MASTER

PDQ-3 - A PROGRAM FOR THE SOLUTION OF THE NEUTRON- DIFFUSION EQUATIONS IN TWO DIMENSIONS ON THE IBM-704

March 1960

CONTRACT AT-11-1-GEN-14

**BETTIS ATOMIC POWER LABORATORY
PITTSBURGH, PENNSYLVANIA**

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PDQ-3

A PROGRAM FOR THE SOLUTION OF THE NEUTRON-DIFFUSION
EQUATIONS IN TWO DIMENSIONS ON THE IBM-704

Report Written By: W. R. Cadwell

Program Written By: W. R. Cadwell
J. P. Dorsey
H. P. Henderson
J. M. Liska
J. T. Mandel
M. C. Suggs

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TABLE OF CONTENTS

	<u>Page No.</u>
I. INTRODUCTION	1
II. PROGRAM DESCRIPTION	2
III. INPUT PREPARATION	11
IV. OUTPUT	23
V. OPERATING INSTRUCTIONS	29
APPENDIX - Sample Problem	34
REFERENCES	56

PDQ-3 solves the few-group, time-independent, neutron-diffusion equations over a rectangular region of the x-y plane. The number of lethargy groups is limited to four, and the number of points of solution is limited to 7500.

PDQ-3

A PROGRAM FOR THE SOLUTION OF THE NEUTRON-DIFFUSION EQUATIONS IN TWO DIMENSIONS ON THE IBM-704

W. R. Cadwell

I. INTRODUCTION

PDQ-3 is a new program in the PDQ series for the IBM-704 computer. It finds a discrete numerical solution over a rectangular region of the x-y plane to the few-group, time-independent, neutron-diffusion equations for a heterogeneous reactor. The program requires a computer with 32,768 words of core storage and at least eight tape units, but drum storage is not used.

PDQ-3 has a number of special features not present in PDQ-2 (Ref 1), but its chief advantage is one of speed. This increased speed is achieved primarily through the use of a line relaxation rather than a point relaxation iteration scheme. Fewer iterations are required in all cases (Ref 2), with the most notable reduction in iterations occurring in cell problems. PDQ-3 does not obsolete PDQ-2, however, since it requires a larger computer and does not treat r-z geometry.

II. PROGRAM DESCRIPTION

Group Equations

The few-group, time-independent, neutron-diffusion equations solved by PDQ-3 are

$$\left\{ -\nabla \cdot [D_i(x) \nabla \phi_i(x)] + [\Sigma_i^a(x) + \Sigma_i^r(x) + D_i(x) B_z^2] \phi_i(x) \right. \\ \left. = \chi_i(x) \frac{\psi(x)}{\lambda} + \Sigma_{i-1}^r(x) \phi_{i-1}(x) \right\}_{i=1}^k \quad (1)$$

Here (x) represents the spatial variables $x-y$; i is the lethargy group index; and k , the index of the thermal group, must be 2, 3, or 4. In addition,

$$\Sigma_0^r(x) = \Sigma_k^r(x) \equiv 0 \quad (2)$$

and

$$\psi(x) = \sum_{i=1}^k v \Sigma_i^f(x) \phi_i(x) \quad (3)$$

The parameters required in these equations may be obtained from the MUFT-4 (Ref 3) and SOFOCATE (Ref 4) programs.

In place of the eigenvalue problem defined by Eqs (1), PDQ-3 may be used to solve the one-group fixed-source problem.

$$-\nabla \cdot [D(x) \nabla \phi(x)] + [\Sigma^a(x) + D(x) B_z^2] \phi(x) = S(x) \quad , \quad (4)$$

where $S(x)$ is a variable input quantity.

Geometry

The rectangular region of solution is composed of a number of subregions called diffusion regions and rod regions. These subregions are separated by interfaces which must be parallel to the outer boundaries of the rectangle.

Within a diffusion region the parameters $D_i(x)$, $\Sigma_i^a(x)$, $\Sigma_i^r(x)$, $\chi_i(x)$, and $v\Sigma_i^f(x)$ are constant. Hence, within such a region, Eqs (1) and (3) reduce to

$$\left\{ -D_i \nabla^2 \varphi_i(x) + \left(\Sigma_i^a + \Sigma_i^r + D_i B_z^2 \right) \varphi_i(x) = \chi_i \frac{\psi(x)}{\lambda} + \Sigma_{i-1}^r \varphi_{i-1}(x) \right\}_{i=1}^k \quad (5)$$

and

$$\psi(x) = \sum_{i=1}^k v\Sigma_i^f \varphi_i(x) \quad (6)$$

Across an interface between two diffusion regions the quantities $\varphi_i(x)$ and $D_i(x) \frac{\partial \varphi_i(x)}{\partial n}$ are continuous.

The neutron flux in one or more lethargy groups is not defined interior to a rod region but satisfies the logarithmic derivative condition

$$\frac{D_i}{\varphi_i(x)} \frac{\partial \varphi_i(x)}{\partial n} = -C_i \quad (7)$$

on its boundary. Here C_i is a positive quantity and the derivative is taken normal to the boundary in the direction of the rod region. In any lethargy groups in which Eq (7) is not applied, the rod region is treated as a diffusion region.

A non-uniform grid of mesh lines is imposed on the rectangular region of solution. Each line in the mesh grid must be parallel to a boundary of the rectangle and must extend from one boundary to the opposite boundary. The intervals between mesh lines must be chosen in such a way that the boundaries of the rectangle and the interfaces between regions exactly coincide with mesh lines. The number of regions in the mesh is limited only by the total number of mesh rectangles. A composition is a set of these regions each of which is

described by the same values of D_i , Σ_i^a , Σ_i^r , χ_i , and $v\Sigma_i^f$. The number of compositions which may be used is restricted to 35.

The discrete solution of Eqs (1) consists of values of $\phi_i(x)$ and $\psi(x)$ at the points of intersection of the mesh grid. The coordinate system is chosen so that the axes coincide with boundary lines of the rectangle. The origin is placed in the upper left corner, the x-axis increases to the right, and the y-axis increases downward. The number of points of solution in the x coordinate direction is limited to 86, the number in the y direction to 173, and the total number of points of solution to 7500.

Along each boundary line of the rectangular region, the flux may be set to zero or the normal derivative of the flux may be set to zero. The latter condition, corresponding to a symmetry axis, is applied half an interval inward from the boundary so that values of the flux on the boundary and on the first interior mesh line are equal. The number of points of solution in the rectangular region is then independent of the boundary conditions applied.

Method of Solution

To obtain a discrete solution, Eqs (1) are approximated by five-point difference equations at the points of intersection of the mesh grid (Ref 5). The matrix equations resulting from this difference approximation are

$$\left\{ A_i \phi_i = \chi_i \xi + B_i \phi_{i-1} \right\}_{i=1}^k, \quad (8)$$

where

$$B_1 \equiv 0 \quad (9)$$

and

$$\lambda \xi = \sum_{i=1}^k v_i \phi_i \quad (10)$$

The matrices A_i , X_i , B_i , and V_i are $N \times N$, where N is the total number of points of solution. The recursive definition

$$L_i = A_i^{-1} (X_i + B_i L_{i-1}), \quad i = 1, 2, \dots, k, \quad (11)$$

where $L_0 \equiv 0$, reduces the matrix problem of Eqs (8)-(10) to

$$\left(\sum_{i=1}^k V_i L_i \right) \underline{\xi} = \lambda \underline{\xi} \quad (12)$$

or

$$T \underline{\xi} = \lambda \underline{\xi} \quad (13)$$

where

$$T = \sum_{i=1}^k V_i L_i. \quad (14)$$

It can be shown (Ref 6) that the largest (in modulus) eigenvalue λ^* of T is positive and simple and that its corresponding eigenvector $\underline{\xi}$ can be chosen to have non-negative components. Hence the power method of iteration,

$$T \underline{\xi}^{(m)} = \lambda^{(m+1)} \underline{\xi}^{(m+1)}, \quad (15)$$

$$\lambda^{(m+1)} = \frac{[\lambda^{(m+1)} \underline{\xi}^{(m+1)} \cdot \lambda^{(m+1)} \underline{\xi}^{(m+1)}]}{[\underline{\xi}^{(m)} \cdot \lambda^{(m+1)} \underline{\xi}^{(m+1)}]}, \quad (16)$$

converges to the eigenvalue λ^* and to a scalar multiple of the eigenvector $\underline{\xi}$.

