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ANALYTICAL CORRELATIONS OF:
PRCF BATCH CORE FUELLED EXPERIMENTS

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Page 3	Line 9	. . . system with the lowest boron concentration in the moderator.
	Line 15 & 16	change "overestimation of k_{eff} " to ". . . error in k_{eff} ".
Page 4	Line 3	". . . size and boron concentration."
Page 5	Footnote	change "molecules" to "gms".
Page 10	Line 24	change "Anillary" to "ancillary"
Page 12	Line 4	change "AUXILLARY" to "AUXILIARY"
Page 15	Line 13	change "28" to "7"
Page 31	Line 25 & 26	change "contributes to overestimating k_{eff} . . ." to ". . . . contributes to the error in k_{eff} . . ."
Page 34	Footnote	change "Matsura" to "Matsuura"

Appendix II

Page 3 Table IV (cont)

Experimental System

	Line 17	add "the" at the end of the line
	Line 18	change "remained" to "remaining"
	Line 19	change "remained" to "remaining"
	Line 22	change "remained" to "remaining"
Page 9	Line 5	change "residual" to "resultant"
Page 11	Line 1	change "spectrum" to "spectra"
	Line 6	". . . . through the cell".
Page 27	Line 7	change "orginal" to "original"
Page 30	Line 12	change "dencity" to "density"
Page 38	Line 7	delete "the" from the line
	Line 10	change "essenticallly" to "essentially"
	Line 15	change "group" to "groups"

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Reactor Technology
Special Distribution

ANALYTICAL CORRELATIONS OF:
PRCF BATCH CORE FUELLED EXPERIMENTS

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M. A. Mannan, S. Matsuura and D. L. Prezbindowski

ABSTRACT

A series of three critical experiments containing 2 wt.% plutonium enriched fuel has been performed in the Plutonium Recycle Critical Facility. These experiments were performed to provide information on some of the physics characteristics of the batch core. Analytical Correlations were made with the data **from** these experiments in order to test the accuracy of calculational methods in their application to plutonium enriched reactor cores. Two computational systems called HTH and **HAMMER** were used in the study. Significant differences in HTH and **HAMMER** calculated results were noted and these differences were determined to result mainly from different epithermal cross-section libraries. The results of this study do not indicate a clear-cut superiority of either HTH or **HAMMER** for plutonium fuelled cores.

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ANALYTICAL CORRELATIONS OF:
PRCF BATCH CORE FUELLED EXPERIMENTS

by

M. A. Mannan, S. Matsuura and D. L. Prezbindowski

I. INTRODUCTION

The Batch Core Experiment (1,2), conducted in the Plutonium Recycle Test Reactor (PRTR) is important with regard to the utilization of plutonium in thermal reactors. The experiment is providing detailed information on the materials and neutronic characteristics of plutonium fuels as a function of irradiation. The experiment is the only irradiation experiment which has been or is being conducted with a full core of plutonium mixed-oxide fuel. Therefore, the reactor behavior is primarily a function of changes occurring in the plutonium fuel. Moreover, this experiment is unique among plutonium utilization experiments in that a 55 element batch core loading was irradiated without extraneous perturbations being made to the core. Moderator level and dissolved boron rather than lumped poisons or control rods were used to control the reactor. Experimental information on the neutronic properties of this core is being gathered in a systematic and detailed manner. Thus this experiment provides a unique opportunity to verify the adequacy of analytical reactor physics design techniques and procedures.

A study of correlating analytical and measured results has begun starting with the experiments, conducted in the Plutonium Recycle Critical Facility (PRCF), which were performed to provide information on some of the physics characteristics of the batch core. (3)

The principal objective of the theoretical analysis of the **PRCF** experiments is to determine the limitations of certain calculational methods in their application to predicting the neutronic behavior of the PRTR Batch Core Experiment. This report presents the results of theory-experiment correlations and the conclusions drawn from these results. A summary of the conclusions of the work is given in Section II. A brief description of the experiments conducted in the **PRCF** is given in Section III. The analytical methodology employed in the study is given in Section IV. The comparison of calculated and experimental results is discussed in Section V. The details of the calculational model are given in Appendix I. The source and effect of analysis errors is discussed in some detail in Appendix II.

II. SUMMARY AND CONCLUSIONS

Calculations were performed to obtain reactor neutronic parameters for comparison with measured values. The reactor systems were UO_2 -2wt % PuO_2 fueled and D_2O moderated. Two computational systems called HTH and HAMMER were used in the study. The comparison between measured results and values calculated using HTH and HAMMER showed discrepancies. Significant differences in HTH and HAMMER calculated results were noted and these differences stem mainly from different epithermal cross section libraries. The calculated values of k_{eff} are higher than measured values by $\sim 0.3\%$ to 1.9% with the largest discrepancy occurring for the highest leakage system. There is a 10% difference between calculated and measured cell spatial lutetium activations in the third fuel ring of the cluster. However, the discrepancy is not large enough to account totally for the discrepancies noted in k_{eff} .

The calculated and experimental reactor flux distributions are in substantial disagreement in the reflector regions. The overestimation of calculated thermal fluxes in the reflector regions contributes to the overestimation of k_{eff} . The calculated values of the moderator temperature coefficient of reactivity are consistently lower than experimental values by about 0.04 milli-k/ $^{\circ}\text{C}$. A sizeable discrepancy (10-50%) occurs between calculated and measured values of the moderator level coefficient of reactivity. It is speculated that this could be due to neglecting the effects of hardware just above the fuel in the calculations. The calculated and measured values of the boron coefficient of reactivity agree to within the uncertainty of the measurement. The calculated worth of a peripheral fuel element is lower than measurement by about 10% for both the 7 and 13 element assemblies.

The calculated values of β/ℓ for the 7, 13, and 28 element cores are lower than the experimental values in all cases, but the correlation improves with increasing core size. Non asymptotic spectrum effects could not alone account for the discrepancy in β/ℓ .

It is evident from this study that it is difficult to attribute superiority for accuracy to either HTH or HAMMER. Moreover the degree of heterogeneity of the experiments is such that the measured data are limited in their applicability as a test of these methods because of the problems in precisely modeling the assemblies. The comparisons also show that agreement in calculated integral parameters such as k_{eff} can result even though large differences exist in the components of k_{eff} . Perhaps much more about the adequacy of these calculational methods would have been learned if the basic cross sections used in HTH and HAMMER were identical. This action is recommended in any future studies such as these.

III. BRIEF DESCRIPTION OF THE PRCF EXPERIMENTS

A. Physical Description

The PRCF critical assemblies consist of the 19 rod cluster fuel elements shown in Figure I. The average cluster rod contains 1549.5 grams of natural uranium and 31.56 grams of plutonium. The plutonium contains 90.62/8.12/1.20/0.06 weight percent of $^{239}\text{Pu}/^{240}\text{Pu}/^{241}\text{Pu}/^{242}\text{Pu}$, respectively. ⁽³⁾ The fuel cladding is made of Zircalloy. The clusters are arranged along with control elements in the 7, 13 and 28 element assemblies on an 8 in. hexagonal pitch as shown in Figure II. Each fuel element is enclosed in an aluminum thimble. The purity of the coolant and moderator are 94% and 99.4% D_2O , respectively. Boric acid enriched in B_{10} was added to the moderator to control excess reactivity. The boron-10 concentrations for the 7, 13 and 28 element assemblies are 0.93, 4.70, and 11.25 parts per million by weight (wppm) of D_2O .^{*} Above the critical assemblies is an aluminum cover plate which varies from 3/4 in. to 2 in. thick. A detailed description of these assemblies is given in Reference 3.

B. Measurements

Experimental values of critical boron content, the worth of loss of the central assembly coolant, and moderator level, boron, and temperature coefficients of reactivity were obtained for all 3 critical assemblies (i.e., 7, 13 and 28 element cores). Measurements of the worth of a peripheral fuel element, and measurements of $\beta_{\text{eff}}/\lambda$ were made for the 7 and 13 element cores. Spatial activation measurements were made in the central cell of the 28 element assembly. Measurements of the activation of gold foils were made in the axial and radial directions for the 7 and 13 element cores and these activations were used to deduce bucklings values.

*weight parts per million (gms boron/ 10^6 molecules of D_2O)

FIGURE I

PRTR Batch Core 19-Rod Cluster Fuel Element

Horizontal Traverse
Mechanism (HTM)

FIGURE II
PRCF Core Geometry

- Initial Critical Loading (6)
- ⊕ Added to Form 7 Element Core
- ▨ Added to Form 13 Element Core
- ▩ Added to Form 28 Element Core
- ⊖ Shutter-Type Cadmium Control Rods
- ⊞ Cadmium Safety Rods
- Vertical Traverse Position
F1
- Cell Traverse Position
F2

IV. ANALYSIS METHODOLOGY

One of the objectives of the Batch Core Experiment is to provide data to test the adequacy of analytical methods in their application to the design of plutonium fueled commercial power reactors. Thus, the approach used in this study was to separate, whenever possible, those problems which are a direct consequence of the use of plutonium as a fuel.

The analytical tools selected for the study are those which have been utilized in analyses and design of both H_2O and D_2O moderated systems containing both uranium and plutonium fuels. The analysis procedures utilized are the standard techniques of multi-group transport theory cell and reflector spectra calculations, reactor cell homogenization, and a few-group two region calculation of reactor flux shapes and multiplication factors using diffusion theory. It was determined that these standard techniques are only partially successful in application to the high leakage cores.

A. Computational Schemes

The two computational schemes used in the analysis of the PRCF experiments are shown in Figure III. The scheme on the left is the HAMMER system from Savannah River Laboratory. ⁽⁴⁾ The scheme on the right is the HTH ⁽⁵⁾ system from BNW, with the additional options of EXTERMINATOR ⁽⁶⁾ to perform 2-D diffusion theory calculations and KINPAR ⁽⁷⁾ to perform kinetics calculations. Although a detailed description of the HAMMER code has been published, a similar description of the HTH system has not; so, for convenience, and in order to explain the approximations and options used in the codes, a brief description of the physics models is given here.

FIGURE III
Computational Schemes used in the Analysis

Scheme Number 1 - HAMMER

HAMMER. HAMMER is primarily a combination of the HAMLET, THERMOS⁽⁸⁾ and FLOG codes. The HAMLET and THERMOS codes calculate multigroup spectra and provide group average (four groups) constants to FLOG, which performs a one-dimensional, multiregion reactor calculation.

HAMLET solves the multigroup Fourier transformed slowing down equations using the B_1 or P_1 approximation to calculate the leakage effects. Heterogeneous calculations of ϵ and p are computed with collision probabilities calculated under the assumption of cosine currents. ZUT and TUZ⁽⁹⁾ are used to calculate the resonance integrals. Some of the other approximations in the code include using the GREULING-GOERTZEL method for energy transfer, neglect of interference between close resonances and resonance overlapping effects, and the narrow resonance (NR) approximation in the moderator. Leakage corrected epithermal cross sections are calculated using an input total buckling.

THERMOS numerically produces a space and energy dependent solution of the integral transport equation. These solutions can be produced for slabs or annular geometry using reflecting boundary conditions. The THERMOS collision probabilities are calculated using the ray tracing technique. The energy dependent flux is calculated in 30 energy groups with the high energy cutoff at 0.625 eV for the HAMMER calculations.

FLOG is a modification of the FOG⁽¹⁰⁾ code and is a one-dimensional, 4-group, multiregion, diffusion theory code. Downscattering is restricted to the next immediate energy group only.

2. Scheme Number 2 - HTH Plus Anillary Methods.

HTH. HTH is a combination of the HRG⁽¹¹⁾, BATTELLE-REVISED-THERMOS⁽¹²⁾ and HFN⁽¹³⁾ Codes. The HRG and BATTELLE-REVISED-THERMOS codes are used to

calculate multigroup spectra and provide the group constants for HFN to perform a reactor calculation.

HRG is a Hanford Revised Version of the GAM-I Code ⁽¹⁴⁾, and solves the slaving down equations in the B_1 or P_1 approximation in 68 fine energy groups. Full group to group scattering matrices are given for few group output. The resonance integral calculations are based upon the methods developed by Adler, Nordheim and Hinman. ^(14.A) The narrow resonance (NR) approximation is used in the moderator region and the NR or Infinite Mass (IM) approximations are made in the fuel region.* The escape probabilities for the resonance calculation are calculated assuming an isolated absorber lump with DANCOFF corrections applied for shadowing effects. Therefore, the 19 rod cluster was approximated by a single rod for the resonance calculation. The mean chord length in the cluster was calculated by assuming an effective rod surface equal to 1.23 times the rubber band surface of the cluster, which is the prescription developed by Pershagen, Anderson and Carlvik ⁽¹⁵⁾. Leakage corrected epithermal cross sections are calculated using an input total buckling.

