Measurement of $D^0 - \bar{D}^0$ Mixing and CP Violation in Two-Body $D^0$ Decays

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We present a measurement of $D^0 - {\overline{D}^0}$ mixing and CP violation using the ratio of lifetimes simultaneously extracted from a sample of $D^0$ mesons produced through the flavor-tagged process $D^{*+} \to D^0\pi^+$, where $D^0$ decays to $K^{\mp}\pi^\pm$, $K^-K^+$, or $\pi^-\pi^+$, along with the untagged decays $D^0 \to K^{\mp}\pi^\pm$ and $D^0 \to K^-K^+$. The lifetimes of the CP-even, Cabibbo-suppressed modes $K^-K^+$ and $\pi^-\pi^+$ are compared to that of the CP-mixed mode $K^{\mp}\pi^\pm$ in order to measure $y_{CP}$ and $\Delta Y$. We obtain $y_{CP} = [0.72 \pm 0.18 \text{(stat)} \pm 0.12 \text{(syst)}]\%$ and $\Delta Y = [0.09 \pm 0.26 \text{(stat)} \pm 0.06 \text{(syst)}]\%$, where $\Delta Y$ constrains possible CP violation. The $y_{CP}$ result excludes the null mixing hypothesis at 3.3$\sigma$ significance. This analysis is based on an integrated luminosity of 468 fb$^{-1}$ collected with the BABAR detector at the PEP-II asymmetric-energy e$^+e^-$ collider.

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I. INTRODUCTION

Several measurements [1–6] show evidence for mixing in the $D^0 - {\overline{D}^0}$ system consistent with predictions of possible Standard Model (SM) contributions [7–11]. These results also constrain many new physics models [12–16]. An observation of CP violation (CPV) in the $D^0 - {\overline{D}^0}$ system at the present experimental sensitivity would provide possible evidence for physics beyond the SM [17–21].

One manifestation of $D^0 - {\overline{D}^0}$ mixing is differing $D^0$ decay time distributions for decays to different CP eigenstates [22]. We present a measurement of charm mixing using the ratio of lifetimes obtained from the decays of neutral $D$ mesons to CP-even and CP-mixed two-body final states. We also present a search for indirect CP violation arising from a difference in $D^0$ and ${\overline{D}^0}$ partial decay widths to CP-even eigenstates. Recently the LHCb Collaboration has reported evidence for CPV in the difference of the time-integrated CP asymmetries in $D^0 \to K^-K^+$ and $D^0 \to \pi^-\pi^+$ decays [23]. This measurement is primarily sensitive to direct CPV. As explained in Appendix A, we are not sensitive to effects of direct CP violation at the level of the result reported by LHCb, and we therefore assume no direct CPV in our baseline model.

We measure the effective $D^0$ lifetimes in three different two-body final states: $K^{\mp}\pi^\pm$, $K^-K^+$, and $\pi^-\pi^+$. We make no distinction between the Cabibbo-favored $D^0 \to K^-\pi^+$ and doubly Cabibbo-suppressed $D^0 \to K^{\mp}\pi^\pm$ modes; in other words, we analyze and describe them together. Given the current experimental evidence indicating a small mixing rate, the lifetime distribution for all two-body final states is exponential to a good approximation. Decays in the $K^{\mp}\pi^\pm$ mode are to a CP-mixed final state, and are assumed to be described by the average $D^0$ width $\Gamma$. The singly Cabibbo-suppressed decays $D^0 (\overline{D}^0)$ to the CP-even $K^-K^+$ and $\pi^-\pi^+$ final states are described by the partial decay rate $\Gamma^+ (\Gamma^-)$, where

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For tagged decays, we reconstruct $D^{*+}$ candidates by combining a $D^0$ candidate with a slow pion track $\pi_\pi^+$, requiring them to originate from a common vertex constrained to the $e^+e^-$ interaction region. We require the $\pi_\pi^+$ momentum to be greater than 0.1 GeV/c in the laboratory frame and less than 0.45 GeV/c in the CM frame. We reject a positron that fakes a $\pi_\pi^+$ candidate by using $dE/dx$ information and veto any $\pi_\pi^+$ candidate that may have originated from a reconstructed photon conversion or $\pi^0$ Dalitz decay. The distribution of the difference $\Delta m$ between the reconstructed $D^{*+}$ and $D^0$ masses peaks near $\Delta m \sim 0.1455$ GeV/$c^2$. Backgrounds are suppressed by retaining only tagged candidates in the range $0.1447 < \Delta m < 0.1463$ GeV/$c^2$.