Further (Ref 6), if

$$\bar{\lambda}^{(m+1)} = \max_j \left[\frac{\lambda^{(m+1)} \xi_j^{(m+1)}}{\xi_j^{(m)}} \right] \quad (17)$$

and

$$\underline{\lambda}^{(m+1)} = \min_j \left[\frac{\lambda^{(m+1)} \xi_j^{(m+1)}}{\xi_j^{(m)}} \right], \quad (18)$$

then

$$\underline{\lambda}^{(m+1)} \leq \lambda^* \leq \bar{\lambda}^{(m+1)}. \quad (19)$$

Hence upper and lower bounds on λ^* may be obtained each iteration, and the iteration process is terminated when the inequality

$$\epsilon^{(m+1)} = \frac{\bar{\lambda}^{(m+1)} - \underline{\lambda}^{(m+1)}}{2\lambda^{(m+1)}} \leq \epsilon^2 \quad (20)$$

is satisfied, where ϵ is an input parameter.

For large problems of the type solved by PDQ-3, it is not practical to calculate the matrix T . Instead, the product $T\underline{\xi}^{(m)}$ is obtained implicitly by solving the equations

$$\left\{ A_{i\varphi_i}^{(m+1)} = \chi_{i\underline{\xi}}^{(m)} + B_{i\varphi_{i-1}}^{(m+1)} \right\}_{i=1}^k \quad (21)$$

for $\varphi_1^{(m+1)}, \varphi_2^{(m+1)}, \dots, \varphi_k^{(m+1)}$ and then evaluating

$$\lambda^{(m+1)} \underline{\xi}^{(m+1)} = \sum_{i=1}^k v_i \varphi_i^{(m+1)}. \quad (22)$$

The process of solving Eqs (21) and (22) and finding $\lambda^{(m+1)}$ via Eq (16) constitutes a single outer iteration.

For each outer iteration performed, a set of inner iterations is required to solve each of Eqs (21). The iteration scheme used is normalized successive

line overrelaxation (Ref 7). The flux residual

$$R^{(n)} = \sum_j |\varphi_j^{(n)} - \varphi_j^{(n-1)}| \quad (23)$$

is calculated each inner iteration, and the iterations are continued until

$$R^{(n)} \leq \delta R^{(1)}, \quad (24)$$

where δ is 0.05 for a cell and 0.10 otherwise.

The use of successive line overrelaxation requires the estimation of overrelaxation factors, ω_i . These factors and corresponding bounds, $\bar{\omega}_i$ and $\underline{\omega}_i$, are obtained by the method described in Ref 7. Estimates of upper and lower bounds on the number of inner iterations required in each group are also calculated. These estimates of $\bar{N}_i$ and $\underline{N}_i$ are found by solving the inequalities

$$(\omega_i - 1)^{\underline{N}_i} \leq \delta \quad (25)$$

and

$$\bar{N}_i (\omega_i - 1)^{\bar{N}_i - 1} \leq \delta. \quad (26)$$

The convergence of the outer iterations is accelerated through the use of Chebyshev polynomials. This results in a source extrapolation

$$\xi_{\star}^{(m)} = \xi_{\star}^{(m-1)} + \alpha^{(m)} [\xi_{\star}^{(m)} - \xi_{\star}^{(m-1)}] + \beta^{(m)} [\xi_{\star}^{(m-1)} - \xi_{\star}^{(m-2)}], \quad (27)$$

where the $\alpha^{(m)}$ and $\beta^{(m)}$ are calculated as in Ref 6. This extrapolated source is multiplied by the renormalization factor

$$\frac{\|\xi^{(0)}\|}{\|\xi^{(m)}\|} \quad (28)$$

to preserve the norm of the original source vector. The use of Chebyshev polynomials requires an estimate of $\bar{\sigma}$, the dominance ratio of the matrix T, which is calculated using a method developed in Ref 8.

Special Features

PDQ-3 has a number of special features not present in PDQ-2. These features are summarized below.

(1) Non-zero values of χ are permitted in the thermal group, and the values of χ may be specified by group only or by group and composition. The latter option should be used only in adjoint calculations.

(2) Either a flat source or a composition-dependent source may be specified in one-group problems, and this source is automatically set to zero in rod regions.

(3) Several options concerning the input flux approximation are provided. This flux may be specified by group only or by group and composition, or it may be taken from the output flux tape of a previous PDQ-2 or PDQ-3 problem.

If the input flux is taken from tape, the problem providing the flux approximation and the current problem must be compatible in number of groups, number of mesh points in each coordinate direction, and boundary conditions.

Under all input flux options, the flux values at the interior points of rod regions are automatically set to zero.

(4) As in PDQ-2, the running time is cut almost in half in the case of problems which are symmetric with respect to the main diagonal. In addition, in PDQ-3, the composition description is automatically reflected about this diagonal so that only half of the mesh need be accurately described.

(5) The flux and source are integrated and averaged over rectangular regions specified in the input.

(6) Pointwise editing of the flux and source is done only in those

groups and over these rectangular regions specified in the input. This selective editing may be done in fixed-point or floating-point form. The pointwise source values are divided by the source average over all fuel compositions to yield a peak-to-average source.

(7) A preliminary input edit may be obtained before running a problem. This includes a complete edit of the input parameters, the composition areas, and a picture of the mesh.

(8) Flux-weighted values of the composition-dependent parameters are automatically calculated and edited at the end of each problem.

Mesh Orientation^{*}

Since PDQ-3 uses a line relaxation iteration, the rate of convergence of the inner iterations may be greatly affected by the mesh orientation chosen. Many PDQ-2 problems which encounter convergence difficulties will converge rapidly if the proper orientation is chosen in PDQ-3. In particular, if there is a coarse mesh in one coordinate direction and a fine mesh in the other, or if there is a point in one coordinate direction at which the ratio of successive mesh intervals is very large, the orientation of the mesh should be chosen carefully.

The mesh lines which are inverted in the PDQ-3 iteration are those parallel to the y coordinate axis. The basis for choosing an orientation is that these mesh lines be as uncoupled as possible. Hence if there is a coarse mesh in one coordinate direction and a fine mesh in the other, the coarse mesh should be in the x direction; if there is a point in one coordinate direction

^{*} This section is adapted from an internal memorandum by Dr. R. S. Varga.

at which the ratio of successive mesh intervals is large, this should be in the y direction.

If a long series of similar problems is to be run, it may be advisable to run the first problem each way to find the better orientation.

III. INPUT PREPARATION

Title Card

A title card must precede the input deck of each problem. This card is used to identify problem output and to indicate the presence of diagonal symmetry.

Columns 1-8 of this card must contain an eight-digit non-zero identification number.

Columns 9-64 may be used for additional identification information, both alphabetic and numeric.

Column 65 must be blank.

Column 66 should contain an "S" if this problem is symmetric with respect to the diagonal through (0,0). Otherwise column 66 must be blank.

Column 67 must be blank.

Columns 68-72 must contain "PDQ03".

Data Cards

All of the input is in fixed-point decimal form, with each data card punched according to a definite pattern. Columns 1-7 and 11 are blank, columns 8-10 contain "DEC", and columns 12-15 contain the card number, followed by a comma in column 16. The data itself begins in column 17 and may extend through column 72. Successive numbers are separated by commas, but no comma is allowed following the last number on a card. The first blank column indicates the end of the information on a card, and any number punched beyond this blank column is ignored. Numbers may be preceded by signs, but only minus signs are necessary.

The card numbering system separates the input into eight series, as follows:

1000 series: Miscellaneous parameters and control information

2000 series: Data by group and composition

3000 series: Flux approximation

4000 series: Mesh intervals in the x coordinate direction

5000 series: Mesh intervals in the y coordinate direction

6000 series: Composition description

7000 series: Integration control information

8000 series: Edit control information

The input deck, preceded by the title card, must be arranged in order of increasing card number.

Since the input is in fixed-point form, it is necessary to indicate which of the numbers are to be converted to floating-point for use in numerical calculations. This is accomplished by including a decimal point in those numbers which are to be converted. (It should be noted that the number zero need never be accompanied by a decimal point.) In particular, the numbers on card 1002, all numbers in the 2000 and 3000 series, and the mesh intervals in the 4000 and 5000 series must contain decimal points; the numbers on cards 1001 and 1003, the row and column numbers in the 4000 and 5000 series, and all numbers in the 6000, 7000, and 8000 series must not contain decimal points. In the following Card Format section, all numbers which must have decimal points are designated by upper-case or Greek letters; all numbers which must not have decimal points are designated by lower-case letters.

The only limitation on the range of input numbers is that contained in the IBM-704 itself. Extreme convergence difficulties and even underflow-overflow situations may arise, however, if values are assigned to input quantities which are well outside the normal range of neutron-diffusion parameters. The number of digits that may be provided in input numbers is completely arbitrary, but no more than eight significant digits can be used

by the computer.

In addition to the specific restrictions listed in the Card Format section below, the total amount of input data may not exceed 13,000 numbers, card numbers included.

