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LTPF: A Linear Theta-Pinch Neutron Source

by

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LTPF: A LINEAR THETA-PINCH NEUTRON SOURCE

Abstract	1
I. INTRODUCTION	1
II. BRIEF DESCRIPTION OF THE LTPF	2
A. Geometry	2
B. Plasma Physics Assumptions	3
C. Neutron Current Requirements	4
III. SCALING RELATIONS FOR THE LTPF	4
A. Parameterizing the Neutron Current	4
1. Plasma Pressure Balance	5
2. Shock-Heating Requirements	5
3. End-loss Time	6
4. D-T Fusion Cross-Section	6
5. Power Supply Considerations	7
B. Deriving the Scaling Relations	8
IV. MAXIMIZING THE NEUTRON CURRENT	12
A. Optimum Ion Temperature	12
B. Optimum Coil and Tube Dimensions	12
C. Optimum E_0	13
D. Optimum Quality Factor and Joule Power Loss	13
E. Optimum End-Loss Parameter	14
V. TWO LTPF POINT DESIGNS	15
A. Point Design A (Regular Sinusoidal Waveform)	15
B. Point Design B (Intermittent Trapezoidal Waveform)	18
VI. POWER SUPPLIES	20
VII. SUMMARY AND CONCLUSIONS	21
Acknowledgments	22
Appendix I - Neutronics Terminology	23
Appendix II - Plasma Heating in the LTPF	25
A. Derivation of Staged-Heating Equations	25
B. Radial Dependence of E_0	28

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Appendix III - Electrical Circuit Parameters	30
A. Parameters for Point Designs A and B	30
B. Capacitor Bank Characteristics (Point Design A)	32
C. Rotating Machinery Characteristics (Point Design B)	33
References	35
Table I - LTPF Parameters Common to Both Point Designs (A and B)	38
Table II - LTPF Parameters for Point Design A (Continuous Sinusoidal Waveform)	39
Table III - LTPF Parameters for Point Design B (Intermittent Trapezoidal Waveform)	40
Figures	41

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ABSTRACT

The linear theta pinch is optimized with respect to maximum neutron current on a sample located between the discharge tube and the compression coil wall. Emphasis throughout is placed on physics and technology considerations which govern the choice of parameters. Technological demands are (hopefully) kept to a minimum. Two "point designs" are developed which are distinguished by their compressional magnetic field (i.e. coil current) waveforms: one is sinusoidal and continuous, the other trapezoidal and pulsed intermittently. Both point designs give an average neutron current of $\sim 5 \times 10^{13}$ n/cm²/s. Both devices are characterized by short lengths (~ 1 m), rapid cycling (2 - 30 kHz), and magnetic mirrors (2 - 5:1) at the ends. A crucial item is the power supply, which is discussed in detail.

1. INTRODUCTION

In this paper the linear theta pinch is treated as a source of 14-MeV neutrons, sometimes called a "FERF". The name FERG (for Fusion Engineering Research Facility) applies to CTR devices whose function is to duplicate, as realistically as possible, the radiation and stress environment of a nuclear fusion reactor core for purposes of component and material testing. A FERG facility is non-existent at present, but its need is generally recognized by the CTR community. In addition to providing a high-energy neutron source of appropriate spectrum and intensity, a FERG would also be expected to simulate the bremsstrahlung radiation, particle first-wall impingement, cyclic stress, high operating temperature, and other factors associated with an operational D-T power plant.

A reactor test facility, or FERG, of the above scope represents an ambitious goal, but it is also an important milestone on the road to fusion power. Without such a facility, the planned operation of an Experimental Power Reactor (EPR) by the year 1992 [1] and the subsequent operation of a Demonstration Power Plant (DPP) by the year 2000 [1] would proceed without reasonable assurances regarding reactor material lifetimes, component interactions, and maintenance downtimes.

We will show that the linear theta-pinch FERG (LTPF) has three important advantages over some other proposed neutron sources: (1) the device can be

designed to produce its maximum output of neutron current [for definitions of neutron transport terminology (flux, fluence, current, etc.), see Appendix I] at machine lengths of the order of 1 meter; hence the unit size will be small, and costs should be low; (2) the linear-pinch is a known and tested system at the sizes to be proposed for the LTPF, hence technology extrapolations can be kept to a minimum; (3) the theta pinch will prepare and confine an actual thermonuclear plasma, permitting various plasma physics effects (e.g. bremsstrahlung, particle-wall interactions) and engineering systems (e.g. hydrogen-isotope fueling, heat removal) which may be important in fusion reactors to be tested.

The LTPF, by definition, is designed to produce 14-MeV neutrons. It does not require "energy-breakeven," as a reactor does, and in fact will be a prolific consumer of electrical power. The tritium-breeding lithium blanket can be eliminated, if desired for simplicity, provided a bottle of tritium is available for fuel. (The tritium consumption is in the one-gram-per-month range).

The basic philosophy followed throughout this study is that rapid development of the LTPF technology -- rapidly-cycled power supplies, sophisticated gas-handling systems, heat removal in enormous quantities, remote operation of a radioactive machine, etc. -- can best be accomplished if the LTPF has a minimum number of technical innovations over conventional theta pinches. Consequently this LTPF design resembles present-day theta-pinch experiments in many ways.

Finally, a word about optimization: the set of variables selected here is not unique, but is important to the specific results obtained. The variables describe the theta pinch coil, the thermonuclear plasma, and the power supply; all three constituents are important in the overall design. They must be represented by a few key variables for the problem to be tractable.

In section II the theta-pinch geometry and the basic LTPF assumptions are introduced. In section III the LTPF scaling laws are derived, and in section IV the LTPF is optimized for maximum neutron current. Two specific point designs are developed in section V. Power supplies are considered in section VI, and finally some conclusions are drawn in section VII. Three Appendices deal with neutronics definitions, the theory of theta-pinch heating as applied to the LTPF, and more about power supplies and circuits.

II. BRIEF DESCRIPTION OF THE LTPF

A. Geometry

The basic geometry of the linear theta pinch is shown in Fig. 1. The compression coil, which may be either single-turn, as shown here, or multiple-turn, carries a current I in the azimuthal (theta) direction, generating a straight uniform magnetic field B which radially confines a long uniform cylinder of high- β D-T plasma. The maximum plasma density n which can be confined at thermonuclear temperatures is in the $10^{16} - 10^{17} \text{ cm}^{-3}$ range, corresponding to a maximum magnetic field B_0 of a few hundred kG. The compression coil may require laminations, to reduce unwanted eddy currents (the skin effect), and a large cooling system will be required, to remove the ohmic (joule) heat.

Figure 2 shows a photograph of an actual linear theta pinch device, the Scylla IV-3 experiment operated at LASL during 1965-69. In this device the

coil bore ($2b_2$) was 10 cm, the length L was 3 meters, the maximum field $B_0 \sim 50$ kG, the ion density $n \sim 2 \times 10^{16} \text{ cm}^{-3}$, the ion temperature $T_i \sim 1.4$ keV, and the electron temperature $T_e \sim 0.4$ keV [2]. When Scylla IV was operated as a 1-meter device, the corresponding parameters were $B_0 \sim 80$ kG, $n \sim 3 \times 10^{16} \text{ cm}^{-3}$, $T_i \sim 3.2$ keV, and $T_e \sim 0.3$ keV (deuterium gas used in all cases) [2]. These parameters are typical of linear theta-pinch operation without separate shock heating.

The ion temperature T_i can be increased into the thermonuclear range (≥ 5 keV) by several methods, including separate shock heating. Shock heating should be much easier to implement than some of the more exotic possibilities, such as electron beam heating, Marshall-gun injection, etc., especially if the shock-heating electric field is kept small. Linear theta pinches are easier to heat than toroidal pinches (either theta or z), which require "fat" plasmas (large a/b) for wall stabilization purposes. Consequently compressional heating can be employed more successfully in linear devices. We will assume separate shock heating in the LTPF, but the shock-heating requirements will be kept deliberately mild (applied E_z 's < 1 kV/cm).

The shock heating coil is shown in Fig. 3 positioned between the ceramic (or fused quartz) discharge tube and the compression coil. The shock-heating coil is electrically insulated from the compression coil as shown.

Sample materials to be irradiated are located between the shock-heating coil and the discharge tube. Approximately 1 cm of space is provided in the design for this purpose. Ports are provided in the compression coil to admit 14-MeV neutrons into the space beyond the coil (see Fig. 1). Neutrons of degraded energy are also available outside the compression coil, having passed through the wall (typically 5-10 cm in thickness).

The LTPF is assumed to have no lithium blanket, although a blanket structure could of course be provided for material-testing purposes, breeding ratio measurements, etc., if desired. The blanket would be located outside the compression coil.

B. Plasma Physics Assumptions.

We will assume a sharp-boundary MHD model for the plasma column with different electron and ion temperatures, T_e and T_i , and make the reasonably good approximation for theta pinches, $\beta = 1$. The electron and ion temperatures and ion density are assumed to be constant after the start of the "burn" phase, and equal to the values attained at the end of the compressional heating phase. This implicitly assumes a low fractional burnup and negligible alpha-particle heating. In general, as the alpha particles are produced during a D-T burn (one alpha particle is produced per 14-MeV neutron), they are thermalized in the plasma by the colder, more dense fuel ions. Concomitantly, the fuel ions are heated by the alpha particles. The net result is an expansion of the high- β plasma against the confining magnetic field, with an associated increase in the plasma ion temperature and a decrease in the ion density. Computer calculations by Oliphant [3], employing numerical solution of the Fokker-Planck equation using time-dependent distribution functions, have shown alpha-particle heating and fuel-ion depletion to be negligible in all parameter ranges of interest for the LTPF (e.g. a burn of less than 500 μs duration in any practical magnetic field). Therefore the plasma temperature, density, and radius will be assumed constant during the LTPF burn, which simplifies the calculations.

C. Neutron Current Requirements

Most D-T fusion reactor designs [4,5] limit the acceptable first-wall loading due to uncollided (14-MeV) neutrons, $\bar{I}_w$, to 1 - 3 MW/m² [6 - 9]. Since each uncollided neutron transports 14.06 MeV (2.25×10^{-12} J) of kinetic energy to the wall, the wall loading is related to the time-averaged uncollided neutron current density $\langle J \rangle$ by a constant:

$$\langle J \rangle = 4.44 \times 10^{13} \bar{I}_w \text{ n/cm}^2/\text{s} \quad (1)$$

for $\bar{I}_w$ in MW/m². Thus a wall-loading of 2 MW/m² corresponds to a neutron current of 8.88×10^{13} n/cm²/s, and this is roughly the value one would expect to encounter in a D-T fusion reactor. Ideally a FERF would produce even higher neutron currents, e.g. 10^{15} n/cm²/s, in order to shorten the time required for substantial material damage effects over that for reactor component lifetimes (estimated at ~ 1 year).

It will be shown that neutron currents in excess of 5×10^{13} n/cm²/s are difficult to generate in an LTPF. Neutron currents of 10^{13} n/cm²/s appear to be fairly readily achievable.

In order to increase the utility of the LTPF, and to reduce projected material-testing times to a practical minimum, we will take as our primary design criterion the maximization of the neutron current $\langle J \rangle$.