To determine the proper time $t$ and its error $\sigma_t$ for each $D^0$ candidate, we perform a combined fit to the $D^0$ production and decay vertices. We constrain the production point to be within the $e^+e^-$ interaction region, which we determine using Bhabha and di-muon events from triggers close in time to any given signal candidate event. We retain only candidates with a $\chi^2$-based probability for the fit $P(\chi^2) > 0.1\%$, and with $-2 < t < 4$ ps and $\sigma_t < 0.5$ ps. For tagged decays, this fit does not incorporate any $\pi_\pi^+$ information in order to ensure that the lifetime resolution models for tagged and untagged signal decays are very similar. The most probable value of $\sigma_t$ for signal events is $\sim 40\%$ of the nominal $D^0$ lifetime [27].

If an event contains a tagged $D^0$ decay, we exclude all untagged $D^0$ candidates from that event in the final sample. For a given final state, when multiple $D^0$ (for the untagged modes) or $D^{*+}$ (for the tagged modes) candidates in an event share one or more tracks, we retain only the candidate with the highest $P(\chi^2)$. The fraction of events with multiple $D^0$ candidates with overlapping daughter tracks is $\ll 1\%$ for all final states.

\section{II. EVENT RECONSTRUCTION AND SELECTION}

We use 468 fb$^{-1}$ of $e^+e^-$ colliding-beam data recorded at, and slightly below, the $Y(4S)$ resonance ($e^+e^-$ center-of-mass [CM] energy $\sqrt{s} \sim 10.6$ GeV) with the \textsc{babar} detector [26] at the SLAC National Accelerator Laboratory PEP-II asymmetric-energy B Factory. To avoid potential bias, we finalize our data selection criteria, as well as the procedures for fitting, extracting statistical limits, and determining systematic uncertainties, prior to examining the results.

We reconstruct charged tracks and vertices with a 5-layer, double-sided silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH). We select $D^0$ candidates by pairing oppositely charged tracks, requiring each track to satisfy particle identification criteria based on specific ionization energy loss ($dE/dx$) from the SVT and DCH, and Cherenkov angle measurements from a ring-imaging Cherenkov detector (DIRC). We then refit the $D^0$ daughter tracks, requiring them to originate from a common vertex. To reduce contributions from $D^0$'s produced via $B$-meson decay to a negligible level, we require each $D^0$ to have momentum in the CM frame $p_{CM} > 2.5$ GeV/c.

\section{III. INVARIANT MASS FITS}

We characterize the $D^0$ invariant mass ($M$) distribution for each of the seven modes with an extended unbinned maximum likelihood fit to $D^0$ and $\bar{D}^0$ samples. We allow the parameters governing the shapes of the probability density functions (PDFs), as well as the expected signal and background candidate yields, to vary in the fits. For the tagged CP-even modes we fit the $D^0$ and $\bar{D}^0$ samples simultaneously, sharing all parameters except for the expected signal and background candidate yields.

We fit the tagged $\pi^-\pi^+$ invariant mass distribution in the fit range $1.82 < M_{\pi\pi} < 1.93$ GeV/$c^2$ using a sum of two Gaussians with independent means and widths for the signal PDF, along with a first-order Chebychev polynomial for the total background.

The fit model for the tagged $K^-K^+$ invariant mass distribution is similar to $\pi^-\pi^+$, except that the fit range is $1.82 < M_{KK} < 1.91$ GeV/$c^2$, and the signal PDF is the sum of two independent Gaussians and a modified...
Gaussian with a power-law tail [28], which aids in better modeling of the lower tail of the distribution.