Card Format

<u>CARD NUMBER</u>	<u>DESCRIPTION</u>
1001	k, n, ss, tt, a, b, c, d k: The number of groups ($1 \leq k \leq 4$). n: The largest composition number for which data is provided in the 2000 series ($1 \leq n \leq 35$). ss: The last column in the mesh ($3 \leq ss \leq 87$). tt: The last row in the mesh [$3 \leq tt \leq 174$; $(ss-1)(tt-1) \leq 7500$]. a: If $a = 0$, row 0 is a zero flux boundary. If $a = 1$, there is a symmetry axis midway between rows 0 and 1. b: If $b = 0$, column 0 is a zero flux boundary. If $b = 1$, there is a symmetry axis midway between columns 0 and 1. c: If $c = 0$, row tt is a zero flux boundary. If $c = 1$, there is a symmetry axis midway between rows tt-1 and tt. d: If $d = 0$, column ss is a zero flux boundary. If $d = 1$, there is a symmetry axis midway between columns ss-1 and ss.

CARD NUMBER

DESCRIPTION

1002

An "S" in column 66 of the title card adds the restrictions $ss = tt$, $a = b$, and $c = d$.

$B_z^2, \lambda_0, \epsilon, \chi_1(S), \chi_2, \chi_3, \chi_4, \bar{\sigma}_0$

B_z^2 : The buckling constant.

λ_0 : The initial approximation to the eigenvalue.

ϵ : The convergence parameter used to terminate the iterations. If $k = 1$, the inner iterations are terminated by

$$R^{(n)} = \max_j \frac{|\varphi_j^{(n)} - \varphi_j^{(n-1)}|}{\psi_j^{(n)}} \leq \epsilon(2 - \omega)$$

where ω is the overrelaxation factor. If $k > 1$, the inner iterations in each group are terminated by

$$R^{(n)} \leq \begin{cases} \delta R^{(1)} & \text{if } \epsilon > 0.005 \\ \epsilon R^{(1)} & \text{if } \epsilon \leq 0.005 \end{cases}$$

where

$$R^{(n)} = \sum_j |\varphi_j^{(n)} - \varphi_j^{(n-1)}|$$

and δ is 0.05 for a cell and 0.10 otherwise. The outer iterations are terminated by

$$\frac{\bar{\lambda}^{(m)} - \underline{\lambda}^{(m)}}{2\lambda^{(m)}} \leq \epsilon^2.$$

If only one outer iteration is to be done,

CARD NUMBERDESCRIPTION

ϵ should be chosen ≤ 0.005 to assure convergence of the inner iterations.

X_1 : The integrals of the fission spectrum by group. If one or more of these values is non-zero, there may be no X values in the 2000 series. If all four values are zero, the X 's must be provided in the 2000 series by group and composition. If $k = 1$, X_1 is replaced by the source S and X_2 , X_3 , and X_4 must be zero. As above, if $S \neq 0$, there may be no source values in the 2000 series; if $S = 0$, the source values must be provided by composition in the 2000 series.

$\bar{\sigma}_0$: The initial approximation to the dominance ratio of the iteration matrix ($0 \leq \bar{\sigma}_0 < 1$). This should be zero unless a good approximation is available from a previous problem.

1003

 P, e_1, e_2, e_3

This card is present if and only if the output flux of a previous problem is to be used as the input flux approximation for this problem. If this card is present there may be no 3000 series.

p : The eight-digit identification appearing in columns 1-8 on the title card of the problem

CARD NUMBERDESCRIPTION

providing the flux approximation. The values of k , ss , tt , a , b , c , and d for problem p and for the present problem must be identical.

e_1 : If $e_1 = 0$, the value of λ_0 on card 1002 is used as the initial eigenvalue approximation. If $e_1 = 1$, the final value of λ in problem p is used.

e_2 : If $e_2 = 0$, the value of $\bar{\sigma}_0$ on card 1002 is used as the initial dominance ratio approximation. If $e_2 = 1$, the final value of $\bar{\sigma}$ in problem p is used.

e_3 : If $e_3 = 0$, values of ω_i , the overrelaxation factors, are calculated for this problem. If $e_3 = 1$, the values of ω_i are taken from problem p .

If problem p is a PDQ-2 problem, none of the checking features or options indicated above are available, and p , e_1 , e_2 , and e_3 may all be zero. Any PDQ-2 flux tape of the proper length will be accepted.

2101

 $D, \Sigma^a, 0, S$

2102

The values of the above parameters for compo-

:

1 through n . This format is used only if $k = 1$.21 n

The value of S is present only if $S = 0$ on card 1002. If $D = 0$, Σ^a must be replaced by C , and

CARD NUMBER

DESCRIPTION

	the logarithmic derivative condition is applied at the boundaries of this composition.
2101	$D_1, \Sigma_1^a, \Sigma_1^r, v\Sigma_1^f, X_1$
2102	The values of the above parameters for group 1,
:	compositions 1 through n. This format is used
21 <u>n</u>	if $k > 1$. The value of X_1 is present if and only if all four X values on card 1002 are zero. If $D_1 = 0$, Σ_1^r and $v\Sigma_1^f$ must be zero and Σ_1^a must be replaced by C_1 . The logarithmic derivative condition is then applied at the boundaries of this composition in this group.
2i01	$D_i, \Sigma_i^a, \Sigma_i^r, v\Sigma_i^f, X_i$
2i02	The values of the above parameters for group i,
:	compositions 1 through n. The value of X_i is
2i <u>n</u>	present if and only if all four X values on card 1002 are zero. If $D_i = 0$, Σ_i^r , Σ_{i-1}^r , and $v\Sigma_i^f$ must be zero and Σ_i^a must be replaced by C_i . The logarithmic derivative condition is then applied at the boundaries of this composition in this group.
2k01	$D_k, \Sigma_k^a, v\Sigma_k^f, X_k$
2k02	As above for the last group with $\Sigma_k^r \equiv 0$ omitted.
:	
2k <u>n</u>	
3000	$\phi_1, \phi_2, \dots, \phi_k$

A constant input flux approximation for each

CARD NUMBERDESCRIPTION

group. This card is optional and may be present only if there is no card 1003. If this card is present there may be no 3000 series below.

3001

 $\phi_1, \phi_2, \dots, \phi_k$

3002

A flux approximation by group for compositions 1 through n. This series is present if and only if there is no card 1003 and no card 3000.

:

30 n

4001

 $H_x, s_1, H_x, s_2, \dots$

4002

:

The sequence of mesh intervals in the x coordinate direction. The first H_x is the mesh interval between columns 0 and s_1 , the second H_x is the interval between columns s_1 and s_2 , and so forth. There may be any number of sets (H_x, s_i) per card and any number of cards in this series, but no set may overlap two cards. The sequence s_i must be strictly increasing, and the last value must equal ss on card 1001.

5001

 $H_y, t_1, H_y, t_2, \dots$

5002

:

The sequence of mesh intervals in the y coordinate direction. The first H_y is the mesh interval between rows 0 and t_1 , the second H_y is the interval between rows t_1 and t_2 , and so forth. There may be any number of sets

CARD NUMBERDESCRIPTION

(H_y, t_i) per card and any number of cards in this series, but no set may overlap two cards. The sequence t_i must be strictly increasing, and the last value must equal tt on card 1001. This series may be omitted if it is identical to the 4000 series. If column 66 of the title card contains an "S", this series must be omitted.

6001

 $c, s_1, s_2, t_1, t_2, \dots$

6002

⋮

The material composition of a problem is described by successively overlaying the mesh with rectangular blocks of specified composition number. Any composition overlay may cover all or part of previous overlays. For each mesh rectangle, the last overlay which includes this rectangle determines its composition number. If column 66 of the title card is blank, every mesh rectangle must be included within at least one of the composition overlays. If column 66 contains an "S", the composition to the left of the diagonal through $(0, 0)$ is automatically reflected about this diagonal. Hence the mesh rectangles lying entirely to the right of the diagonal may be described inaccurately or not described at all. It is not necessary that every composition

CARD NUMBERDESCRIPTION

c , $1 \leq c \leq n$, appear in the mesh.

c : The number of the composition to be overlaid on the rectangular block described by the following four words ($1 \leq c \leq n$).

s_1 : The left-hand boundary of the rectangle.

s_2 : The right-hand boundary of the rectangle ($0 \leq s_1 < s_2 \leq ss$).

t_1 : The upper boundary of the rectangle.

t_2 : The lower boundary of the rectangle ($0 \leq t_1 < t_2 \leq tt$).

There may be any number of sets

(c , s_1 , s_2 , t_1 , t_2) per card and any number of cards in this series, but no set may overlap two cards.

7001

g , s_1 , s_2 , t_1 , t_2 , ...

7002

⋮

The integrals of the flux and source are automatically calculated for each composition.