III. SCALING RELATIONS FOR THE LTPF

A. Parameterizing the Neutron Current

For a D-T plasma with Maxwellian ions, the fusion reaction rate is given by [10]

$$R = n_D n_T \overline{\sigma v}(T) \text{ reactions/cm}^3/\text{s} , \quad (2)$$

where $\overline{\sigma v}(T)$ is the Maxwell-averaged fusion cross section, T is the ion temperature, and n_D and n_T are the ion densities for deuterium and tritium, respectively. For a 50/50 D-T mixture, $n_D = n_T = n/2$, the number of neutrons produced per cm of plasma column in a burning pulse of duration τ_b sec is (a = plasma radius)

$$\# \text{ neutrons} = \frac{\pi}{4} a^2 n^2 \overline{\sigma v}(T) \tau_b \text{ cm}^{-3} . \quad (3)$$

For pulsed operation with cycle time τ_c (τ_c = number of seconds between successive burning pulses), the time-averaged uncollided neutron current density, $\langle J \rangle$, at the sample radius b_s (see Fig. 3) is obtained by dividing by $2\pi b_s \tau_c$:

$$\langle J \rangle = \frac{a^2 n^2 \overline{\sigma v}(T)}{8 b_s \tau_c} \frac{\tau_b}{\tau_c} \text{ n/cm}^2/\text{s} . \quad (4)$$

Our units are a and b_s in cm, n in cm⁻³, $\overline{\sigma v}(T)$ in cm³/s, and τ_b and τ_c in seconds. In the interest of brevity we will refer to $\langle J \rangle$ henceforth as

the "neutron current."

Equation (4) is not in a useful form for maximizing $\langle J \rangle$, since a , b_s , n , T , τ_b , and τ_c cannot all be specified independently. In addition to such obvious constraints as $b_s > a$, $\tau_c > \tau_b$, etc., more subtle constraints arise involving conflicting physics requirements, shock-heating considerations, strength-of-materials limitations, availability of electrical power, etc. Some of these constraints are not apparent by inspection of Eq. (4).

The proper choice of independent variables in the LTPF problem is a crucial step, and a subjective one. In this and following sections we will systematically develop a parameterization and a set of design equations for the LTPF which have been found by trial-and-error to be the most useful. Other parameterizations are certainly possible, but the writer prefers the independent variables used in this study.

We will need the following relations:

1. Plasma Pressure Balance. In SI units the pressure balance relation is

$$nk (T_e + T_i) = \frac{B_0^2}{2\mu_0} \quad (5)$$

For a ratio of electron temperature to ion temperature equal to λ ($T_e/T_i = \lambda \leq 1$ for theta pinches), and assuming $\beta = 1$, the density can be written

$$n = 2.48 \times 10^{23} \frac{B_0^2}{(1 + \lambda) T_i} \text{ cm}^{-3}, \quad (6)$$

where n is in cm^{-3} for B_0 in kG and T_i in keV. For notational convenience we will drop the subscript "i" on the ion temperature, and the symbol T will refer to the ion temperature in the remainder of this report.

2. Shock-Heating Requirements. Shock heating followed by adiabatic compression is the traditional method of heating plasmas in theta pinches [11-13]. The theory of "separate" (the term "separate" as used here applies to a separate power supply) shock heating has been developed in detail elsewhere [14,15]. The theory has been modified to include unequal electron and ion temperatures in Appendix II. The result can be expressed in the following equation:

$$E_\theta = 0.244 \left(\frac{1 + \lambda}{2} \right)^{1/2} \left(\frac{a}{b_1} \right)^{7/3} B_0 T^{1/2} \quad (8)$$

(E_θ in kV/cm for a , b_1 in cm, B_0 in kG, and T in keV), where E_θ is the electric field (applied at radius b_1), B_0 is the final magnetic compression field, T is the final ion temperature (after compression), a/b_1 is the equilibrium compression ratio, and $\lambda = T_e/T_i$. Solving for the plasma radius a yields

$$a = 1.83 \left(\frac{2}{1 + \beta} \right)^{3/14} \left(\frac{E_\theta}{B_0} \right)^{3/7} \frac{b_1}{T^{3/14}} \quad \text{cm} \quad (9)$$

(kV/cm, kG, cm, keV) in terms of the tube radius, temperature, applied fields.

3. End-Loss Time. Particle end-loss is an important effect in linear theta pinches, and the shorter the device the shorter the useful confinement time for a given temperature. The minimum plasma length is found by equating the required confinement time, τ_b , to the characteristic end loss time, τ_{EL} . Using the definitions $\dot{N} = -N/\tau_{EL}$ and $\dot{N} = -2\Gamma$, where N is the total number of particles originally in the plasma and Γ is the rate at which particles are lost from either end, the end loss time τ_{EL} can be written in the form [16]

$$\tau_{EL} = \frac{L}{2} \left(\frac{m_i}{2kT} \right)^{1/2} (2\sqrt{\pi} \eta_m) \quad (10)$$

where L is the total length of the pinch, m_i is the mass of an "average" D-T ion (4.2×10^{-27} kg), and η_m is a "mirror parameter" which gives the ratio of total particles incident on one end of the machine to particles lost from that end. For a low- β device without mirrors, $\eta_m = 1$. If strong mirrors are applied, then $\eta_m \sim R$, where R is the applied mirror ratio. In high- β devices there is a self-mirroring effect even without applied mirrors. Several calculations of η_m have been made in this case, and are reviewed in ref [16]. An analytic form, $\eta_m = 2R/(1 + \sqrt{1 - \beta})$, derived by Freidberg [17] for the case of a diffuse boundary plasma, gives slightly more optimistic results than numerical computations of η_m [16]. We will use Freidberg's analytic result in the form

$$\eta_m = \frac{2 \alpha R}{1 + \sqrt{1 - \beta}} \quad (11)$$

where α is an empirical coefficient which serves two important functions: it allows Eq. (10) to be brought into line with experimentally-measured end-loss times (α is typically 1/2); and it allows the machine length to be chosen larger than the minimum value dictated by end-loss considerations, if desired (i.e. choose $\alpha < 1/2$).

Substituting Eq. (11) into Eq. (10), letting $\tau_{EL} = \tau_b$ and $\beta = 1$, yields the plasma length:

$$L = 7.75 \times 10^6 \frac{\tau_b T^{1/2}}{\alpha R} \quad (12)$$

(L in cm for τ_b in seconds, T in keV).

4. D-T Fusion Cross-Section. The D-T fusion cross-section $\overline{\sigma v}(T)$ is a function of ion temperature only. It has the experimental temperature dependence

shown in Fig. 4, taken from Controlled Thermonuclear Reactions by Glasstone and Lovberg, Von Nostrand Reinhold, 1962. Also shown, for comparison are the cross-sections for the D-D and D-H₂ reactions. The experimental D-T curve can be approximated by [10]

$$\overline{\sigma v}(T) = \frac{3.68 \times 10^{-12}}{T^{2/3}} \exp\left(-\frac{19.94}{T^{1/3}}\right) \text{ cm}^3/\text{s} \quad (13)$$

(T in keV). In the 5-10 keV temperature range of interest to the LTPF (see below), a better fit is obtained if Eq. (13) is multiplied by a factor of 1.35. Thus we will use

$$\overline{\sigma v}(T) = \frac{5.0 \times 10^{-12}}{T^{2/3}} \exp\left(-\frac{19.94}{T^{1/3}}\right) \text{ cm}^3/\text{s} \quad (14)$$

5. Power Supply Considerations. Substituting Eqs. (6), (9), (12), and (14) into Eq. (4) eliminates n , a , τ_D , and $\overline{\sigma v}(T)$, respectively, yielding for the neutron current [12],

$$\begin{aligned} \langle J \rangle &= 4.15 \times 10^7 \left(\frac{2}{1+\lambda}\right)^{17/7} \frac{a R b_1^2}{b_s} \frac{E_\theta^{6/7} B_o^{22/7}}{\tau_c} L \\ &\times \left\{ T^{-151/42} \exp\left(-\frac{19.94}{T^{1/3}}\right) \right\} \text{ n/cm}^2/\text{s} \quad (15) \end{aligned}$$

Equation (15) shows the necessity of high-field operation for obtaining the highest values of neutron current; τ_c is also an obviously crucial parameter.

An example will serve to illustrate some points. Assume $B_o = 300$ kG, $\lambda = 1/2$, $R = 1$ (no applied mirrors), $\lambda = 1$ (equal electron and ion temperatures), $E_\theta = 2$ kV/cm (the design point for LASL's staged theta-pinch experiment), $b_1 = 10$ cm, and $b_s = 11$ cm. The maximum value of $\langle J \rangle$ occurs for $T = 6.3$ keV (set $\partial(\langle J \rangle)/\partial T = 0$). Assuming $T = 6.3$ keV, and requiring $\langle J \rangle = 10^{14}$ n/cm²/s yields the linear relationship between length and cycle time,

$$L = 1.75 \times 10^5 \tau_c (\text{s}) \quad \text{cm} \quad (16)$$

Either L or τ_c may be arbitrarily chosen, but not both. For example if $\tau_c = 1$ s, $L = 1750$ m; if $\tau_c = 1$ ms, $L = 1.75$ m; etc. Let us denote the $L = 1.75$ km case as the "long" case and the $L = 1.75$ m case as the "short" case for purposes of illustration. Some important principles can be deduced from these two cases.

The magnetic energy in the coil bore, E_M (neglecting for the present the penetration of field into the coil wall), is given by

$$E_M = \left(\pi b_2^2 \right) \left(\frac{B_o^2}{2\mu_o} \right) L \quad (17)$$

in SI units, or

$$E_M = 1.25 \times 10^{-8} b_2^2 B_o^2 L \quad \text{MJ} \quad (18)$$

(B_o in kG and b_2 and L in cm). Substituting $b_2 = 13$ cm, $B_o = 300$ kG, and $L = 1.75 \times 10^5$ cm (the "long" case) yields $E_M = 33$ GJ. The steady-state power transfer is $E_M/\tau_c = 33$ GW.

In the "short" case, E_M is reduced by a factor of one thousand, to 33 MJ. The steady-state power transfer, E_M/τ_c , is still 33 GW, however, since the transfer time τ_c is reduced in exact proportion to the length L , and L determines the amount of energy transferred. Also, if a certain fraction of the energy is lost per pulse in joule heating, say 10%, then in either the "short" case or the "long" case the same amount of power is dissipated in the circuit. For a 10% loss, that make-up power is 3.3 GW in our example.

The following general observations can be made: (1) The power required by the LTPF is independent of the particular combination of length and cycle time chosen, but if the "short" mode (rapid cycling, short length) is chosen the energy requirement is much reduced, and thereby, presumable, the capital cost. (2) The dissipated power (i.e. the power consumed from the line) is potentially so large for neutron currents of 10^{14} n/cm²/s that very careful consideration must be given to reducing the dissipated power per unit of neutron current. (3) Heat removal problems will tend to be severe, particularly in short coils. Since the total joule loss in the circuit is fixed (for a given $\langle J \rangle$ and Q), shorter coils will have greater heat densities.

These considerations lead us to identify the power consumption and the circuit quality factor as primary variables in the LTPF problem. Predictions of neutron current can be examined realistically only when power-supply characteristics are folded into the scaling equations.

B. Deriving the Scaling Equations

Define a quality factor "Q" for the circuit such that

$$Q = \frac{E_M}{E_J} \quad (19)$$

where E_M is the energy transferred into the compression coil per cycle, and E_J is the energy lost per cycle as joule heat. If P is defined as the steady-state average power loss in the circuit, then

$$P = \frac{E_J}{\tau_c} \quad , \quad (20)$$

and from Eqs. (19) and (20),

$$Q = \frac{E_M}{P\tau_c} \quad (21)$$

Eliminating E_M between Eqs. (21) and (18) yields the machine length in terms of the parameter QP :

$$L = 8.00 \times 10^7 \frac{(QP) \tau_c}{b_2^2 B_o^2} \text{ cm} \quad (22)$$

(L in cm for QP in MW, τ_c in seconds, b_2 in cm, and B_o in kG).

