ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

A NEW TECHNIQUE FOR INVESTIGATING FEEDBACK
AND STABILITY OF A NUCLEAR REACTOR

by

Ivar A. Engen

EBR-II Project

Based on a thesis presented in partial fulfillment
of the requirements for the degree of
Master of Science
in the Graduate School of
University of Idaho

June 1970

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ABSTRACT

The rod-drop technique for feedback and stability analysis of Experimental Breeder Reactor II was evaluated in an attempt to develop a reliable, less expensive method of analysis compared to the established oscillation technique.

Oscillation experiments determined system feedback in the frequency domain through use of a Fourier analysis technique. The rod-drop experiments determined the feedback in the time domain through use of an inverse-kinetics computation utilizing linearized, one-group-energy, space-independent, kinetics equations. Data for the computations were obtained from an ion-chamber signal and other signals that monitored the position of the reactivity perturbation mechanism.

The two techniques were compared by developing mathematical models representing the feedback. Models developed from rod-drop experimental results were transformed to the frequency domain for direct comparison with the feedback determined by the oscillation technique.

Two models developed from rod-drop experiments and one from the oscillation experiments were compared extensively and show that the two techniques determine similar models for representing reactor feedback.

Physical limitations of both perturbation mechanisms limited the conclusions that could be drawn. However the rod-drop technique showed sufficient promise to warrant its continued use, particularly for feedback analysis at high reactor power levels, where the oscillation mechanism suffered restrictions on its operation.

I. INTRODUCTION

Stability characteristics of nuclear reactors are investigated to predict the performance of a reactor when operated at a significant power level. In such investigations, a reactivity-oscillation technique is often used to evaluate the reactor's transfer function, and from the transfer function the stability characteristics are obtained. Although this method presents a relatively easy analysis problem, it would be desirable to develop other less costly methods for stability analysis of nuclear reactors. The oscillation method requires a significant amount of reactor operating time to complete and the mechanical portion of the system is complex and costly. One method that offers a saving in reactor operating time and equipment cost is the rod-drop method. Both the oscillation and the rod-drop methods have been used at Experimental Breeder Reactor II (EBR-II) at the National Reactor Testing Station, Idaho. This study compares the two methods and presents the advantages found in the rod-drop method.

A. The Problem

1. Statement of the Problem

The purposes of this study were (a) to evaluate the rod-drop technique for the stability analysis of EBR-II, (b) to compare and evaluate results obtained from the oscillation and rod-drop methods, (c) to analyze and, if possible, explain major differences in the results from the two techniques, and (d) to develop a mathematical model exhibiting the stability characteristics of the reactor.

2. Importance of the Study

Stability analysis of nuclear reactors is necessary to ensure the safety of the operating personnel and the reactor facility. Completing the analysis quickly and at a reasonable cost is desirable in both experimental and commercial reactor applications.

Mechanical failure of the first oscillation mechanism installed in EBR-II emphasized the need for a secondary or additional method for stability analysis. The rod-drop technique was thought to be a reasonable alternative and was used for stability analysis until a second oscillation mechanism could be installed in the reactor. With both systems installed in the reactor, the relative worths of the two analysis techniques could be investigated. Members of the EBR-II Analysis and Test Section undertook this study to establish the relative worths of the two techniques.

B. Description of EBR-II

EBR-II is a sodium-cooled, uranium-fueled fast reactor, which is controlled by the addition or subtraction of fissionable material. The

reactor is divided into three regions: the core, the inner blanket, and the outer blanket. Each region is composed of a number of subassemblies, each about 5 ft long, hexagonal in cross section, and containing a bundle of metal pins. The dimension across the flats of the hexagonal cross section is 2.2 in.

Nominally, the subassemblies in the core region are of three types: driver fuel, control, and safety. The central section of each of these types contains a bundle of enriched-uranium fuel pins, approximately 13 in. long. There are 91 stainless-steel-clad enriched-uranium fuel pins in each fuel element and 61 in each control and safety rod. Recently, a fourth type of subassembly, which contains experimental fuels or structural materials for irradiations testing, has become an important part of EBR-II core loadings. The experimental fuels have included such diverse types as mixed uranium and plutonium metallic alloys, oxides, and carbides. The reactivity worth of a fueled experimental subassembly has generally been less than that of a driver-fuel subassembly.

When the reactor is shut down, the control and safety rods, which move vertically, are kept in the down position so that their fueled sections are outside the core to maintain a highly subcritical condition. For reactor operation, the safety rods are fully raised into the core, and then the control rods are partially raised to make the reactor critical and bring it to the desired power level. For shutdown, both control and safety rods are dropped out of the core. There are always two safety rods in the core loading, but there may be 10 to 12 ordinary control rods, since the control-rod positions can be used for the installation of up to two experimental mechanisms. For this study, two such positions were used to accommodate an oscillation mechanism and a special stainless-steel control rod.

At the time of this study, subassemblies in the inner-blanket region contained stainless-steel pins. The purpose of this stainless-steel blanket was to enhance neutron economy by reflecting some of the neutrons escaping from the core back into the core, making them available for fissioning, and thereby utilizing the fuel more efficiently.

The outer-blanket subassemblies contain stainless-steel-clad natural-uranium pins. Some fissioning occurs here, but the main purposes of the outer blanket are to breed ^{239}Pu and to act as a neutron reflector.

The reactor is immersed in a large tank of liquid sodium, which is circulated upwards through the reactor at 18,000 gpm for heat removal. Maximum design power production is 62.5 MWt. The configuration of the reactor is shown in Fig. 1.

NOTE: CONTROL ROD NO. 1 CONTAINS
SST CONTROL ROD.
CONTROL ROD NO. 8 CONTAINS
OSCILLATOR ROD.

KEY: D--DRIVER FUEL
C--CONTROL-ROD NUMBER
S--SAFETY-ROD NUMBER
R--SST REFLECTOR
X--EXPERIMENT NUMBER

Fig. 1. EBR-II Core and Inner Blanket; Row 7, Run 26B

C. Definition of Terms

1. Feedback

A system's response which affects or modifies the system's driving function is, by definition, feedback. A response that acts to enhance the driving function is called positive feedback, and one that acts to inhibit the driving function is called negative feedback. Feedback may be composed of both positive and negative components, which modify the driving function on different time scales.

2. Transfer Function

A system's transfer function is the complex-plane ratio of the system's response function to its driving function. For a feedback control system, a block diagram can be used to illustrate the transfer-function concept.

In Fig. 2, let $I(s)$ be the driving function, $O(s)$ the response function, $G_0(s)$ the open-loop transfer function, and $H(s)$ the feedback function of a linear system. The open-loop transfer function is obtained when the feedback is zero or disconnected and is expressed as $O(s)/I(s) = G_0(s)$. With nonzero feedback, the driving function applied to the system is $I(s) - O(s)H(s)$ and response is $G_0(s)[I(s) - O(s)H(s)]$. The closed-loop transfer function, $G(s)$, is then expressed as

$$G(s) = O(s)/I(s) = G_0(s)/[1 + G_0(s)H(s)].$$

Fig. 2
System Block Diagram

The open-loop transfer function of a reactor is called the zero-power transfer function because it is valid only for very low-power operation of the reactor, where only nuclear characteristics of the core determine the transfer function. At any significant power level, the reactor transfer function includes feedback and the closed-loop model is used to characterize it.

3. Zero Power

In this study, a zero-power level of the reactor was taken to be operating at 500 kWt. It is assumed that feedback at this power level is negligible and the zero-power transfer function describes the reactor adequately. This seems a reasonable assumption since feedback effects are considered to be temperature-dependent and at 500 kWt, the vertical temperature differential across the reactor is only 1 or 2°F. Thus, temperature effects are negligible.

4. Stability

Stability is defined as in the Nyquist sense.¹ The relationship between feedback and the open- and closed-loop transfer functions is given in the definition of transfer functions. The denominator, $1 + G_0(s)H(s)$, may be expressed as the ratio of two factored polynomials in s , the Laplace transform variable $i\omega$. The roots of the numerator are called zeros, and the roots of the denominator are called poles of the expression. If any of the poles occur in the right half of the complex plane, the system is unstable. A more complete discussion of stability appears in Chapter II.

5. Reactivity

Reactivity (ρ) is the difference between the neutron multiplication rate of the reactor and unity. It is expressed as $\rho = 1 - 1/(k_{eff})$.

where k_{eff} is the effective reactor multiplication or ratio of neutron formation to neutron loss in the reactor.

The units of reactivity used in this study are the dollar (\$), and the cent (¢); one-hundred cents equal one dollar. A dollar of reactivity is the amount of reactivity associated with a reactor being critical on prompt neutrons alone.

Feedback-function amplitude is expressed as the ratio $(\Delta k/k)/(\Delta n/n)$. Dividing this ratio by the delayed neutron fraction (β) gives reactivity in dollars.

6. Linear System

A linear system is a physical system which can be described by a set of simultaneous, ordinary, linear differential equations with constant coefficients.

7. Time Constant

A time constant is the time required for a transient effect to attain the fraction, $1 - \exp(-1)$, of its final value.

8. Power-reactivity Decrement

The power-reactivity decrement is the change in reactivity required to change the reactor's power from one level to another.

9. Power Coefficient

The power coefficient of the reactor is defined to be the derivative of the power-reactivity decrement.

D. Organization of Remainder of Report

Chapter II discusses the reactor transfer function, the feedback function, and reactor stability and presents a frequency-domain calculation of the zero-power transfer function.

Succeeding chapters contain the experimental procedures and results for the oscillation and rod-drop techniques; each technique is covered separately.

A separate chapter is devoted to the comparison and analyses of results from the two methods, a summary, and conclusions from analyses of the results. Suggestions for improvement in the techniques and for further investigation are presented. Appendixes contain methods of computation and tabulated data.

II. DETERMINATION OF FEEDBACK AND STABILITY ANALYSIS

Measurement of the reactor transfer function allows the determination of whether the reactor is a stable system. The open-loop, or zero-power, transfer function can be computed from nuclear constants, and, with measurement of the at-power transfer function, values of the feedback can be calculated. This is usually done by using an oscillation mechanism to perturb the reactor slightly, in a cyclic manner, to allow determination of the transfer function for selected frequencies. If the feedback can be determined in some other manner, it is not necessary to use a frequency-domain driving function for an assessment of reactor stability.