Integrals are also calculated in those groups and over those rectangular regions specified in this series. If such a rectangle includes a symmetry axis, the region outside this axis is ignored.

g : The number of the group to be integrated over the rectangular region described by the following four words

CARD NUMBERDESCRIPTION

$(1 \leq g \leq k + 1)$. Group $k + 1$ is the source.

s_1 : The left-hand boundary of the rectangle.

s_2 : The right-hand boundary of the rectangle
 $(0 \leq s_1 < s_2 \leq ss)$.

t_1 : The upper boundary of the rectangle.

t_2 : The lower boundary of the rectangle
 $(0 \leq t_1 < t_2 \leq tt)$.

There may be any number of sets

(g, s_1, s_2, t_1, t_2) per card, but no set may overlap two cards, and the total number of sets may not exceed 100. The sequence of values of g must be non-decreasing. If there is no integration control information, a card 7001 must be provided with no comma following the card number.

8001 $\pm g, s_1, s_2, t_1, t_2, \dots$

8002 Pointwise editing is done only in those groups
:
: and over those rectangular regions specified in
this series.

$\pm g$: The number of the group to be edited over the rectangular region described by the following four words
 $(1 \leq |g| \leq k + 1)$. Group $k + 1$ is the source. If g is positive, the editing is done in fixed-point with three digits

CARD NUMBERDESCRIPTIONEXAMPLE

to the right of the decimal point and up to four digits to the left. If g is negative, the editing is done in floating-point with a two-digit exponent and a three-digit mantissa.

s_1 : The left-hand boundary of the rectangle.

s_2 : The right-hand boundary of the rectangle
($1 \leq s_1 \leq s_2 \leq ss - 1$).

t_1 : The upper boundary of the rectangle.

t_2 : The lower boundary of the rectangle
($1 \leq t_1 \leq t_2 \leq tt - 1$).

There may be any number of sets

($\pm g, s_1, s_2, t_1, t_2$) per card, but no set may overlap two cards, and the total number of sets may not exceed 100. The sequence of values of $|g|$ must be non-decreasing.

If there is no edit control information, a card 8001 must be provided with no comma following the card number.

9000

A control card indicating the end of the input. The comma following the card number is optional.

IV. OUTPUT

Input Edit

The edit of the input includes the number of groups and compositions; the outer boundaries of the mesh and the boundary conditions; values of B_z^2 , λ_0 , ϵ , and $\bar{\sigma}_0$; values of D , Σ^a , Σ^r , $v\Sigma^f$, and χ by group and composition; the sequence of mesh intervals in each coordinate direction; and the composition areas (with areas outside symmetry axes excluded).

Picture Edit

The result of the composition overlay is edited in picture form. The two-digit composition number is shown for each mesh rectangle, and a dotted line is placed along each interface.

Integration Edit

The integration edit consists of the flux and source integrals and averages for each composition and for each rectangular region specified in the input. These integrals are obtained by calculating the flux and source integrals for each mesh rectangle and taking appropriate sums of these.

The integrated flux for mesh rectangle r , bounded by columns $s-1$, and s , rows $t-1$ and t , is

$$\tilde{\phi}_{1,r} = \frac{h_s h_t}{4} (\phi_{1,s-1,t-1} + \phi_{1,s-1,t} + \phi_{1,s,t-1} + \phi_{1,s,t}) . \quad (29)$$

If the mesh rectangle is bisected by a symmetry axis, only that part of the rectangle interior to the symmetry axis is considered, and the flux values on the symmetry axis are taken equal to those on the first interior mesh line.

The integrated source for mesh rectangle r is

$$\tilde{S}_r = \sum_{i=1}^k (v\Sigma^f)_{i,r} \tilde{\phi}_{i,r} . \quad (30)$$

The source normalization factor to be used in the pointwise edit is also calculated. This factor is given by

$$\bar{S} = \frac{\sum_c \tilde{S}_c}{\sum_c A_c}, \quad (31)$$

where the sums are taken over all compositions c such that $\tilde{S}_c \neq 0$.

Flux-Weight Edit

This edit provides flux-weighted values of the composition-dependent parameters for each group. The calculations performed depend upon whether the problem is a cell ($a = b = c = d = 1$) or a non-cell.

In a non-cell problem the following quantities are calculated for each group:

$$\overline{3\Sigma_i^{tr}} = \frac{\sum_f \frac{\tilde{\varphi}_{i,f}}{D_{i,f}}}{\sum_f \tilde{\varphi}_{i,f}} \quad (32)$$

$$\overline{D_i} = \frac{\sum_f D_{i,f} \tilde{\varphi}_{i,f}}{\sum_f \tilde{\varphi}_{i,f}} \quad (33)$$

$$\overline{\Sigma_i^a} = \frac{\sum_f \Sigma_{i,f}^a \tilde{\varphi}_{i,f}}{\sum_f \tilde{\varphi}_{i,f}} \quad (34)$$

$$\overline{\Sigma_i^r} = \frac{\sum_f \Sigma_{i,f}^r \tilde{\varphi}_{i,f}}{\sum_f \tilde{\varphi}_{i,f}} \quad (35)$$

$$\overline{v\Sigma_i^f} = \frac{\sum_f v\Sigma_{i,f}^f \tilde{\phi}_{i,f}}{\sum_f \tilde{\phi}_{i,f}} \quad (36)$$

$$\overline{B_i^2} = \frac{1}{D_i} \left\{ \frac{1}{\lambda} \left(\frac{\sum_f \chi_{i,f} \tilde{S}_f}{\sum_f \tilde{\phi}_{i,f}} \right) + \frac{\sum_f \Sigma_{i-1,f}^r \tilde{\phi}_{i-1,f}}{\sum_f \tilde{\phi}_{i,f}} - \left(\overline{\Sigma_i^a} + \overline{\Sigma_i^r} + \overline{D_i B_z^2} \right) \right\} \quad (37)$$

$$\overline{B_i^2} = \overline{3\Sigma_i^{tr} D_i B_i^2} \quad (38)$$

The symbol $\sum_f$ indicates that the sums are taken over all fuel compositions. Composition c is a fuel composition if $v\Sigma_{i,c}^f \neq 0$ for some i .

In a cell problem the calculations distinguish between a group i in which $D_{i,c} \neq 0$ for any c (non-log group) and a group i in which $D_{i,c} = 0$ for some c (log group). The quantities calculated in a non-log group are:

$$\overline{3\Sigma_i^{tr}} = \frac{\sum_c \frac{\tilde{\phi}_{i,c}}{D_{i,c}}}{\sum_c \tilde{\phi}_{i,c}} \quad (39)$$

$$\overline{D_i} = \frac{\sum_c D_{i,c} \tilde{\phi}_{i,c}}{\sum_c \tilde{\phi}_{i,c}} \quad (40)$$

$$\overline{\Sigma_i^p} = \frac{\sum_r \Sigma_{i,r}^a \tilde{\phi}_{i,r}}{\sum_c \tilde{\phi}_{i,c}} \quad (41)$$

$$\overline{\Sigma_i^{c+f}} = \frac{\sum_n \Sigma_{i,n}^a \tilde{\phi}_{i,n}}{\sum_c \tilde{\phi}_{i,c}} \quad (42)$$

$$\overline{\Sigma_i^a} = \overline{\Sigma_i^p} + \overline{\Sigma_i^{c+f}} \quad (43)$$

$$\overline{\Sigma_i^r} = \frac{\sum_c \Sigma_{i,c}^r \tilde{\varphi}_{i,c}}{\sum_c \tilde{\varphi}_{i,c}} \quad (44)$$

$$\overline{v\Sigma_i^f} = \frac{\sum_c v\Sigma_{i,c}^f \tilde{\varphi}_{i,c}}{\sum_c \tilde{\varphi}_{i,c}} \quad (45)$$

The symbol $\sum_c$ indicates that the sums are taken over all compositions, $\sum_r$ indicates that the sums are taken over all rod compositions, and $\sum_n$ indicates that the sums are taken over all non-rod compositions. Composition c is a rod composition if $D_{i,c} = 0$ for some i .