The usefulness of this formulation is that nearly all of the important LTPF equations can be formulated in terms of QP , and both Q and P are relatively easy to estimate in a practical circuit.

Eliminating L from Eqs. (22) and (14) yields

$$\frac{\tau_b}{\tau_c} = 10.3 \frac{\alpha R(QP)}{b_2^2 B_o^2 T^{1/4}} \quad (23)$$

for the duty factor in terms of QP .

The quantity τ_c/τ_b is an important design parameter which we will take as an independent variable, along with QP . Rearranging Eq. (23) yields the magnetic field,

$$B_o = 3.21 \frac{[\alpha R(QP)(\tau_c/\tau_b)]^{1/2}}{b_2 T^{1/4}} \text{ kG} \quad (24)$$

(B_o in kG for QP in MW, b_o in cm, T in keV).

If the applied mirror ratio R is greater than unity, the maximum field, $B_{\max}$, occurs in the mirror throat with magnitude $B_{\max} = R B_o$. Hence, from Eq. (24),

$$B_{\max} = 3.21 \frac{[\alpha R^3(QP)(\tau_c/\tau_b)]^{1/2}}{b_2 T^{1/4}} \text{ kG} \quad (25)$$

Substituting B_o from Eq. (24) and L from Eq. (22) into Eq. (15) for $\langle J \rangle$ yields the neutron current:

$$\langle J \rangle = 1.26 \times 10^{16} \left(\frac{2}{1+\lambda} \right)^{\frac{17}{7}} [\alpha R(QP)]^{\frac{11}{7}} \frac{b_1^2 E_\theta^{\frac{6}{7}} (\tau_c/\tau_b)^{\frac{4}{7}}}{b_2^{\frac{22}{7}} b_s} \left\{ T^{-\frac{163}{42}} \exp\left(-\frac{19.94}{T^{1/3}}\right) \right\} \text{ n/cm}^2/\text{s} \quad (26)$$

Combining Eqs. (9) and (24) yields the plasma radius:

$$a = 1.11 \left(\frac{2}{1 + \lambda} \right)^{3/14} \frac{1}{T^{3/28}} \frac{b_1 b_2^{3/7} E_\theta^{3/7}}{[\alpha R(QP)(\tau_c/\tau_b)]^{3/14}} \text{ cm} . \quad (27)$$

The gyroradius (or Larmor radius) of a charged particle having temperature T and charge q in a magnetic field B_0 is given by

$$r_L = \frac{(2m_i kT)^{1/2}}{qB_0} \quad (28)$$

in SI units. In a D-T plasma with a given ion temperature, the largest gyroradius is associated with tritons. Substituting for the triton mass ($m_T = 5.006 \times 10^{-27}$ kg) and proton charge ($q = 1.602 \times 10^{-19}$ C) yields

$$r_L = 7.24 \frac{T^{1/2}}{B_0} \text{ cm} \quad (29)$$

(r_L in cm for T in keV, B_0 in kG). Combining Eqs. (24), (27), and (29) yields the ratio of plasma radius to gyroradius in our variables:

$$\frac{a}{r_L} = 0.492 \left(\frac{2}{1 + \lambda} \right)^{3/14} \frac{b_1 E_\theta^{3/7}}{b_2^{4/7} T^{6/7}} [\alpha R(QP)(\tau_c/\tau_b)]^{2/7} . \quad (30)$$

The criterion $a/r_L \geq 1$ must be satisfied at all times for physical feasibility [18].

The Lawson parameter $n\tau$ furnishes another plasma physics criterion and provides a useful quantitative measure of the difficulty of the proposed confinement system. From Eqs. (6) and (24), the ion density is

$$n = 1.28 \times 10^{14} \left(\frac{2}{1 + \lambda} \right) \frac{\alpha R(QP)(\tau_c/\tau_b)}{b_2^2 T^{3/2}} \text{ cm}^{-3} . \quad (31)$$

Multiplying by τ_b yields

$$n\tau_b = 1.28 \times 10^{14} \left(\frac{2}{1 + \lambda} \right) \frac{\alpha R(QP) \tau_c}{b_2^2 T^{3/2}} \text{ cm}^{-3} \text{ s} . \quad (32)$$

These relationships form the basic set of scaling equations for the LTPF. For convenience they are collected below:

LTPF Scaling Equations

$$\langle J \rangle = 1.26 \times 10^{16} \left(\frac{2}{1+\lambda} \right)^{17/7} [\alpha R(QP)]^{11/7} \frac{b_1^2 E_\theta^{6/7} (\tau_c / \tau_b)}{b_2^{22/7} b_s} \times \left\{ T^{-163/42} \exp \left(- \frac{19.94}{T^{1/3}} \right) \right\} \quad \text{n/cm}^2/\text{s} \quad (33a)$$

$$B_o = 3.21 \frac{[\alpha R(QP) (\tau_c / \tau_b)]^{1/2}}{b_2 T^{1/4}} \quad \text{kG} \quad (33b)$$

$$B_{\max} = R B_o = 3.21 \frac{[\alpha R^3(QP) (\tau_c / \tau_b)]^{1/2}}{b_2 T^{1/4}} \quad \text{kG} \quad (33c)$$

$$E_M = (QP) \tau_c \quad \text{MJ} \quad (33d)$$

$$L = 7.75 \times 10^6 \frac{T^{1/2} \tau_b}{\alpha R} \quad \text{cm} \quad (33e)$$

$$a = 1.11 \left(\frac{2}{1+\lambda} \right)^{3/14} \frac{1}{T^{3/28}} \frac{b_1 b_2^{3/7} E_\theta^{3/7}}{\alpha R(QP) (\tau_c / \tau_b)} \quad \text{cm} \quad (33f)$$

$$\frac{a}{r_L} = 0.492 \left(\frac{2}{1+\lambda} \right)^{3/14} \frac{b_1 E_\theta^{3/7}}{b_2^{4/7} T^{6/7}} [\alpha R(QP) (\tau_c / \tau_b)]^{2/7} \quad (33g)$$

$$n = 1.28 \times 10^{14} \left(\frac{2}{1+\lambda} \right) \frac{\alpha R(QP) (\tau_c / \tau_b)}{b_2^2 T^{3/2}} \quad \text{cm}^{-3} \quad (33h)$$

$$n \tau_b = 1.28 \times 10^{14} \left(\frac{2}{1+\lambda} \right) \frac{\alpha R(QP) \tau_c}{b_2^2 T^{3/2}} \quad \text{cm}^{-3} \cdot \text{s} \quad (33i)$$

(b_1, b_2, b_s in cm, E_θ in kV/cm, T in keV, P in MW, τ_b and τ_c in s, α, R , and Q dimensionless).

IV. MAXIMIZING THE NEUTRON CURRENT

We seek to maximize the neutron current, Eq. (33a), by optimizing the parameters α , R , QP , T , etc., subject to reasonable plasma physics and engineering assumptions, and subject to the constraint that all the other scaling relations in Eq. (33) should give reasonable results. The independent parameters in Eq. (33) have been selected to permit just this type of optimization approach.

A. Optimum Ion Temperature

The dependence of $\langle J \rangle$ on the ion temperature is the bracketed expression in Eq. (33a), which we denote as $F(T)$:

$$F(T) = T^{-163/42} \exp\left(-\frac{19.94}{T^{1/3}}\right). \quad (34)$$

The function $F(T)$ is plotted in Fig. 5. The broad maximum is ~ 2 keV in width, and the temperature at the peak, found by setting $\partial F/\partial T = 0$, is 5.0 keV. The maximum neutron current will thus occur, other things being equal, when the ion temperature is 5.0 keV, although small departures (~ 1 keV) in either direction will produce negligible decreases in the neutron current. For self-consistency it will be necessary to show that the other quantities in Eq. (33) are not adversely affected by this choice of temperature; it will become apparent later that this condition is satisfied. Thus we choose $T = 5.0$ keV.

B. Optimum Coil and Tube Dimensions

The dependence of $\langle J \rangle$ in Eq. (33a) on internal radius dimensions is given by

$$G(b) = \frac{b_1^2}{b_s b_2^{22/7}} \quad (35)$$

(see Fig. 3). It is important that the coil radius b_2 be kept as small as possible (note the $22/7$ power dependence). The coil radius b_2 exceeds the tube radius b_1 by an amount Δ (Fig. 3),

$$b_2 - b_1 = \Delta, \quad (36)$$

and Δ should also be kept as small as possible since it represents wasted magnetic volume; note that $\langle J \rangle$ scales approximately as $1/(1 + \Delta/b_1)^{22/7}$. The space Δ must accommodate the wall of the discharge tube, the sample to be irradiated, the shock-heating coil, and any necessary electrical insulation. A reasonable conjecture is that Δ will be about 2 cm.

The radius b_s defining the sample location may be written $b_s = b_1 + \epsilon$, where ϵ is small compared to b_1 . If we make the further assumption that ϵ is small compared to Δ (discharge tubes have thin walls), then the optimum tube radius, found by setting $\partial G/\partial b_1 = 0$, is

$$(b_1)_{\text{opt}} \sim \frac{\Delta}{22/7 - 1} = 0.47 \Delta \quad . \quad (37)$$

For $\Delta = 2$ cm, therefore, $(b_1)_{\text{opt}} = 0.93$ cm and $b_2 = 2.93$ cm. These values are unfortunately not realistic for efficient shock heating in view of the measured sheath widths of 1-2 cm in typical theta pinches [19]. In the Scylla IV-1 theta pinch, which had a tube radius of 4.4 cm, the measured ion temperature was > 3 keV (cf section III and ref [20]). A choice of $b_1 = 5$ cm ($b_2 = 7$ cm) would therefore seem to assure efficient shock heating in the LTPF. If we further assume that the test sample radius b_s is 0.5 cm greater than b_1 , we arrive at:

$$\begin{aligned} \Delta &= 2 \text{ cm} \\ b_1 &= 5 \text{ cm} \\ b_s &= 5.5 \text{ cm} \\ b_2 &= 7 \text{ cm} \end{aligned} \quad . \quad (38)$$

C. Optimum E_θ

The shock-heating electric field E_θ will be taken as 1.0 kV/cm (total). In practice a fraction of E_θ (which can be a substantial fraction in the case of LTPF) will be generated by the dB/dt of the magnetic compression field, $B(t)$ (see discussion in Appendix III).

The value of 1.0 kV/cm may be compared with 0.51 kV/cm measured in the Scyllac 5-meter sector experiment [21], 0.70 kV/cm measured in the Scylla I-B theta pinch [19], and 1.1 kV/cm measured in the Scylla IV-1 device [20]; in each of these cases E_θ was produced by the primary capacitor bank (no separate shock heating). Experiments with separate shock heating have traditionally produced higher fields (2-3 kV/cm) [13, 22, 23]. The voltage at the shock coil radius b_{SH} is given by $(b_{SH}/b_1)(2\pi b_{SH})E_\theta = 53$ kV for $b_{SH} = 6.5$ cm. The stored energy in the shock-heating circuit is small ($< 1\%$ compared to the compression circuit), and we will neglect it.

D. Optimum Quality Factor and Joule Power Loss

Since $\langle J \rangle$ in Eq. (33a) scales as $(QP)^{11/7}$, both Q and P should be chosen as large as possible. The maximum realistic Q value for the circuit will depend in detail on the power supply (see discussions in section VI and Appendix III), but in any case will probably be much less than 100. A measured circuit Q of ~ 20 (5% decrement per cycle) was obtained by the Culham Group in a dynamic stabilization experiment on a theta pinch, using special high- Q Toby Deutschmann capacitors [24]. For present purposes we will assume an overall Q of 15 for the LTPF.