The BATTELLE-REVISED-THERMOS code performs the same function in HHH as THERMOS does in HAMMER and they are very similar codes. The main differences in problem size between BATTELLE-REVISED-THERMOS and THERMOS (HAMMER) are , that the former has a maximum of 30 space points and 8 mixtures whereas the latter has a maximum of 20 space points and 5 mixtures. The main differences in the method of computation are that BATTELLE-REVISED-THEWS uses information from the transport kernel directly to calculate the diffusion coefficient rather than the flux gradient and is capable of treating void regions. The thermal energy cutoff used in BATTELLE-REVISED-THERMOS is at 0.683 eV, whereas it is at 0.625 eV in THERMOS (HAMMER).

* A more recent version of the code removes some of these approximations.

The HFN Code is a one-dimensional, multigroup, multiregion diffusion theory code. It accepts full group to group transfer matrices and computes both direct and adjoint fluxes.

AUXILLARY MEIHODS Other analytical methods were utilized to evaluate specific effects. The EXTERMINATOR code was used to evaluate the effects of assuming one dimensional theory solutions of the reactor problems. The KINPAR code was used to calculate the kinetics parameters β_{eff} and λ to compare with measured values.

The EXTERMINATOR code is a two dimensional multigroup, multiregion diffusion theory code. Direct and adjoint fluxes, activation traverses, effective delayed neutron fraction β_{eff} , average prompt neutron lifetime λ , and perturbation coefficients can be claculated using this code.

The KINPAR code ~~was~~ developed specifically for calculating the ratio of the effective delayed neutron fraction, β_{eff} , to the average prompt neutron lifetime, λ . The fluxes and their adjoints are taken from the 1-D diffusion code HFN. KINPAR evaluates the following perturbation theory expressions for β_{eff} and λ :

$$\beta_{\text{eff}} = \frac{\sum_{m=1}^M \int_V \left[\sum_{j=1}^J (\nu \Sigma_f)_j^m \phi_j \right] \cdot \left[\beta^m \cdot \sum_{j=1}^J x_j^D \phi_j^* \right] dv}{\sum_{m=1}^M \int_V \left[\sum_{j=1}^J (\nu \Sigma_f)_j^m \phi_j \right] \cdot \left[\beta^m \cdot \sum_{j=1}^J x_j^D \phi_j^* + (1-\beta^m) \cdot \sum_{j=1}^J x_j^P \phi_j^* \right] dv},$$

and

$$\ell = \frac{\int_V \sum_{j=1}^J \left(\frac{\phi_j^* \phi_j}{V_j} \right) dv}{\sum_{m=1}^M \int_V \left[\sum_{j=1}^J (\nu \Sigma_f)_j^m \phi_j \right] \left[\sum_{j=1}^J x_j^P \phi_j^* \right] dv}$$

where

β_{eff} = total effective delayed neutron fraction

M = total number of fissionable species present

$\nu \Sigma_f^m$ = macroscopic nu sigma fission for energy group j for material m

ϕ_j = real flux for energy group j

ϕ_j^* = adjoint flux for energy group j

x_j^D = fraction of delayed fission neutrons emitted into energy group j

x_j^P = fraction of prompt fission neutrons emitted into energy group j

β^m = total fraction of delayed fission neutrons emitted by material m

V_j = average velocity of neutrons in j energy group

J = number of energy groups for which fluxes are to be calculated

$\int_V$ = integration over volume of entire reactor.

Since the systems under study contain a contribution of delayed neutrons resulting from the photoneutron effect in D_2O , the value of β^m for U^{235} and Pu^{239} is increased by the value γ , called the photoneutron production efficiency; hence

$$\beta_{D_2O}^{235} \text{ system} = \beta^{235} (1+\gamma), \text{ etc.} \quad (3)$$

where γ is defined to be the ratio of photoneutron production in the given reactor to the number produced in a pure D_2O system containing a point source. (16)
The value was taken as 0.22. (17)

B. Special problems in the Analysis

While the PRCF experiments were performed using the same fuel and lattice pitch as in the PRIR BATCH CORE, there were some special considerations that had to be given to the analysis of the PRCF experiments. As a result, care had to be exercised in drawing conclusions concerning the adequacy of the HAMMER and HTH methods when applied in a straight forward manner to correlating the PRCF measurements.

1. Control

As indicated in Figure II, the PRCF has positions for 3 control rods and 3 safety rods. The three safety rods were not inserted for the experiments which are analyzed in this report. The control rods are of the shutter type so that the control rod assembly has a considerable negative worth even when the rods were "fully withdrawn." In the analysis of these experiments, the effects of the control rods were not calculated due to the inherent difficulty in calculating a shutter type element, and also because the control rod calculation would contribute little insight into the analysis of the PRIR batch core which is rodless. Instead, the PRCF experiments were calculated in a

rodless condition and the measured integral worth of the control rods **was** added to the experimental values of k_{eff} to provide values for comparison with calculation.

2. Leakage

The effectiveness of the reflector was not identical for all systems. The best reflected system was the 7 element core, and the poorest reflected system was the 28 element core. The major difference was the **top** reflector thickness: 10.75 in., 6.25 in., and 0.23 in., for the 7, 13, and 28 element cores, respectively. Another trouble area of considerable importance is the effect of leakage on the spectrum and the non circular core-reflector interface. This is an area of "non-asymptotic" spectra in which infinite medium or infinite repeating array cell calculations are not valid. Six cells of the 28 element assembly are at the core-reflector radial interface. Also, the axial leakage is difficult to calculate accurately in these assemblies due to the complicated hardware and geometries above and below the core and the axial asymmetry. Immediately below the active fuel in the 19 rod cluster is an insulating depleted UO_2 pellet and then a zirconium bottom bracket. At about the same height (7.75 in. above the calandria tank) is a 1/2 in. thick aluminum grid plate. Beyond the grid plate is D_2O reflector and aluminum thimble, and beneath the thimble is the calandria tank. The bottom of the active fuel is 9 in. above the calandria tank. Although there is some back scattering from the tank and its supporting material, a vacuum boundary condition was assumed at the reflector-calandria interface. Above the active fuel is a depleted UO_2 pellet and then a voided fission-gas plenum containing an inconel spring. Above the plenum is an upper aluminum grid plate and cover over the entire assembly. The D_2O moderator level for the standard 13

element case was at 40.34 in. above the active fuel or in the plenum region. However, D_2O moderator level accounts for only 80% of the cross sectional area above the active core. D_2O -filled aluminum-thimbles containing the fuel elements covers the other 20%. Back scatter from the D_2O filled aluminum-thimbles and aluminum cover plate were not represented in the 2-D diffusion theory calculation because of the complicated transport nature of the problem. A vacuum boundary condition was assumed at the D_2O moderator level. Axial bucklings derived from gold foil traverses were prescribed for the one dimensional diffusion theory calculations. The measured buckling is essentially the thermal buckling. The results of a calculation which are shown in Table I proved that the thermal leakage dominates the axial neutron leakage. Thus, the measured buckling was assumed to be an adequate prescription to account for these complicated effects.

3. Cluster Geometry

Another well known area of difficulty is the two dimensional nature of the fuel cluster and the necessity of defining an approximate model to represent the cluster. For the HAMMER calculation and the HTH calculation an annular or one-dimensional approximation to the 19 rod cluster is made. A section of the cell and the approximation are shown on Figure IV. The approximation was defined so that the cross sectional area of fuel and clad adjacent to the line connecting rod centers are transformed into annulae adjacent to the line. Coolant between lines connecting rod centers is similarly transformed into an included annulus. The annular approximation conserves the cross sectional area of fuel, clad and coolant between fuel ring center lines and also the cross sectional area of the moderator. As mentioned previously, the HRG portion of the HTH calculation assumes a pseudo one dimensional model in accounting for spatial effects.

TABLE I

CALCULATED GROUP DEPENDENT FRACTIONAL LEAKAGES
FOR THE THIRTEEN ELEMENT ASSEMBLY

<u>Group</u>	<u>Top Leakage</u>	<u>Bottom Leakage</u>	<u>Radial Leakage</u>	<u>Total</u>
1	0.0001585446	0.0000552798	0.0000043495	0.000218174
2	0.0000348281	0.0000120101	0.0000009507	0.0000477889
3	0.0002767196	0.0001165138	0.0000141706	0.000407404
4	0.0012037078	0.000720649	0.0003558150	0.002280172
5	0.0002266614	0.0001429321	0.0000770715	0.000446665
(Thermal) 6	<u>0.0038916831</u>	<u>0.004482825</u>	<u>0.015793105</u>	<u>0.024167613</u>
Total	<u>0.005792144</u>	<u>0.005302107</u>	<u>0.016245463</u>	<u>0.02756781</u>

FIGURE IV
Annular Approximation to the 19-Rod Cluster

V. RESULTS OF THE CORRELATION

Comparisons of calculated lattice cell parameters using HAMMER and HTH were made to gain some insight as to the causes of any difference in the results of the theory-experiment correlations.

A. Theory-Theory Correlations

1. Comparison of HTH and HAMMER Cell Parameters

The lattice cell parameters for two neutron energy groups (thermal and nonthermal) were calculated for all assemblies and are compared in Table II. Differences are noted between the parameters obtained from HTH and HAMMER. The most significant differences are in τ (~37%) and p (~7%). The effect of these differences in multiplication values is shown in Table III. As shown, the values of k_{∞} calculated using HAMMER are consistently lower than the HTH values. However, the effective multiplication values are closer because the differences in values of k_{∞} and the leakage parameters, namely, τ , compensate.

2. Comparison of Spectral Averaged Cross Sections

Additional insight into the causes of these differences can be gained by examining the spectrum averaged cross sections themselves and these quantities are compared in Tables IV and V for the fissile and fertile materials. In Table IV, the thermal values are compared and as shown the differences are slight. Since the same basic cross sections were used in the THERMOS portion of the HTH and HAMMER calculations, the differences are attributable to differences in the limits of the integrand (i.e., HAMMER = 0.625 eV, whereas HTH = 0.683 eV) and/or to differences between the internals of these codes. In Table V the epithermal average values are

TABLE II

COMPARISON OF THERMAL AND NONTHERMAL PARAMETERS

Assembly (# Elements)	Calculated Parameter									
	ηf_1		ηf_2		p_1		$\tau (\text{cm}^2)$		$L^2 (\text{cm}^2)$	
	HTH	HAMMER	HTH	HAMMER	HTH	HAMMER	HTH	HAMMER	HTH	HAMMER
7	0.6317	0.65278	1.6949	1.73070	0.7743	0.7244	179.8	138.0	73.00	73.260
13	0.6291	0.64498	1.6029	1.61670	0.7781	0.7290	172.6	128.0	68.43	69.570
28	0.6254	0.63040	1.4641	1.47724	0.7828	0.7395	168.8	121	62.37	65.387

where: $\eta f_1 = (\nu \bar{\Sigma}_f / \bar{\Sigma}_a)_1$ = nonthermal,

$$\tau = (D / \bar{\Sigma}_r)_1,$$

$$\eta f_2 = (\nu \bar{\Sigma}_f / \bar{\Sigma}_a)_2 = \text{thermal}$$

$$L^2 = (D / \bar{\Sigma}_a)_2.$$

$$p_1 = \left(\bar{\Sigma}_{s_1 \rightarrow 2} / \bar{\Sigma}_{r_1} \right)$$

TABLE III

COMPARISON OF MULTIPLICATION VALUES

Assembly (# Elements)	k_∞		Geometrical Buckling (M^{-2})	k_{eff}^{***}	
	HTH	HAMMER		HTH	HAMMER
7	1.4548	1.41767	30.70*	1.0225	1.0248
13	1.3868	1.35550	23.19**	1.0193	1.0208
28	1.2819	1.25740	14.15*	1.0110	1.0045

* Estimated Value

** Measured Value

*** 1D Calculation

TABLE IV

ONE GROUP THERMAL FISSION AND CAPTURE CROSS SECTIONS
FOR THE FUEL CELL WITH 4.64 wppm B₁₀ FROM HAMMER AND HTH

Material	HTH Spectral Average Cross Sections (barns)			HAMMER Spectral Average Cross Sections (barns)		
	$\bar{\sigma}_c$	$\bar{\sigma}_f$	$\bar{\alpha}$	$\bar{\sigma}_c$	$\bar{\sigma}_f$	$\bar{\alpha}$
²³⁵ U	63.89	368.34	0.17345	64.81	377.62	0.17346
²³⁸ U	1.8535	0.0		1.8366	0.0	
²³⁹ Pu	280.1	617.11	0.45388	283.27	625.19	0.45309
²⁴⁰ Pu	223.68	0.0		220.84	0.0	
²⁴¹ Pu	265.57	754.83	0.351827	270.199	767.44	0.35208
²⁴² Pu	20.331	0.0		20.332	0.0	

TABLE V

ONE GROUP EPITHERMAL FISSION AND CAPTURE CROSS SECTIONS
FOR THE FUEL CELL WITH 4.64 wppm B₁₀ FROM HAMMER AND HTH

Material	HTH			HAMMER		
	Spectral Average Cross Sections			Spectral Average Cross Sections		
	$\bar{\sigma}_c$	$\bar{\sigma}_f$	$\bar{\alpha}$	$\bar{\sigma}_c$	$\bar{\sigma}_f$	$\bar{\alpha}$
²³⁵ U	6.844	12.63	0.5417	6.953	13.80	0.5040
²³⁸ U	0.8046	0.04371		0.9567	0.07791	
²³⁹ Pu	5.872	11.50	0.5104	9.257	14.42	0.6418
²⁴⁰ Pu	97.67	0.2362		155.1	0.5253	
²⁴¹ Pu	3.459	18.24	0.1897	17.66	29.43	0.6001
²⁴² Pu	53.68	0.2266		53.38	0.4573	

compared. Here very large differences are noted for almost all isotopes and indicate the reason for the differences noted in p and ηf , in Table II. Such large differences in spectral average values of $\bar{\sigma}_c$ for ^{238}U and α for ^{239}Pu must be the result of different differential cross section data for these isotopes rather than differences between HRG and HAMMER theories. Thus the results of theory-experiment correlations presented in the following sections are not a strict comparison of HTH and HAMMER theory but rather a collective comparison of theory and cross sections.