A. The Rod-oscillation Method for Determining Power Transfer Function and Feedback

In general, the reactor transfer function describes the reactor's response to a change in the neutron population in the reactor, or, equivalently, a change in power level. Owing to feedback in the reactor system, the transfer function at power levels above zero power should differ from the zero-power transfer function. The feedback effects are due to temperature characteristics of fuel, coolant, and structural materials in the reactor and its environment.

For the oscillation technique of measuring the transfer function, small sinusoidal power oscillations are induced in the reactor. For such small oscillations, the relationship between the open- and closed-loop transfer functions and the feedback is that given in the definitions in Section I.C. The feedback, $H(s)$, can be readily separated from the expression for the closed-loop transfer function:

$$H(s) = G(s)^{-1} - G_0(s)^{-1}, \quad (1)$$

where

$G(s)$ = closed-loop transfer function,

$G_0(s)$ = open-loop transfer function,

and

s = Laplace transform variable $i\omega$, because the Laplace transforms converge in the entire right half-plane and on the imaginary axis.

To determine values for $H(s)$, the closed-loop transfer function is measured, the open-loop transfer function is computed, and the difference between their reciprocals is found. Since these quantities are defined in the complex plane, each has an amplitude and phase angle associated with it.

B. The EBR-II Open-loop Transfer Function

The zero-power transfer function depends on the nuclear characteristics of the reactor core. The general time-domain forms of the space-independent differential equations describing the reactor's kinetic behavior are given on page 109 of Ref. 2, pages 46-48 of Ref. 3, and page 322 of Ref. 4:

$$\frac{dn(t)}{dt} = \frac{\Delta k(t)}{\ell^*} n(t) - \sum_{i=1}^6 \frac{dC_i(t)}{dt} \quad (2)$$

and

$$\frac{dC_i(t)}{dt} = \frac{\beta_i n(t)}{\ell^*} - \gamma_i C_i(t), \quad (3)$$

where

$C_i(t)$ = neutron-precursor concentration,

$n(t)$ = neutron concentration at time t ,

$\Delta k(t)$ = excess reactivity, a small change in reactivity,

β_i = i th delayed-neutron fraction,

γ_i = i th neutron-precursor concentration,

and

ℓ^* = neutron-generation time.

The reactor is considered to be a linear system which requires constant coefficients in the differential equations describing its behavior. The linearization is accomplished by considering $n(t)$ and $C_i(t)$ to be composed of steady-state and transient components

$$n(t) = n_0 + \Delta n(t), \text{ and } C_i(t) = C_{i0} + \Delta C_i(t).$$

Substitution into Eqs. 2 and 3 results in

$$\frac{dn(t)}{dt} = \frac{d\Delta n(t)}{dt} = \frac{\Delta k(t)n_0}{\ell^*} - \sum_{i=1}^6 \frac{d\Delta C_i(t)}{dt} \quad (4)$$

and

$$\frac{dC_i(t)}{dt} = \frac{d\Delta C_i(t)}{dt} = \frac{\beta_i}{\ell^*} [n_0 + \Delta n(t)] - \gamma_i [C_{i0} + \Delta C_i(t)].$$

However, for the steady-state case,

$$\frac{dC_i(t)}{dt} = 0 = \frac{\beta_i}{\ell^*} n_0 - \gamma_i C_{i0}.$$

Therefore,

$$\frac{dC_i(t)}{dt} = \frac{d\Delta C_i(t)}{dt} = \frac{\beta_i}{\ell^*} \Delta n(t) - \gamma_i \Delta C_i(t), \quad (5)$$

where the derivatives of constants have been dropped and second-order terms have been neglected as being small in comparison with other terms. Equations 4 and 5 are Laplace transformable as shown by Schultz.²

$$s\Delta n(s) = \frac{n_0}{\ell^*} \Delta k(s) - s \sum_{i=1}^6 \Delta C_i(s)$$

and

$$s\Delta C_i(s) = \frac{\beta_i}{\ell^*} \Delta n(s) - \gamma_i \Delta C_i(s).$$

These equations can be combined to form

$$\frac{\Delta n(s)}{\Delta k(s)} = \frac{n_0}{s \left[\ell^* + \sum_{i=1}^6 \frac{\beta_i}{s + \gamma_i} \right]}. \quad (6)$$

This is the zero-power transfer function defined in the frequency domain. The response function is $\Delta n(s)$, and the driving function is $\Delta k(s)$.

The delayed-neutron constants for the EBR-II zero-power transfer function are given in Table I. Equation 6 was used to compute the zero-power transfer function. Tabulated values are given in Appendix A. The transfer function is plotted in Fig. 3.

TABLE I. One-energy-group Delayed-neutron Constants for EBR-II

i	Delayed-neutron Fraction, β_i	Decay Constant, $\gamma_i, \text{sec}^{-1}$
1	0.00025842	0.0127
2	0.00151386	0.0317
3	0.00137510	0.1150
4	0.00300787	0.3110
5	0.00104661	1.4000
6	0.00024178	3.8700

$$\beta = \sum \beta_i = 0.00744364; \ell^* = 1.5555 \times 10^{-7}.$$

Fig. 3
Computed EBR-II Open-loop
Transfer Function

C. Feedback Determination by the Rod-drop Technique

In the rod-drop experiments, the reactivity of the reactor was changed by dropping a special stainless steel control rod out of the reactor core, causing a sudden drop in the neutron population or, equivalently, a sudden drop in the power of the reactor. The reactivity change, as a function of time, $\Delta k(t)$, was found by inverse-kinetics computation with Eqs. 2 and 3. This involved solving the equations for $\Delta k(t)$ as a function of $n(t)$. The solution is presented in Appendix B. Measuring $n(t)$ during the rod-drop experiments allowed the computation of $\Delta k(t)$. The feedback reactivity was then found by comparing the results obtained from the zero-power experiments with those from the experiments performed during power operation.

Initially, the feedback was defined by the relation

$$FB_p(t) = \Delta k_p(t) - \Delta k_0(t), \quad (7)$$

where

$FB_p(t)$ = feedback reactivity in the time domain for power level p ,

$\Delta k_p(t)$ = excess reactivity computed from the rod drop at power level p ,

and

$\Delta k_0(t)$ = excess reactivity computed from the rod drop at zero power.

This formulation for the feedback reactivity seemed reasonable, since it was assumed that any feedback at zero power was negligible and that any differences between the zero-power and power results were due to feedback in the system.

Problems in matching the initial points of zero-power and power results showed a need for an improved technique. If the zero-power values are not matched to the power values correctly, some physically unrealizable values of the feedback occur.

Experimental group members devised methods of calibrating the special steel rod as a function of vertical position and obtaining rod position as well as neutron level for each rod drop, at the same time improving the method for computing $FB_p(t)$.⁵ This is explained in Chapter IV.

D. Reactor Stability

The assessment of reactor stability requires an analysis of Eq. 1, sometimes called the Nyquist stability criterion. The denominator of this equation approaches zero as the product $G_0(s)H(s)$ approaches $(-1, i0)$ in the complex plane; thus the system gain or transfer function tends toward infinity, and the system is unstable. $G_0(s)$ always takes on values in the right half-plane; therefore $H(s)$ must be the contributor of values that could cause the product to appear in the left half-plane. If the product equals the instability point $(-1, i0)$, $H(s) = -G_0(s)^{-1}$. This allows an examination of the behavior of $H(s)$ to determine whether the system is absolutely or conditionally stable.

Third-quadrant values of $H(s)$ indicate conditional stability, since the phase lag of the product is the sum of the phase lags of $G_0(s)$ and $H(s)$ and could approach $-\pi$, where the magnitude of $H(s)$ would determine how close to unity at π radians the product lies. If the magnitude of the feedback is approximately proportional to reactor power, in theory, a power level at which the reactor will become absolutely unstable may be indicated. It is necessary that the phase lag of the product approach $\pm\pi$ radians for assessment of such a power level.

Any time the feedback lies in the third quadrant there is cause for concern.

The limiting value $-G_0(s)^{-1}$ is shown in Fig. 4.

Fig. 4
 $-G_0^{-1}$, Theoretical Reactor
 Feedback Values for Absolute
 Instability

III. THE ROD-OSCILLATION TECHNIQUE FOR MEASURING THE REACTOR TRANSFER FUNCTION

The oscillation technique is well established and documented in the literature (see page 250 of Ref. 4; also see Refs. 6 and 7). However, a description of the system and technique peculiar to the EBR-II application is necessary for the interpretation of the results obtained.

A. The Experimental Procedure

The oscillation mechanism provided rotation of a vertical rod of boron carbide, a neutron absorber, about the central axis of the control-rod thimble in which the oscillator was located. The off-center boron-carbide rod was balanced by an aluminum-oxide rod. A variable-speed drive permitted constant-speed rotation at any frequency between 0.001 and 10 Hz. This variable-speed drive was connected to the oscillator rod by a shaft supported by bearings at several positions along its length of approximately 29 ft. For cooling, sodium coolant was circulated upward through the oscillator.

Rotation of the oscillator rod varied the neutron absorption rate cyclically, maximum absorption occurring with the vertical poison rod nearest the core centerline, and minimum absorption a half-cycle later with the rod at its greatest distance from the core centerline. Revolving the rod at constant frequency and waiting until transient effects die out produces an oscillating reactivity change, approximately sinusoidal, in the reactor. Elimination of transient effects implies analysis of the "steady-state" reactor.

The effect of the reactivity perturbation was seen by obtaining an electrical output signal from an ion chamber near the core. Angular position of the oscillator was obtained from two sine-cosine potentiometers attached to the variable-speed drive so that their angular frequency was identical to that of the oscillator shaft. The sine and cosine functions

generated by these potentiometers had a constant phase angle with respect to inserted reactivity, thereby allowing a constant angular correction to these functions to produce a zero-degree lag between the functions and the reactivity inserted.

The three signals--ion chamber, sine function, and cosine function--allowed the computation of Fourier coefficients for the oscillating component of power once the steady-state component of the ion-chamber output was electronically subtracted out.

The three signals were transmitted over 500-ft lines to an IBM-1620 computer system for reduction and analysis. The system included a NAVCOR analog-to-digital (A/D) converter, which changed the signals to digital form.