The maximum amount of power P which can be taken from the line depends on the availability and cost of the power, and the difficulty of removing joule heat from the compression coil, where most of P will be dissipated. To address the first point, the Paducah gaseous diffusion plant for uranium enrichment in Kentucky, one of the largest power consumers in the country, uses 2550 MW at full capacity [25]. The LTPF will have to get by on much less. The crucial limitation is probably the maximum tolerable heat load in the coil.

For purposes of illustration assume a coil length L of 1.5 meters, a coil radius b_2 of 7 cm, and a coil thickness of 9 cm; the corresponding volume of metal is $\sim 10^5 \text{ cm}^3$. Assuming an optimistic tolerable heat load of 3 kW/cm^3 [26], the maximum tolerable power loss is $10^5 \times 3 \times 10^{-3} = 300 \text{ MW}$. For purposes of this study we will assume $P = 300 \text{ MW}$. The power supply must therefore be capable of handling $QP = 4500 \text{ MVA}$, of which a fraction Q^{-1} , or 7%, is lost per cycle.

E. Optimum End-Loss Parameter

The quantity α [see Eqs. (11) and (12)] is an empirical end-loss coefficient with a measured value of ~ 0.5 in linear Scyllac end-loss experiments [2]. Since $\langle J \rangle$ in Eq. (33a) scales as $\alpha^{11/7}$, α should be chosen as large as possible. [Note that if the length of the compression coil, L , is increased beyond its minimum required value (i.e. if α is artificially assigned values < 0.5), $\langle J \rangle$ will decrease]. We will assume $\alpha = 0.5$.

The remaining parameters in Eq. (33a) (λ , R , τ_c/τ_b , τ_c) cannot be specified without knowledge of the $B(t)$ waveform, which is the basis for the two point designs developed in the next section.

Substituting $T = 5.0 \text{ keV}$, $b_1 = 5.0 \text{ cm}$, $b_s = 5.5 \text{ cm}$, $b_2 = 7.0 \text{ cm}$, $E_0 = 1 \text{ kV/cm}$, $Q = 15$, $P = 300 \text{ MW}$, and $\alpha = 0.5$ into Eq. (33) yields the following LTPF scaling equations "optimized" for maximum neutron current:

$$\langle J \rangle = 3.91 \times 10^{11} \left(\frac{2}{1 + \lambda} \right)^{17/7} R^{11/7} (\tau_c/\tau_b)^{4/7} \quad \text{n/cm}^2/\text{s} \quad (39a)$$

$$B_0 = 14.6 R^{1/2} (\tau_c/\tau_b)^{1/2} \quad \text{kG} \quad (39b)$$

$$B_{\text{max}} = 14.6 R^{3/2} (\tau_c/\tau_b)^{1/2} \quad \text{kG} \quad (39c)$$

$$E_M = 4500 \tau_c \quad \text{MJ} \quad (39d)$$

$$L = 3.46 \times 10^7 \frac{\tau_b}{R} \quad \text{cm} \quad (39e)$$

$$a = 2.06 \left(\frac{2}{1 + \lambda} \right)^{3/14} \frac{1}{[R(\tau_c/\tau_b)]^{3/14}} \quad \text{cm} \quad (39f)$$

$$\frac{a}{r_L} = 1.85 \left(\frac{2}{1 + \lambda} \right)^{3/14} [R(\tau_c/\tau_b)]^{2/7} \quad (39g)$$

$$n = 5.23 \times 10^{14} \left(\frac{2}{1 + \lambda} \right) R(\tau_c/\tau_b) \quad \text{cm}^{-3} \quad (39h)$$

$$n\tau_b = 5.23 \times 10^{14} \left(\frac{2}{1+\lambda} \right) R \tau_c \quad \text{cm}^{-3} \cdot \text{s} \quad (39i)$$

None of the scaling equations, at this point, yield physically unreasonable results. In particular, since $\lambda = T_e/T_i \leq 1$ in a theta pinch, and both the mirror ratio R and duty factor τ_c/τ_b are ≥ 1 by definition, then $a/r_L \geq 1.85$ from Eq. (39g), and the gyroradius criterion $a/r_L \geq 1$ is satisfied for any possible choice of the remaining four parameters.

Ultimate magnetic field strengths will be discussed in section V.A, where 300 kG is arbitrarily chosen for the upper limit in both point designs. This and other LTPF parameters which are common to both point designs are given for reference in Table I.

V. TWO LTPF POINT DESIGNS

The parameters remaining to be specified in Eq. (39) for the LTPF scaling relations are the ratio of electron to ion temperature λ , the applied mirror ratio R , the duty factor τ_c/τ_b , and the cycle time τ_c . The ratio of electron to ion temperature depends on thermal conduction to the ends of the device, and hence on the plasma density n and length L , both of which depend on R , τ_c/τ_b , and τ_c (or τ_b) [Eq. (39)].

In order to deal with these variables, we must consider specific coil-current (i.e. magnetic field) waveforms. A variety of waveforms are (in principle) available from the power supply. We will begin by considering the simplest class of waveforms, sinusoids, as the basis for Point Design A. Trapezoidal waveforms will be considered in Point Design B.

The point designs presented here are preliminary and certainly incomplete. Extensive engineering studies will be necessary to determine whether these designs are truly feasible. The main purpose at this time is to illustrate salient design features, identify important problem areas, and define parameter spaces where the LTPF is most likely to operate.

A. Point Design A (Regular Sinusoidal Waveform)

Two important features of sinusoid waveforms are (1) their relative ease of generation (e.g. by a ringing capacitor bank) and (2) their relatively slow risetimes (a sine wave is essentially all "risetime", with zero time-duration at the peak). This last point has important consequences for plasma end-loss during the compressional heating phase of the discharge.

A variety of sinusoids are possible, such as the two examples shown in Fig. 6. Figure 6a shows a "simple" sinusoid,

$$B(t) = B_0 \sin(\omega t) \quad (40)$$

and Fig. 6b shows a sinusoid superimposed on a steady D.C. bias field of magnitude $B_0/2$:

$$B(t) = B_0 \left[\frac{1 + \sin(\omega t)}{2} \right] \quad (41)$$

Note that the presence of the bias field reduces the cycle rate by a factor of 2 for the same angular frequency ω , which can be a disadvantage in terms of $\langle J \rangle$. An advantage, in principle at least, is that the D.C. component of field, being steady, might be generated by an external superconducting (solenoid) magnet which would involve no joule losses (but non-zero refrigeration losses).

The end-loss problem may be stated as follows. The plasma end-loss calculation, Eqs. (10)-(12), assumes a uniform plasma column of ion temperature T . This is equivalent to neglecting the heating phase of the pinch, during which T is increasing, compared to the burning phase, when T is assumed constant. In order for the end-loss calculations to be valid, therefore, the plasma must reach its operating temperature (5 keV in this case) in a reasonable fraction of the sine-wave quarter-cycle. If the burn starts too soon, T will not be constant at 5 keV during the burn due to the continuing compression. If the burn starts too late, much of the plasma will escape at high temperature before neutron production begins and the assumed ion density will be too large.

Define the "risetime", τ_r , as the time to the start of the burn (5 keV), as shown in Fig. 6, and not as the time to peak field. Then

$$2 \tau_r = \tau_c - \tau_b \quad . \quad (42)$$

Solving for the duty factor τ_c/τ_b yields

$$\frac{\tau_c}{\tau_b} = 1 + 2 \left(\frac{\tau_r}{\tau_b} \right) \quad . \quad (43)$$

If we take into account the fact that T is, on the average, substantially lower during τ_r than during τ_b , then negligible plasma end-loss during τ_r , compared to τ_b , requires roughly that $\tau_r \lesssim \tau_b$. Equation (43) then yields

$$\frac{\tau_c}{\tau_b} \lesssim 3 \quad . \quad (44)$$

Since $\langle J \rangle$ scales as $(\tau_c/\tau_b)^{4/7}$ [Eq. (39a)], the constraint in Eq. (44) is a serious matter.

Substituting $\tau_c/\tau_b = 3$ into Eq. (39a) yields

$$\langle J \rangle = 7.33 \times 10^{11} \left(\frac{2}{1+\lambda} \right)^{17/7} R^{11/7} \quad \text{n/cm}^2/\text{s} \quad . \quad (45)$$

Noting that $0 < \lambda \leq 1$ for a theta pinch, it is clear that very strong mirrors are required ($R \gg 1$) if $\langle J \rangle$ is to even approach values of 10^{14} n/cm²/s.

The maximum acceptable mirror ratio is determined by the maximum acceptable magnetic field in the mirror throat. From Eq. (39c) this mirror field is given by

$$B_{\max} = 25.3 R^{3/2} \quad \text{kG} \quad (46)$$

for $\tau_c/\tau_b = 3$. Figure 7 shows the yield points of some possible coil materials plotted as a function of the magnetic field strength [18]. The corresponding magnetic pressure (ordinate of Fig. 7) is given by [12]

$$p_m = 0.588 B^2 \quad \text{psi} \quad (47)$$

for B in kG. If we assume $B_{\max} = 300$ kG ($p_m = 53,000$ psi), then from Eq. (46), $R = 5.2$. The magnetic field in the unmirrored central region is $B_0 = 57.7$ kG, from Eq. (39b). These numbers may be compared with the linear Scyllac experimental mirror values [2], $B_0 \sim 60$ kG, $R \sim 2.5-3$, $B_{\max} = 250$ kG (maximum test value)[27].

Strong mirrors can induce $m = 1$ instabilities in a theta-pinch plasma [2], and the question arises whether the intended thermonuclear burn in the LTPF can be completed in the available time if the plasma is unstable. The prospects for a successful confinement are obviously enhanced if the burn time (and hence the cycle time) is made very short.

An inspection of Eq. (39) shows that if the cycle time τ_c is chosen as short as possible, the machine length L , magnetic energy E_M , and Lawson parameter $n\tau_b$ are all reduced to their minimum practical values. This circumstance has its roots in physics considerations but here is a consequence of our particular choice of parameters.

A lower limit on τ_c is set by another technological constraint, namely the high voltages associated with rapidly-rising magnetic fields. As a compromise, and also to take advantage of well-known theta-pinch operating regimes, we will assume $\tau_c = 30$ μs , and hence $\tau_b = 10$ μs .

The remaining parameter in Eq. (39) is λ . Substituting $\tau_b = 10$ μs and $R = 5.2$ into Eq. (39e) yields the machine length L as 66.5 cm, excluding mirrors. In short machines such as this the ratio of electron to ion temperature can be quite low, which is a distinct advantage for generating large neutron currents. Basically this occurs because pressure balance (for a given B_0) can be satisfied at higher ion densities than if electrons and ions had equal temperatures.

We can estimate the electron temperature in a theta pinch from a model by Morse [2,28], who assumes a balance between heating from the ions and cooling by thermal conduction to the ends. In our units his equation (8) is

$$T_e = 1.08 \times 10^{-8} \left(\frac{T_i n^2 L^2}{A_i} \right)^{1/5} \quad \text{keV.} \quad (48)$$

Here T_i is the ion temperature in keV, n is the ion density in cm^{-3} , L is the machine length in cm, A_i is the atomic weight ($= 2.5$ for D-T), and we have assumed $\ln \Lambda = 13$. In order to evaluate Eq. (48) we must know the density n , which depends on λ [Eq. (39h)], which in turn depends on T_e . However, based on past experience with short theta pinches, we may begin by estimating $T_e \sim 250$ eV (i.e. $\lambda = 0.05$ for $T_i = 5.0$ keV), and start an iteration. It is

helpful to re-write Eq. (48) as

$$\lambda = \frac{T_e}{T_i} = 1.08 \times 10^{-8} \left(\frac{n_L^2}{A_i T_i^4} \right)^{1/5} . \quad (49)$$

Substituting $\lambda = 0.05$ into Eq. (39h) gives $n = 1.55 \times 10^{16}$, and thus $\lambda \approx 0.0398$ from Eq. (49). This new value for λ is substituted into Eq. (39h), and after several iterations we obtain $\lambda = 0.040$, corresponding to an electron temperature of 200 eV.