The quantities calculated in a log group are:

$$\overline{3\Sigma_i^{tr}} = \frac{\sum_n \frac{\tilde{\varphi}_{i,n}}{D_{i,n}}}{\sum_n \tilde{\varphi}_{i,n}} \quad (46)$$

$$\overline{D_i} = \frac{\sum_n D_{i,n} \tilde{\varphi}_{i,n}}{\sum_n \tilde{\varphi}_{i,n}} \quad (47)$$

$$\overline{\Sigma_i^p} = \frac{1}{\lambda} \left(\frac{\sum_n \chi_{i,n} \tilde{S}_n}{\sum_n \tilde{\varphi}_{i,n}} \right) + \frac{\sum_n \Sigma_{i-1,n}^r \tilde{\varphi}_{i-1,n}}{\sum_n \tilde{\varphi}_{i,n}} - \left(\overline{\Sigma_i^{c+f}} + \overline{\Sigma_i^r} + \overline{D_i B_z^2} \right) \quad (48)$$

$$\overline{\Sigma_i^{c+f}} = \frac{\sum_n \Sigma_{i,n}^a \tilde{\varphi}_{i,n}}{\sum_n \tilde{\varphi}_{i,n}} \quad (49)$$

$$\overline{\Sigma_i^a} = \overline{\Sigma_i^p} + \overline{\Sigma_i^{c+f}} \quad (50)$$

$$\overline{\Sigma_i^r} = \frac{\sum_n \Sigma_{i,n}^r \tilde{\phi}_{i,n}}{\sum_n \tilde{\phi}_{i,n}} \quad (51)$$

$$\overline{v\Sigma_i^f} = \frac{\sum_n v\Sigma_{i,n}^f \tilde{\phi}_{i,n}}{\sum_n \tilde{\phi}_{i,n}} \quad (52)$$

$$\overline{D_i} = \frac{\overline{D_i}}{1 + 3 \overline{\Sigma_i^p} \overline{D_i}} \quad (53)$$

$$\overline{3\Sigma_i^{tr}} = \overline{3\Sigma_i^{tr}} + \overline{3\Sigma_i^p} \quad (54)$$

At the end of the last group, a g factor is calculated via

$$g = \frac{\sum_n A_n}{\sum_c A_c} \quad (55)$$

Pointwise Edit

Pointwise values of the flux and source are edited in a row and column array over the rectangular regions specified in the input.

The pointwise source is given by

$$S_{s,t} = \frac{1}{\bar{S}} \sum_{i=1}^k \phi_{i,s,t} \frac{\sum_{q=1}^4 v\Sigma_{i,s,t,q}^f}{\sum_{q=1}^4 n_{s,t,q}} \quad (56)$$

Here $\bar{S}$ is the source normalization factor calculated via Eq (31), q refers to the quadrant at the point (s,t), and $n_{s,t,q} = 1$ if quadrant q contains a

fuel composition ($v_{i,c}^f \neq 0$ for some i), $n_{s,t,q} = 0$ otherwise. If the point is interior to a non-fuel composition, the source is set to zero.

V. OPERATING INSTRUCTIONS

Card Reader

Use the 72-72 card reader board.

On-Line Printer

Use either the SHARE-2 or the GLOUT-2 printer board and either 72-column or 120-column paper.

Card Punch

Use a standard SHARE punch board which makes use of sense exit 1 to effect offset gangpunching.

Off-Line Printer

Set the carriage control switch to PROGRAM and use 120-column paper.

Tapes

Mount the instruction tape on logical unit 1 and mount blanks on units 2-8. If the input flux approximation is to be taken from tape, mount this tape on unit 10.

Sense Lights

During the iterations, the sense lights are used to indicate the group being iterated.

Sense Switches

1) Up: Normal.

Down: Computer will stop at end of current iteration. Remove problem for later restart or START to force an edit of this iteration.

2) Up: Normal.

Down: Restart.

3) Not used.

4) Up: Normal.

Down: Preliminary input edit.

5,6) Up: Normal.

Down: Used to control flux merging as indicated by on-line printout.

Starting Procedure

Ready the input deck (no blank cards) in the card reader, CLEAR, and LOAD TAPE. When the computer stops with a select on the card reader, depress the start key on the card reader to read in the remaining cards.

Restart Procedure

A restart may be done at any time during the running of a problem. Without changing any tapes, rewind tape 1, depress sense switch 2, CLEAR, and LOAD TAPE.

If a problem is restarted without depressing sense switch 2, the computer stops with a select on the card reader. CLEAR, depress the start key on the card reader to clear the select, and follow the restart procedure above.

Removing a Problem - Force Edit

To remove a problem from the computer on short notice, rewind, remove, and label tapes 2-8. To restart the problem, remount these tapes and follow the restart procedure.

During the iterations it is more efficient to remove a problem by first depressing sense switch 1. This causes the computer to stop at the end of the current iteration. Remove and label tapes 2-7. To restart the problem, remount these tapes, mount a blank on unit 8, and follow the restart procedure.

Instead of removing the problem when the computer stops on sense switch 1, the operator may START to force an edit of the current iteration. At the end of the problem, remove and label tapes 2-7 and print tape 8. If further iterations are desired, remount tapes 2-7, mount a blank on unit 8, and follow

the restart procedure.

Decimal Output - Preliminary Input Edit

The title card is used to identify the first page of on-line output and each page of off-line output. This title is also printed on-line at every restart.

The beginning of each routine is indicated by an on-line printout. During the iteration routine, a separate page is devoted to each iteration to indicate the progress of the problem. Error indications and instructions to the operator may be printed at any time.

All output for off-line printing is written on tape 8. Three files are generated during the running of a problem. The first file contains an edit of the input, the composition areas, and a picture of the mesh; the second file contains integrated flux and source values and flux-weighted parameters; and the third file contains pointwise values of the flux and source.

The three output files are normally written on tape 8 at the end of a problem. If sense switch 4 is depressed at the beginning of the problem, however, the first of these files is written immediately after the input is read and the computer then stops. Remove and label tapes 2, 3, and 6 and print tape 8. This allows the problem originator to examine these edits before proceeding with the problem. To restart, remount tapes 2, 3, and 6, mount blanks on units 4, 5, 7, and 8, and follow the restart procedure.

The decimal output of several problems cannot be accumulated on tape 8 since this tape is used for temporary storage.

Input Flux

If an input deck contains a card 1003, the input flux approximation is taken from the tape on unit 10. This tape must be one of three types: a PDQ-2

output flux tape, a PDQ-3 output flux tape, or a PDQ-3 flux merge tape.

Card 1003 may contain no significant information if the output flux of a PDQ-2 problem is to be used. The problem originator must identify to the operator the tape to be mounted. It is extremely important that the proper tape be used since this cannot be checked. It is also important that the problem providing the flux approximation and the current problem be compatible (same number of groups, same number of mesh points in each coordinate direction, and same external boundary conditions).

If the output flux of a PDQ-3 problem is to be used, card 1003 contains the eight-digit identification of this problem. Either the output flux tape of this problem or a flux merge tape containing the output flux of this problem may be mounted. If a merge tape is used, this tape is searched for the required information. In either case the problem providing the flux approximation and the current problem are checked for compatibility.

Output Flux - Flux Merge

At the end of the iterations, the number of the unit containing the output flux tape is printed on-line. This tape may be removed at the end of the problem and used to provide an input flux approximation for a subsequent problem.

To decrease the tape storage required for this purpose, the information on this tape may be added to a flux merge tape. This is done at the end of the problem and is under operator control. If a merge is desired, mount a blank tape or a tape containing previous merges on unit 10. An on-line printout indicates the sense switch settings to distinguish between a blank and a merge tape. If a blank tape has been mounted, the current output flux is placed on this tape as the first problem of a new merge. If a merge tape has been mounted, the current output flux is placed at the end of this merge. In either

case an on-line listing is provided of the identifications of all problems on the merge tape. The number of problems on a merge tape may not exceed 50.

Card Output

Every tape check sum or redundancy tape test error causes a single card to be punched. Rows 9R and 8R contain the check sum from tape and the computed check sum (9R and 8R both zero indicates a redundancy tape test error), row 2R contains the record and file numbers in the address and decrement, and row 0R contains the RTB or WTB instruction.

Instruction Tape Preparation

To write an instruction tape, ready a blank tape 1, ready the PDQ-3 deck (WBPQ3000-WBPQ3837) in the card reader, CLEAR, and LOAD CARDS. After the tape is written, it is rewound and the computer stops at 0343. The instruction tape contains one file of 20 records.

Program Stops

- 00006: Error loading binary tape loader from tape 1. LOAD TAPE to try again.
- 00077: Error loading record of instructions from tape 1. START to try again.
- 77777: Reason for stop has been printed on-line.

APPENDIX

Sample Problem

The sample problem in this appendix is identical to the sample problem in Ref 1. The first page is a listing of the input deck, the next eight pages are the on-line output, and the following 12 pages are the off-line output. The running time of this problem is 0.10 hours.