The amplitude and phase of the function represented by the data were computed from the relations

$$|F(\omega)| = \left[\frac{2}{\text{Re } F(\omega)} + \frac{2}{\text{Im } F(\omega)} \right]^{1/2} \quad (8)$$

$$\theta = \arctan [\text{Im } F(\omega)/\text{Re } F(\omega)] - \phi, \quad (9)$$

where

$$\text{Re } F(\omega) = \frac{K1}{N} \sum_{i=1}^N I(t_i) \cos (\omega t_i + \phi),$$

$$\text{Im } F(\omega) = \frac{K2}{N} \sum_{i=1}^N I(t_i) \sin (\omega t_i + \phi),$$

ω = frequency of oscillation, radians/sec,

$I(t_i)$ = i th ion-chamber output sample,

t_i = i th sample time,

ϕ = angular position offset at $\omega = 0$, between the oscillator and inserted reactivity,

N = number of points sampled, determined from sample rate, period of oscillation and number of cycles sampled,

and

$K1, K2$ = normalizing factors obtained by normalizing the real and imaginary components to the calculated zero-power transfer function at the highest practical frequency. The K 's are functions of measurement-system gain, magnitude of the steady-state power component, and total β (delayed-neutron fraction).

According to Kaplan (see page 165 of Ref. 8), determination of the Fourier coefficients of a function completely determines the function. Consequently, if there is a relationship between the coefficients corresponding to two functions, knowledge of the relationship at one frequency point allows the coefficients to be normalized or scaled to each other at that point. The assumption is made here that there is no feedback affecting the transfer function when the oscillation frequency is sufficiently high. Therefore, the open- and closed-loop transfer functions are identical at such a frequency, and the measured transfer function may be normalized to the computed open-loop transfer function.

The normalizing was performed at the highest frequency attained by the oscillator rod under the assumption that a sufficiently high frequency could be attained to make feedback effects negligible. This normalizing should produce a frequency function (transfer function) identical to the computed zero-power transfer function over any frequencies measured at zero power (500 kW), and deviating from the zero-power transfer function at significant power levels for frequencies much less than the frequency where the normalizing is performed. The measurement system is described in Ref. 9.

B. System Error Assessment

Before any oscillation experiments were performed, errors inherent in the system were considered. Possible sources of error considered were (a) the disturbance function, (b) the neutron detector, (c) signal transmission and digitizing, and (d) the computing technique.

Determination of the transfer function by the oscillation technique requires that the disturbance function be measurable and repeatable in a cyclic manner. The reactivity worth of the disturbance must be a constant function of angular position. Testing the oscillator showed it to have a natural mechanical resonance at slightly above 9 Hz. This would seem to make data taken very near the resonance frequency highly questionable; therefore the upper frequency limit on the oscillator was set at 8.8 Hz. Even here the data might be questioned since no method was devised to assess the effect at frequencies near the resonance frequency.

It was discovered that the oscillator rubbed when rotated at low frequencies with the reactor at a power level of 30 MWt or more. With the increased power, the rubbing started at higher frequencies. A probable explanation for this rubbing is that the oscillator rod or the stainless steel containment of the oscillator was bowing because of the temperature gradient across them. The neutron-flux shape along the core midplane is approximately a sinusoid, peaking at the centerline of the reactor. This means that at a control-rod position, such as that of the oscillator, there is a marked neutron-flux gradient across the position, which in turn

implies a significant temperature gradient. Thus, at low frequencies the oscillator poison rod would absorb neutrons at a rate producing a marked temperature gradient across the rod. The hottest side was that nearest the core centerline, causing the rod to bow toward the core centerline and increasing the absorption rate. As the poison rod passed through its most reactive position and moved away from the core centerline, the neutron-absorption rate would decrease and the "cold" side of the rod would be rotated toward the core centerline; this tended to equalize the temperature through the rod and relieve the bowing as it passed through its least reactive position farthest from the core centerline.

This explanation requires bowing to become more pronounced as reactor power is increased, because the poison rod is fixed at the ends and the increased neutron-absorption rate at higher power levels produces more heat in the rod. Due to the time constant of the material of the poison rod, the reactivity worth of the rod is a function of frequency. A low frequency would allow pronounced heating, which could not be compensated for by the coolant flow through the rod. At higher power levels, the frequency at which bowing became pronounced would be higher, though not necessarily in proportion to the power increase. The reactivity worth of the rod could possibly be a function of frequency and power, making determination of the transfer function difficult if not impossible. To minimize the possibility for bowing to interfere with the experimental results, it was decided to limit oscillation tests at power levels of 22.5, 30.0, and 41.5 MWt to frequencies above 0.002, 0.08, and 0.2 Hz, respectively. These lower limits on frequency seemed to be a reasonable compromise if any worthwhile data were to be obtained at the higher power levels.

It was not possible to assess an error associated with the neutron detector, since there was no in-pile method available to determine detection characteristics of the ion chamber; however, the environment of the detector and its position relative to the reactor core were believed to minimize any error.

Signal transmission and digitizing were treated as a single entity for error assessment. The NAVCOR A/D converter was controlled by the IBM-1620 computer. Maximum time skew between channels was 160 μ sec. The sample rate limitation imposed by the system was the time required by the IBM-1620 to do necessary computations between each set of data points as the data were accumulated.

The time required for the computer to perform necessary operations on the sampled data limited the sampling rate to about six samples per cycle of oscillation at frequencies approaching 9 Hz. Lindorff has shown that a sampling rate that is over twice the highest frequency component of the sampled signal is sufficient to permit recovery of the signal from its samples.¹⁰ In practice, about 2200 data points were necessary

to ensure repeatability of the results within 0.5% of the average amplitude and 0.5° of the average phase shift. To ensure good repeatability, about 3000 data samples were taken at each frequency of oscillation.

Errors introduced in the computation were primarily due to the number of significant digits associated with each numerical quantity. The digitized data consisted of values containing four significant digits; computed values contained eight digits. The maximum overall significance in a computed value was then contained in four significant digits.

C. Measurement of EBR-II Zero-power Transfer Function

The first major test of the system for oscillation test analysis was the measurement of the zero-power transfer function. The measured values had to closely match the calculated values to give confidence in measurement of the closed-loop transfer function and the computed feedback in tests to follow.

The reactor was operated at 500 kWt until equilibrium conditions were assured. Then, for each frequency of interest, the oscillator rod was rotated long enough to allow the reactor to reach an equilibrium oscillation in power and allow the taking of data.

A small difference was found, primarily at the higher frequencies, between the measured and calculated values for the zero-power transfer function. Correction factors were determined empirically and applied to the measured values as follows:

$$|F(\omega)|_c = |F(\omega)| \cdot [1 + (f/65.1)^2]^{1/2}$$

and

$$\theta_c = \theta + \arctan(f/153),$$

where the subscript c denotes the corrected value, and f is frequency of oscillation in cycles per second (Hz). The other symbols are as defined for Eqs. 8 and 9.

The source of the error that necessitated this correction could not be determined.

After the corrections were applied to the measured open-loop transfer function and phase, the maximum amplitude difference between measured and calculated values was $\pm 0.25\%$ of the calculated value, and the maximum phase difference was $\pm 0.23^\circ$. The measured and computed values are displayed in Fig. 5, where the smooth curves represent the calculated zero-power transfer function and phase lag. Values are tabulated in Appendix A.

Fig. 5
Measured and Computed EBR-II
Open-loop Transfer Function

The excellent agreement obtained gave confidence in the method of analyzing the data, and permitted the use of calculated values for the open-loop transfer function when calculating the feedback.

D. Measurement of EBR-II Closed-loop Transfer Function and Calculation of Feedback

The EBR-II closed-loop transfer function was measured at the fairly well separated power levels of 22.5, 30.0, and 41.5 MWt, so that a possible relationship between feedback and power level might be examined. It was thought possible that some power- or temperature-dependent feedback effects might appear only after a threshold power level was attained, and it was desired to separate as many feedback components as practicable. Once a mathematical model of the feedback was constructed, an estimate could be made as to whether the feedback was linearly related to power.

The frequency limitations on the oscillator reduced the value of the 30.0- and 41.5-MW data severely; however, the 22.5-MW oscillation experiments produced enough data to permit development of a feedback model.

The values of the closed-loop transfer function, the feedback, and the corresponding calculated zero-power transfer function are tabulated in Appendix A. Figures 6-8 display the closed-loop transfer functions

Fig. 6. EBR-II Open-loop and 22.5-MWt Closed-loop Transfer Function

Fig. 7. EBR-II Open-loop and 30.0-MWt Closed-loop Transfer Function

Fig. 8
EBR-II Open-loop and 41.5-MWt Closed-loop Transfer Function

compared with the computed open-loop transfer function. The feedback corresponding to the closed-loop transfer functions are shown in Figs. 9-11. Feedback values were obtained by using Eq. 1.

Fig. 9. EBR-II 22.5-MWt Frequency-domain Feedback. (Numbers in parentheses indicate frequency in Hz.)

Fig. 10. EBR-II 30.0-MWt Frequency-domain Feedback. (Numbers in parentheses indicate frequency in Hz.)

Fig. 11. EBR-II 41.5-MWt Frequency-domain Feedback. (Numbers in parentheses indicate frequency in Hz.)

An error of $\pm 1\%$ in the measured $G(s)$ will result in an error of the order of $\pm 20\%$ in $H(s)$ when $G_0(s)$ and $G(s)$ differ by about 5% of $G(s)$. It may be difficult to determine $H(s)$, at low frequency and high power, to a greater degree of accuracy than $\pm 5\%$ if the measurement errors of the previous section hold.

E. Development of Feedback Models

In choosing a mathematical model of the feedback, we considered the reactor to be a stable linear system in which the power output reaches

a steady-state condition. All feedback components were assumed to be first order so that the feedback could be represented by a summation of terms:

$$H(s) = \sum_i \frac{A_i \exp(-T_i s)}{1 + \tau_i s}, \quad (10)$$

where

T_i = time delay of the i th term,

τ_i = time constant of the i th term of the function,

A_i = magnitude of the i th feedback component,

and

$s = j\omega$; ω is angular velocity; again the assumptions on the system ensure convergence of the Laplace transform in the entire right half-plane and on the imaginary axis.

If it is assumed that the feedback can be expressed as the ratio of two complex polynomials in factored form, then relative amplitude constants (A_i) can be estimated by equating the model and the ratio of polynomials and evaluating the residue at each pole.¹¹ To accomplish this, the time constants must be determined.