Substituting $\tau_c/\tau_b = 3$, $R = 5.2$, $\tau_c = 30 \mu s$, $\tau_b = 10 \mu s$, and $\lambda = 0.040$ into Eq. (39) yields the "Point Design A" parameters given in Table II. Various neutronics quantities of interest are also listed (see Appendix I for definitions).

Note that, because of its small τ_c/τ_b ratio, the "A" device contains plasma at or near thermonuclear temperature during much of its cycle; consequently refueling during the short cycle time may be a problem. The "burned" fuel (alpha particles plus fuel ions) will be able to escape along magnetic field lines during the course of normal end loss, and hence "flushing" the device may be unnecessary. Fuel may be injected either radially or axially. If axial injection is used, Marshall guns [29] are the obvious candidates. The plasma would be "pre-heated" in Marshall guns, possibly eliminating the need for separate shock heating. However, radial injection of "cold" fuel ions (~ 0.1 eV), via peripheral gas jets, might be a more attractive alternative.

The daily rate of tritium consumption can be estimated from

$$R = m_T t_d S \quad \text{g/day} , \quad (50)$$

assuming one tritium is consumed for each 14-MeV neutron produced, where m_T is the mass of a triton in grams (5.006×10^{-24} g), t_d is the number of seconds per day (86,400), and S is the total neutron source strength, 1.10×10^{17} n/s from Table II. Substituting, we find $R = 0.048$ g/day, or ~ 1.4 g/month. At present commercial prices of $\$0.75/\text{curie} = \$7,500/\text{g}$ [30, 31], the monthly tritium fuel bill would be $\sim \$11$ k.

B. Point Design B (Intermittent Trapezoidal Waveform)

The slow risetime associated with a sinusoidal $B(t)$ waveform can be eliminated if the waveform is made more "flat-topped," or trapezoidal, as shown in Fig. 8. An examination of Eq. (43) shows that τ_c/τ_b can become large in the case of continuous, or regular, waveforms (e.g. Fig. 6), only when $\tau_c/\tau_b \gg 1$, which is unacceptable from end-loss considerations. However if the waveform is made intermittent, as shown in Fig. 8, τ_c/τ_b can be made essentially arbitrarily large, by increasing the zero-field time between pulses, while keeping τ_b , and thereby the machine length, short. The intermittent pulses needn't be trapezoidal, but they can be, since rotating machinery can generate trapezoids easily, and rotating machinery will likely be needed for the intermittent wavetrain. Intermittent pulses are needed in order to depart significantly from the continuous-sinusoid case.

Once the constraint on τ_c/τ_b is removed [Eq. (44)], the problem becomes one of maximizing $\langle J \rangle$ in Eq. (39a), subject to the technology constraint that $B_{\max}$ [Eq. (39c)] not exceed 300 kG. Substituting $B_{\max} = 300$ kG into Eq. (39c) yields $R^3(\tau_c/\tau_b) = 422$. Solving for τ_c/τ_b ,

$$\frac{\tau_c}{\tau_b} = \frac{422}{R^3} \quad (51)$$

and substituting Eq. (51) into Eq. (39a) yields the neutron current in the form

$$\langle J \rangle = 1.24 \times 10^{13} \left(\frac{2}{1 + \lambda} \right)^{17/7} \frac{1}{R^{1/7}} \quad \text{n/cm}^2/\text{s} \quad (52)$$

The maximum $\langle J \rangle$ occurs when $R = 1$. Thus we arrive at the general theorem that whenever the duty factor τ_c/τ_b is limited by the maximum allowable magnetic field, and not by particle end loss (i.e. the $B(t)$ waveform), then the maximum neutron current is obtained with an applied mirror ratio of unity. The corollary follows that, with no magnetic mirrors, high field operation is implied, since $B_0 = B_{\max}$ along the entire plasma column when $R = 1$.

Due to the $R^{-1/7}$ dependence of $\langle J \rangle$, the penalty in diminished neutron current is small if R should be chosen greater than unity. For example, if $R = 2$, $\langle J \rangle$ is reduced by $\sim 10\%$ over the $R = 1$ case, whereas τ_c/τ_b is reduced by a factor of 8, from 422 to 53. With $B_{\max} = 300$ kG and 2:1 mirrors, B_0 is reduced from 300 to 150 kG. The plasma length L is also reduced by a factor of 2. These two factors taken together reduce E_M , the magnetic energy, by a factor of 8 for a given burn time, since $E_M \propto \tau_c = (\tau_c/\tau_b)(\tau_b)$. Thus it seems appropriate, considering the broad picture, to choose an operating point slightly below the "maximum $\langle J \rangle$ " point, e.g. $R = 2$ instead of $R = 1$, which will substantially ease the technology demands in this point design. With $R = 2$ we obtain $\langle J \rangle = 1.12 \times 10^{13} [2/(1 + \lambda)]^{17/7} \text{ n/cm}^2/\text{s}$.

It should be noted that the choice of an operating point given above was greatly facilitated by the way in which the LTPF problem is formulated here; the "trade-offs" became fairly clear.

The arguments given under section V.A. for choosing τ_b (and hence τ_c , which is related to τ_b by a constant) as short as possible remain valid. Hence we choose $\tau_b = 10 \mu\text{s}$ as before. Thus $\tau_c = 53\tau_b = 530 \mu\text{s}$ ($f = 1.89$ kHz), and $L = 1.73$ cm. The electron temperature is next found by the iteration procedure given in section V-A, with the result $T_e = 610$ eV, $\lambda = 0.122$, and $n = 9.87 \times 10^{16} \text{ cm}^{-3}$.

With a burn time τ_b of $10 \mu\text{s}$, the risetime can also be $10 \mu\text{s}$ [i.e. $\tau_r/\tau_b \approx 1$] without causing excessive end-loss.

Table III lists various design and neutronics parameters for Point Design B.

Due to the long pauses between burning pulses in Point Design B, refueling the device with fresh D-T gas can probably be accomplished by conventional gas flow from the ends. A rough estimate of the required filling time can be obtained by calculating $L/2c_s$, where $L/2$ is the machine half-length and c_s is the sound speed in the filling gas,

$$c_s = \left(\frac{\gamma kT}{M} \right)^{1/2} \quad (53)$$

in SI units. Here γ is the ratio of specific heats [$\gamma = (f + 2)/f = 7/5$ for diatomic molecules, where $f = 5$ is the number of degrees of freedom of the molecule], k is Boltzmann's constant ($k = 1.38 \times 10^{-23}$ joule/°K), and M is the average molecular mass [$M = (2A_1)m_H$, where A_1 is the atomic weight -- 5 for an equal mixture of D-T molecules -- and m_H is the mass of a hydrogen ion, 1.672×10^{-27} kg]. If we assume a filling temperature of 0.1 eV (1140°K), we find that $c_s = 1624$ m/s, and hence the filling time, $L/2c_s$, is 533 μ s. This filling time is comparable to the cycle time of 530 μ s. [Note that the r.m.s. speed of a molecule is $(3kT/M)^{1/2}$ at temperature T ; the sound speed calculated above is thus smaller than the r.m.s. speed of the molecules by a factor $(\gamma/3)^{1/2} = 0.683$ in the present case, which allows a little leeway].

The average power output of 14-MeV neutrons is the neutron source strength (S) multiplied by the energy per neutron (2.25×10^{-12} J). For $S = 2.73 \times 10^{17}$ n/s (Table III), $P_n = 0.617$ MW.

VI. POWER SUPPLIES

The power supply for the LTPF is a crucial item, and the details of its performance and cost will depend heavily on the particular type of power supply used as well as on which LTPF design is chosen.

For an overall circuit Q of 15 and an assumed power consumption P of 300 MW, the power supply must be capable of handling $QP = 4500$ MVA of oscillating power at frequencies of ~ 33 kHz (Point Design A) or ~ 1.9 kHz (Point Design B). The transferred energy in the two cases is ~ 0.13 MJ (design A) and ~ 2.4 MJ (design B). Either frequency can be obtained with a ringing capacitor bank, but a capacitor bank is presumably not a viable candidate to produce the widely-intermittent trapezoidal current pulses required in Point Design B. A homopolar device is not able to operate at 33 kHz, and probably not at 1.9 kHz [32]. Conventional rotating machinery (i.e. an MG set) could operate at 1.9 kHz without difficulty, and could also produce the desired waveform in Point Design B; however two large machines will be required instead of one, since power taken from the line (at 60 Hz) would drive the motor of the MG set, while the generator would drive the theta pinch (at 1.9 kHz). The cost of the motor and generator has been estimated at \$50/kW installed [33], or \$100/kW for the pair. MG sets are not usually employed in circumstances where the power factor of the load is far from zero, as it would be here; consequently problems involving the returning 93.3% of the electrical power (for a Q of 15) at the end of each pulse can be anticipated. If the MG set has to be costed on the basis of 4500 MVA instead of 300 MW, the cost would be prohibitive ($\sim$ \$450 M).

The magnitude of the electric current and other circuit parameters for Point Design B are calculated in Appendix III.

For design A the power supply could be either a capacitor bank or a solid-state oscillator, either of which could produce a sinusoidal current at 33 kHz. Since the capacitor bank will, in any case, have to be "pumped" with 300 MW of power to replenish the joule losses each cycle, probably the capacitor bank and oscillator circuit could be used to best advantage together. The capacitor bank and theta-pinch coil can be connected as a resonant RLC tank circuit which will appear as a purely resistive (zero power-factor) load to the driving oscillator. In this case the oscillator would be rated at 300 MW, and for an estimated cost of \$50/kW installed [33], would

cost $\sim$ \$15 M.

The capacitor bank can be costed on the basis of its energy storage, 135 kJ (Table II). The cost of the high-voltage, low-inductance, 2-MJ Scylla IV-P bank, which utilizes Scyllac-type components, was $\sim$ \$2 M in 1975 [34], or $\sim$ \$1/J installed. Adding a factor of 10 for the increased duty factor in the LTPF yields \$10/J installed, or a total of \$1.35 M. The major cost will thus be associated with the solid-state driver.

As an economy measure, a prototype, or proof-of-concept device, could be built and operated without the solid-state driver. In this case the capacitor bank should ring down in $\sim$ 15 cycles, for a circuit Q of 15. This number of cycles should be adequate to test a variety of system components, such as the gas-handling system, and also allow for a comparison of the measured neutron yield vs. theory.

A further discussion of power supplies and circuit parameters is given in Appendix III.

VII. SUMMARY AND CONCLUSIONS

In previous sections we have examined the possibility of using the linear theta pinch as the basis for a 14-MeV neutron source and Fusion Engineering Research Facility (FERF). The approach has been to express the neutron current and other quantities of interest in the problem (e.g. the machine length, magnetic field energy, ratio of plasma radius to ion gyroradius, etc.) in terms of carefully chosen independent variables which describe the plasma parameters, the theta-pinch coil, and the power supply which furnishes the coil current. The choice of independent variables used in this study is not unique, but has been found by trial-and-error to permit a systematic optimization of the linear theta-pinch FERG (LTPF) for maximum 14-MeV neutron production, while also realistically incorporating the limitations imposed by the finite (and expensive) power supply. The basic philosophy has been to restrict technological innovations to a minimum, so as to take full advantage of established theta-pinch techniques and experience. Emphasis throughout has been placed on the physics and technology considerations which govern the choice of device parameters. The optimum plasma ion temperature, for instance, is shown to be 5.0 keV for maximum neutron current.