B. Theory-Experiment Correlations

1, Lutetium Activation Traverses

The central cell in the 28-element assembly contained foils of ^{175}Lu and ^{176}Lu oxide in aluminum oxide. Measurements of the activation of bare and cadmium covered foils were made to determine the foil activation due to thermal neutrons. The foils in the fuel rods were oriented perpendicular to the fuel rod axis so that their activation is an average over the cross section area of the fuel. The foils in the moderator were oriented perpendicular to the fuel cell radius. The foil activations were corrected for flux depression and peaking by the method reported in Reference (18). Briefly, the activation of the foil, A_{foil} , is calculated from the formula:

$$(A_{\text{foil}} - A_0)/A_{\text{foil}} = t (\Sigma_{\text{medium}} - \Sigma_{\text{foil}}) \ln(L/t)$$

where A_0 is the foil activation that would occur if the flux in the foil were the same as the flux in the medium, L and t are the diameter and thickness of foil respectively, and Σ_{medium} and Σ_{foil} are macroscopic absorption cross sections of the medium and foil, respectively. This formula is valid under the restriction that $\lambda_0 \Sigma_{\text{fuel}}$ and $\lambda_0 \Sigma_{\text{medium}} > 1$, where λ_0 is the neutron track length in the body.

The activations were then normalized so that the foil activation in the cell central rod is unity. The calculated lutetium activations were generated using THERMOS with a white (isotropic return) boundary condition rather than a reflection condition. The lutetium activations obtained were then corrected for the curvature in the base flux and normalized so that the average activation in the cell central rod is unity. The calculated and experimental lutetium 176 and 175 activations are shown on Figures V and VI, respectively.

FIGURE V
PRCF Fuel Cell LU176 Activation Traverse

FIGURE VI
PRCF Fuel Cell LU175 Activation Traverse

The calculated and experimental lutetium 176 activations correlate quite well except in the third fuel ring where the calculated activations are about 10-15% higher. The uncertainty in the experimental activations has been calculated to be $\pm 3\%$ (Reference 3), so that the discrepancy between calculation and experiment in the third fuel ring in Figures V and VI is probably significant while the discrepancy (24%) in the moderator of Figure VI probably is not. This discrepancy in the third fuel ring would contribute to the generally high k_{eff} calculated in this study. A comparison of the ^{175}Lu and ^{176}Lu activation results helps to separate the error in energy from that in space of the thermal flux calculation. Lu-176 has a resonance at 0.142 eV so that its activation is sensitive to errors in the energy dependent flux. Lu-175 has essentially a $1/v$ cross section so that it is insensitive to errors in the energy dependent flux. The fact that the ^{176}Lu and ^{175}Lu activations exhibit the same kinds of discrepancies indicates that they are caused by errors in the spatial dependence rather than the energy dependence of the flux. Additional evidence from the results of Monte Carlo calculations⁽¹⁹⁾ for the PRCF 28-element central cell support this conclusion and point toward the cause of the discrepancy being the annular approximations made, (i.e., limited to one dimension).

2. Radial and Axial Power Distributions

Experimental and calculated radial and axial flux distributions are compared on Figures VII and VIII for the 13 element core. Gold foils were used to obtain the experimental activation. No depression or peaking corrections were applied to the experimental values. The infinite dilute radial and axial gold activations were calculated using EXTERMINATOR with the few-group constants calculated using HTH.

With the calculated and experimental power distributions normalized at the peak power point, the calculated values agree to within ~6% over most of the active fuel region of the core. The greatest differences in axial and radial traverse correlation exist in the reflector region where the calculated values are much higher (~60%) than experiment. There is also an asymmetry difference between measured and calculated values in the axial directions. Above the core height center line the measured points are consistently above the calculated points. The reverse is evident below the core height center-line. It is difficult to determine what these differences mean in terms of neutron leakage; however, multiplication discrepancies could be caused by these differences.

There are a number of factors both experimental and analytical which could contribute to the observed discrepancies. These are:

- As noted in Figure VIII, the measured data does not fall on a smooth curve. As shown in Figure II, the radial foil holding tube passes near a control assembly and a safety rod guide tube. The absorption due to the safety rod guide tube is relatively smaller than that of control rod assembly. The absorption difference causes an asymmetry in the activation traverse in the reflector region. The calculated

FIGURE VII

Comparison of Calculated and Experimental Axial Activation Traverses
for 13-Element Core

FIGURE VIII
Comparison of Calculated and Experimental Radial Activation Traverses
for 13-Element Core

traverse was obtained by calculating a core without a control rod assembly and by homogenizing the safety rod guide tube with the reflector. Furthermore, the usual radial symmetry condition was assumed in the calculation. Kouts has reported⁽²⁰⁾ on results of other experiments conducted at Brookhaven which demonstrate the ambiguity in traverse measurements performed in heterogeneous rod assemblies perpendicular to the rod axis.

- The foil holding tube for the radial traverse was placed in the middle of the moderator region between fuel elements (Figure III). As a result the fluxes seen by detector foils within the active core region are not too different from the fluxes in the adjacent reflector, and the experimental traverse shows little peaking in the reflector. A homogeneous core with cell-averaged parameters is used in the calculation. Therefore, fluxes obtained are cell average fluxes. A simple correction for the heterogeneity effect was applied to the calculated flux traverses in the reflector region. The calculated values were multiplied by the ratio of the average thermal flux in the cell to the average thermal flux in the moderator of the cell obtained from the thermalization calculations. Figure VIII shows that the corrected activation decreases the discrepancy between the calculated and experimental traverses in the reflector region, but a substantial discrepancy still remains. The calculated axial traverse in the bottom reflector shows a peak which is not seen experimentally as was the case with radial reflector. This overestimation of calculated thermal fluxes in the reflector regions undoubtedly contributes to overestimating k_{eff} in the 7 and 13 element cores and indicates that the standard homogenization model is not appropriate for these small and highly heterogeneous assemblies.

- The moderator level used in the bare axial and radial foil irradiation was 2.25 inches higher than used in the just critical experiment.

This increased moderator level compensated the negative worth of the foils and foil holders. The calculated activation was obtained using fluxes from the standard critical experiment calculations. This difference in moderator level does not significantly affect the radial and axial activation rates.

3. Effective Multiplication Factors

The effective multiplication factor of a core is one of the most common and appropriate quantities to compare in any calculation-experiment correlation work. The values of k_{eff} for the 7, 13 and 28 element cores were calculated using different energy group structures. The multi-energy group structures used are shown diagrammatically in Figure IX. The 6 group structure is similar to the 4 group structure except that it has a group spanning the energy range of the delayed neutrons (2nd group) and a group spanning the energy range of the low lying plutonium resonances (5th group). The comparison of measured and calculated values is given in Table VI.

The measurements of k_{eff} had been carried out under the condition that all safety rods were fully withdrawn, and all control rods were in the full open position. These control rods were shutter type, which consisted of fixed parts and movable parts repeating every 6 inches. Even though they were in the full open position, about half of the total control rod worth was left in the core. The actual measured quantities were the moderator level heights, and the ^{10}B concentrations in the moderator. The calculations took into account the

FIGURE IX
Energy Group Structure

TABLE VI
COMPARISON OF EXPERIMENTAL AND CALCULATED k_{eff}

	<u>Number of Energy Groups</u>	<u>Effective Multiplication Factor</u>		
		<u>7 Element 0.93 wppm ^{10}B</u>	<u>13 Element 4.46 wppm ^{10}B</u>	<u>28 Element 11.25 wppm ^{10}B</u>
Experiment	--	1.009 1.011*	1.006	1.002 1.007*
Hammer (1-D)	4	1.0218	1.0208	1.0093
HTH (1-D)	4	--	1.0222	--
	6	1.0225	1.0193	1.0110
	17	1.0218	1.0190	--
Exterminator (2-D)	6	1.0297	1.0240	1.0133

*J. Lauby, S. Matsura April 16, 1969

moderator level, the ^{10}B concentration and the guide tubes for the safety rods, but did not represent the residual control rod worths. In order to make the experimental values of k_{eff} comparable to the calculated ones, the residual worth of all control rods was estimated from the experiments, and added to the experimental k_{eff} .

From the correlation in Table VI, the following points are indicated:

- All calculated values of k_{eff} are higher than the experimental values by the amount of about 0.3% to 1.9% $\Delta k/k$,
- These discrepancies are related to core size (i.e., largest for the 7 element core and smallest for the 28 element core).
- The results of the various 1D calculations agree with each other within $\pm 0.2\%$ $\Delta k/k$, in spite of the internal differences in the calculations and differences in the numbers of energy groups used.
- Among the calculated methods, the 2D calculation gives higher values of k_{eff} than the 1D calculation does.

4. Reactivity Coefficients

Temperature Coefficient - The temperature coefficient of reactivity of the 13 element core with 4.64 wppm B_{10} in the moderator was measured by increasing the core temperature in step increments from 20°C to 70°C. The temperature increases were accomplished by pumping the moderator through an external heater and back to the reactor tank. Sufficient time elapsed after each heating step to allow all of the core components to come to a nearly uniform temperature. The reactivity changes were inferred from the observed changes in the critical moderator height via the measured moderator level coefficient of reactivity.

The temperature coefficient for the 13 element core was calculated by the HAMMER code. Contributions to the temperature coefficient considered in the calculational model include: expansion of the aluminum grid plate, change in the density and temperature of the scattering kernel for D_2O , and Doppler broadening of the resonances for the uranium and plutonium isotopes. The k_{eff} was calculated for 20°, 60° and 90°C only. The temperature coefficient curve was determined by differing these values of k_{eff} .

The experimental and calculated results are compared in Figure X. The calculated results are consistently lower than the experimental values by about .04 mk/C°. The experimental value of the coefficient is first positive from 20 to 28° and then becomes negative.

Moderator Level Coefficient - The results of an experiment-theory correlation of the moderator level coefficient for the 13 element core are given in Figure XI. The calculations were performed using EXTERMINATOR with R-Z geometry. The sizeable discrepancy (10 ~ 50%) between calculation and experiment may be caused by a number of inadequacies in the calculational model. For example:

- The spectra used to average cross sections for the plenum and upper reflector regions are infinite medium spectra while the actual spectra near the top of the active fuel are non-asymptotic.
- a The proper calculation of the Moderator Level Coefficient depends upon a correct calculation of reaction rate near the moderator level. Diffusion theory is known to be inaccurate near boundaries and sources.

FIGURE X

Comparison of Calculated and Measured Temperature Coefficients of 13-Element Core

FIGURE XI
Moderator Level Coefficient of Reactivity
for 13-Element Core

- The streaming of neutrons from the top of the active fuel through the fission gas plenum containing an Inconel spring is a complicated neutron transport phenomenon. This phenomenon is not properly accounted for in the calculation of multigroup diffusion coefficients by THERMOS-HRG. The axial diffusion of neutrons in the plenum region is probably underestimated by THERMOS-HRG.