A Bode plot of the amplitude (decibels versus $\log_{10}$ frequency) will have a slope change of -20 db per decade change in frequency associated with each pole of the function; also, a slope change of +20 db per decade change in frequency will be associated with each zero of the function. By making a straight-line approximation to the Bode plot, one can estimate the time constants of the system. The straight-line segments must differ in slope by ± 20 db per decade of frequency change for the first-order, simple-pole model proposed. The time constants are determined by noting the frequency at which the curve deviates by 3 db from each straight-line segment. The time constant is equal to the reciprocal of the frequency if the frequency is in radians per second. This method was used to estimate the poles and zeros for the model of $H(s)$ corresponding to the feedback obtained from oscillations at 22.5 MWt. However, the straight-line approximation was too arbitrary to give good results without more knowledge of the model parameters.

An examination of Fig. 9, the Nyquist plot of $H(s)$ obtained from the 22.5-MWt experiments, reveals some properties of any model that approximates measured values. The feedback in the fourth quadrant resembles a semicircle; the amplitude is small for high frequency ($\omega \approx 40$ rad/sec) and increases as ω is reduced until $\omega \approx 1.2$ where $H(s)$ crosses the real axis. The shape of $H(s)$ implies that at high frequency the magnitude of the sum of the A_i is much less than the sum of $A_i\omega\tau_i$, which is negative. As frequency decreases, the imaginary component increases negatively until $\omega \approx 4$; then it decreases to zero for $\omega \approx 1.2$. When $H(s)$ crosses the real axis, the sum of the products $A_i\omega\tau_i$ must be zero, according to Eq. 10; therefore there are both positive and negative components in the model. At very low frequency, the feedback approaches the real axis from the positive side. It is assumed that the real axis is not crossed a second time and the imaginary component again vanishes for $\omega = 0$. The behavior of the imaginary component of $H(s)$ can be attributed to terms of Eq. 10 in which the A_i are nearly equal in magnitude and opposite in sign and the τ_i differ appropriately. The effect of any time delay (phase rotation) associated with the model must be accounted for as well. This effect was used primarily for refinement of the model, so the frequency correspondence between the model and measured feedback was improved.

Combining the observed implications of the measured feedback (Nyquist plot) and the feedback amplitude allowed approximation of time constants which could be used in modeling $H(s)$. The relative magnitudes of the A_i remained to be determined.

To estimate the relative magnitude of each feedback component in the partial-fraction expansion of the model, defined by Eq. 10, the residue at each pole was computed. (See pages 150-159 of Ref. 8.) The expression for the model is analytic in the complex domain, except at the isolated singularities (the poles). A model (Model A) and the experimentally determined feedback at 22.5 MWt are shown in Fig. 12. The agreement between the model and the experimental results seems quite good, except at low frequency where the feedback measurement error is most likely at a maximum.

Fig. 12
Reactor Feedback Model A and Frequency-domain Feedback at 22.5 MWt. (Numbers in parentheses indicate frequency in Hz.)

Model A for the 22.5-MWt feedback is expressed by

$$H(s) = \frac{0.0645}{1 + 0.2s} + \frac{0.0269 \exp(-0.5s)}{1 + 0.4s} - \frac{0.0269}{1 + s} - \frac{0.0631 \exp(-s)}{1 + 5s},$$

where the A_i are expressed in cents. Linear scaling of the model to 30.0 and 41.5 MWt was accomplished by multiplying Model A by the ratios 30.0/22.5 and 41.5/22.5, respectively. Prompt components appear to scale fairly well, as evidenced by the good fit over the high-frequency portion of the 30.0- and 41.5-MWt feedback data. The scaled-up model deviates significantly from the measured feedback at the lower frequencies, however. The feedback components with longer time constants evidently are not linearly increasing functions of power level. Tabulated values of Model A for 22.5 MWt are given in Appendix C, and the results of scaling are shown in Figs. 13 and 14. Each decade of frequency (Hz) is indicated on the curves representing Model A.

Association of model parameters with physical reactor components required an estimate of the time constants that could be associated with the various materials and regions of the reactor core and inner blanket.

The computer code ARGUS¹² was used to simulate the temperature response of some of the major reactor components. A step increase in power was simulated, and the time constant was chosen to be the time at which the temperature of a specific material had reached 63% of its equilibrium temperature. Transport delay times were also available from the program. The results are given in Table II.

Fig. 13. Model A Scaled to 30.0 MWt with Frequency-domain Feedback at 30.0 MWt. (Numbers in parentheses indicate frequency in Hz.)

Fig. 14. Model A Scaled to 41.5 MWt with Frequency-domain Feedback at 41.5 MWt. (Numbers in parentheses indicate frequency in Hz.)

TABLE II. EBR-II Time Constants and Transport Delay Times

Material	Time Constant, sec	Delay Time, sec
Fuel	0.15-0.4	0
Coolant	0.15-0.35	0
Stainless Steel	1.0-10.0	0-4

Time components of Model A agree quite well with these results and give some support to the model determined above. Time constants of less than 1 sec were associated with fuel and coolant; the longer time constants were associated with structural material (stainless steel), particularly in the inner-blanket region where Rows 6 and 7 of the reactor grid contained a large mass of stainless steel. This steel was subject to a large temperature differential because of the high heat rate of fuel in the core region and the lower heat rate of the inner blanket region. In addition, the coolant flow rate of the inner blanket was much less than that of the core; therefore the inner side of Row 6 could be expected to be at a higher temperature than Row 7. It appeared that subassembly bowing with a variable reactivity effect was possible under these conditions; however, no evaluation of such an effect was attempted at the time of this study.

IV. THE ROD-DROP TECHNIQUE FOR MEASURING FEEDBACK

Rod-drop experiments were used to monitor the prompt and overall system feedback following mechanical failure of the first EBR-II oscillator. During Run 24, the reciprocating-type oscillator had been used for transfer-function measurement. Ball-bearing failure in the section of the mechanism inside the reactor vessel ended the usefulness of the device and forced reliance on rod-drop experiments to investigate the behavior of the reactor. Before each run, the rod-drop experiment was performed with a special stainless steel control rod. The rod was dropped out of the core to cause a power decrease, and the system reactivity was computed. (See page 242 of Ref. 4.) Results were compared with results obtained from earlier experiments to determine if any gross differences in the system reactivity were apparent immediately after the rod drop or for several seconds thereafter. When a second oscillator was available, it was decided to obtain mathematical models from both the rod-drop and oscillator techniques and to evaluate the rod-drop method for determining a reactor feedback function.

A. Experimental Procedure

The special stainless steel control rod used for the rod-drop experiments was installed in place of one of the ordinary control rods. The control system for the rod-drop experiments allowed the steel rod to be dropped without dropping the remaining control rods. The major physical difference between the steel rod and the standard control rods was that the steel rod contained stainless steel pins instead of enriched-uranium pins. This resulted in a reactivity worth for the steel rod of approximately 10% of that of a standard control rod. Dropping the steel rod out of the reactor core resulted in a prompt power drop of about 4 to 5% and an exponential decay of power following the prompt drop. Power data were recorded during and following the rod drop for about 10 sec.

Early rod-drop experiments produced only power and reactivity data. To determine the feedback reactivity, it was necessary to subtract out the reactivity worth of the steel rod, and, as mentioned in Chapter II, matching the zero-power worth of the rod with the reactivity obtained from rod drops at power was difficult. Consequently, a method for calibrating the worth of the steel rod as a function of vertical rod position was developed. This necessitated recording rod position as well as power during the experiment. The computation of feedback reactivity was changed so that the feedback was computed by the same computer code that computed system reactivity. This method for handling the data was developed by Hyndman and Nicholson.⁵

The data-acquisition system used was essentially the same as that for the oscillator experiments; it was calibrated in the same manner. Signals available to the NAVCOR analog-to-digital (A/D) converter were

(a) rod position, (b) ion-chamber output proportional to reactor power, and (c) an initiate signal which started the collection of digitized values of the first two signals by the IBM-1620 computer. Data were collected at 200 samples per sec for about 10 sec. The storage capacity of the computer limited the amount of data collected to about 4000 data points.

The rod-drop experiment was relatively simple to perform. The reactor was operated at a selected delayed-critical power level for sufficient time to establish an equilibrium or steady-state condition where transient effects were negligible. The steel rod was then dropped, completing its downward motion in about 280 msec. Data were collected, starting at 0.5 sec before the drop and ending 10 sec after the drop.

Feedback was computed by subtracting the reactivity inserted by the rod drop from the system reactivity following the rod drop. Experiments performed at zero power calibrated the reactivity worth of the steel rod as a function of rod position. During succeeding experiments at power, rod-position and power traces were obtained; then system reactivity was computed by using inverse-kinetics calculations, and the inserted rod worth from position and calibration data was subtracted to give the feedback reactivity.

Usually three rod drops were performed and the results averaged to obtain a set of values for the feedback reactivity. Averaging served to minimize the effect of noise in the data.

B. System Error

Errors associated with the data-acquisition system were assumed to be minimized by calibrating the system gain with a standard signal so that the system gain for each experiment was known. It was assumed that the noise in the data produced greater error than any introduced by the A/D converter. The signal-to-noise (S/N) ratio was about 100 to 1 for rod-drop experiments at power, and about 50 to 1 for the zero-power experiments.

Errors introduced by (a) trapezoidal integration in the inverse-kinetics calculation, and (b) the assumption that power was linear between sample points, were small in comparison with the error due to noise in the data. Pointwise averaging of feedback obtained from three experiments and the data-sampling rate of 200 samples/sec tended to minimize error due to noise.

C. Experimental Results

The results obtained from rod-drop experiments at 22.5, 30.0, and 41.5 MWt for one reactor loading (Run 26B) are shown in Figs. 15-17. Corresponding power traces are also shown. Plots are from results that

have been subjected to an averaging process to reduce the number of points to be plotted. The time step between the computed values was 0.005 sec, and the time step between plotted values was 0.01 sec for power data and 0.02 sec for feedback-reactivity data.

Fig. 15. Reactor Feedback and Normalized Power during Rod Drop at 22.5 MWt

Fig. 16. Reactor Feedback and Normalized Power during Rod Drop at 30.0 MWt

Fig. 17
Reactor Feedback and Normalized Power during Rod Drop at 41.5 MWt

An adjustment of the calibrated rod worth was necessary to obtain an initial value of the feedback near zero. The adjustment consisted of multiplication of the rod calibration by the factor that made the initial feedback approximately zero. This adjustment seemed to be an increasing function of power and may have been the result of a calibration error corresponding to a slight change in rod worth between zero power and higher power levels. Adjustment factors are given in Table III.

