{\overleftarrow{\partial}_{\varepsilon_{\mu}(l)} \overrightarrow{\partial}_{\sigma_{\mu}(l)} x_2^l} e^{i\sigma_{\mu}(l) \sqrt{\frac{2}{l}} k_{3\mu}} \Bigg|_{\alpha=\beta=\dots=\sigma=0}$$

$$= \int_0^1 dy dx_1 dx_2 y^{-1+m_\delta^2} x_1^{-1+k_1^2+m_\pi^2} x_2^{-1+(k_1+k_2)^2+m_\rho^2}$$

$$\times \left[(1-y^2 x_1^2 x_2^2)^{-k_3^2} (1-y^2 x_1^2 x_2)^{-2k_2 k_3} (1-y^2 x_1^2)^{-k_2^2} (1-y^2 x_1 x_2)^{-2k_1 k_3} \right.$$

$$\left. \times (1-y^2 x_1)^{-2k_1 k_2} (1-y^2)^{-k_1^2} (1-x_1 x_2)^{2k_1 k_3} (1-x_1)^{2k_1 k_2} (1-x_2)^{2k_2 k_3} \right]$$

Now, because the power of each factor in the square bracket in the integrand is much smaller than 1 (Bev^2) near the physical regions and the region of integration is $0 \sim 1$, we neglect the integration variables in the integrand the powers of which are larger than 2 in the Taylor expansion of each factor. Finally, inserting the numerical value of each meson mass after integrating each term and neglecting the terms including the π meson kinetic energies the powers of which are larger than 2, we obtain Eq. (5).

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FIG. 1

FIG. 2. $A(\eta, +, -, 0)$

FIG. 3 $A(+, \eta, -, 0)$

FIG. 4 $A(+, -, \eta, 0)$

FIG. 5 $A(+, -, 0, \eta)$