An important source of error is the neglect of back scattering from hardware, D₂O coolant, and a 3/4 in. to 3 in. thick aluminum plate which covers the PRCF. All of the calculations of moderator level coefficient were made assuming a vacuum boundary at the moderator level. The inclusion of the effect of back scattering would decrease the calculated values of moderator level coefficient and the discrepancy between calculation and experiment.

It is evident from the complicated nature of the axial neutron transport and the contribution of reflectors beyond the moderator level that the accurate calculation of the moderator level coefficient requires at least some correction for the effect of the complicated geometry and reflection of neutrons from the top cover plate and D₂O filled thimble.

Boron Coefficient - A comparison of experimental and calculated (by HAMMER) boron coefficient of reactivity for the 7, 13, and 28 element assemblies is shown in Figure XII. All of the calculated values lie within the uncertainty of the experimental points. This evidence suggests that HAMMER does an adequate job of calculating boron worth for these assemblies. This might be expected since the $1/v$ shape of σ_a for boron makes the calculation of boron worth independent of the flux spectrum.

FIGURE XII
Boron Coefficient of Reactivity

5. Worth of a Peripheral Fuel Element

The reactivity worth of the addition of a peripheral fuel element was measured for the 7 and 13 element cores with 0.93 and 4.64 wppm B_{10} in the moderator, respectively. The element reactivity worth was inferred from changes in the critical moderator height and/or boron concentration via the measured curves of boron coefficient and moderator level coefficient of reactivity.

The calculated values of the peripheral fuel element worth were obtained using HAMMER. The volume of one additional fuel cell was added as an annulus to the basic homogeneized core. The cell constants of the added cell were the same as for the basic core. The difference between the effective multiplication factors of these cores is taken as the worth of the element. A comparison of calculated and experimental values is given in Table VII. The calculated result for the 13 element case is 10.4% lower and falls within the experimental uncertainty, whereas the calculated result for the 7 element case is 11% lower and falls outside the experimental uncertainty.

6. Kinetic Parameters

The ratio of the average delayed neutron fraction to the average prompt neutron lifetime (β_{eff}/ℓ) was measured for the 7, 13 and 28 element cores with 0.93, 4.64 and 11.25 wppm B_{10} , respectively, in the moderator using the reactor noise technique. Signals from two channels (ionization chambers) were recorded on magnetic tapes for future cross correlation analysis. The present results represent the frequency analysis of a single channel only.

Values of β_{eff} and ℓ were calculated using KINPAR with direct and adjoint fluxes calculated with HFN. The delayed neutron fractions used in KINPAR for the plutonium and uranium isotopes are:

TABLE VII

**WORTH OF A
PERIPHERAL FUEL ELEMENT**

<u>Cluster Number</u>	^{10}B in D_2O <u>(wppm)</u>	Worth of a Cluster (milli-k)	
		<u>Measured</u>	<u>Calculated</u>
8	0.93	24.0 $\pm$ 1.0	21.35
14	4.64	12.0 $\pm$ 2.0	10.75

$$\begin{aligned}
\beta_{235} &= 0.0065 \\
\beta_{236} &= 0.0000 \\
\beta_{238} &= 0.0157 \\
\beta_{239} &= 0.0021 \\
\beta_{240} &= 0.0026 \\
\beta_{241} &= 0.0051 \\
\beta_{242} &= 0.0000
\end{aligned}$$

Values for β_{eff} and λ were calculated using the 6 and 17 energy group structure shown on Figure IX. The delayed neutrons fall in the energy range 235. to 490. KeV, which is the energy range of the 2nd group of the 6 group structure and the 4th group of the 17 group structure. The calculated and experimental results are shown in Table VIII. The calculated values of $\beta_{\text{eff}}/\lambda$ are lower than the measured values in all cases and the correlation improves with increasing core size. In the 28 element case the calculated value lies within the experimental uncertainty. In the 7 and 13 element case the calculated values are outside the experimental uncertainty. The main contributor to the error in the calculation of $\beta_{\text{eff}}/\lambda$ is the calculated neutron lifetime, since β_{eff} is relatively insensitive to changes in boron concentration and core size. The calculated 6 and 17 group values of $\beta_{\text{eff}}/\lambda$ agree to within 1% for the 13 element core. It appears that the 6 energy groups are adequate for the calculation of $\beta_{\text{eff}}/\lambda$.

The Effect on $\beta_{\text{eff}}/\lambda$ due to the Nonasymptotic Spectrum and the Change of Leakage Rate - The error in the calculated value of β/λ due to the nonasymptotic spectrum effect was estimated by utilizing a modified value of Σ_f in the expression used for calculating β/λ in KINPAR. The modified value of Σ_f was determined by calculations designed to approximate the non-asymptotic spectrum. The result for the 13 element core showed that the relative change of β/λ due to the transient spectrum was about 0.34%.

TABLE VIII

COMPARISON OF MEASURED AND CALCULATED
KINETIC PARAMETERS

B ¹⁰ CONCENTRATION (wppm)	NUMBER OF FUEL ELEMENTS	CALCULATED β/ℓ SEC ⁻¹				MEASURED ₁ β/ℓ SEC ⁻¹
		6 GROUPS		17 GROUPS		
		ℓ	β/ℓ	ℓ	β/ℓ	
0.93	7	6.947-04	4.67	7.018-04	4.63	5.0 ± 0.3
4.64	13	3.797-04	8.56	3.823-04	8.50	9.4 ± 0.5
11.25	28	3.233-04	12 57		----	12.9 ± 0.7

The neutron lifetime λ is essentially determined by the average neutron speed, the average cross-section and the nonleakage probability. The effect on λ due to the change of leakage rate of neutrons was estimated by the simple point reactor model. The resultant effect for the difference of leakage between the cylindricalized core and the assembly of the hexagonal cells was estimated to be about 0.3% to 1.0%. Therefore, the discrepancies between the measured and calculated value of β/λ cannot be explained in terms of the spectrum and leakage effects alone. These studies also indicate that the measurements of β/λ in small critical assemblies are much more applicable to large reactors than k_{eff} because β/λ is relatively insensitive to spectrum and leakage effects.

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APPENDIX I

DETAILS OF THE 7, 13 AND 28
ELEMENT CORE CALCULATIONAL MODELS

APPENDIX I. DETAILS OF THE 7, 13 AND 28 ELEMENT CORE CALCULATIONAL MODELS

A. INTRODUCTION

Included in this appendix is specific information regarding the calculational models used in the analysis. The dimensions, number densities, and other details provided are intended to enable the reader to duplicate the analyses performed, if comparable computer codes are used.

B. HOMOGENIZED CORE MODELS FOR THE 7, 13 AND 28 ELEMENT CORES

1. 7 Element - 1-D

<u>Region</u>	<u>Distance of Outer Boundary From Core Center (Cm)</u>	<u>Region Description</u>
A	28.227	Active Fuel Region, Fuel + Moderator + Coolant
B	46.505	Reflector Region, D ₂ O + Al + B ₁₀
C	91.44	Reflector Region, D ₂ O + B ₁₀

Homogenized Number Densities												
Isotope	U235	U238	Pu239	Pu240	Pu241	Pu242	H ₁	H ₂	O	Zr	Al	B ₁₀
Region												
A	1.00516 +00	1.0893 +00	2.0718 +00	2.0649 +00	3.0774 +00	1.0815 +00	0.0000 +00	5.70093 -02	3.0190 -02	7.0412 +00	2.0855 +00	14.95857 -08
B	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	6.3693 -02	3.1846 -02	0.0 +00	4.7419 -04	5.9294 -08
C	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	6.648 -02	3.324 -02	0.0 +00	0.0 +00	6.1889 -08

2. 13 Element - 1-D

<u>Region</u>	<u>Distance of Outer Boundary From Core Center(Cm)</u>	<u>Region Description</u>
A	38.4675	Active Fuel Region Fuel + Moderator + Coolant
B	46.5050	Reflector Region D ₂ O + Aa + B ₁₀
C	91.44	Reflector Region D ₂ O + B ₁₀

Homogenized Number Densities												
Isotope	U235	U238	Pu239	Pu240	Pu241	Pu242	H ₁	H ₂	O	Zr	Al	B ₁₀
Region												
A	1.0061 -05	1.3893 -03	2.5718 -05	2.2949 -06	3.3774 -07	1.6815 -08	0.0 +00	5.7469 -02	3.1590 -02	7.3112 -04	2.9855 -03	2.4737 -07
B	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	6.0906 -02	3.0453 -02	0.0 +00	9.4838 -04	2.8302 -07
C	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	6.648 -02	3.324 -02	0.0 +00	0.0 +00	3.089 -07

3. 13 Element 2-D

Homogenized Number Densities												
Isotope	U235	U238	Pu239	Pu240	Pu241	Pu242	H ₁	H ₂	O	Zr	Al	B ₁₀
Region												
A	1.0661 +00	1.0893 +00	2.0318 +00	2.0049 +00	3.0774 +00	1.0815 +00	0.0 +00	5.0269 -02	3.0890 -02	7.3112 -04	2.9855 -03	2.4737 -07
B	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	6.0906 -02	3.0453 -02	0.0 +00	9.0838 -04	2.0302 -07
C	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	6.648 -02	3.324 -02	0.0 +00	0.0 +00	3.089 -07
D	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	5.750 -02	2.8750 -02	8.4394 -04	2.9870 -03	2.4737 -07

Isotope	Cr	Ni
D	1.3213 -05	5.6978 -05

4. 28 Element - 1-D

<u>Region</u>	<u>Distance of Outer Boundary From Core Center (Cm)</u>	<u>Region Description</u>
A	28.227	Standard Fuel Region Fuel + Moderator + Coolant
B	59.4024	Modified Fuel Region Fuel + Moderator + Coolant + Absorber
C	91.44	Reflector Region D ₂ O - B ₁₀

Homogenized Number Densities												
<u>Isotope Region</u>	U235	U238	Pu239	Pu240	Pu241	Pu242	H ₁	H ₂	O	Zr	Al	B ₁₀
A	1.0061 -05	1.3893 1 - 03	2.5718 1 - 05	2.2949 -06	3.3774 -07	1.6815 -08	0.0 +00	5.7469 -02	3.159 -02	7.3112 -04	2.9855 -03	5.9977 -07
B	8.0638 -05	1.0356 -05	2.0503 -05	2.00680 -06	2.0352 -07	1.0813 -08	0.0 +00	5.0239 -02	3.0268 -02	6.0473 -04	2.0495 -03	5.0977 -07
C	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	0.0 +00	6.648 -02	3.324 -02	0.0 +00	0.0 +00	7.490 - 0 7

C. MIXTURES USED IN THE STANDARD
FUEL THERMOS CELL

Mixture:	Fuel	Clad	Coolant	Al Thimble	D ₂ O B ₁₀
Isotope					
H ₂	4.158-02	4.23-02	6.648-02	6.02-02	6.648-02
B					**
O			3.324-02		3.324-02
Al					***
Zr					
U235	1.4654-04				
U238	2.0234-02				
Pu239	3.7457-04				
Pu240	3.3424-05				
Pu241	4.9191-06				
Pu242	2.4491-07				

** = 6.1889-08, 3.089-07, and 7.4902-07 for the 7, 13 and 28 element cores.

*** = 2.8716-04 for the 28 element modified fuel; 0.0 for all other fuel.

Miscellaneous Information			
Core	Axial Buckling	Top Reflector Thickness	Temp.
7 Element	2.93-04 Cm ⁻²	27.3 Cm	20°C
13 Element	3.138-04 Cm ⁻²	15.88 Cm	20°C
28 Element	3.6893-04 Cm ⁻²	.58 Cm	20°C

APPENDIX II

INVESTIGATION OF SOURCE AND EFFECT OF ANALYSIS ERRORS

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INVESTIGATION OF THE EFFECT OF THE

APPENDIX II. INVESTIGATION OF SOURCE AND EFFECT OF ANALYSIS ERRORS

The discrepancies noted in this study between the calculated results and the experimental results are considered to have a combination of causes:

- Errors introduced by approximating the experimental system in order to satisfy the constraints of the calculational methods
- Errors due to the shortcomings of the theory utilized
- Errors due to the inherent uncertainty of the cross section data

The cross-section data used in any nuclear analysis have an implicit error due to the uncertainty of the experimental and analytical methods used to deduce their values. The effect and extent of the cross-section data uncertainty were not explicitly investigated in this study; nevertheless, its presence is not to be neglected.

The effect of the errors due to approximating the experimental system and due to theoretical deficiencies was investigated in some detail, and the results are presented in the following sections. In most cases the effect of the particular error investigated is related to the ultimate effect on the calculated values of k_{eff} .