Our studies show that neutron currents of $\sim 5 \times 10^{13}$ n/cm²/s can be generated on a test sample located at a radius of ~ 5.5 cm, between the discharge tube and the compression coil wall (i.e. inside the separate shock-heating coil), in either of two point designs. The two point designs are distinguished primarily by their magnetic-field (i.e. coil-current) waveforms, B(t). In Point Design A, B(t) is sinusoidal and continuous, such as might be generated by a resonant LC circuit. In this design the machine length is 67 cm, the magnetic field energy is 135 kJ, and the operating frequency (rep rate) is 33 kHz. Strong ($\sim 5:1$) magnetic mirrors are applied at the ends to reduce particle end loss. For a maximum magnetic field of 300 kG in the mirror regions, the central (unmirrored) field strength is ~ 60 kG.

In Point Design B, B(t) is trapezoidal and intermittently pulsed, such as might be produced by a motor-generator set. In this design the machine length is 1.7 m, the magnetic field energy is 2.2 MJ, and the rep rate is 1.9 kHz. The magnetic field in the central (unmirrored) region is 150 kG, and 2:1 mirrors are used at the ends.

The question of power supplies for the LTPF is obviously a crucial one, and will require further study. In particular, since the assumed power loss

in both point designs is 300 MW for an overall circuit Q of 15, methods for reducing the coil and lead resistance, such as layering or litzing the conductors, should be considered. Since such coils are necessarily complex, they will be subject to new strength and cost considerations. Heat removal, for example, may be more difficult. However if the circuit Q can be increased to 30, the same LTPF performance and machine parameters will be obtained for a power loss of 150 MW, since Q and P appear in the scaling laws only as a product. To put it another way, the LTPF parameters depend on the power transfer in the circuit, and not on the power loss per se. If the power loss can be reduced by increasing the circuit Q , the heat removal problem will be correspondingly less severe. The size and cost of the power supply will also be reduced.

A second technology area which will require further study is the matter of refueling the plasma chamber with fresh D-T gas each cycle. Flushing the chamber of spent gas should be easier than refueling, due to the high temperature of the particles (~ 5 keV) after the burn. Normal particle end loss will suffice to remove much of the burned fuel. The burn-up fraction will be very small for a burn time of $10\ \mu\text{s}$ ($\ll 1\%$), and therefore most of the tritium will have to be recovered for future use. Injection of new fuel may be either radial or axial. Due to the shorter injection distance, radial fueling may be easier in a high rep rate case, such as Point Design A. If axial injection is used, Marshall guns are obvious candidates in the case of high rep rate. The highly directed velocities (of order 10^7 - 10^8 cm/s) [29] will be necessary for filling an approximately 1-meter-long device in the available few μs . In addition, thermalization of the directed plasma energy might eliminate the need for separate shock heating to pre-heat the plasma. In the case of Point Design B, the long interval (~ 0.5 ms) between pulses should allow filling the plasma chamber with neutral gas via conventional gas-dynamic flow from the ends.

From a plasma physics point of view, no fundamental obstacles to the LTPF concept have been encountered. The projected plasma parameters are familiar, and the confinement times are short ($10\ \mu\text{s}$); the corresponding values of the Lawson parameter, $n\tau_b$, are 1.6×10^{11} and 9.9×10^{11} s/cm³ for Point Designs A and B, respectively. The principle plasma physics uncertainties relate to the projected high rep rates, and to end-effects problems such as impurities, plasma-wall interactions, etc. Magnetic divertors at the ends might be required to lessen these effects. The presence of magnetic mirrors at the ends introduces regions of unfavorable field curvature which can induce $m = 1$ instabilities [2]. This effect is not expected to be important on the $10\text{-}\mu\text{s}$ time scale projected for the LTPF confinement.

As a very crude estimate, the cost of developing an LTPF based on the principles presented here might be in the range of \$20M-\$30M. The largest expense is expected to be associated with the development of a suitable power supply.

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APPENDIX I - NEUTRONICS TERMINOLOGY

The neutron-transport terminology used in the FERF or reactor field can be confusing to the casual reader because of sometimes inconsistent or conflicting usage. For this reason we begin by defining some of the terms appearing in this report (based in part on ref [26]).

1. Neutron Current Density. The neutron current density, often abbreviated to current when the implication is obvious, is the net number of neutrons crossing a unit area, oriented in some given direction, in unit time. The current may be thought of as the net sum of all neutrons crossing a small test disk from both sides in any direction. Since the disk has a vector unit normal, the current as defined here is a vector, even though it includes neutrons with random directions. The current is expressed as neutrons per cm^2 per sec, or $\text{n/cm}^2/\text{s}$, and in this report has the symbol J .

2. Neutron Flux Density. The neutron flux density, often abbreviated to flux when the implication is obvious, is the product of neutron density and velocity. The flux is a scalar, and although usually expressed as neutrons per cm^2 per sec ($\text{n/cm}^2/\text{s}$), might be less confusing if expressed as $(\text{n/cm}^3) \times (\text{cm/s})$. The flux may be thought of as the sum of all neutrons entering from any direction a small test sphere having unit projected area. In this report its symbol is ϕ .

3. Neutron Fluence Density. The neutron fluence density, often abbreviated to fluence when the implication is obvious, is the time integral of the flux. The fluence density is a scalar with units of n/cm^2 . The total fluence is the integral of the fluence density, or simply the total neutrons. In this report fluence density has the symbol F .

4. Uncollided Neutrons. The term "uncollided" refers to primary neutrons, as differentiated from back-scattered neutrons. Uncollided (or source or primary) neutrons approach the first wall from the plasma side only, and in the case of a D-T burning FERF have an energy of $14.06 \text{ MeV} \pm$ a plasma ion thermal energy component.

The average neutron energy is increased somewhat because of the kinetic energy associated with the relative motion of the reacting ions, and this effect has been taken into account in arriving at 14.06 MeV for the center-of-mass neutron energy for the D-T reaction. In addition to this effect, which slightly shifts the location of the peak, a broadening of the neutron spectrum will occur due to the distribution of relative ion velocities. The full width at half maximum (fwhm) is related to the ion kinetic temperature by the equation [35]

$$\text{fwhm (MeV)} = 0.177 \sqrt{kT(\text{keV})} \quad . \quad (\text{I-1})$$

The width of this distribution is much larger than kT for most cases of interest. For example, if $kT = 5 \text{ keV}$, the width of the 14-MeV neutron peak

is 0.40 MeV, or a factor of eighty greater than kT. (Note that an INS of the wind-tunnel type does not produce this type of spectrum).

5. Total Source Strength. As used in this report, the total source strength, S , for neutrons refers to the integral of uncollided neutron current over any surface enclosing the source. Source strength is expressed in units of neutrons per sec (n/s), and may be thought of as the rate of 14-MeV neutron production for the entire plasma column.

6. "Average" vs. "Instantaneous" Values. In a pulsed neutron source such as the LTPF, the neutrons are produced intermittently. From the end of one burn phase until the start of the next, the neutron fluence remains constant. The neutron current and flux may be described either by their instantaneous values, which are assumed constant during the burn, or by their average values, which are time-averages over a complete cycle (i.e. the time between successive burning pulses). In this report time-averaged quantities are denoted by angle brackets, i.e. $\langle J \rangle$.

7. Neutron Current vs. Neutron Flux. Using the above definitions, it follows that the uncollided neutron current on the first wall of an LTPF is numerically equal to the uncollided flux only in the special case of purely radial neutron velocity. In any actual device the production of neutrons with purely radial velocity will not occur; there will always be an axial neutron component. The closest approach to a purely radial velocity would be if the plasma were an infinite line source, in which case the ratio of uncollided current to uncollided flux (J/ϕ) can be shown to be [36] $2/\pi = 0.64$ (i.e. the average value of $\cos \theta$ as θ goes from 0 to $\pi/2$). If the neutron velocity were purely isotropic (as would be caused, for example, if the plasma existed as a thin annulus, or sheath, adjacent to the wall), the ratio (J/ϕ) would be [36] $1/2$. In an actual theta pinch, the uncollided current and uncollided flux are related by a factor lying somewhere between 0.5 and 0.64; for present purposes we will assume $J/\phi = 0.5$.

8. "Total" Flux vs. "Uncollided" Flux. The total neutron flux (ϕ_T) contains a contribution from back-scattered neutrons having energies less than 14 MeV. A proper calculation of the total flux requires complex computer programs which take into account non-radial neutron velocities, scattering cross sections for the various wall materials, multiple energy groups for the neutrons, etc. Bulk material damage effects are probably more closely related to the total neutron flux at a point than to the uncollided flux. Neutronics calculations for the RTPR blanket [6] indicate that the total neutron flux is about 10 times the uncollided flux in a fusion reactor. We will assume the same ratio here.

In this report we will be concerned primarily with calculating the uncollided neutron current.

APPENDIX II - PLASMA HEATING IN THE LTPF

A. Derivation of Staged-Heating Equations

Staged heating in theta pinches has been discussed in detail elsewhere [13-15]. Those treatments have usually assumed equal temperatures for the electrons and ions after the "equilibration" phase (prior to the start of adiabatic compression). The staged-heating equation [Eq. (7)] is derived below in SI (MKS) units for the case of unequal electron and ion temperatures.

We begin by assuming that at $t = 0$ the implosion of a cylinder of gas is initiated by applying E_θ to the boundary of radius b . The initial gas atom density is n_a . Following the implosion the plasma is fully ionized and the electrons and ions settle down to a state characterized by n_s , B_s , a_s , T_{es} , and T_{is} . The plasma is then further compressed to a final state n , B , a , T_e , and T_i . We will use the so-called "free-expansion" model of shock heating [15], in which the gas is shocked by a two-stage process. An initial piston field of magnitude $B_s/\sqrt{2}$ is suddenly removed at time t , when the sheath has advanced to position $a/b = 1/3$. A larger field, B_s , is then reapplied at $3/2 t$, when all ions have advanced to the opposite wall, reflecting them with velocity $2 v$. When pressure equilibrium is achieved, the radius a_s is related to the starting radius by the dimensionless ratio x_s ; that is,

$$a_s = x_s b. \quad (\text{II-1})$$

The value of x_s depends upon complicated particle dynamics which take place during the implosion. Numerical simulations [15] give $x_s = 0.76$ for the free-expansion case, whereas simple kinetic calculations [15] give $x_s = 0.76$. We will eventually use $x_s = 0.76$, but for the present x_s is retained as a free parameter.