Examination of the plotted results reveals that for a short time during or following the rod drop there is no feedback. This does not seem likely; however, no completely satisfactory explanation was found. The most likely explanation is that there is some error in the calibration obtained for the steel rod

TABLE III. Rod-calibration Adjustments

Power, MWt	Adjustment Factor
22.5	1.0
30.0	1.02
41.5	1.04

and that the sampling rate is not fast enough to adequately define reactor power during the time the rod falls, so that reactivity and feedback reactivity cannot be computed with sufficient accuracy during this time. In addition, it cannot be shown that the reactivity worth of the rod remains constant as reactor power is increased.

The method used to compute reactivity and feedback reactivity is presented in Appendix B.

D. Feedback Models

Feedback models were assumed to have the form of Eq. 10, which is repeated here for convenience:

$$H(s) = \sum_i \frac{A_i \exp(-T_i s)}{1 + \tau_i s}$$

The time-domain form of this function was considered to be

$$H(t) = \sum_i \frac{A_i \exp(T_i/\tau_i)}{\tau_i} \int_0^{t-T_i} [1 - P(t')] \exp\left[-\frac{(t-t')}{\tau_i}\right] dt', \quad (11)$$

where

τ_i = time constant,

T_i = time delay,

$P(t')$ = normalized power,

and

A_i = amplitude constant proportional to change in power.

The time-domain form can be found by considering that the magnitude of a feedback component is primarily dependent on the change in power at any time t . Values of prior times decay with the appropriate time constant and must be integrated to time t . Terms delayed in time ($T_i \neq 0$) represent transport delays within the reactor system which delay the contribution of such terms until $t > T_i$.

To determine values of the model, the A_i , τ_i , and T_i must be estimated and values of $H(t)$ computed for comparison with measured values. Examination of a plot of the feedback obtained from a rod-drop experiment allows the first estimation of these values.

Experience has shown that the model can be developed by estimating the components of the first term, the term with the smallest time constant

and little or no time delay; once it is established, successive terms can be determined and added until the measured values are approximated by the model.

The time constant of the first term was chosen to be the time required for the initial rise in the measured feedback to reach 63% of its saturation value, and A_1 was then approximately the saturation value divided by the prompt power change. Assuming no time delay, this one-term model was used in Eq. 11, and both the measured and modeled feedback were plotted for comparison; necessary adjustments were made in the amplitude constant, A_1 . The procedure was repeated until this term approximated the measured feedback for about 1 sec.

A slight improvement in the approximation of the prompt feedback was attained by considering the prompt feedback to be composed of two components with time constants of 0.2 and 0.4 sec, respectively.

Referring to the 22.5-MWt feedback in Fig. 15, it was assumed that at 1-2 sec a negative term was causing the feedback to level off, and at 5 or 6 sec a positive term began to increase the feedback magnitude. The negative term was estimated to be slightly less in amplitude than the prompt term or terms, with a time constant on the order of 1 to 2 sec. In the manner described above, successive terms were estimated and added to the model until a good approximation to the measured feedback was obtained. Each additional term required some small adjustment of the preceding terms. In some cases, use of time delay in conjunction with a reduced time constant provided a better fit.

This procedure is quite time-consuming, and it is necessary to have a computer-plotter to plot measured feedback and computed model values for comparison if a model is to be determined in any reasonable time. Experience in fitting the model to the measured values shortens the time appreciably.

Models B and C, determined in this manner, and the feedback computed from rod-drop experiments at 22.5 MWt are shown in Fig. 18.

Fig. 18
Time-domain Reactor Feedback Models B
and C with 22.5-MWt Feedback

The frequency domain expressions for Models B and C are:

Model B

$$H(s) = \frac{0.0603}{1 + 0.2s} + \frac{0.0548}{1 + 0.4s} - \frac{0.096 \exp(-0.2s)}{1 + 2s} + \frac{0.0658 \exp(-0.5s)}{1 + 5s} - \frac{0.0603}{1 + 10s}.$$

Model C

$$H(s) = \frac{0.0548}{1 + 0.2s} + \frac{0.0479 \exp(-0.03s)}{1 + 0.4s} - \frac{0.053545}{1 + 2.5s} - \frac{0.0452 \exp(-s)}{1 + 16s}.$$

Values for the 22.5-MWt models are tabulated in Appendix C.

Note that these models are not unique. Small variations in the parameters of the model may also produce very similar models; however, such small variations are not likely to imply contradictory physical phenomena in the reactor system. In many cases, a large negative component and a large positive component with similar time constants could have been chosen so that they would almost cancel each other and so produce a relatively minor change in the amplitude of the model. This seemed physically unrealizable in the reactor system; therefore, the modeling procedure was felt to describe the feedback function reasonably well.

As with the oscillator model, the terms with short time constants (<1 sec) correspond to metal fuel and coolant feedback; the longer time constants are associated with structural effects, such as bowing in the sixth or seventh row, which is possibly due to the temperature differential at the edge of the core. A compaction-expansion effect which is a function of power may be a result of this temperature differential; however, this study does not encompass enough experimental data to evaluate such a possibility. Prompt components of the model seem to scale fairly well with power. This is to be expected, since these components represent fuel and coolant effects and the reactor-core temperature differential is nearly linear with power. Other components apparently do not scale linearly, for the feedback at higher powers is underestimated by the scaled-up 22.5-MWt model. This is a desirable condition as it implies that, as power is increased, the negative terms in the assumed model have relatively less effect than the positive terms. The positive terms represent negative feedback effects and the negative terms represent positive feedback effects with respect to the definition of negative feedback. The results from scaling 22.5-MWt Models B and C up to 30.0 and 41.5 MWt are shown in Figs. 19 and 20.

Frequency-domain plots of these models show fairly good agreement with the feedback obtained from the oscillator experiments for frequencies in the range 0.03-6.0 Hz. Below this range, the feedback amplitude is

overestimated, particularly by Model B. The phase angle of the feedback is best approximated by Model C, except for frequencies within the range 0.04-0.7 Hz. Tabulated values of Models B and C are given in Appendix C. The Nyquist plots of the models with the 22.5-, 30.0-, and 41.5-MWt oscillator-determined feedback are shown in Figs. 21-23.

Fig. 19. Time-domain Reactor Feedback Models B and C Scaled to 30.0 MWt with 30.0-MWt Feedback

Fig. 20. Time-domain Reactor Feedback Models B and C Scaled to 41.5 MWt with 41.5-MWt Feedback

Fig. 21. Reactor Feedback Models B and C with Frequency-domain Feedback at 22.5 MWt. (Numbers in parentheses indicate frequency in Hz.)

Fig. 22. Scaled Reactor Feedback Models B and C with Frequency-domain Feedback at 30.0 MWt. (Numbers in parentheses indicate frequency in Hz.)

Fig. 23

Scaled Reactor Feedback Models B and C with Frequency-domain Feedback at 41.5 MWt. (Numbers in parentheses indicate frequency in Hz.)

V. REACTOR STABILITY AND COMPARISON OF METHODS

Evaluation of the two experimental techniques used in this study requires a comparison of the resulting models. Limitations on the oscillator experiments force the comparison to be made on the basis of experiments conducted with the reactor operating at 22.5 MWt.

A. Reactor Stability

Conditional stability of the reactor is exhibited by the models obtained from both experimental methods. The feedback $H(s)$ takes on values, for all models, in the third quadrant. Presence of negative terms and time delay in the models both give rise to the possibility of an unstable condition should the magnitude of the negative terms or the time delay become great enough.

Amplitude magnification and angular phase addition necessary to make the measured 22.5-MWt feedback and the three models respectively equal to $-G_0(s)^{-1}$ were computed over the frequency range 0.001-9.0 Hz. The amplitude magnification was the value $[|G_0(s)| |H(s)|]^{-1}$ and the phase adjustment was the minimum angular rotation that made the phase of $H(s)$ equal to that of $-G_0(s)^{-1}$. The results indicate that the system represented by the measured feedback and models would be absolutely unstable for a tenfold increase in feedback with a phase change of about 180° . The increase must be greater (except for Model B at low frequency) with a more moderate phase modification. Assuming the results are valid, the reactor could be absolutely unstable at a power level of about 225 MWt or almost four times the maximum design power level of 62.5 MWt. Models B and C agree with Model A and the measured feedback. This fact gives reassurance that the rod-drop technique is well worth further investigation. Tabulated amplification and phase addition are given in Appendix C.

Continued experiments using either the oscillator or rod-drop method would give ample warning of a tendency toward instability. The feedback models developed from the Run 26B experiments do not show $H(s)$ to be near $-G_0(s)^{-1}$ for 22.5-MWt reactor operation, and the measured feedback at higher powers indicated increased stability, although prediction of feedback above 41.5 MWt was not attempted. Models corresponding to feedback at higher power as well as models corresponding to the feedback at reduced power should be developed for each power run, since the character of the reactor is modified by both replacement of subassemblies and irradiation effects.

B. Comparison of Methods

Nyquist plots of models obtained from both types of experiments have the same general shape through the fourth quadrant. Proceeding from the origin, the high-frequency portion extends down along the negative

imaginary axis, then with decreasing frequency tends toward the positive real axis in a somewhat circular path. After the real axis is crossed, differences become apparent; the rod-drop models tend back toward the real axis more sharply than the oscillator model, which tends toward the origin as frequency approaches zero. None of the models approach the value of the power coefficient for this power level (about 0.006); however, this value is not known to sufficient accuracy to require discarding any of the models. Frequency-power limits on the oscillator experiments make both high- and low-frequency portions of the model somewhat questionable. Determination of feedback by normalizing the measured transfer function to the open-loop transfer function at a high-frequency point may inject error, either because the feedback is not negligible at that point or because the measured transfer function is in error as an effect of the resonance known to be present in the oscillator mechanism at frequencies greater than 9 Hz. Either error would affect the amplitude and phase of the feedback. In addition, it cannot be shown that the worth of the oscillator is independent of frequency, or that the worth of the stainless rod drop is independent of the power level of the reactor. The behavior of feedback computed for the time that the stainless rod is in motion during the rod-drop experiment is somewhat questionable too, since the feedback is believed to appear immediately after the rod drop begins.