A. Errors Due to the Approximation of the Experimental System

Uncertainties in physical data, such as dimensions, weights, etc., and the approximations utilized in modeling the experimental assembly are itemized in Table IX. Some estimates have been made of the errors in the correlation which resulted from these uncertainties and approximations. Some simple methods have been used in calculating the contribution of individual approximations to the total error.

One feasible method to test the validity of the cylindrical approximation for the 19 rod cluster fuel is to compare the reaction rates in the cylindricalized fuel element with those in the cluster fuel. This could be performed by comparing the calculated activation traverse or the power distribution in

TABLE IX
Uncertainties and Approximations in Experiment and Calculation

Item	Experimental System	Calculating Procedure
Fuel Element		
Construction	19 rods cluster with A ₂ thimble	Center rod and co-axial annuli
Fuel Material	Vibratory compacted oxide particles	Homogeneous oxide
Fuel Density	9.45 ± 0.09 gr/cm ³	9.322 gr/cm ³
Fuel Composition Pu/Pu + U	1.9962± wt. %	1.9961 w/o
239Pu	90.62 ± 0.08 wt. %	90.620
240Pu	8.12 ± 0.081 wt. %	8.120
241Pu	1.2 ± 0.0015 wt. %	1.200
Moderator		
Composition D ₂ O	99.75 ± 0.50 %	100% D ₂ O
B ¹⁰ (7 element core)	0.93 ± 0.05 WPPM	0.931 WPPM
" (13 element core)	4.64 ± 0.05 WPPM	4.647 WPPM
" (28 element core)	11.25 ± 0.05 WPPM	11.267 WPPM
Unit Cell		
Standard cell (core central region)	Hexagonally shaped cell	Cylindricalized cell with appropriate B ²
Peripheral cell (core boundary region)	Undefined and more moderated cell	Neglected
Core Configuration		
Plan view	Heterogeneous core bounded with Undefined core-reflector boundary	Cylindricalized homogeneous core Neglecting the unevenness of the core surface and the spectrum change at the core-reflector boundary.
Horizontal reflector thickness (poisoned D ₂ O)	63.2/cm ~ 32.04 cm ($\frac{ID}{2}$ of core tank - equivalent core radius)	63.21 cm ~ 32.04 cm

TABLE IX (contd)

Item	Experimental System	Calculating Procedure
Vertical view	Consisting of following regions from top to bottom. upper grid plate process tube containing D ₂ O region upper hardware region F.P. plenum region Depleted UO ₂ region PuO ₂ -UO ₂ region Depleted UO ₂ region lower hardware region process tube lower grid plate Moderator level changing through these regions Completely immersed	1 D Calculation Neglecting regionwise description and using vertical buckling obtained from gold foil traverse for all energy groups and all horizontal regions. 2 D Calculation Using three kinds of vertical regions plenum region PuO ₂ -UO ₂ region lower reflector region Neglecting anisotropic neutron diffusion in both cases.
Safety rods	Absorber: completely withdrawn Guide tube and void: left in core	Guide tube and void: homogenized to cell.
Control rods	Shutter type All measurements were done at full open position. The reactivity worths of remained part were measured for 13 element core. The remained rod worths of 7 and 28 element cores were estimated from the result of 13 element core. Estimated remained worth 7 element core: 11.0 ± 0.4 mk 13 element core: 5.6 ± 0.2 mk 28 element core: 7.0 ± 0.2 mk	Neglected

the cluster fuel with the experimental values. The comparison between the calculated and the measured activation traverse by Lu foil for the 28 element core has been discussed previously. A relatively higher calculated reaction rate in the outer fuel ring could partially explain the generally higher calculated values of k_{eff} .

There could be some difference of self-shielding characteristics between the homogeneous fuel material, as used in the calculation, and the vibratory compacted particle material, as used in the experiment, even if their smeared density is exactly same. No attempt to confirm the validity of the homogeneous fuel approximation has been made for the PRCF fuels.

Every component of the actual critical facility has some tolerance in its density, composition, content, geometrical dimension, etc., whereas every quantity in the calculation is definite. These tolerances of the components would bring some ambiguities into the atomic number densities of the homogenized regions of the calculational representation. Ultimately, these ambiguities contribute to the error in the calculated effective multiplication factor. The effect on k_{eff} caused by these tolerances in the important components were estimated for the 13 element core by using the results of calculations using HAMLET-THERMOS and EXTERMINATOR. In this estimation, only the ambiguities in the Σ_a and the $\nu\Sigma_f$ were taken into account. Those in D were not, because the diffusion coefficient is primarily dependent upon the density of D_2O , and is relatively insensitive to the densities of other components. In addition, the change in reactivity due to a given relative change of D is about one tenth that caused by an equal relative change of Σ_a and $\nu\Sigma_f$. The ambiguities in the Σ_a and $\nu\Sigma_f$ of a component were assumed linearly proportional to the atomic number density; in other words, the ambiguity $\Delta\Sigma_i$ was estimated as

$\Sigma_i = \Delta N_i / N$, where N_i was the cell averaged atomic density for isotope i , ΔN_i was the ambiguity of N_i , and Σ_i was the cell averaged macroscopic cross-section for isotope i . This assumption was not necessarily accurate because of the self shielding characteristics of Σ , but still it was considered sufficient to estimate the approximate amount of reactivity ambiguity. EXTERMINATOR gave the reactivity coefficient, $\Delta k_{\text{eff}} / k_{\text{eff}}$ AX. Then the reactivity ambiguity due to the tolerance of component was finally obtained as the product of $\Delta \Sigma$ and the reactivity coefficient. The results are shown in Table X.

B. Errors Due to the Shortcomings of the Theory

1. Buckling Prescription in Cell Codes

The constants used in the diffusion calculation were obtained by a cell calculation based on the assumption of an infinite repeating cell system. In this process, it was found that the buckling prescription in the cell codes was important to the calculation of k_{eff} . For example, it could be worth as much as 1% in k_{∞} for the fuel region in the 13 element core. Therefore, the total buckling, i.e., the summation of the horizontal and the vertical buckling, was prescribed for the calculation of the fuel region constants. The buckling prescription for the calculation of the reflector region constants was found to be not as important as it was for the fuel; therefore, infinite medium, zero leakage calculations were prescribed for the reflector constants.

2. Spatial Meshpoint Allocation

The next problem in a cell calculation is how to allocate spatial meshpoints within the limitations of the code. A comparison was performed using HAMMER on the 13 element core to find the effect in the cell spectrum, flux distribution, cross-section and multiplication factor due to the difference of mesh prescription. This cell consisted of PuO_2 - UO_2 mixed oxide fuel, zircalloy clad, pure heavy water coolant, aluminum thimble and poisoned heavy

TABLE X

UNCERTAINTY IN k_{eff} DUE TO TOLERANCE OF EXPERIMENTAL SET-UP

X-1: EFFECT OF UNCERTAINTY OF FUEL DENSITY (13 ELEMENT CORE)

Energy Group	$\Delta\Sigma_a$	$\frac{1}{K} \frac{\Delta K}{\Delta\Sigma_a}$	$\frac{\Delta K}{K}$	$\Delta v\Sigma_f$	$\frac{1}{K} \frac{\Delta K}{\Delta v\Sigma_f}$	$\frac{\Delta K}{K}$	Total
1	$\pm 1.149 \times 10^{-5}$	± 10.498	$\pm 1.207 \times 10^{-4}$	$\pm 1.822 \times 10^{-5}$	± 10.243	$\pm 1.865 \times 10^{-6}$	$\pm 0.658 \times 10^{-4}$
2	$\pm 5.083 \times 10^{-6}$	± 24.448	$\pm 1.245 \times 10^{-4}$	$\pm 1.942 \times 10^{-6}$	± 23.171	$\pm 4.49 \times 10^{-5}$	$\pm 0.796 \times 10^{-4}$
3	$\pm 3.214 \times 10^{-5}$	± 42.539	$\pm 1.369 \times 10^{-3}$	$\pm 1.898 \times 10^{-5}$	± 37.751	$\pm 7.16 \times 10^{-4}$	$\pm 0.653 \times 10^{-3}$
4	$\pm 1.247 \times 10^{-4}$	± 63.052	$\pm 7.860 \times 10^{-3}$	$\pm 2.009 \times 10^{-4}$	± 45.057	$\pm 9.05 \times 10^{-3}$	$\pm 1.19 \times 10^{-3}$
Total			$\pm 9.47 \times 10^{-3}$			$\pm 9.997 \times 10^{-3}$	$\pm 0.53 \times 10^{-3}$

The degree of uncertainty of other cores is considered to be about the same.

X-2: EFFECT OF UNCERTAINTY OF B^{10} CONCENTRATION

Core	Reactivity Co-effi- cient of B^{10}	Uncertainty in ΔB^{10}	Uncertainty in $\Delta K/K$
7 Elements	$-3.2 \times 10^{-3} \Delta K/KPPM$	± 0.05	$\pm 1.6 \times 10^{-3}$
13 Elements	-2.0×10^{-3}	± 0.05	$\pm 1.0 \times 10^{-3}$
28 Elements	-1.2×10^{-3}	± 0.05	$\pm 0.6 \times 10^{-3}$

X-3: TOTAL UNCERTAINTY OF MEASURED K_{eff} DUE TO THE AMBIGUITY
IN THE FUEL DENSITY, B^{10} CONCENTRATION, WATER LEVEL,
VERTICAL BUCKLING AND RESIDUAL RODS WORTH

Core	$\Delta K/K$
7 Elements	$\pm 2.0 \times 10^{-3}$
13 Elements	$\pm 1.5 \times 10^{-3}$
28 Elements	$\pm 1.3 \times 10^{-3}$

water moderator. One prescription used 17 meshpoints of the total 20 points inside the thimble. The fuel, the clad and the coolant were treated as separate regions. The other prescription used 15 meshpoints inside the aluminum thimble. The clad and the coolant were homogenized and treated as one region to save two meshpoints. The number of meshpoints in the fuel regions was identical for these two prescriptions. Three meshpoints were used in the former prescription and five in the latter for the moderator region.

This comparison indicated the following results:

- Differences of fast flux $\phi(u)$ ($u < 16.59$) at comparable meshpoints were less than 0.6% through the whole lethargy range;
- No significant difference was observed on the average thermal neutron velocity through the cell;
- Differences in $n(v)v$ in the fuel region were less than 1% for almost all velocity points, and differences of a few percent were observed at the extremely low velocity side and around the 0.3 ev resonance of ^{239}Pu ;
- In the moderator region, differences of about 2% in $n(v)v$ were observed over a rather wide velocity range around the modal velocity;
- Microscopic cross-sections of each fuel material in the thermal region did not differ by more than 0.1%;
- The largest difference in microscopic cross-section was observed for the clad, but the amount was just about 1%;
- Differences of cell microscopic cross-sections of each fuel material in thermal region were about 1.5%, in spite of the above mentioned small differences in the region microscopic cross-sections. This was essentially due to the difference in the integrated thermal flux in the moderator region and the method of determining all average microscopic cross-sections in THERMOS by volume integration over the entire cell;

- Differences of macroscopic cross-sections of other materials in the thermal region were too small to affect the multiplication factor;
- Differences of all macroscopic cross-sections in the epithermal and fast energy range were quite small;
- The residual difference of effective multiplication factor due to the different ways of mesh allocation was 3.3 mk.

3. Unit Cell Definition and Nonasymptotic Spectrum Effects

One of the most important considerations in a cell calculation is the definition of the unit cell of the lattice. Fuel elements are usually arranged to form a rectangular or a triangular lattice with a definite pitch, which is the distance from the center of a fuel element to that of the next. A unit cell is defined by this lattice pitch. This process assumes tacitly an infinitely repeated lattice.