The plasma ion density after the implosion event is

$$n_s = \frac{n_a}{x_s^2} \quad (\text{II-2})$$

(for particle conservation). The magnetic field and electric field are related by $\vec{E} = \vec{V} \times \vec{B}$; since $B = B_s/\sqrt{2}$ during the acceleration of the ions,

$$E_\theta = \left(\frac{B_s}{\sqrt{2}} \right) U_s \quad (\text{II-3})$$

The "effective" piston velocity in Eq. (II-3) is $U_s/\sqrt{2}$. During the first reflection the ion velocity is doubled to $\sqrt{2} U_s$. During the second reflection the ion velocity is doubled once more, to $2\sqrt{2} U_s$. The momentum-balance equation, $p = 1/2 m v^2$, takes the form

$$\frac{B_s^2}{2\mu_0} = 4 m_i n_a U_s^2 . \quad (\text{II-4})$$

Eliminating U_s between Eq. (II-3) and Eq. (II-4) yields

$$E_\theta = \frac{B_s^2}{4(\mu_0 m_i n_a)^{1/2}} . \quad (\text{II-5})$$

If the plasma is adiabatically compressed from state "s", the temperature is related to the plasma volume by [ref. (10), p 124]

$$\frac{T}{T_s} = \left[\frac{(\text{Vol.})_s}{\text{Vol.}} \right]^{2/f} = \left(\frac{a_s}{a} \right)^{4/f} , \quad (\text{II-6})$$

where f = number of degrees of freedom for the particles. For $f = 3$,

$$\frac{T}{T_s} = \left(\frac{a_s}{a} \right)^{4/3} . \quad (\text{II-7})$$

The relationship between the plasma temperature and confining magnetic field in an adiabatic compression is [ref (10), p 126]:

$$\frac{T}{T_s} = \left(\frac{B}{B_s} \right)^{2(\gamma-1)/\gamma} \quad (\text{II-8})$$

where $\gamma = (f + 2)/f$ is the ratio of specific heats. Equation (II-8) is only valid in the case $\beta = 1$, i.e. no trapped internal field. For $f = 3$,

$$\frac{T}{T_s} = \left(\frac{B}{B_s} \right)^{4/5} . \quad (\text{II-9})$$

Eliminating T/T_s between Eq. (II-7) and Eq. (II-9) yields

$$\frac{a}{a_s} = \left(\frac{B}{B_s} \right)^{-3/5} . \quad (\text{II-10})$$

Conservation of particles requires

$$n_s a_s^2 = n a^2 . \quad (\text{II-11})$$

Equations (II-10) and (II-11) yield

$$\frac{n}{n_s} = \left(\frac{B}{B_s} \right)^{6/5} . \quad (\text{II-12})$$

Collecting results, the adiabatic laws for a $\beta = 1$, $f = 3$ compression are

$$\left. \begin{aligned} \frac{T}{T_s} &= \left(\frac{B}{B_s} \right)^{4/5} \\ \frac{a}{a_s} &= \left(\frac{B}{B_s} \right)^{-3/5} \\ \frac{n}{n_s} &= \left(\frac{B}{B_s} \right)^{6/5} \end{aligned} \right\} . \quad (\text{II-13})$$

Dividing a and a_s in Eq. (II-13) by b yields

$$B_s = \left(\frac{x}{x_s} \right)^{5/3} B \quad (\text{II-14})$$

where $x = a/b$. Particle conservation requires

$$n_a = x^2 n \quad (\text{II-15})$$

and pressure balance at $\beta = 1$ yields

$$n = \frac{B^2}{2 \mu_0 (1+\lambda) kT} \quad (\text{II-16})$$

($\lambda = T_e/T_i = T_e/T$). Substituting n from Eq. (II-16), n_a from (A-15), and B_s from (II-14) into Eq. (II-5) yields for E_θ

$$E_\theta = \left(\frac{1+\lambda}{8m_i} \right)^{1/2} \left(\frac{a}{b} \right)^{7/3} \left(\frac{1}{x_s} \right)^{10/3} B_o (kT)^{1/2}, \quad (\text{II-17})$$

which is the desired result.

Assuming $x_s = 0.76$ and $m_i = (m_D + m_T)/2 = 4.1756 \times 10^{-27}$ kg, and setting $B = B_o$ and $b = b_1$ yields in our notation

$$E_\theta = 0.244 \left(\frac{1+\lambda}{2} \right)^{1/2} \left(\frac{a}{b_1} \right)^{7/3} B_o T^{1/2} \quad (\text{II-18})$$

(E_θ in kV/cm for B_o in kG and T in keV). In the case of equal electron and ion temperature, $\lambda^0 = 1$ and Eq. (II-18) reverts to the form given in ref [37].

B. Radial Dependence of E_θ .

The radial dependence of E_θ is obtained by integrating Ampere's law, $\nabla \times \vec{E} = - \partial \vec{B} / \partial t$, over the open cross-sectional area of the coil:

$$\int_{\text{coil area}} \nabla \times \vec{E} \cdot d\vec{A} = - \int_{\text{coil area}} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} \quad (\text{II-19})$$

Stokes' law is used to transform the left side of Eq. (II-19) into a line integral,

$$\oint \vec{E} \cdot d\vec{\ell} = - \int_{\text{coil area}} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} \quad (\text{II-20})$$

The line integral is taken around a circular path of radius r centered on the tube axis:

$$\int_0^{2\pi} E_{\theta} (rd\theta) = - \int_0^r \frac{\partial B_z}{\partial t} (2\pi r dr) . \quad (\text{II-21})$$

If we assume B_z is spatially uniform within r , then E_{θ} is constant on the boundary, by symmetry, and the integration is trivial.

$$E_{\theta} (2\pi r) = - \frac{\partial B_z}{\partial t} (\pi r^2) \quad (\text{II-22})$$

thus

$$E_{\theta} = - \frac{r}{2} \frac{\partial B_z}{\partial t} \quad (\text{II-23})$$

in SI units.

If $B(t) = B_0 \sin \omega t$ (cf Section V), then $\frac{\partial B_z}{\partial t} = \omega B_0 \cos \omega t$ and E_{θ} has its maximum value at $t = 0$. In our usual units the maximum value of E_{θ} is

$$E_{\theta} = - \frac{10^{-8}}{2} r \omega B_0 \quad (\text{II-24})$$

(E_{θ} in kV/cm for r in cm, ω in s^{-1} , and B_0 in kG).

If $B(t) = B_0 (t/\tau_r)$ (i.e. a trapezoidal waveform), then in our units,

$$E_{\theta} = - \frac{10^{-8}}{2} r \frac{B_0}{\tau_r} . \quad (\text{II-25})$$

APPENDIX III - ELECTRICAL CIRCUIT PARAMETERS

A. Parameters for Point Designs A and B

Coil Current. Neglecting magnetic mirrors, the maximum coil current I is related to the maximum magnetic field B_0 by the solenoid formula, $B_0 = 4\pi NI/10 L$, where L is the coil length in cm, N is the number of coil turns, and B_0 is in gauss for I in amperes. Thus

$$I = \frac{10 B_0 L}{4\pi N} \quad (\text{III-1})$$

$$= \begin{cases} \frac{3.05}{N} & \text{MA} & (\text{design A}) \\ \frac{20.7}{N} & \text{MA} & (\text{design B}) \end{cases}$$

using values from Tables II and III.

Coil Inductance. The inductance of an N turn solenoid of length L and radius b_2 is ($\mathcal{L}$ in nH for dimensions in cm) [38]

$$\mathcal{L}_{\text{coil}} = \frac{4\pi^2 N^2 b_2^2 L}{L^2 + 4b_2^2} \quad (\text{III-2})$$

$$= \begin{cases} 27.9 N^2 & \text{nH (A)} \\ 11.1 N^2 & \text{nH (B)} \end{cases}$$

Coil Resistance. The electrical resistance R of a multi-turn solenoid having total conductor length ℓ and conductor cross-sectional area A can be found from $R = \eta \ell / A$, where η is the conductor electrical resistivity. For an N -turn solenoid of length L , average conductor radius $\bar{b}$, and skin depth δ , we find $\ell \sim 2\pi N \bar{b}$, $A \sim L \delta / N$, and thus

$$R \simeq \frac{2\pi \bar{b} \eta N^2}{L \delta} \quad (\text{III-3})$$

(R in ohms for dimensions in cm and resistivity in ohm-cm). The skin depth δ of a conductor is given by (SI units)

$$\delta = \sqrt{\frac{2\eta}{\mu_0 \omega}} \quad (III-4)$$

For the resistivity we take $\eta = 4 \times 10^{-6} \Omega\text{-cm}$ [39], the value for commercial Amzirc (copper-zirc) alloy at a temperature of 300°C . For design A ($f = 33 \text{ kHz}$), $\omega = 2.1 \times 10^5 \text{ s}^{-1}$ and $\delta = 0.6 \text{ mm}$; for design B ($f = 1.9 \text{ kHz}$), $\omega = 1.2 \times 10^4 \text{ s}^{-1}$ and $\delta = 2.3 \text{ mm}$. For $b \sim b_2 = 7 \text{ cm}$ the coil resistance is

$$R = \begin{cases} 4.4 \times 10^{-5} \text{ N}^2 \Omega & (A) \\ 4.4 \times 10^{-6} \text{ N}^2 \Omega & (B) \end{cases} \quad (III-5)$$

Joule Losses. Neglecting eddy currents, the joule losses in the compression coil are due to the transport current I:

$$E_J = \int_0^{\tau_c} I^2(t) R dt \quad (III-6)$$

$$= \begin{cases} \frac{3}{2} I^2 R \tau_b = 6.14 \text{ kJ} & (A) \\ 5/3 I^2 R \tau_b = 31.4 \text{ kJ} & (B) \end{cases}$$

by direct integration of the design A and design B waveforms (see Tables II and III). Note that the joule losses are independent of the numbers of turns in the coil.

Circuit Q. The quality factor Q for the coil, defined as (energy transferred to the coil per cycle)/(energy lost in the coil per cycle), forms an upper bound for the circuit Q, which is necessarily smaller.

Using Eq. (III-6) and Tables II and III, we find

$$Q_{\text{coil}} = \begin{cases} 135 \text{ kJ}/6.14 \text{ kJ} = 22 & (A) \\ 2.39 \text{ MJ}/31.4 \text{ kJ} = 76 & (B) \end{cases} \quad (III-7)$$

These values for the coil-Q are compatible with our assumption that the circuit-Q for LTPF is ~ 15 .

If the waveform shown in Fig. 6b had been chosen for point design A instead of the waveform in Fig. 6a, the joule losses in Eq. (III-6) would have been $9/8 I^2 R_{th} = 4.61$ kJ and the coil Q would have been 29 instead of 22. Finally, if the DC bias field of Fig. 6b were generated by a lossless superconducting coil, the joule losses in Eq. (III-6) would have been $3/8 I^2 R_{th} = 1.54$ kJ and the coil Q would have been 88 instead of 22. In this latter case, however, refrigeration losses should also be included to make a valid comparison.

B. Capacitor Bank Characteristics (Point Design A).

Energy Transfer Efficiency. If we define the transfer efficiency α as the fraction of the bank energy (or voltage) transferred to the load, α can be written as

$$\alpha = \frac{L_{coil}}{L_{coil} + L_{source}} \quad (III-8)$$

where L_{source} refers to all the circuit inductance not in the load coil. For example, in the Scyllac 8-m toroidal sector experiment [21] L_{coil} was ~ 3 nH and L_{source} was ~ 0.5 nH, giving $\alpha = 0.86$ [40]. If we assume a transfer efficiency of, say, 80% for the LTPF, then Eq. (III-8) requires

$$\begin{aligned} L_{source} &= \frac{1-\alpha}{\alpha} L_{coil} \\ &= 6.98 N^2 \text{ nH} \quad (A) . \end{aligned} \quad (III-9)$$

If the design-A coil were 1 m long instead of 66.5 cm, L_{coil} would be reduced to $\sim 18.6 N^2$ nH and L_{source} in Eq. (III-9) to $\sim 4.64 N^2$ nH. The equivalent 1-m values for the Scyllac 8-m sector experiment (actual length = 8.4 m) are $L_{source} = 4.2$ nH/m and $L_{coil} = 25$ nH/m.

We conclude that the LTPF does not require a bank of lower inductance than Scyllac, even for the 'worst' case of $N = 1$. A single-turn coil will certainly be the simplest to fabricate, from the viewpoints of both mechanical strength and cost. Therefore the logical choice, at least for prototype development, is $N = 1$. For a single-turn coil the relevant circuit values become: $I = 3.05$ MA, $R = 0.44 \mu\Omega$, $L_{coil} = 27.9$ nH, and $L_{source} = 6.98$ nH (for 80% transfer efficiency).