1 DEC 1001,2,5,25,25,1,1,0,0
 2 DEC 1002,1,0009,1,1,05,1,0,0,0,0
 3 DEC 2101,1.62,0000,041,0000
 4 DEC 2102,2.00,0000,000,0000
 5 DEC 2103,1.73,0042,013,0063
 6 DEC 2104,1.73,0033,014,0046
 7 DEC 2105,1.70,0600,000,0000
 8 DEC 2201,0.238,012,000
 9 DEC 2202,1.000,005,000
 10 DEC 2203,0.433,117,207
 11 DEC 2204,0.434,093,156
 12 DEC 2205,0.000,100,000
 13 DEC 3001,0200,400.
 14 DEC 3002,1100,300.
 15 DEC 3003,1100,200.
 16 DEC 3004,1400,200.
 17 DEC 3005,1200,000.
 18 DEC 4001,1.6,1.2,1.9,0.8,11,21,15,1.7,21,12,0.8,22,4.5,25
 19 DEC 6001,1,0,25,0,25,2,0,16,0,16,2,0,10,0,22,3,1,9,17,21
 20 DEC 6002,4,1,5,11,15,3,5,9,11,15,3,11,15,11,15,4,1,9,1,9
 21 DEC 6003,5,0,1,6,14,5,0,4,9,11,5,6,14,9,11,5,9,11,6,14
 22 DEC 6004,5,0,1,0,4
 23 DEC 7001,1,0,25,0,25,2,0,25,0,25,3,0,25,0,25
 24 DEC 8001,1,1,24,1,24,2,1,24,1,24,3,1,24,1,24
 25 DEC 9000

12345678 SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-179

S PD003

BEGIN COEFFICIENT ROUTINE

BEGIN ITERATION ROUTINE

OUTER ITERATION 2

GROUP 1 R(1) = +106305+05 NO. IT. = 7 R(N) = +102305+04

GROUP 2 R(1) = +243133+04 NO. IT. = 5 R(N) = +163569+03

MAX. = +106983+01 LAMBDA = +100640+01 MIN. = +939661+00

EPS. = +646709-01 RENORM. = +100785+01

OUTER ITERATION 1

GROUP 1 MAX. = +168415+01 OMEGA = +164727+01 MIN. = +149997+01
 NO. IT. PERFORMED = 15 N MAX. = 13 N MIN. = 6

GROUP 1 R(1) = +162303+06 NO. IT. = 9 R(N) = +906296+04

GROUP 2 MAX. = +146629+01 OMEGA = +145155+01 MIN. = +115745+01
 NO. IT. PERFORMED = 15 N MAX. = 7 N MIN. = 3

GROUP 2 R(1) = +258071+05 NO. IT. = 5 R(N) = +215759+04

MAX. = +161185-01 LAMBDA = +101583+01 MIN. = +649284+00

EPS. = +473780-00 RENORM. = +103110+01

OUTER ITERATION 3

GROUP 1 R(1) = +274279+04 NO. IT. = 9 R(N) = +162908+03

GROUP 2 R(1) = +626510+03 NO. IT. = 5 R(N) = +389202+02

MAX. = +101343+01 LAMBDA = +996487+00 MIN. = +978862+00

EPS. = +173446-01 RENORM. = +100277+01

OUTER ITERATION 4

GROUP 1 R(1) = +959860+03 NO. IT. = 9 R(N) = +762823+02

GROUP 2 R(1) = +312537+03 NO. IT. = 5 R(N) = +251451+02

MAX. = +100073+01 LAMBDA = +993863+00 MIN. = +985508+00

EPS. = +765995-02 RENORM. = +100153+01

ALPHA = +125936+01 BETA = +000000+00 SIGMA = +424406+00

OUTER ITERATION 5

GROUP 1 R(1) = +572839+03 NO. IT. = 8 R(N) = +390798+02

GROUP 2 R(1) = +214610+03 NO. IT. = 5 R(N) = +182488+02

MAX. = +994936+00 LAMBDA = +992602+00 MIN. = +987166+00

EPS. = +391426-02 RENORM. = +100061+01

ALPHA = +131715+01 BETA = +376437-01

OUTER ITERATION 6

GROUP 1 $R(1) = +274912+03$ NO. IT. = 8 $R(N) = +168179+02$

GROUP 2 $R(1) = +113102+03$ NO. IT. = 5 $R(N) = +101922+02$

MAX. = $-993062+00$ LAMBDA = $+992164+00$ MIN. = $+989546+00$

EPS. = $-177177-02$ RENORM. = $+100000+01$

BINARY OUTPUT ON TAPE 6

BEGIN INPUT EDIT ROUTINE

BEGIN PICTURE EDIT ROUTINE

BEGIN INTEGRATION EDIT ROUTINE

BEGIN FLUX WEIGHT EDIT ROUTINE

BEGIN POINTWISE EDIT ROUTINE

PROBLEM COMPLETED - PRINT 3 FILES ON TAPE 8

BEGIN FLUX MERGE ROUTINE

1. START IF NO FLUX MERGE IS DESIRED
2. DEPRESS SENSE SWITCH 5 AND START IF FLUX MERGE IS DESIRED
AND TAPE 10 CONTAINS PREVIOUS MERGES
3. DEPRESS SENSE SWITCH 6 AND START IF FLUX MERGE IS DESIRED
AND THIS IS THE FIRST MERGE ON TAPE 10

NO FLUX MERGE

12345678 SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-179

S PDQ03

INPUT EDIT

PAGE 1

2 GROUPS 5 COMPS SS= 25 TT= 25 B.C.=1,1,0,0

B SQUARE -.000900 LAMBDA 1.000000 EPSILON .050000 SIGMA .000000

GROUP	COMP	D	SIGMA A	SIGMA R	NU SIG F	CHI
1	1	1.620000	.000000	.C41000	.000000	1.000000
1	2	2.000000	.000000	.C00000	.000000	1.000000
1	3	1.730000	.004200	.C13000	.006300	1.000000
1	4	1.730000	.003300	.C14000	.004600	1.000000
1	5	1.730000	.060000	.C00000	.000000	1.000000
2	1	.238000	.012000		.000000	.000000
2	2	1.030000	.005000		.000000	.000000
2	3	.433000	.117000		.207000	.000000
2	4	.434000	.093000		.156000	.000000
2	5	.000000	.100000		.000000	.000000

MESH	COL	MESH	COL	MESH	COL	MESH	COL	MESH	COL
1.600000	1	2.100000	9	.800000	11	2.100000	15	1.000000	17
2.100000	21	.800000	22	4.500000	25				

COMPOSITION AREAS

+154197+04 +179560+03 +493920+03 +423360+03 +964800+02

```
      1 1 1 1 1 1 1 1 1 2 2 2 2 2 2
0 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5
0 .....
  .05 05 05 05.02 02.05 05 05 05 05 05 05.02 02 02 02 02 02 02.01 01 01.
1 .....
  .05.04 04 04 04 04 04 04 04.05 05.04 04 04 04.02 02.03 03 03 03.02.01 01 01.
2 .....
  .05.04 04 04 04 04 04 04 04.05 05.04 04 04 04.02 02.03 03 03 03.02.01 01 01.
3 .....
  .05.04 04 04 04 04 04 04 04.05 05.04 04 04 04.02 02.03 03 03 03.02.01 01 01.
4 .....
  .02.04 04 04 04 04 04 04 04.02 02.04 04 04 04.02 02.03 03 03 03.02.01 01 01.
5 .....
  .02.04 04 04 04 04 04 04 04.02 02.04 04 04 04.02 02.03 03 03 03.02.01 01 01.
6 .....
  .05.04 04 04 04 04 04 04 04.05 05.03 03 03 03.02 02.03 03 03 03.02.01 01 01.
7 .....
  .05.04 04 04 04 04 04 04 04.05 05.03 03 03 03.02 02.03 03 03 03.02.01 01 01.
8 .....
  .05.04 04 04 04 04 04 04 04.05 05.03 03 03 03.02 02.03 03 03 03.02.01 01 01.
9 .....
  .05 05 05 05.02 02.05 05 05 05 05 05 05.02 02 02 02 02 02 02.01 01 01.
10 .....
  .05 05 05 05.02 02.05 05 05 05 05 05 05.02 02.01 01 01 01 01 01 01 01 01.
11 .....
  .05.04 04 04 04.03 03 03 03.05 05.03 03 03 03.02.01 01 01 01 01 01 01 01 01.
12 .....
  .05.04 04 04 04.03 03 03 03.05 05.03 03 03 03.02.01 01 01 01 01 01 01 01 01.
13 .....
  .05.04 04 04 04.03 03 03 03.05 05.03 03 03 03.02.01 01 01 01 01 01 01 01 01.
14 .....
  .02.04 04 04 04.03 03 03 03.02 02.03 03 03 03.02.01 01 01 01 01 01 01 01 01.
15 .....
  .02 02 02 02 02 02 02 02 02 02 02 02 02 02 02 02.01 01 01 01 01 01 01 01 01.
16 .....
  .02 02 02 02 02 02 02 02 02 02 02 02 02.01 01 01 01 01 01 01 01 01 01 01 01.
17 .....
  .02.03 03 03 03 03 03 03 03.02.01 01 01 01 01 01 01 01 01 01 01 01 01 01 01.
18 .....
  .02.03 03 03 03 03 03 03 03.02.01 01 01 01 01 01 01 01 01 01 01 01 01 01 01.
19 .....
  .02.03 03 03 03 03 03 03 03.02.01 01 01 01 01 01 01 01 01 01 01 01 01 01 01.
20 .....
  .02.03 03 03 03 03 03 03 03.02.01 01 01 01 01 01 01 01 01 01 01 01 01 01 01.
21 .....
  .02 02 02 02 02 02 02 02 02 02 02 02 02.01 01 01 01 01 01 01 01 01 01 01 01.
22 .....
  .01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01.
23 .....
  .01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01.
24 .....
  .01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01.
25 .....
```