This assumption is appropriate for cells sufficiently far from the reflector-core boundary. However, the unit cell at the core-reflector interface is not well defined. The fuel bundles on the boundary are surrounded by much more moderator than the interior fuel; therefore, the spectrum in these cells should be softer than that in central cells of the core. On the other hand, the spectrum in the reflector close to the fuel region should be rather harder than that in the reflector far from the fuel region, but there is no established method of representing the non asymptotic flux in these regions by the definition of a core-reflection boundary cell. The calculated results shown in Table VI do not include any effect due to this transient spectrum change at the core-reflector boundary. In these calculations, the cores were basically assumed to consist of a standard fuel cell region and an asymptotic reflector region (i.e. infinitely large reflector region). The transient

spectrum effects on k_{eff} were estimated by the following two methods:

- a) The core was assumed to consist of three regions, i.e. a standard fuel cell region, a boundary region and a asymptotic reflector region. The boundary region is composed of a mixture of standard fuel cells and a part of the reflector. Since it was not obvious how much reflector should be added to the fuel cell for the boundary region, a set of parametric cell calculations were performed with a varying reflector volume included in the boundary region. From the results of these calculations, the changes of spectrum, microscopic and macroscopic cross-sections, and other constants in the cell due to the change of moderator to fuel volume ratio were observed. The effect on the effective multiplication factor caused by introducing the boundary region was estimated by FLOG. (10)
- b) The core was assumed to consist of four regions, i.e. standard fuel cell region, boundary fuel cell region, boundary reflector region and asymptotic reflector region. The moderator to fuel volume ratio was not changed, but the changes of cross-section due to the transient spectrum at core-reflector boundary were taken into account for both regions of boundary fuel cell and reflector by using the results of above mentioned method. The resultant reactivity effect was estimated by a perturbation method.

The effects due to the change of moderator to fuel volume ratio were studied for the 13 element core by HAMMER. The volume ratio of standard fuel cell was 12.6. Three kinds of additional cells were selected parameterically to represent the boundary region. Their volume ratios were 18.4, 24.6 and 30.1, respectively.

The thermal neutron flux spectrum of a standard cell is shown in Figure XII.A. It shows the shift of the modal neutron velocities as a function of position in the cell, and the deep spectrum depression rapidly disappears outside the aluminum thimble. One peculiar point in Fig. XII.A is the $n(v)v$ has almost the same value at the thermal cut-off energy for every spatial point through cell. This came from the assumption of the flat source in HAMMER. On the other hand, Figures XIII, and XIV which contain the epithermal and fast neutron spectrum, show the spatial dependence of the neutron flux at the thermal cutoff energy. This inconsistency would be removed if the spatially dependent slowing down flux were used as a source to THERMOS, or if the thermal cutoff energy is raised up to 2 or 3 ev (i.e., about 15 in lethargy) where the spatial dependence of flux is small. The changes of thermal neutron flux spectrum are shown in Figures XV, XVI, XVII, and XVIII. Points of interest are the small change of the modal neutron velocity, and the substantial change in the ratio of the peak value of the Maxwellian distribution to the value of the $1/E$ tail with the change in volume ratio. From this result it appears that the primary determinant of the modal neutron velocity is the absorption cross section, while the volume ratio is the primary determinant of the relative height of the $1/E$ tail, for these systems. Figure XIX shows the changes of average thermal neutron velocities. The spatial change was caused by some combination of the change of modal neutron velocity and the change of relative height of the $1/E$ tail, but the change with volume ratio was mainly effected by relative height of $1/E$ tail. The change of fast neutron spectrum with spatial points and volume ratios is shown in Fig. XIII and Fig. XIV. They show that the flux in the intermediate energy range had small spatial dependence, but the epithermal and the fast neutron flux had large spatial dependencies. The change of volume ratio made a large change in ratio of fission energy neutron spectrum to intermediate energy neutron spectrum.

FIGURE XII.A
Thermal Flux Spectrum in 19-Rod Cluster (THERMOS);
Moderator-to-Fuel Volume Ratio = 12.6

FIGURE XIII
Fast Spectrum in 19-Rod Cluster;
Moderator-to-Fuel Volume Ratio = 12.6

FIGURE XIV
Fast Spectrum in 19-Rod Cluster;
Moderator-to-Fuel Volume Ratio = 30.1

FIGURE XV
Change of Thermal Spectrum in 19-Rod Cluster

FIGURE XVI
Change of Thermal Spectrum in 19-Rod Cluster

FIGURE XVII
Change of Thermal Spectrum in 19-Rod Cluster

FIGURE XVIII
Change of Thermal Spectrum in 19-Rod Cluster

FIGURE XIX
Change of Average Velocity Distribution in Lattice
with Change in Moderator Thickness

The changes in cross-sections and lattice constants with volume ratios are shown in Table XI through Table XVII. As expected, thermal microscopic absorption and fission cross-sections of several isotopes, which had almost $1/v$ characteristics, had similar fractional changes while plutonium isotopes showed an entirely different trend because of their large non $1/v$ characteristics. These changes in the thermal region mainly came from these changes in the relative height of the modal neutron peak to $1/E$ tail, but not from the changes in modal velocity. From these results, the differences of thermal microscopic cross-sections between the standard fuel cell and the well moderated fuel cell can be determined. The large changes in the thermal macroscopic cross-sections were caused by the averaging process in the cell. But the changes in $v\Sigma_f/\Sigma_a (= \eta f)$ are much more significant to the reactivity effect. The changes in cross-section caused the thermal utilization to decrease with increasing volume ratio. Although the resonance integrals and self-shielding factors had only small changes with volume ratio, the resonance escape probability increased enough to compete with the decrease the thermal utilization factor with increasing moderator volume. As a whole, the curve of k_∞ vs volume ratio was convex. It's maximum value occurred at volume ratio 18.4. The value of k_∞ at volume ratios 12.6 and 24.6 was about the same. Large changes were not found in the fast neutron microscopic cross sections because of the relatively small change of the fast spectrum shape. The changes of the fast macroscopic cross-sections as in the case of the thermals were mainly caused by the averaging process in the cell. As suggested in Figures XIII and XIV broad energy group fluxes 1&2 were much larger inside the aluminum thimble than in the moderator. This decreased the weight of moderator region in cell averaging the cross-sections, and resulted in expectionally small changes of the macroscopic cross-section in the highest energy groups

TABLE XI
Cluster Averaged Cross-Sections (Thermal)

Element	Element	*VRR=1.6	VR = 18.4		VR = 2.6		VR = 30.1	
		Output	Output	ifference (%)	Output	ifference (%)	Output	ifference (%)
σ	^{235}U	4.3880 $\pm$ 02	4.5977	+ 4.97	4.7166	+ 7.68	4.7918	+ 9.40
	^{238}U	1.8335 $\pm$ 00	1.9148	+ 4.34	1.9582	+ 6.70	1.9857	+ 8.20
	^{239}Pu	9.0880 $\pm$ 02	9.0217	- 0.65	8.9902	- 0.99	8.9714	- 1.20
	^{240}Pu	2.2007 $\pm$ 02	2.2410	+ 1.50	2.2598	+ 2.36	2.2721	+ 2.91
	^{241}Pu	1.0336 $\pm$ 03	1.0659	+ 2.83	1.0816	+ 4.34	1.0916	+ 5.31
	^{242}Pu	2.0331 $\pm$ 01	2.1197	+ 4.34	2.16578	+ 6.70	2.1982	+ 8.20
	^{91}Zr	1.2660 $\pm$ 01	1.3176	+ 4.51	1.3487	+ 6.98	1.3685	+ 8.55
	^{27}Al	1.7007 $\pm$ 01	1.7819	+ 4.38	1.82227	+ 6.77	1.8488	+ 8.30
	^{10}B	2.9449 $\pm$ 03	3.0825	+ 4.50	3.15322	+ 6.87	3.2947	+ 8.31
	^{235}U	3.7332 $\pm$ 02	3.9199	+ 5.02	4.02222	+ 7.76	4.08370	+ 9.49
σ_f	^{239}Pu	6.2448 $\pm$ 02	6.2634	+ 0.24	6.27719	+ 0.37	6.27780	+ 0.47
	^{241}Pu	7.6566 $\pm$ 02	7.8925	+ 2.95	8.01141	+ 4.53	8.09907	+ 5.53
	^{241}Pu	9.0770 $\pm$ 02	9.5254	+ 5.02	9.77739	+ 7.75	9.93314	+ 9.49
$\nu\sigma_f$	^{239}Pu	1.7940 + 03	1.7983	+ 0.24	1.80007	+ 0.37	1.80024	+ 0.47
	^{241}Pu	2.2762 + 03	2.3433	+ 2.95	2.37794	+ 4.53	2.40021	+ 5.53
	^{241}Pu	9.0770 $\pm$ 02	9.5254	+ 5.02	9.77739	+ 7.75	9.93314	+ 9.49
Σ	^{235}U	1.6005 - 03	1.0855	- 32.18	8.09982 04	- 44.41	6.39995	- 60.02
	^{238}U	9.2592 - 04	6.2422	- 32.58	4.64416	- 44.87	3.56619	- 60.45
	^{239}Pu	8.4807 - 03	5.4427	- 35.82	3.94446	- 55.49	3.06626	- 63.89
	^{240}Pu	1.8400 - 04	1.2067	- 34.42	8.84477 5	- 55.91	6.02208	- 62.39
	^{241}Pu	1.2714 - 04	8.4469 - 05	- 33.56	6.23326	- 55.98	4.39937	- 61.51
	^{242}Pu	1.2406 - 07	8.3637 - 08	- 32.58	6.21191	- 44.87	4.00064	- 62.17
	^{91}Zr	4.2349 - 05	2.8588	- 32.49	2.12280	- 44.75	1.58802	- 60.32
	^{27}Al	4.0933 - 04	2.7925	- 31.78	2.09922	- 44.89	1.55595	- 59.46
	^{10}B	8.1444 - 04	8.8658	+ 8.86	9.24481	+ 11.55	9.8819	+ 16.42
	^{235}U	1.3639 - 03	9.2544 04	- 32.15	6.90043	- 44.38	5.5581	- 59.98
Σ_f	^{239}Pu	5.8360 - 03	3.7797	- 35.23	2.75519	- 55.85	2.4431	- 63.28
	^{241}Pu	9.4034 - 05	6.2548	- 33.48	4.61178	- 55.89	3.52171	- 61.43
	^{241}Pu	9.4034 - 05	6.2548	- 33.48	4.61178	- 55.89	3.52171	- 61.43
Σ_f	^{235}U	3.3142 - 03	2.2488	- 32.15	1.67778	- 44.38	1.32163	- 59.98
	^{239}Pu	1.6755 - 02	1.0852	- 35.23	7.90008 03	- 55.85	6.15529	- 63.28
	^{241}Pu	2.7919 - 04	1.8570	- 33.48	1.37710	- 55.89	1.07169	- 61.43
$\nu\Sigma_f$	^{235}U	2.0707 + 00	2.0717	+ 0.05	2.07723	+ 0.08	2.07725	+ 0.09
	^{239}Pu	1.9757 + 00	1.9939	+ 0.92	2.00029	+ 1.38	2.00090	+ 1.69

Ratio

TABLE XII
THERMAL LATTICE CONSTANTS

Const.	*VR = 12.6	VR = 18.4		VR = 24.6		VR = 30.1	
	Output	Output	Difference (%)	Output	Difference (%)	Output	Difference (%)
2	7.0430 + 01	1.0157 + 02	+ 44.21	1.3169	+ 86.98	1.6045	+ 127.81
D	8.8786 - 01	8.7114	- 1.88	8.6239	- 2.87	8.5723	- 3.45
Σa	1.2621 - 02	8.5889 - 03	- 31.95	6.5569	- 48.05	5.3472	- 57.63
$\nu \Sigma f$	2.0348 - 02	1.3287 - 02	- 34.70	9.7157 - 03	- 52.25	7.5869	- 62.71
$\eta f = \nu \Sigma f / \Sigma a$	1.6122	1.5470	- 4.04	1.4818	- 8.09	1.4189	- 11.99
Σf	7.2939 - 03	4.76773 - 03		3.48855 - 03		2.72519 - 03	

TABLE XIII
RESONANCE INTEGRAL AND SELF-SHIELDING FACTOR

Item	Element	VR = 12.6	VR = 18.4		VR = 24.6		VR = 30.1	
		Output	Output	Difference (%)	Output	Difference (%)	Output	Difference (%)
Resonance Integrals	²³⁵ U	2.0505 + 02	2.0505	0.00	2.0540	+ 0.17	2.0558	+ 0.26
	²³⁸ U	1.2692 + 01	1.2689	- 0.02	1.2721	+ 0.23	1.2722	+ 0.24
	²³⁹ Pu	1.5310 + 02	1.5310	0.00	1.5337	+ 0.18	1.5352	+ 0.27
	²⁴⁰ Pu	2.9388 + 03	2.9389	+ 0.00	2.9402	+ 0.05	2.9402	+ 0.05
	²⁴¹ Pu	3.6070 + 02	3.6070	0.00	3.6070	0.00	3.6076	+ 0.02
	²⁴² Pu	1.0056 + 03	1.0056	0.00	1.0063	+ 0.07	1.0062	+ 0.06
	⁹¹ Zr	5.3417 - 01	5.3276	- 0.25	5.3377	- 0.07	5.3217	- 0.37
Self-Shielding Factors	²³⁵ U	9.2480 - 01	9.2480	0.00	9.2638	+ 0.17	9.2717	+ 0.26
	²³⁸ U	4.7319 - 02	4.7306	- 0.03	4.7426	+ 0.23	4.7430	+ 0.23
	²³⁹ Pu	6.9982 - 01	6.9984	+ 0.00	7.0107	+ 0.18	7.0175	+ 0.28
	²⁴⁰ Pu	3.5388 - 01	3.5389	0.00	3.5405	+ 0.05	3.5405	+ 0.05
	²⁴¹ Pu	9.9693 - 01	9.9693	0.00	9.9693	0.00	9.9708	+ 0.02
	²⁴² Pu	9.9178 - 01	9.9178	0.00	9.9242	+ 0.06	9.9239	+ 0.06
	⁹¹ Zr	7.7644 - 01	7.7440	- 0.26	7.7587	- 0.07	7.7353	- 0.37