Bank Energy. With a transfer efficiency of $\alpha = 80\%$, the bank energy is greater than the coil energy by 25%:

$$E_{bank} = \frac{E_m}{\alpha} \quad (III-10)$$

$$= 169 \text{ kJ} \quad (\text{A})$$

Bank Capacity. The bank capacity is chosen to give the correct resonant frequency for the total circuit:

$$C = \frac{1}{\omega^2 \mathcal{L}_T} \quad (\text{III-11})$$

$$= 650 \text{ } \mu\text{F} \quad (\text{A})$$

where $\mathcal{L}_T = \mathcal{L}_{\text{coil}} + \mathcal{L}_{\text{source}} = 34.9 \text{ nH}$ and $\omega = 2.1 \times 10^5 \text{ s}^{-1}$ for design A.

Bank Voltage. The bank voltage is chosen to give the desired bank energy. Thus

$$V = \left(\frac{2 E_{\text{bank}}}{C} \right)^{1/2} \quad (\text{III-12})$$

$$= 22.8 \text{ kV} \quad (\text{A})$$

Shock-Heating Requirements. The shock-heating electric field E_θ has been specified as 1.0 kV/cm, at a radius $b_1 = 5.0 \text{ cm}$. The corresponding applied voltage at the 6.5 cm radius of the shock coil is 53 kV (see Sec. IV-C). Since the compression bank alone supplies over 40% of this voltage at time $t = 0$ [Eq. (III-12)], the required E_θ from the shock-heating circuit can be correspondingly reduced. Using Eq. (II-24), from Appendix II, we find

$$E_\theta = \frac{-10^{-8}}{2} r \omega B_o \quad (\text{III-13})$$

$$= 0.30 \text{ kV/cm} \quad (\text{A})$$

at $r = 5 \text{ cm}$, due to the compression bank alone. Therefore the shock-heating coil need supply only $1 \text{ kV/cm} - 0.3 \text{ kV/cm} = 0.7 \text{ kV/cm}$ - a very mild requirement.

C. Rotating Machinery Characteristics (Point Design B).

Maximum rotor speed. The maximum tip speed of the spinning rotor will be limited by strength-of-materials considerations (centrifugal force) to around 200-400 m/s [41]. An additional factor of two can be gained in the relative speed between rotor and stator if rotor and stator are both counter-rotated at the maximum tip speed [41]. However this feat is technologically difficult

to accomplish, particularly with large machines. If we assume, conservatively, that 300 m/s is the maximum tip speed, a current pulse 30 μ s wide would require a pole-piece width of ~ 1 cm. Such widths imply rather narrow air gaps, and consequently expensive construction. The technology is probably within the state of the art. Rotating machinery has traditionally been reliable in steady-state service. A 1-MW, 10 kHz inductor-alternator generator has been successfully operated by Westinghouse (cost $\sim$ \$50/kW) [33].

Output Voltage. The output voltage required from the generator is approximately

$$V = \mathcal{L}_T \frac{dI}{dt} \quad . \quad (\text{III-14})$$

Assuming $I(t) = I(t/\tau_r)$, $0 < t < \tau_r$ (i.e. for a trapezoid) then $dI/dt = I/\tau_r$. For $I = 20.7$ MA, $\tau_r = 10$ μ s, and $\mathcal{L}_T = 14$ nH (11 nH for the coil inductance and a transfer efficiency of 80%), we find $V = 30$ kV (B).

Shock-Heating Requirements. For a linearly rising magnetic field, the contribution to E_θ from the compression field alone is given by Eq. (II-25), Appendix II:

$$E_\theta = \frac{-10^{-8}}{2} \quad r \quad \frac{B_o}{\tau_r} \quad (\text{III-15})$$

$$= 0.38 \text{ kV/cm (B)}$$

assuming $r = 5$ cm, $B_o = 150$ kG, and $\tau_r = 10$ μ s. The shock-heating coil therefore has to supply $(1.0 \text{ kV/cm} - 0.38 \text{ kV/cm}) = 0.62 \text{ kV/cm}$ (a total of 33 kV) at a radius of 6.5 cm in design B - a value comparable to present-day theta pinches without separate shock coils. Thus the shock-heating requirements are mild.

Power-Factor Considerations. The largely inductive load means that a large fraction of the power transferred per cycle (4500 MVA in this design) will be returned to the power supply. When the power supply is a generator, the load-current buildup occurs at the expense of shaft power, most of which is returned during the current-decay phase. In order to keep large reactive-power oscillations from the electrical grid, an energy storage system such as a mechanical flywheel may be used [42]. In order to have synchronous operation, the stored energy in the flywheel should greatly exceed the energy transferred per cycle to the compression coil. In Point Design B, speed constancy of the generator to within $\sim 1\%$ requires a flywheel energy storage of $\sim 100 E_M = 240$ MJ, which is not difficult. With this scheme the electrical power delivered to the motor-generator-flywheel set need not exceed 300 MW.

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TABLE I

LTPF PARAMETERS WHICH ARE COMMON TO BOTH POINT DESIGNS (A AND B)

<u>Quantity</u>	<u>Symbol</u>	<u>Value</u>
Beta	β	1
Ion temperature	T_i (or T)	5.0 keV
Compression Coil Radius	b_2	7.0 cm
Shock-Coil Radius	b_{SH}	6.5 cm
Test-Sample Radius	b_s	5.5 cm
Discharge Tube Radius	b_1	5.0 cm
Electric Field	E_θ	1.0 kV/cm
Quality Factor	Q	15
Power Consumption	P	300 MW
End-Loss Coefficient	α	0.5
Maximum Magnetic Field	B_{max}	300 kG

TABLE II

LTPF PARAMETERS FOR POINT DESIGN A
(CONTINUOUS SINUSOIDAL WAVEFORM)THETA-PINCH PARAMETERS

<u>Quantity</u>	<u>Symbol</u>	<u>Value</u>
Duty factor	τ_c/τ_b	3
Mirror magnetic field	$B_{\max}$	300 kG
Unmirrored magnetic field	B_θ	57.7 kG
Mirror ratio	R	5.2
Burn time	τ_b	10 μ s
Cycle time	τ_c	30 μ s
Rise time	τ_r	10 μ s
Cycle frequency	f	33 kHz
T_e/T_i	λ	0.040
Electron temperature	T_e	200 eV
Stored magnetic energy	E_M	135 kJ
Plasma length	L	66.5 cm
Plasma radius	a	1.32 cm
Plasma radius/gyroradius	a/r_L	4.67
Ion density	n	$1.57 \times 10^{16} \text{ cm}^{-3}$
Lawson Parameter	$n\tau_b$	$1.57 \times 10^{11} \text{ cm}^{-3}\text{-s}$

NEUTRONICS PARAMETERS

<u>Quantity</u>	<u>Symbol</u>	<u>Value</u>
Average primary neutron current	$\langle J \rangle$	$4.78 \times 10^{13} \text{ n/cm}^2\text{-s}$
Average primary neutron flux	$\langle \phi \rangle \sim 2 \langle J \rangle$	$9.56 \times 10^{13} \text{ n/cm}^2\text{-s}$
Average total neutron flux	$\langle \phi \rangle_T \sim 10 \langle \phi \rangle$	$9.56 \times 10^{14} \text{ n/cm}^2\text{-s}$
Instantaneous total neutron flux	$\phi_T = (\tau_c/\tau_b) \langle \phi \rangle_T$	$2.87 \times 10^{15} \text{ n/cm}^2\text{-s}$
Primary neutron fluence/pulse	$F_p = (\tau_c) \langle J \rangle$	$1.43 \times 10^9 \text{ n/cm}^2\text{/pulse}$
Primary neutron fluence/day	$F_d = 86,400^a \langle J \rangle$	$4.13 \times 10^{18} \text{ n/cm}^2\text{/day}$
Primary neutron fluence/yr.	$F_y = (3.154 \times 10^{7b}) \langle J \rangle$	$1.51 \times 10^{21} \text{ n/cm}^2\text{/yr}$
Primary neutron source strength	$S = (2\pi b_s L) \langle J \rangle$	$1.10 \times 10^{17} \text{ n/s}$

^a number of seconds per day^b number of seconds per year

TABLE III

LTPF PARAMETERS FOR POINT DESIGN B
(INTERMITTENT TRAPEZOIDAL WAVEFORM)THETA-PINCH PARAMETERS

<u>Quantity</u>	<u>Symbol</u>	<u>Value</u>
Duty factor	τ_c/τ_b	53
Minor magnetic field	$B_{\max}$	300 kG
Unmirrored magnetic field	B_o	150 kG
Mirror ratio	R	2
Burn time	τ_b	10 μ s
Cycle time	τ_c	530 μ s
Rise time	τ_r	10 μ s
Cycle frequency	f	1.9 kHz
T_e/T_i	λ	0.122
Electron temperature	T_e	610 eV
Stored magnetic energy	E_M	2.39 MJ
Plasma length	L	173 cm
Plasma radius	a	7.86 cm
Plasma radius/gyroradius	a/r_L	7.9
Ion density	n	$9.87 \times 10^{16} \text{ cm}^{-3}$
Lawson parameter	$n\tau_b$	$9.87 \times 10^{11} \text{ cm}^{-3}\text{s}$

NEUTRONICS PARAMETERS

<u>Quantity</u>	<u>Symbol</u>	<u>Value</u>
Average primary neutron current	$\langle J \rangle$	$4.57 \times 10^{13} \text{ n/cm}^2\text{-s}$
Average primary neutron flux	$\langle \phi \rangle \sim 2 \langle J \rangle$	$9.14 \times 10^{13} \text{ n/cm}^2\text{-s}$
Average total neutron flux	$\langle \phi \rangle_T \sim 10 \langle \phi \rangle$	$9.14 \times 10^{14} \text{ n/cm}^2\text{-s}$
Instantaneous total neutron flux	$\phi_T = (\tau_c/\tau_b) \langle \phi \rangle_T$	$4.84 \times 10^{16} \text{ n/cm}^2\text{-s}$
Primary neutron fluence/pulse	$F_p = (\tau_c) \langle J \rangle$	$2.42 \times 10^9 \text{ n/cm}^2\text{/pulse}$
Primary neutron fluence/day	$F_d = 86,400^a \langle J \rangle$	$3.95 \times 10^{18} \text{ n/cm}^2\text{/day}$
Primary neutron fluence/yr	$F_y = (3.154 \times 10^7)^b \langle J \rangle$	$1.44 \times 10^{21} \text{ n/cm}^2\text{/yr}$
Primary neutron source strength	$S = (2\pi b_s L) \langle J \rangle$	$2.73 \times 10^{17} \text{ n/s}$

^a number of seconds per day^b number of seconds per year

Fig. 1. Linear theta-pinch geometry.

Fig. 2. Photograph of the Scylla IV-3 device, a 3-meter linear theta pinch at LASL.

Fig. 3. Cross-sectional view of LTPF, showing test sample location and shock-heating coil.

Fig. 4 Curves of $\bar{\sigma}v(T)$ for D-T, D-D (total) and D-H³_e fusion reactions [taken from Ref. 10].

Fig. 5. Temperature dependence, $F(T)$, of the 14-MeV neutron current through a test sample located at radius b_s .

Fig. 6. Two possible sinusoidal waveforms: (a) $B(t) = B_0 \sin \omega t$; (b) $B(t) = B_0(1 + \sin \omega t)/2$.

Fig. 7. Yield points of some possible coil materials as a function of magnetic field strength.

Fig. 8. Illustrating an intermittent trapezoidal magnetic field waveform.