Despite the above-mentioned faults and inconsistencies, the two methods produced remarkably similar results, and it is difficult to select one method as being superior on the basis of these experiments. Considering the present capabilities and limitations of the two methods, the rod-drop method is more useful in that data may be obtained at higher power levels and the overall behavior of the mechanism and the computed results are consistent with other rod-drop experiments. The rod-drop experiment produced stability data very similar to that of the oscillator tests; however, both methods need further evaluation. Too little oscillator data can be taken at the higher power levels to enable a judgment as to the worth of that mechanism in its present form.

Continued use will be made of both methods, since the results thus far justify continued investigation of the rod-drop technique. The oscillator technique is already well established. Rod-drop experiments typically require less than an hour for performing a set of three experiments at a given power level. Oscillator experiments require several hours at a given power level to obtain a set of transfer-function measurements covering three or more decades in frequency. The reduction of cost is considerable with the rod drop method, due to reduction of experiment time as well as reduction of the complexity of the experimental mechanism. For this reason alone, continued use and evaluation of the method are justified.

C. Desirable Improvements in Feedback-measurement System

Desirable oscillator improvements would be (1) extensions of the angular-velocity range above and below the present range, (2) reduction of the possibility that the oscillator reactivity worth is a function of power or frequency, and (3) development of an oscillator mechanism that has a long life span under irradiation. It is possible that all but the last of these improvements might be approached by designing the active or poison section of the rod so that the reactivity worth is a minimum, consistent with a sufficiently high signal-to-noise ratio during experiments. Improved cooling of the active section of such an oscillator is being investigated in an attempt to reduce the likelihood of bowing effects.

Some possible improvements to the rod-drop experiments are the reduction of rod-drop time and the use of positive, as well as negative, reactivity insertions. A method for effectively decreasing the rod-drop time is being investigated. This involves changing the configuration of the rod so that during the initial period of slow rod motion, negative reactivity insertion is compensated by a small, equal positive insertion. After the rod reaches a significant velocity, the inserted reactivity is all negative. Using a follower with proper reactivity characteristics could possibly reduce the reactivity-insertion time by as much as one-fourth to one-third of the present 280 msec.

Data-acquisition-system improvements already implemented include an increased memory capacity for the IBM-1620 computer and a method whereby the data-sampling rate is decreased after the first few seconds of a rod-drop experiment, enabling the acquisition of power data over longer time periods.

An improved computing system is planned for the near future, allowing faster sampling rates and much greater data-reduction capability. At present, the rod-drop experimental results require a large off-site computer to do the inverse-kinetics computations. The elimination of volumes of intermediate data, saving of time, and the possibility of increased sensitivity in measurements offer opportunities to expand and refine some of the techniques now used. The modeling technique, in particular, needs to be improved in order to reduce the amount of time spent in developing models that fit the observed feedback. Computer programs could be developed to aid in the determination of models if adequate criteria for the models can be determined.

APPENDIX A

Tabulated Oscillation Data

Tabulated values of the results obtained from the oscillator experiments are given in Tables A.1-A.8. Tabulated feedback values are converted to those plotted in the study by dividing the feedback by 0.00744.

TABLE A.1. Computed Open-loop Transfer Function

Frequency, Hz	Amplitude, $(\Delta n/n)/(\Delta k/k)$	Phase, deg	Frequency, Hz	Amplitude, $(\Delta n/n)/(\Delta k/k)$	Phase, deg
0.0010	1837.80	-77.972	0.35	143.23	-8.9852
0.0015	1279.60	-73.135	0.40	141.85	-8.2009
0.0020	1006.70	-69.081	0.45	140.78	-7.5450
0.0025	845.66	-65.643	0.50	139.93	-6.9875
0.0030	739.68	-62.685	0.60	138.66	-6.0890
0.0035	664.88	-60.115	0.70	137.77	-5.3942
0.0040	609.39	-57.871	0.80	137.13	-4.8396
0.0045	566.62	-55.907	0.90	136.65	-4.3862
0.0050	532.62	-54.184	1.00	136.28	-4.0085
0.0060	481.77	-51.338	1.50	135.29	-2.7909
0.0070	445.14	-49.116	2.00	134.89	-2.1352
0.0080	417.06	-47.348	2.50	134.70	-1.7291
0.0090	394.50	-45.910	3.00	134.59	-1.4545
0.0100	375.76	-44.711	3.50	134.52	-1.2573
0.0150	313.27	-40.585	4.00	134.48	-1.1092
0.0200	276.51	-37.781	4.50	134.45	-0.9943
0.0250	251.90	-35.512	5.00	134.43	-0.9028
0.0300	234.18	-33.546	6.00	134.40	-0.7670
0.0350	220.83	-31.788	7.00	134.38	-0.6719
0.0400	210.46	-30.190	8.00	134.37	-0.6023
0.0450	202.22	-28.729	9.00	134.37	-0.5497
0.0500	195.56	-27.391	10.00	134.36	-0.5091
0.0600	185.56	-25.047	20.00	134.34	-0.3676
0.0700	178.49	-23.087	30.00	134.34	-0.3704
0.0800	173.27	-21.447	40.00	134.34	-0.4095
0.0900	169.26	-20.067	50.00	134.33	-0.4630
0.1000	166.08	-18.897	60.00	134.33	-0.5237
0.1500	156.35	-14.999	70.00	134.33	-0.5886
0.2000	151.01	-12.723	80.00	134.33	-0.6561
0.2500	147.51	-11.143	90.00	134.33	-0.7253
0.3000	145.04	-9.942	100.00	134.32	-0.7957

TABLE A.2. Computed and Measured Open-loop Transfer Function

Frequency, Hz	Computed Amplitude, ($\Delta n/n$)/($\Delta k/k$)	Computed Phase, deg	Measured Amplitude, ($\Delta n/n$)/($\Delta k/k$)	Measured Phase, deg
8.803	1.34370E+02	-5.54000E-01	1.34370E+02	-5.54000E-01
6.156	1.34401E+02	-7.45000E-01	1.34509E+02	-6.88000E-01
4.009	1.34481E+02	-1.10000E-00	1.34233E+02	-1.02600E-00
1.973	1.34895E+02	-2.15100E-00	1.35070E+00	-2.21900E-00
0.974	1.36329E+02	-4.09100E-00	1.36158E+02	-3.86000E-00
0.798	1.37097E+02	-4.84000E-00	1.37194E+02	-4.76800E-00
0.603	1.38580E+02	-6.06500E-00	1.38629E+02	-6.09000E-00
0.499	1.39893E+02	-7.00000E-00	1.40102E+02	-7.05000E-00
0.400	1.41805E+02	-8.02050E-00	1.41934E+02	-8.08200E-00
0.301	1.44948E+02	-9.92300E-00	1.44950E+02	-9.80200E-00
0.200	1.50968E+02	-1.27260E+01	1.51069E+02	-1.27170E+01
0.099	1.66226E+02	-1.89630E+01	1.66521E+02	-1.90040E+01

TABLE A.3. 22.5 MWt Transfer Function for Run 26B

Frequency, Hz	Open-loop Amplitude, ($\Delta n/n$)/($\Delta k/k$)	Open-loop Phase, deg	Closed-loop Amplitude, ($\Delta n/n$)/($\Delta k/k$)	Closed-loop Phase, deg
6.21072	1.34401E+02	-7.43957E-01	1.34338E+02	-2.52498E-01
4.05057	1.34482E+02	-1.09595E-00	1.34241E+02	-4.64583E-01
2.50105	1.34703E+02	-1.72812E-00	1.34403E+02	-4.49441E-01
1.50223	1.35289E+02	-2.78674E-00	1.34808E+02	-1.25909E-00
1.15661	1.35853E+02	-3.52912E-00	1.34025E+02	-1.57920E-00
0.96477	1.36401E+02	-4.13384E-00	1.34560E+02	-1.85357E-00
0.79985	1.37136E+02	-4.83999E-00	1.34043E+02	-2.39816E-00
0.58380	1.38837E+02	-6.21813E-00	1.33116E+02	-3.44777E-00
0.50102	1.39915E+02	-6.97667E-00	1.33110E+02	-4.51868E-00
0.39948	1.41869E+02	-8.20790E-00	1.33634E+02	-5.79899E-00
0.30027	1.45033E+02	-9.93589E-00	1.35071E+02	-7.97105E-00
0.20023	1.50994E+02	-1.27150E+01	1.39924E+02	-1.17877E+01
0.20027	1.50990E+02	-1.27133E+01	1.38974E+02	-1.13321E+01
0.19980	1.51029E+02	-1.27306E+01	1.38941E+02	-1.04134E+01
0.13980	1.57822E+02	-1.56108E+01	1.46604E+02	-1.48458E+01
0.09955	1.65213E+02	-1.89458E+01	1.53841E+02	-1.92900E+01
0.09953	1.66216E+02	-1.89470E+01	1.52087E+02	-1.88693E+01
0.08016	1.73200E+02	-2.14226E+01	1.58301E+02	-2.20575E+01
0.08014	1.73209E+02	-2.14252E+01	1.56158E+02	-2.21753E+01
0.08013	1.73212E+02	-2.14266E+01	1.58898E+02	-2.20212E+01
0.06063	1.85039E+02	-2.49115E+01	1.67990E+02	-2.61025E+01
0.06064	1.85034E+02	-2.49101E+01	1.71378E+02	-2.55255E+01
0.04070	2.09182E+02	-2.99755E+01	1.91389E+02	-3.07485E+01
0.04071	2.09174E+02	-2.99741E+01	1.91909E+02	-3.10898E+01
0.01985	2.77374E+02	-3.78529E+01	2.55994E+02	-4.04656E+01
0.01985	2.77401E+02	-3.78552E+02	2.55298E+02	-3.98952E+01
0.00987	3.77936E+02	-4.48496E+01	3.49441E+02	-4.49636E+01
0.00987	3.77926E+02	-4.48490E+01	3.51002E+02	-4.67775E+01
0.00782	4.21600E+02	-4.76364E+01	3.98403E+02	-5.00926E+01
0.00567	4.96201E+02	-5.21772E+01	4.66393E+02	-5.33643E+01