*VR = Moderator-to-Fuel Volume Ratio

TABLE XIV
Fast Few Group Data

Group	Item	VR = 12.6	VR = 18.4		VR = 24.6		VR = 30.1	
		Output	Output	Difference (%)	Output	Difference (%)	Output	Difference (%)
Group 1 (~ 10)	Σ_a	.002422	.002387	- 1.45	.002373	- 2.02	.002367	- 2.27
	Σ_R	.117695	.118666	+ 0.83	.119067	+ 1.17	.119282	+ 1.35
	Σ_f	.001734	.001707	- 1.56	.001697	- 2.13	.001692	- 2.42
	$\nu\Sigma_f$	.004549	.004474	- 1.65	.004445	- 2.28	.004431	- 2.59
	P	.979833	.980278	+ 0.05	.980456	+ 0.06	.980546	+ 0.07
	τ	11.645	11.539	- 0.91	11.496	- 1.28	11.473	- 1.48
	D	1.857129	1.854469	- 0.14	1.85347	- 0.20	1.853028	- 0.22
Group 2 (~ 25)	Σ_a	.000831	.000727	- 12.52	.000679	- 18.29	.000652	- 21.54
	Σ_R	.030398	.031200	+ 2.64	.031565	+ 3.84	.031770	+ 4.51
	Σ_f	.000109	.000098	- 10.09	.000092	- 15.60	.000090	- 17.43
	$\nu\Sigma_f$	.000306	.000275	- 10.13	.000260	- 15.03	.000252	- 17.65
	P	.973375	.977241	+ 0.40	.978949	+ 0.57	.979883	+ 0.67
	τ	50.045	49.190	- 1.71	48.816	- 2.46	48.611	- 2.87
	D	1.199423	1.202293	+ 0.24	1.203590	+ 0.35	1.204327	+ 0.41
Group 3 (~ 54)	Σ_a	.004060	.002946	- 27.44	.002345	- 42.24	.001977	- 51.31
	Σ_R	.013439	.014776	+ 9.95	.015497	+ 15.31	.015935	+ 18.57
	Σ_f	.000844	.000609	- 27.84	.000482	- 42.89	.000405	- 52.01
	$\nu\Sigma_f$	.002354	.001699	- 27.82	.001345	- 42.86	.001131	- 51.95
	P	.768000	.833776	+ 8.56	.868570	+ 13.10	.889616	+ 15.84
	τ	121.342	119.282	- 1.70	118.273	- 2.53	117.698	- 3.00
	D	1.247606	1.242122	- 0.44	1.239235	- 0.67	1.237516	- 0.81
3 2 1	$\nu\Sigma_f/\Sigma_a$	1.8782	1.8743	- 0.21	1.8732	- 0.27	1.8720	- 0.33
	$\nu\Sigma_f/\Sigma_a$	0.3682	0.3783	+ 2.74	0.3829	+ 3.99	0.3865	+ 4.97
	$\nu\Sigma_f/\Sigma_a$	0.5798	0.5767	- 0.53	0.5736	- 1.07	0.5721	- 1.33

TABLE XV

INTERMEDIATE INTEGRAL

group	item	VR = 12.6	VR = 4		VR = 6		VR = 30.1	
		Output	output	diff. (%)	output	diff. (%)	output	diff. (%)
group 1 (1~10)	ϕ	6.05343	6.00855	- 0.74	5.99017	- 1.05	5.98031	- 1.21
	$\Sigma_a \phi$	.014664	.014345	- 2.18	.014217	- 3.05	.014153	- 3.48
	$\Sigma_f \phi$	.010495	.010259	- 2.25	.010165	- 3.14	.010119	- 3.58
	$\nu \Sigma_f \phi$	.027538	.026885	- 2.37	.026627	- 3.31	.026498	- 3.78
	S	.753200	.753200	- 0.	.753200	- 0.	.753200	- 0.
	J	.541417	.536633	- 0.88	.534704	- 1.24	.533695	- 1.43
group 2 (11~25)	ϕ	28.19892	27.64289	- 1.97	27.39599	- 2.85	27.25910	- 3.33
	$\Sigma_a \phi$	.023447	.020086	-14.33	.018595	-20.69	.017780	-24.17
	$\Sigma_f \phi$	.003069	.002696	-12.15	.002531	-17.53	.002440	-20.50
	$\nu \Sigma_f \phi$	.008633	.007593	-12.05	.007133	-17.38	.006879	-20.32
	S	.246610	.246610	- 0.	.246610	- 0.	.246610	- 0.
	J	1.628893	1.600596	- 1.74	1.588010	- 2.51	1.581044	- 2.94
group 3 (26~54)	ϕ	42.04312	41.87117	- 0.41	41.75325	- 0.69	41.67991	- 0.86
	$\Sigma_a \phi$	.170684	.123340	-27.74	.097909	-42.64	.082412	-51.72
	$\Sigma_f \phi$	.035473	.025495	-28.13	.020133	-43.24	.016897	-52.37
	$\nu \Sigma_f \phi$	.098980	.071133	-28.13	.056168	-43.25	.047138	-52.38
	S	.000190	.000190	- 0.	.000190	- 0.	.000190	- 0.
	J	2.526156	2.504766	- 0.85	2.491906	- 1.36	2.484080	- 1.67

TABLE XVI

MISCELLANEOUS DIFFUSION THEORY PARAMETERS

	VR = 12.6	VR = 18.4		VR = 24.6		VR = 30.1	
Item	Output	Output	Diff. (%)	Output	Diff. (%)	Output	Diff. (%)
P	7.3018-01	7.9680	+ 9.12	8.3193	+ 13.93	8.5313	+ 16.84
$\nu(1-pf)$	1.7465-01	1.3602	-22.12	1.1562	- 33.80	1.0342	- 40.78
τ	1.2602+02	1.2414	- 1.49	1.2320	- 2.24	1.2266	- 2.67
B_m^2	1.7867-03	1.6148	- 9.62	1.3570	- 24.05	1.1100	- 37.87
k_∞	1.3555+00	1.3720	+ 1.22	1.3511	- 0.32	1.3158	- 2.93
k_{eff} at B_g^2	9.1912-01	8.8125	- 4.12	8.2526	- 10.21	7.6789	- 16.45
ϕ_1	6.1019+00	6.0718	- 0.49	6.0764	- 0.42	6.0886	- 0.22
ϕ_2	2.8910+01	2.8556	- 1.22	2.8636	- 0.95	2.8818	- 0.32
ϕ_3	4.4555+01	4.5173	+ 1.39	4.6308	+ 3.93	4.7482	+ 6.57
ϕ_4	4.2190+01	6.6855	+58.46	9.2967	+120.35	12.0214	+184.93

TABLE XVII

EFFECT ON K_{eff} DUE TO THE INTRODUCTION OF A BOUNDARY REGION
(13 Element Core)

Case	Regions (radius: cm)				K_{eff}
	Standard	Boundary	$\text{D}_2\text{O-B}^{10}\text{-Al}$	$\text{D}_2\text{O-B}^{10}$	
Original Case No. 132 01	38.47		46.51	91.44	1.0208
No. 131 ²⁾	38.47		46.51	91.44	1.0175
No. 132	28.23	41.87	46.51	91.44	1.0221
No. 133	28.23	45.02	46.51	91.44	1.0229
No. 134	28.23	47.96		91.44	1.0206

- 1) " $\text{D}_2\text{O-B}^{10}\text{-Al}$ " region is a mixed region of poisoned moderator and aluminum sheath for the safety rod guide.
- 2) No. 131 is essentially the same calculation as the original case except the meshpoint distribution in the cell calculation. Volume ratios of 18.4, 24.6 and 30.1 are used for the boundary region in cases of No. 132, 133 and 134, respectively.

with volume ratio. The changes of diffusion coefficient were very small through the whole energy range, since the diffusion coefficient was essentially determined by the transport cross-section and the transport cross-section of these systems were mostly determined by the moderator.

The resultant effect on the effective multiplication factor due to introducing the core-reflector boundary region was estimated by two methods using the above mentioned cross-sections. In the original calculation, the core was considered to consist of three region i.e., the standard fuel cell region, the $D_2O-B^{10}-Al$ region, and the D_2O-B^{10} reflector region. The second region was the mixed region of reflector and safety rod sheaths. In the first method, (method a), to find the effect of the core reflector boundary region, the core was divided into four regions. The central region consisted of 7 fuel cells, and the asymptotic fuel cell constants were used. The boundary region consisted of 6 peripheral fuel cells and a part of $D_2O-B^{10}-Al$ region. The third and fourth regions were the $D_2O-B^{10}-Al$ region and the reflector region, respectively. Calculated results are shown in Table XVII. It shows that the effective multiplication factor at first increases as the boundary region is introduced, because the standard cell is undermoderated. This result seems to be reasonable, but does not explain the difference of the measured value and the originally calculated value shown in Table VI.

The method (b) consisted of two steps. The first was to estimate the cross-section in the boundary fuel cell region and in the boundary reflector region respectively. The second was to calculate the resultant reactivity effect by a perturbation formula. In this method, the core was approximated as shown in Fig. XX. Any cell which had an interface with a different region was treated to be a boundary cell. The cells numbered 2 and 3 in Fig. XX were boundary fuel cells and boundary reflector cells, respectively, while 1 and 4 represent the

1. STANDARD FUEL CELL
2. BOUNDARY FUEL CELL
3. BOUNDARY REFLECTOR CELL
4. ASYMPTOTIC REFLECTOR CELL

FIGURE XX
Approximated Core Configuration

asymptotic fuel cell and asymptotic reflector respectively. The cross-sections of the boundary fuel cells were assumed to have some intermediate value in between that of the standard fuel cell and the well moderated fuel cell (asymptotic fuel cell). The cross-sectional difference $\Delta\sigma$ between the standard fuel cell and the boundary fuel cell was expressed as follows,

$$\Delta\sigma = A (\sigma_{\text{asymptotic}} - \sigma_{\text{standard}}),$$

where $\Delta\sigma$: difference of microscopic cross-section between standard fuel cell and boundary fuel cell.

A: fractional parameter ($0 \leq A \leq 1$),

$\sigma_{\text{asymptotic}}$: microscopic cross-section of a well moderated fuel cell,

σ_{standard} : microscopic cross-section of a standard fuel cell.

It is a very difficult problem to identify the quantity of A, because there is no well defined method determining A. A number of methods to determine A could be suggested. One of these is based on the assumption that the average spectrum in the boundary region is representative of the spectrum intermediate between that of the standard cell and that of the well moderated cell. The other assumes that the spectrum in the boundary cells can be determined by the fraction of surface area which faces the standard fuel cells and moderator. In other words, A is determined as $S_r / (S_r + S_f)$, where S_r and S_f were the number of faces which are adjacent to the reflector and the standard fuel cells, respectively. The former assumption was selected and A was tentatively decided to be 0.5; because, (1) the changes of microscopic cross-section between the standard cell and the well moderated cell were monotonic and not so large (about 10% at the maximum), (2) the difference of final results in k_{eff} due to the different methods of determining A was estimated less than 2.5 milli k. The microscopic

cross-sections of the well moderated fuel cell were used to produce the cross-sections used in method (a). The changes of macroscopic cross-sections due to the changes of microscopic cross-sections were estimated for every energy group by

$$\Delta \overline{\Sigma^x} = \sum_{j=1}^m \frac{\sum_{i=1}^n N_{ij} \Delta \sigma_{ij}^x \phi_j \Delta V_j}{\sum_{j=1}^n \phi_j \Delta V_j}$$

where $\Delta \overline{\Sigma^x}$: change of cell averaged macroscopic cross-section of reaction x (x is absorption or fission),

$\sum_{j=1}^m$: summation over composite isotopes,

$\sum_{i=1}^n$: summation over spatial mesh points,

ϕ_j : relative flux at mesh j,

ΔV_j : volume element of mesh j,

N_{ij} : atomic density of isotope i at mesh j,

$\Delta \sigma_{ij}$: difference of microscopic cross-section between the boundary cell region and the standard cell region.