12345678 SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-179

S PDG03

INTEGRATION EDIT

PAGE 3

GROUP 1 FLUX

COMPOSITION INTEGRALS

+304401+06 +200407+06 +568152+06 +587410+06 +116457+06

COMPOSITION AVERAGES

+197410+03 +111610+04 +115029+04 +138750+04 +120705+04

S1	S2	T1	T2	INTEGRAL	AVERAGE	AREA
0	25	0	25	+177682+07	+649594+03	+273529+04

12345678 SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-179

S PDQ03

INTEGRATION EDIT

PAGE 4

GROUP 2 FLUX

COMPOSITION INTEGRALS

+554807+05 +478686+05 +100573+06 +748111+05 +837118+04

COMPOSITION AVERAGES

+359804+03 +266589+03 +203622+03 +176708+03 +867659+02

S1	S2	T1	T2	INTEGRAL	AVERAGE	AREA
0	25	0	25	+786430+06	+287513+03	+273529+04

12345678 SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-179

S PD003

INTEGRATION EDIT

PAGE 5

SOURCE

COMPOSITION INTEGRALS

+000000+00 +000000+00 +243980+05 +142725+05 +000000+00

COMPOSITION AVERAGES

+000000+00 -000000+00 +493966+02 +339489+02 -000000+00

SOURCE NORMALIZATION FACTOR = +422669+02

S1	S2	T1	T2	INTEGRAL	AVERAGE	AREA
0	25	0	25	+387706+05	+141742+02	+273529+04

12345678	SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-179			S PDQ03	FLUX WEIGHT EDIT		PAGE 6
GROUP	3 SIGMA TR	D	SIGMA A	SIGMA R	NU SIGMA F	B SQ	B SQ PRIME
1	+578035+00	+173000+01	-374250-02	+135083-01	+543583-02	+104754-01	+104754-01
2	+230720+01	+433427+00	-106763+00	+000000+00	+185246+00	-400748-01	-400749-01

12345678. SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-179												S PDQC3	GROUP 1	LAMBDA .992164	PAGE 7
1	2	3	4	5	6	7	8	9	10	11	12				
1	1249.283	1304.543	1358.394	1411.888	1424.661	1397.096	1328.879	1261.334	1193.444	1179.050	1182.857	1223.837			
2	1304.543	1372.277	1425.995	1463.665	1473.856	1447.515	1394.117	1325.900	1245.495	1226.132	1233.683	1284.001			
3	1358.394	1425.995	1475.860	1505.378	1511.038	1488.268	1441.180	1375.426	1293.972	1273.712	1280.934	1328.424			
4	1411.888	1463.665	1505.878	1530.639	1533.118	1511.531	1468.129	1400.181	1339.855	1325.083	1326.755	1357.427			
5	1434.661	1473.856	1511.038	1533.118	1534.239	1512.660	1470.560	1414.175	1356.800	1346.765	1345.399	1364.060			
6	1397.096	1447.515	1488.268	1511.531	1512.660	1489.995	1445.708	1385.155	1317.391	1303.395	1305.782	1333.869			
7	1328.879	1394.117	1441.180	1468.129	1470.560	1445.708	1396.768	1329.087	1246.239	1225.348	1231.126	1270.198			
8	1261.334	1325.900	1375.426	1408.181	1414.175	1385.155	1329.087	1256.772	1171.407	1149.413	1152.837	1187.061			
9	1193.444	1245.495	1293.972	1339.855	1356.800	1317.391	1246.239	1171.407	1092.979	1072.685	1070.023	1086.931			
10	1179.050	1225.132	1273.712	1325.083	1346.765	1303.395	1225.348	1149.413	1072.685	1054.301	1047.503	1057.507			
11	1182.857	1233.683	1280.934	1326.755	1345.399	1305.782	1231.126	1152.837	1070.023	1047.503	1042.186	1051.819			
12	1223.837	1284.001	1328.424	1357.427	1364.060	1333.869	1270.198	1187.061	1086.931	1057.507	1051.819	1060.268			
13	1267.824	1324.842	1361.609	1379.702	1373.852	1349.743	1289.584	1205.142	1097.672	1064.360	1055.128	1050.985			
14	1318.970	1355.983	1379.227	1386.630	1373.583	1347.348	1288.357	1205.668	1100.400	1066.737	1049.520	1016.420			
15	1353.938	1369.091	1377.873	1374.581	1337.531	1320.952	1260.444	1176.626	1069.102	1030.619	998.463	922.321			
16	1359.435	1369.741	1374.228	1367.153	1345.779	1305.850	1243.622	1156.800	1041.924	996.656	950.485	851.973			
17	1361.036	1370.477	1371.982	1361.742	1337.024	1294.633	1231.255	1142.295	1015.118	957.818	893.995	771.914			
18	1339.153	1346.775	1345.997	1330.461	1300.662	1254.552	1188.831	1095.334	948.071	877.288	797.791	645.838			
19	1271.451	1281.805	1276.839	1258.487	1226.271	1178.645	1112.115	1016.959	862.918	788.450	705.299	545.897			
20	1148.247	1160.874	1155.297	1136.267	1104.086	1057.509	993.179	901.633	753.254	681.700	603.467	455.200			
21	945.544	951.677	947.665	930.218	901.172	859.191	801.199	720.324	599.564	543.989	483.899	366.214			
22	851.823	854.444	848.276	832.084	805.148	765.988	711.504	635.847	530.568	484.287	436.795	333.654			
23	401.276	395.434	394.174	384.591	369.927	349.423	322.301	288.011	247.193	230.949	214.683	173.778			
24	158.568	157.457	154.872	150.576	144.336	135.941	125.288	112.495	98.082	92.443	86.792	72.230			

12345678 SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-179

S PDQ03

GROUP 1

LAMBDA .992164

PAGE 8

	13	14	15	16	17	18	19	20	21	22	23	24
1	1267.824	1318.970	1353.938	1359.435	1361.036	1339.153	1271.451	1148.247	945.544	851.823	401.276	158.568
2	1324.842	1355.983	1369.091	1369.741	1370.477	1348.775	1281.805	1160.874	953.677	854.444	399.434	157.457
3	1361.609	1379.227	1377.873	1374.228	1371.982	1345.997	1276.839	1155.297	947.665	848.276	394.174	154.872
4	1379.702	1386.630	1374.581	1367.153	1361.742	1330.461	1258.487	1136.267	930.218	832.084	384.591	150.576
5	1378.852	1378.583	1357.531	1345.779	1337.024	1300.662	1226.271	1104.086	901.172	805.148	369.927	144.336
6	1349.743	1347.348	1320.952	1305.850	1294.633	1254.552	1178.645	1057.509	859.191	765.988	349.423	135.941
7	1289.584	1288.357	1260.444	1243.622	1231.255	1188.831	1112.115	993.179	801.199	711.504	322.301	125.288
8	1205.142	1205.668	1176.626	1156.800	1142.295	1095.334	1016.959	901.633	720.324	635.847	288.011	112.495
9	1097.672	1100.400	1069.102	1041.924	1015.118	948.071	862.918	753.254	599.564	530.568	247.193	98.082
10	1064.360	1066.737	1030.619	996.656	957.818	877.288	788.450	681.700	543.989	484.287	230.949	92.443
11	1055.128	1049.520	998.463	950.485	893.995	797.791	705.299	603.467	483.899	436.795	214.683	86.792
12	1050.985	1016.420	922.321	851.973	771.914	645.838	545.897	455.200	366.214	333.654	173.778	72.230
13	1024.300	967.165	846.860	765.831	675.956	537.265	436.009	354.216	282.855	257.820	138.880	59.058
14	967.165	891.894	758.462	674.783	584.952	449.105	353.590	280.927	221.766	201.638	110.413	47.722
15	846.860	758.462	633.103	561.569	485.375	368.953	286.379	224.380	175.384	159.025	87.615	38.265
16	765.831	674.783	561.569	499.927	435.562	332.845	257.979	201.372	156.851	142.067	78.384	34.361
17	675.956	584.952	485.375	435.562	386.411	298.546	231.731	180.540	140.270	126.944	70.093	30.818
18	537.269	449.105	368.953	332.845	298.546	234.702	183.490	143.068	110.949	100.333	55.427	24.466
19	436.009	353.590	286.379	257.979	231.731	183.490	144.210	112.693	87.412	79.046	43.690	19.332
20	354.216	280.927	224.380	201.372	180.540	143.068	112.693	88.211	68.478	61.939	34.261	15.185
21	282.855	221.766	175.384	156.851	140.270	110.949	87.412	68.478	53.197	48.129	26.647	11.824
22	257.820	201.638	159.025	142.067	126.944	100.333	79.046	61.939	48.129	43.548	24.119	10.707
23	138.880	110.413	87.615	78.384	70.093	55.427	43.690	34.261	26.647	24.119	13.380	5.948
24	59.058	47.722	38.265	34.361	30.818	24.466	19.332	15.185	11.824	10.707	5.948	2.647