TABLE A.4. 22.5-MWt Feedback for Run 26B

Frequency, Hz	Amplitude, $(\Delta k/k)/(\Delta n/n)$	Phase, deg	Real Part	Imaginary Part
6.21072	6.39332E-05	-8.63349E+01	4.08690E-06	-6.38025E-05
4.05057	8.30959E-05	-7.99647E+01	1.44798E-05	-8.18244E-05
2.50105	1.66687E-04	-8.31938E+01	1.96543E-05	-1.65512E-04
1.50223	1.99175E-04	-8.03740E+01	3.33054E-05	-1.96371E-04
1.15661	2.71445E-04	-6.57432E+01	1.11517E-04	-2.47480E-04
0.96477	3.10395E-04	-6.81565E+01	1.15489E-04	-2.88110E-04
0.79985	3.56504E-04	-5.82301E+01	1.87703E-04	-3.03089E-04
0.58380	4.71501E-04	-4.41425E+01	3.38354E-04	-3.28374E-04
0.50102	4.81991E-04	-3.49721E+01	3.94959E-04	-2.76266E-04
0.39948	5.30964E-04	-2.81177E+01	4.68300E-04	-2.50235E-04
0.30027	5.64431E-04	-1.67902E+01	5.40369E-04	-1.63046E-04
0.20023	5.35669E-04	2.44962E-01	5.35664E-04	2.29018E-06
0.20027	5.96321E-04	-4.19686E-00	5.94722E-04	-4.36409E-05
0.19980	6.40145E-04	-1.43079E+01	6.20289E-04	-1.58200E-04
0.13980	5.85901E-04	6.54325E-00	5.82084E-04	6.67653E-05
0.09955	4.85295E-04	2.35598E+01	4.44842E-04	1.93975E-04
0.09953	5.58992E-04	1.80320E+01	5.31537E-04	1.73034E-04
0.08016	5.47537E-04	2.87673E+01	4.79961E-04	2.63504E-04
0.08014	6.35379E-04	2.90062E+01	5.55682E-04	3.08098E-04
0.08013	5.23849E-04	2.85881E+01	4.59982E-04	2.50666E-04
0.06063	5.61007E-04	3.76516E+01	4.44171E-04	3.42696E-04
0.06064	4.34825E-04	3.31960E+01	3.63863E-04	2.38069E-04
0.04070	4.49525E-04	3.89966E+01	3.49363E-04	2.82874E-04
0.04071	4.40935E-04	4.32765E+01	3.21024E-04	3.02270E-04
0.01985	3.46318E-04	6.87934E+01	1.25274E-04	3.22866E-04
0.01985	3.39566E-04	6.20988E+01	1.58899E-04	3.00093E-04
0.00987	2.15827E-04	4.63606E+01	1.48946E-04	1.56193E-04
0.00987	2.23015E-04	7.03096E+01	7.51421E-05	2.09975E-04
0.00782	1.73234E-04	8.60194E+01	1.20257E-05	1.72816E-04
0.00567	1.35811E-04	7.12677E+01	4.36155E-05	1.28617E-04

TABLE A.5. 30.0 MWt Transfer Function for Run 26B

Frequency, Hz	Open-loop Amplitude, $(\Delta n/n)/(\Delta k/k)$	Open-loop Phase, deg	Closed-loop Amplitude, $(\Delta n/n)/(\Delta k/k)$	Closed-loop Phase, deg
4.02251	1.34484E+02	-1.10309E-00	1.34275E+02	-9.64883E-02
1.99498	1.34899E+02	-2.13992E-00	1.34126E+02	-4.68658E-01
0.89671	1.36666E+02	-4.39944E-00	1.34630E+02	-1.04103E-03
0.59219	1.38746E+02	-6.15035E-00	1.31881E+02	-2.24451E-00
0.40652	1.41702E+02	-8.10849E-00	1.31116E+02	-4.56194E-00
0.20175	1.50866E+02	-1.26590E-01	1.35368E+02	-1.03226E+01
0.09757	1.66795E+02	-1.91640E+01	1.48925E+02	-1.81991E+01

TABLE A.6. 30.0-MWt Feedback for Run 26B

Frequency, Hz	Amplitude, $(\Delta k/k)/(\Delta n/n)$	Phase, deg	Real Part	Imaginary Part
4.02251	1.31244E-04	-8.43509E+01	1.29191E-05	-1.30607E-04
1.99498	2.21017E-04	-7.75427E+01	4.76761E-05	-2.15813E-04
0.89671	4.88953E-04	-6.02013E+01	2.42987E-04	-4.24302E-04
0.59219	6.28172E-04	-4.91587E+01	4.10803E-04	-4.75227E-04
0.40652	7.28565E-04	-3.22496E+01	6.16171E-04	-3.88768E-04
0.20175	8.10699E-04	-9.14675E-00	8.00394E-04	-1.28872E-04
0.09757	7.27260E-04	1.02198E+01	7.15722E-04	1.29033E-04

TABLE A.7. 41.5-MWt Transfer Function for Run 26B

Frequency, Hz	Open-loop Amplitude, $(\Delta n/n)/(\Delta k/k)$	Open-loop Phase, deg	Closed-loop Amplitude, $(\Delta n/n)/(\Delta k/k)$	Closed-loop Phase, deg
4.07902	1.34480E+02	-1.08883E-00	1.34189E+02	2.84245E-01
1.98246	1.34906E+02	-2.15263E-00	1.34105E+02	-8.64840E-02
1.49922	1.35292E+02	-2.79188E-00	1.34180E+02	-2.86904E-01
0.99243	1.36306E+02	-4.03458E-00	1.33187E+02	-4.82834E-01
0.79887	1.37141E+02	-4.84487E-00	1.33032E+02	-1.04630E-00
0.59829	1.38681E+02	-6.10200E-00	1.31911E+02	-1.77460E-00
0.47889	1.40269E+02	-7.21199E-00	1.31750E+02	-2.80068E-00
0.39631	1.41946E+02	-8.25351E-00	1.30393E+02	-3.64862E-00
0.30175	1.44971E+02	-9.90460E-00	1.32121E+02	-5.95521E-00
0.19963	1.51044E+02	-1.27371E-01	1.34634E+02	-1.00428E+01

TABLE A.8. 41.5-MWt Feedback for Run 26B

Frequency, Hz	Amplitude, $(\Delta k/k)/(\Delta n/n)$	Phase, deg	Real Part	Imaginary Part
4.07902	1.79118E-04	-8.44288E+01	1.73892E-05	-1.78272E-04
1.98246	2.71718E-04	-7.95041E+01	4.94976E-05	-2.67171E-04
1.49922	3.30195E-04	-7.77714E+01	6.99396E-05	-3.22703E-04
0.99243	4.91033E-04	-6.72729E+01	1.89706E-04	-4.52907E-04
0.79887	5.39970E-04	-6.24138E+01	2.50050E-04	-4.78584E-04
0.59829	6.69788E-04	-5.25505E+01	4.07273E-04	-5.31738E-04
0.47889	7.30115E-04	-4.58802E+01	5.08277E-04	-5.24139E-04
0.39631	8.59315E-04	-3.75132E+01	6.81620E-04	-5.23275E-04
0.30175	8.35505E-04	-2.86998E+01	7.32868E-04	-4.01229E-04
0.19963	8.71718E-04	-1.08745E+01	8.56063E-04	-1.64456E-04

APPENDIX B

Inverse-kinetics Determination of System Reactivity

System reactivity was computed during the rod-drop experiment by using the customary one-group, space-independent kinetics equations which are linearized to comply with the assumption that the system is linear.

The procedure begins with the equations

$$\frac{dn(t)}{dt} = \frac{\Delta k(t)(1 - \beta) - \beta}{\ell^*} n(t) + \sum_{i=1}^6 \gamma_i C_i(t) \quad (B.1)$$

and

$$\frac{dC_i(t)}{dt} = \frac{1 + \Delta k(t)}{\ell^*} \beta_i n(t) - \gamma_i C_i(t). \quad (B.2)$$

Equation B.1 can be rewritten, by adding and subtracting Eq. B.2 after summing on i ,

$$\begin{aligned} \frac{dn(t)}{dt} &= \frac{\Delta k(t)(1 - \beta) - \beta}{\ell^*} n(t) + \sum_{i=1}^6 \gamma_i C_i(t) - \sum_{i=1}^6 dC_i(t) \\ &\quad + \sum_{i=1}^6 \frac{1 + \Delta k(t)}{\ell^*} \beta_i n(t) \sum_{i=1}^6 \gamma_i C_i(t). \end{aligned}$$

Collecting terms results in the form of $dn(t)/dt$ given in Chapter II,

$$\frac{dn(t)}{dt} = \frac{\Delta k(t)n(t)}{\ell^*} - \sum_{i=1}^6 \frac{dC_i(t)}{dt}. \quad (B.1')$$

Linearize Eq. B.2 by neglecting the product $\Delta k(t)\beta_i n(t)$, which is small compared with $\beta_i n(t)$; this results in the form of $dC_i(t)/dt$ given in Chapter II,

$$\frac{dC_i(t)}{dt} = \frac{\beta_i n(t)}{\ell^*} - \gamma_i C_i(t). \quad (B.2')$$

Let $n(t)$ and $C_i(t)$ be comprised of a steady-state and a transient component, respectively:

$$n(t) = n_0 + \Delta n(t)$$

and

$$C_i(t) = C_{i0} + \Delta C_i(t).$$

Consider Eq. B.2' for the steady-state case,

$$\frac{d[C_{i0} + \Delta C_i(t)]}{dt} \approx \frac{\beta_i[n_0 + \Delta n(t)]}{\ell^*} - \gamma_i[C_{i0} + \Delta C_i(t)],$$

or

$$0 \approx \frac{\beta_i n_0}{\ell^*} - \gamma_i C_{i0}, \quad (\text{B.3})$$

because the $\Delta n(t)$ and $\Delta C_i(t)$ are zero for the steady-state case, and the derivative of a constant is zero.