For the boundary reflector region, processes similar to the above mentioned were used to estimate the differences of macroscopic cross-section between the asymptotic reflector region and the boundary reflector region. According to perturbation theory, reactivity effects caused by changes of cross-section are estimated by

$$\frac{1}{k_{\text{eff}}} \frac{\Delta k_{\text{eff}}}{\Delta \Sigma_g^a} = - \frac{\int \phi_g \phi_g^* dr}{E}$$

$$\frac{1}{k_{\text{eff}}} \frac{\Delta k_{\text{eff}}}{A v \Sigma_g^f} = \int \phi_g \phi_1^* \frac{dr}{E} \quad (\text{where } \chi_1 = 1.00; \text{ all fission neutrons appear in group 1})$$

$$\left\{ \frac{\Delta k}{k_{\text{eff}}} \right\}_{\text{total}} = \sum_{g=1}^n \left\{ \frac{\Delta k}{k} \right\}_g$$

where $\Delta k/k$: reactivity change due to cross-sectional change,

$\Delta \Sigma_g^a$: change of absorption cross-section of energy group g ,

$\Delta v \Sigma_g^f$: change of nu - sigma fission of energy group g ,

ϕ_g and ϕ_g^* : basic flux and adjoint flux of energy group g ,

$\int dr$: volume integral over the region where the cross sectional changes were caused.

n : total number of energy groups g .

E is defined as,

$$E = \frac{1}{k_{\text{eff}}} \int_{\text{core}} \sum_{g=1}^n (v \Sigma_g^f) \phi_g \phi_1^* dr \quad (\text{where } \chi_1 = 1.00).$$

In the original calculation, the core was cylindricalized as shown in Fig. XX, then the reactivity effect was obtained by applying the above mentioned formula to regions 2 and 3. ϕ_g and ϕ_g^* obtained from HFN output were used. It was found from the estimation for the 13 element core that the main effect on the multiplication factor came from changes in thermal neutron range, and the amount was a few milli k . Only the changes in the thermal neutron range were used to estimate the transient spectrum effect for the 7 and 28 elements cores. The final results are shown in Table XVIII. It shows that the transient spectrum effect decreased as the core size increases, and this effect can be neglected within error of a few milli k for a sufficiently large core.

TABLE XVIII

SPECTRUM AND LEAKAGE CORRECTION TO k_{eff}

PRCF Core	Calculation Method(a)	Calculated, k_{cal} (not corrected)	Experimental, k_{ex} (b)	Difference, Δk $= k_{cal} - k_{exp}$	Boundary Spectrum Correction, Δk_s	Surface Leakage Correction, Δk_L	Corrected, k'_{cal}	Difference, $\Delta k'$ $= k'_{cal} - k_{exp}$
7	1	1.0248		+0.014			1.0186	+0.008
	2	1.0226	1.011	+0.012	+0.0049	-0.0111	1.0164	+0.005
	3	1.0297		+0.019			1.0235	+0.013
13	1	1.0208		+0.015			0.9989	-0.007
	2	1.0193	1.006	+0.013	+0.0035	-0.0254	0.9974	-0.009
	3	1.0249		+0.019			1.0030	-0.003
28	1	1.0093		+0.002			1.0045	-0.003
	2	1.0010	1.007	+0.004	+0.0010	-0.0058	1.0062	-0.001
	3	1.0133		+0.006			1.0085	+0.002

(a) Calculation Method 1 = HAMMER 1D-4 groups, 2: HHH ID-6 groups,
3 = EXTERMINATOR 2D-6 groups.

(b) These values were revised from previously reported values by the result of
rechecking the experimental data of control rod worths.

4. Cylindrical Approximation to A Hexagonal Pitch Core Boundary

Another important problem in the original calculation process seems to exist in the cylindrical approximation of the reactor core. The actual PRCF cores were assemblies of cluster fuel elements, and had no definite core radius. On the other hand, the cylindrical core prescription was applied in the calculation process under the assumption of a core radius which give an equivalent core volume. The PRCF cores were basically constructed of hexagonal lattice cells. Therefore, an equivalent radius of a core which consisted of N fuel elements with a lattice pitch 2ℓ was determined to be $\ell\sqrt{2N\sqrt{3}/\pi}$. Since the core is an assembly of hexagonal cells, the cylindrical approximation of the core may underestimate the neutron leakage rate from the core surface, because a surface area of a cylinder is smaller than a surface area of an assembly of hexagons with the same volume. This underestimation of neutron leakage results in an overestimation of the effective multiplication factor of a core.

The influence of this leakage effect on the multiplication factor of the PRCF cores was estimated by a simple method. The process of this estimation consisted of two steps. The first step was to estimate the difference of the effective surface area between a cylinder and an assembly of hexagons from the standpoint of neutronic leakage. The next was to convert the difference of effective surface area or the difference of leakage rate to the difference of effective multiplication factor. The first step was carried out under the following assumptions;

- i) The surface of the active core can be divided into four categories of surfaces, i.e. S_0 , S_1 , S_2 , and S_3 , as shown in Figure XXI.

FIGURE XXI
Surface of an Assembly of Hexagons

- ii) The neutron flux and gradient of the neutron flux at the surface of hexagonal assembly have the same values at the surface of a cylinder with an equivalent radius.
- iii) A conception of "surface effectiveness of neutron leakage" can be introduced for these surfaces,
- iv) If these "surface effectivenesses" are $f_0, f_1, f_2,$ and f_3 for S_0, S_1, S_2 and S_3 , respectively, the effective surface area of the assembly of hexagons is determined by $S_{\text{eff}} = S(af_0 + bf_1 + cf_2 + df_3)$ where S is the geometrical area of one faces of hexagon, ((a + b + c + d) equals the total number of hexagonal faces around the active core), a,b,c and d are integers determined by the shape of the assembly.

The most basic problem of this step was to determine the effectiveness factors: f_0, f_1, f_2 and f_3 . The effectiveness was basically defined as the ratio of the effective angle in which neutrons were able to escape out the core to π . For instance, neutrons which are escaping at any position on S_0 for any direction θ , $0 \leq \theta \leq \pi$, can entirely escape out of the core. This same angular interval is characteristic of a cylinder. Therefore, the effectiveness of S_0 was considered to be unity compared with a cylinder.

Neutrons which are escaping at x_1 on S_1 can escape without reentering the core, only when the directions of flight are in the range of $\alpha_1 \leq \theta \leq \pi$. Neutrons which are leaving at x_1 in the $(\pi - \alpha_1)$ direction have some probability to collide in the trapezoid $P_1 P_2 P_3 P_4$ inside the core. The non-collision probability can be expressed as $\exp(-\ell(\theta) \cdot \Sigma_g^r)$, where $\ell(\theta)$ is the cord length inside the trapezoid for the θ direction, and Σ_g^r is the neutron removal cross-section of the g energy group neutron. Then the effectiveness at x_1 is expressed as

$$f(x_1) = \frac{1}{\pi} \left\{ \alpha_1(x_1) + \int_0^{\pi - \alpha_1(x_1)} d\theta \left[\frac{\sum_{g=1}^n \frac{\phi_g^* \phi_g}{\sum_{g=1}^n \phi_g^* \phi_g} e^{-\ell(\theta) \Sigma_g^r} \right] \right\}$$

The average of $f(x_1)$ with x_1 was calculated, and the average was used as the effectiveness of the surface S_1 . i.e.

$$f_1 = \frac{1}{P_0 P_1} \int_{P_0}^{P_1} f(x_1) dx_1$$

Some approximations were applied for ease of calculation to obtain f_1 . At first, the average of $\alpha_1(x_1)$ with x_1 was obtained. The mean cord length $\bar{\ell}$ of the trapezoid was used instead of $\ell(0)_0$ finally f_1 was obtained as:

$$f_1 = \frac{\bar{\alpha}_1}{\pi} + \left(1 - \frac{\bar{\alpha}_1}{\pi}\right) \left\{ \frac{\sum_{g=1}^n \phi_g \phi_g^*}{\sum_{g=1}^n \phi_g \phi_g^*} e^{-\bar{\ell} \sum_g^r} \right\}$$

where ϕ_g and ϕ_g^* were the direct and adjoint flux at the surface. The approximation of $\alpha(x)$ by $\bar{\alpha}$ did not seem to result in much ambiguity of the final result, because the intervals of $\alpha(x)$ with x were not large; for instances 90° to 120° for the surfaces S_2 and S_3 and 120° to 150° for the surface S_1 . The same procedures were applied for the other surfaces to determine their effectiveness.

The reactivity effect due to the difference of core surface area between cylindrical core and assembly of hexagons was obtained by using a critical equation. Neutron leakage from a core is expressed as follows,

$$\text{Leakage from a core} = \int_{\text{core surface}} D \nabla \phi_n dS = \int_{\text{core volume}} D \nabla^2 \phi dv$$

If the $\nabla \phi_n$ at the surface, and D are constant, the left hand side of the above equation = $D \cdot (\nabla \phi_n)_S \cdot S$;

the right hand side of the above equation = $-DB^2 \int_{\text{core volume}} \phi dv = -DB^2 \bar{\phi} V$

where

$D(\nabla\phi_n)_s$: $D(\nabla\phi_n)$ evaluated at the surface

S : core surface area

V : core volume

B^2 : buckling

ϕ : volume averaged flux in the core.

Then, the buckling B^2 can be written as follows,

$$B^2 = - \frac{D(\nabla\phi_n)_s S}{D\phi V}$$

If D , ϕ , V , $(\nabla\phi_n)_s$ are maintained at the original values, and only S changes to S_{eff} , a new buckling B_{eff}^2 corresponding to S_{eff} is determined by

$$B_{eff}^2 = - \frac{D(\nabla\phi_n)_s S_{eff}}{D\phi V}$$

Therefore, B_{eff}^2 is obtained as $S_{eff} B^2/S$. The resultant reactivity effect due to the change of buckling was estimated by using the critical equation of point reactor model: $k_{eff} = k_{\infty}/(1+B^2M^2)$. The quantities k_{∞} , M^2 and original k_{eff} and B^2 were obtained from HAMMER output. The final leakage effects for 7, 13 and 28 elements PRCF core are shown in Table XVIII.

The estimation of the effects of the annularization of the irregular core-reflector interface and the non asymptotic spectra on k_{eff} can be made by methods different from those used here. For example, one could represent the irregular core surface of the PRCF experiments using a diffusion theory code with a triangular mesh option. One could estimate the effects of the irregular core reflector interface by comparing the results of this code with those of a 1-D annularized approximation. Similarly, one could estimate the effect of the non asymptotic spectra on k_{eff} by defining non asymptotic regions on either side of the interface and recalculate using 1-D diffusion theory. The principle advantage

of the methods used in this study (aside from the saving in computer time) is that since each step is performed by hand, a much deeper insight into the detailed processes which contribute to these effects is gained than is possible when only the final results are supplied by a finite difference code.

5. Buckling Prescription in Diffusion Theory Codes

In the one dimensional calculation (HTH, HAMMER), one effective vertical buckling was assumed in the a core for all energy groups and for all geometrical regions. This vertical buckling was obtained from a vertical activation traverse by gold foils. The buckling obtained from the gold activation traverse is essentially a thermal neutron buckling. The experimental ambiguity in the vertical buckling was estimated to be about 0.4%. This could bring about 1 mk of ambiguity in the calculated k_{eff} . The error committed in the k_{eff} calculation by using a thermal buckling in all groups should not be large. The two dimensional calculation (EXTERMINATOR) indicated that the leakage fraction of the first two group was about one tenth of total leakage in vertical direction. Considering that the magnitude of adjoint functions were about the same for all energy groups, neglecting the energywise buckling prescription seems to bring little ambiguity in the k_{eff} calculation.

C. Conclusions

The preceeding discussion suggests the following conclusions:

- The 1D method with measured vertical buckling is suitable to analyze a large core within accuracy of 5 milli k;
- For a small core, agreement within 10 nk between measurement and calculation can not be obtained without appropriate correction for the core-reflector transient spectrum effect and the leakage effect due to the unevenness of core surface;

- No clear cut superiority is evident between HTH and HAMMER, and between energy group schemes;
- a A number of problem areas in the calculational model region are the joining region of the thermal spectrum and the epithermal spectrum, the error introduced by the cylindrical approximation of the 19 rod cluster fuel element, and the relation between thermalization and the low lying plutonium resonances.

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