12345678 SAMPLE PROBLEM FOR INCUSION IN WAPD-TM-179

S PD003

GROUP 2

LAMBDA .992164

PAGE 9

	1	2	3	4	5	6	7	8	9	10	11	12
1	101.066	129.319	144.339	161.216	175.507	160.011	141.421	125.359	97.310	.000	94.255	119.883
2	129.319	166.556	184.553	195.354	159.859	193.358	180.604	161.274	124.336	.000	120.059	153.555
3	144.339	184.553	202.770	211.423	213.571	208.984	198.003	177.953	137.638	.000	132.543	167.814
4	161.216	195.354	211.423	218.725	220.071	215.903	205.769	186.697	148.941	143.837	145.248	172.523
5	175.507	199.859	213.571	220.071	221.076	216.973	207.216	189.470	157.953	152.895	152.082	163.307
6	160.011	193.358	208.984	215.903	216.973	212.685	202.486	183.100	143.667	135.459	130.804	145.293
7	141.421	180.604	198.003	205.769	207.216	202.486	191.821	172.058	132.535	.000	104.701	130.146
8	125.359	161.274	177.953	186.697	189.470	183.100	172.058	153.625	118.082	.000	90.622	115.142
9	97.310	124.336	137.638	148.941	157.953	143.667	132.535	118.082	91.303	.000	70.390	89.399
10	.000	.000	.000	143.837	152.895	135.459	.000	.000	.000	.000	.000	.000
11	94.255	120.059	132.543	145.248	152.082	130.804	104.701	90.622	70.390	.000	69.009	88.471
12	119.883	153.555	167.814	172.523	153.307	145.293	130.146	115.142	89.399	.000	88.471	113.906
13	132.896	168.313	181.573	182.959	163.758	152.861	141.848	129.978	105.894	.000	108.692	136.789
14	148.380	175.151	194.372	183.879	171.159	157.381	152.138	151.838	157.363	168.639	167.541	187.682
15	163.909	171.465	174.179	172.800	166.411	161.894	167.595	188.587	232.947	248.215	266.198	323.827
16	165.461	168.652	159.998	168.546	164.402	162.739	171.135	198.526	258.540	281.115	314.436	384.367
17	166.870	166.949	157.150	165.592	162.506	161.973	171.074	201.889	285.463	325.406	460.713	597.120
18	176.454	168.631	155.277	162.695	160.245	160.251	169.775	209.112	342.452	392.246	574.719	780.927
19	205.432	181.536	180.709	176.732	173.645	173.237	182.916	225.697	372.856	423.831	603.216	810.618
20	277.157	248.388	238.854	233.139	226.256	225.831	232.978	271.353	409.213	456.200	609.219	777.301
21	437.117	431.297	423.891	415.144	405.343	397.061	397.035	420.411	499.066	531.545	628.097	721.511
22	499.135	494.113	486.351	476.376	464.722	454.064	451.137	470.304	536.251	575.005	635.629	696.224
23	767.738	762.425	750.788	732.468	707.343	676.101	640.476	603.393	568.099	555.045	541.851	501.629
24	434.171	430.892	423.728	412.491	397.062	377.603	354.595	328.833	301.266	290.468	279.504	249.905

12345678 SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-179

S PDQ03

GROUP 2

LAMBDA .992164

PAGE 10

	13	14	15	16	17	18	19	20	21	22	23	24
1	132.896	148.380	163.909	165.461	166.870	176.454	205.432	277.157	437.117	499.135	767.738	434.171
2	168.313	175.151	171.465	168.652	166.949	168.631	187.536	248.388	431.297	494.113	762.425	430.892
3	181.573	184.372	174.179	169.998	167.150	165.277	180.709	238.854	423.891	486.351	750.788	423.728
4	182.959	183.879	172.800	168.546	165.592	162.695	176.732	233.139	415.144	476.376	732.468	412.491
5	169.758	171.159	166.411	164.402	162.506	160.245	173.645	228.256	405.343	464.722	707.343	397.062
6	152.861	157.381	161.894	162.739	161.973	160.251	173.237	225.831	397.061	454.064	676.101	377.603
7	141.848	152.138	167.595	171.135	171.074	169.775	182.916	232.978	397.035	451.137	640.476	354.595
8	129.973	151.838	188.587	198.526	201.889	209.112	225.697	271.353	420.411	470.304	603.393	328.833
9	105.894	157.363	232.947	258.540	285.463	342.452	372.856	409.273	499.066	536.251	568.099	301.266
10	.000	168.639	248.215	281.115	325.406	392.246	423.831	456.200	531.545	575.005	555.045	290.468
11	108.692	167.541	266.198	314.436	460.713	574.719	603.216	609.219	628.097	635.629	541.851	279.504
12	136.789	187.682	323.827	384.367	597.120	780.927	810.618	777.301	721.511	696.224	501.629	249.905
13	160.187	210.896	359.545	422.200	633.807	821.182	843.370	791.828	709.647	674.778	451.394	219.621
14	210.896	258.592	400.798	459.660	643.583	797.809	802.818	740.840	651.436	614.922	396.069	189.793
15	359.545	400.798	488.269	532.082	663.883	756.695	735.972	565.673	576.370	541.169	340.631	161.574
16	422.200	459.660	532.082	579.502	682.523	736.612	701.599	527.764	539.639	505.482	315.357	148.990
17	633.807	643.583	663.883	682.523	709.784	715.873	666.336	589.483	503.112	470.184	291.023	137.008
18	821.182	797.809	756.695	736.612	715.873	661.356	589.893	509.902	428.910	398.998	243.486	113.914
19	843.370	802.818	735.972	701.599	666.336	589.893	511.411	433.987	360.509	334.012	201.365	93.702
20	791.828	740.840	665.673	627.764	589.483	509.902	433.987	363.285	298.736	275.852	164.583	76.225
21	709.647	651.436	576.370	539.639	503.112	428.910	360.509	298.736	243.702	224.419	132.724	61.214
22	674.778	614.922	541.169	505.482	470.184	398.998	334.012	275.852	224.419	206.470	121.730	56.060
23	451.394	396.069	340.631	315.357	291.023	243.486	201.365	164.583	132.724	121.730	70.993	32.511
24	219.621	189.793	161.574	148.990	137.008	113.914	93.702	76.225	61.214	56.060	32.511	14.843

[illegible]

12345678 SAMPLE PROBLEM FOR INCLUSION IN WAPD-TM-179

S PDQ03

SOURCE

LAMBDA .992164

PAGE 12.

	13	14	15	16	17	18	19	20	21	22	23	24
1	.623	.691	.752	.000	1.020	1.064	1.196	1.529	2.282	.000	.000	.000
2	.755	.794	.782	.000	1.022	1.027	1.110	1.389	2.254	.000	.000	.000
3	.818	.831	.793	.000	1.023	1.010	1.075	1.342	2.217	.000	.000	.000
4	.825	.830	.787	.000	1.014	.995	1.053	1.311	2.172	.000	.000	.000
5	.907	.913	.890	.000	.995	.979	1.033	1.282	2.119	.000	.000	.000
6	.950	.972	.990	.000	.986	.972	1.024	1.264	2.073	.000	.000	.000
7	.897	.937	1.009	.000	1.021	1.009	1.062	1.289	2.064	.000	.000	.000
8	.816	.923	1.099	.000	1.159	1.187	1.257	1.463	2.166	.000	.000	.000
9	.682	.935	1.300	.000	1.549	1.818	1.955	2.117	2.534	.000	.000	.000
10	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
11	.690	.977	1.453	.000	.000	.000	.000	.000	.000	.000	.000	.000
12	.827	1.071	1.723	.000	.000	.000	.000	.000	.000	.000	.000	.000
13	.937	1.177	1.887	.000	.000	.000	.000	.000	.000	.000	.000	.000
14	1.177	1.399	2.076	.000	.000	.000	.000	.000	.000	.000	.000	.000
15	1.887	2.076	2.486	.000	.000	.000	.000	.000	.000	.000	.000	.000
16	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
17	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
18	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
19	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
20	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
21	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
22	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
23	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
24	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

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