Now, Eq. B.2' can be rewritten using Eq. B.3:

$$\frac{d[\Delta C_i(t)]}{\ell^*} \approx \frac{\beta_i \Delta n(t)}{\ell^*} - \gamma_i \Delta C_i(t). \quad (\text{B.2''})$$

Linearize Eq. B.1' by rewriting with the Δ -notation of Eq. A.2'' and neglecting the product $\Delta k(t)\Delta n(t)$, which is small in comparison with $\Delta k(t)n_0$, and $dn_0 = 0$:

$$\frac{d[\Delta n(t)]}{dt} \approx \frac{\Delta k(t)n_0}{\ell^*} - \sum_{i=1}^6 \frac{d[\Delta C_i(t)]}{\ell^*}, \quad (\text{B.1''})$$

Equation B.2'' can be solved by application of the integration factor $\exp(\gamma_i t)$ and evaluation of the integral in the resulting expression by trapezoidal integration:

$$\begin{aligned} \Delta C_i(t) &\approx \frac{\beta_i}{\ell^*} \exp(-\gamma_i t) \int_{t_0}^t \Delta n(x) \exp(\gamma_i x) dx + \Delta C_i(t) \exp[-\gamma_i(t - t_0)] \\ &\approx \frac{\beta_i}{\ell^*} \exp(-\gamma_i t) \frac{\Delta t}{2} [\Delta n(t_0) \exp(\gamma_i t_0) + \Delta n(t) \exp(\gamma_i t)] \\ &\quad + \Delta C_i(t) \cdot \exp(-\gamma_i \Delta t); \quad \Delta t = t - t_0. \end{aligned}$$

Rearrange terms to give

$$\begin{aligned}\Delta C_i(t) &\approx \frac{\beta_i}{\ell^*} \frac{\Delta t}{2} [\Delta n(t_0) \exp(-\gamma_i \Delta t) + \Delta n(t) \exp(0)] + \Delta C_i(t) \exp(-\gamma_i \Delta t) \\ &\approx \frac{\beta_i}{\ell^*} \frac{\Delta t}{2} [\exp(-\gamma_i \Delta t) \Delta n(t_0) + \Delta n(t)] + \Delta C_i(t_0) \exp(-\gamma_i \Delta t).\end{aligned}$$

Substitute this expression for $\Delta C_i(t)$ in Eq. B.2", then substitute the resulting expression for $d\Delta C_i(t)/dt$ into Eq. B.1":

$$\begin{aligned}\frac{d\Delta n(t)}{dt} &\approx \frac{\Delta k(t)}{\ell^*} [n_0 + \Delta n(t)] - \sum_{i=1}^6 \frac{\beta_i \Delta n(t)}{\ell^*} \\ &\quad + \frac{\Delta t}{2} \sum_{i=1}^6 \frac{\gamma_i \beta_i}{\ell^*} [\exp(-\gamma_i \Delta t) \cdot \Delta n(t_0) + \Delta n(t)] \\ &\quad + \sum_{i=1}^6 \Delta C_i(t) \exp(-\gamma_i \Delta t).\end{aligned}$$

Multiply this relation by $\ell^* (\approx 10^{-7})$ and neglect any term in which ℓ^* remains as a coefficient after reducing to lowest terms. The result is

$$0 \approx \Delta k(t)[n_0 + \Delta n(t)] - \beta \Delta n(t) + \frac{\Delta t}{2} \sum_{i=1}^6 \gamma_i \beta_i [\exp(-\gamma_i \Delta t) \Delta n(t_0) + \Delta n(t)],$$

where

$$\beta = \sum_{i=1}^6 \beta_i.$$

Solve this expression for $\Delta k(t)$ to give

$$\Delta k(t) \approx \frac{\beta \Delta n(t)}{n_0 + \Delta n(t)} - \frac{\Delta t}{2[n_0 + \Delta n(t)]} \sum_{i=1}^6 \gamma_i \beta_i [\exp(-\gamma_i \Delta t) \Delta n(t_0) + \Delta n(t)].$$

Multiply this expression by β^{-1} to obtain reactivity with units of dollars,

$$\Delta k(t)(\text{dollars}) \approx \frac{\Delta n(t)}{n_0 + \Delta n(t)} - \frac{\Delta t}{2[n_0 + \Delta n(t)]} \cdot \sum_{i=1}^4 \frac{\beta_i}{\beta} \gamma_i [\exp(-\gamma_i \Delta t) \Delta n(t_0) + \Delta n(t)]. \quad (\text{B.4})$$

This expression was used in the study to evaluate the rod-drop worth at zero power and the system reactivity during rod-drop experiments at power. The $\Delta n(t)$ were obtained by normalizing the ion-chamber output to unity for the steady-state condition before the rod drop. The appropriate values on the right side of the relation were computed from the normalized power data by considering that Eq. B.4 represents the system reactivity at time t ; the system reactivity is zero for $t = 0$. Pointwise computation of the system reactivity injected by the rod drop is computed from rod position at each time point, and the value is subtracted from the system reactivity, leaving a value of system feedback reactivity.

Nuclear constants used are those given in Table I.

APPENDIX C

**Tabulated Model Values and Adjustment
to Equal the Instability Point**

Numerical values computed for the feedback Models A, B, and C are given in Table C.1. The adjustment of the models and the measured feedback necessary to make them respectively equal to the feedback value associated with absolute instability are shown in Table C.2.

TABLE C.1. Computed Values of ZLS-MMM Models A, B, and C

Frequency, Hz	Model A		Model B		Model C	
	Amplitude, cents	Phase, deg	Amplitude, cents	Phase, deg	Amplitude, cents	Phase, deg
0.001	0.27	39.5	2.59	6.0	0.71	49.8
0.002	0.50	71.0	2.59	11.4	1.21	60.4
0.005	0.73	74.1	2.73	16.0	1.73	61.6
0.008	1.21	74.6	3.10	22.1	2.66	57.9
0.008	1.09	71.6	3.71	25.3	3.72	50.2
0.010	2.33	69.0	4.07	25.3	4.23	45.7
0.020	4.13	55.5	5.02	20.4	5.64	32.9
0.030	5.35	44.4	5.32	19.1	6.39	27.2
0.050	6.63	29.6	6.07	21.1	7.44	20.1
0.080	7.42	17.0	7.42	18.3	8.37	11.6
0.100	7.68	11.4	8.07	14.3	8.69	6.8
0.200	8.04	-6.9	9.10	-3.5	8.96	-10.2
0.300	7.74	-19.6	8.94	-15.2	8.53	-21.7
0.500	6.57	-37.3	7.74	-30.0	7.31	-37.7
0.800	4.92	-50.7	6.34	-42.6	5.70	-52.3
1.000	4.31	-54.6	5.67	-48.1	4.94	-58.0
2.000	2.46	-74.8	3.79	-70.0	2.80	-78.0
3.000	1.64	-84.4	2.34	-81.5	1.89	-87.9
5.000	0.89	-93.6	1.17	-81.1	1.07	-100.2
8.000	0.45	-92.7	0.96	-98.9	0.56	-112.4
9.000	0.38	-88.8	0.71	-91.9	0.46	-115.3

TABLE C.2. Amplitude Magnification and Phase Rotation Making Models and Feedback Equal ($-G_0^{-1}$)

Frequency, Hz	Model A		Model B		Model C		Measured (Hz)	
	Amplitude ×	Phase, deg, +	Amplitude ×	Phase, deg, +	Amplitude ×	Phase, deg, +	Amplitude ×	Phase, deg, +
0.001	26.3	-161.5	2.9	-108.0	10.3	-151.9		
0.002	26.5	-182.0	5.2	-122.4	11.0	-171.4		
0.003	24.6	-191.5	6.7	-133.4	10.5	-179.0		
0.004	22.6	-197.0	7.6	-141.7	9.9	-182.4		
0.005	20.8	-200.5	8.1	-147.9	9.5	-183.7		
0.006	19.3	-202.6	8.4	-152.5	9.1	-184.0	15.3	-199.1
0.007	18.1	-203.7	8.6	-155.7	8.9	-183.6		
0.008	17.0	-204.3	8.7	-158.0	8.7	-182.9	12.5	-218.4
0.009	16.1	-204.5	8.7	-159.6	8.5	-182.0		
0.010	15.4	-204.3	8.8	-160.6	8.4	-181.1	11.9	-192.2
0.020	11.7	-197.7	9.7	-162.6	8.6	-175.1	10.3	-217.1
0.030	10.7	-190.9	10.8	-165.6	9.0	-173.7		
0.040	10.4	-185.8	11.3	-170.1	9.2	-173.2	11.0	-191.0
0.050	10.4	-182.2	11.3	-173.8	9.2	-172.7		
0.060	10.4	-179.5	11.1	-176.0	9.2	-172.1	11.0	-190.1
0.070	10.4	-177.4	10.7	-176.9	9.3	-171.2		
0.080	10.4	-175.6	10.4	-176.9	9.3	-170.2	10.2	-187.4
0.090	10.5	-174.0	10.2	-176.3	9.3	-169.1		
0.100	10.5	-172.5	10.0	-175.5	9.3	-168.0	11.6	-181.8
0.200	11.1	-160.3	9.8	-163.8	9.9	-157.1	11.2	-165.7
0.300	12.0	-150.4	10.5	-154.8	10.9	-148.3	12.4	-153.3
0.400	13.2	-142.3	11.4	-148.1	11.9	-142.2	12.3	-133.7
0.500	14.6	-135.7	12.4	-143.0	13.1	-135.3	14.8	-138.1
0.600	16.3	-130.6	13.4	-138.9	14.4	-130.4	15.3	-129.6
0.700	18.1	-126.9	14.4	-135.5	15.8	-126.3		
0.800	19.9	-124.4	15.5	-132.6	17.2	-122.9	22.0	-116.9
0.900	21.5	-122.7	16.4	-130.1	18.6	-119.9		
1.000	22.9	-121.3	17.4	-127.8	20.0	-117.4	23.6	-107.7
2.000	40.4	-103.0	26.9	-107.9	35.5	-99.8		
3.000	60.8	-94.1	42.6	-97.0	52.8	-90.6		
4.000	84.4	-88.8	59.8	-92.4	71.8	-84.1	64.0	-98.9
5.000	111.8	-85.4	85.5	-93.0	92.8	-78.8		
6.000	143.8	-84.0	85.7	-101.7	116.5	-74.3	117.0	-92.9
7.000	180.6	-84.4	95.8	-98.1	144.9	-70.4		
8.000	220.6	-86.7	104.4	-90.4	176.2	-66.9		
9.000	259.7	-90.6	140.5	-87.6	215.1	-64.1		

ACKNOWLEDGMENTS

The encouragement, support, and use of facilities of Argonne National Laboratory are greatly appreciated. This report could not have been done without the support of the Laboratory and a number of Laboratory employees including my thesis advisor, R. W. Hyndman, and my supervisor, R. R. Smith.

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