The signal PDF for the untagged $K^- K^+$ mode and for both tagged and untagged $K^\pm \pi^\pm$ modes is a sum of three independent Gaussians; the background is modeled using a second-order Chebychev polynomial. The mass fit range is $1.82 < M_{KK} < 1.91 \text{ GeV/c}^2$ for the untagged $K^- K^+$ mode, $1.81 < M_{K\pi} < 1.92 \text{ GeV/c}^2$ for the untagged $K^\pm \pi^\pm$ mode, and $1.80 < M_{K\pi} < 1.93 \text{ GeV/c}^2$ for the tagged $K^\pm \pi^\pm$ mode. In these modes, we do not distinguish $D^0$ from $D^*$ candidates, and therefore determine only the total signal and total background yields, in addition to the signal and background shape parameters.

The reconstructed $D^0$ invariant mass distributions and the fit results are shown in Fig. 1, together with a plot of the corresponding normalized Poisson pulls [29].

IV. SIGNAL AND SIDEBAND REGIONS

For the lifetime fit, we determine the regions in two-body invariant mass that maximize signal significance, minimize systematic effects due to backgrounds, and minimize the effect of the correlation between $D^0$ invariant mass and proper time. We refer to these regions as the lifetime-fit mass regions. Based on these studies, the optimal lifetime-fit mass region is $34 \text{ MeV/c}^2$ wide for all tagged modes and untagged $K^\pm \pi^\pm$ events, $1.847 < M < 1.881 \text{ GeV/c}^2$. Because of the smaller signal-to-background ratio for the untagged $K^- K^+$ events, the lifetime-fit mass region for this mode is only $24 \text{ MeV/c}^2$ in width, $1.852 < M < 1.876 \text{ GeV/c}^2$. For the tagged modes, a mass difference sideband $0.151 < \Delta m < 0.159 \text{ GeV/c}^2$ is used, along with a low (high) invariant mass sideband, $1.819 (1.890) < M < 1.839 (1.910) \text{ GeV/c}^2$. The low (high) mass sideband used for the untagged modes, $1.810 (1.899) < M < 1.830 (1.919) \text{ GeV/c}^2$, is displaced from the tagged sideband in order to reduce the signal component there. The signal purities in the lifetime-fit mass regions range from $\sim 75\%$ for the untagged $K^- K^+$ sample to $\sim 99.8\%$ for the tagged $K^\pm \pi^\pm$ events.

We classify $D^0$ candidate decays in the lifetime-fit mass region as follows: $D^0$ signal decays; misreconstructed-charm decays, i.e., those in which the candidate-$D^0$ daughter tracks are decay products of a non-signal weak charm decay; and random combinatorial background. Table I gives the composition of the misreconstructed-charm backgrounds expected from simulated events [30] in each final state.

V. LIFETIME FIT

The lifetimes are determined from an extended unbinned maximum likelihood fit to $t$ and $\sigma_t$ for candidates in the lifetime-fit mass region. All modes are fit simultaneously using shared signal resolution function param-

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FIG. 1: The reconstructed two-body invariant mass distributions for the seven modes. The vertical lines show the lifetime-fit mass region, defined in Sec. IV. The shaded regions are the background contributions. The normalized Poisson pulls for each fit are shown under each plot; “unt” refers to the untagged datasets.
TABLE I: Expected composition (in %) of the misreconstructed-charm backgrounds. Only misreconstructed-charm background channels that have > 1% contribution in at least one signal mode are listed. For the tagged modes, the yields are the sum of the separate $D^0$ and $\bar{D}^0$ tags.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Tagged Modes</th>
<th>Untagged Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 \rightarrow X\ell\nu$</td>
<td>15.4</td>
<td>10.3</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^-\pi^+$</td>
<td>80.8</td>
<td>14.9</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi^0\pi^+K^+$</td>
<td>1.1</td>
<td>70.3</td>
</tr>
<tr>
<td>$D^+ \rightarrow \pi^+\pi^+K^- \leq 1$</td>
<td>2.9</td>
<td>$\leq$</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^+K^-$</td>
<td>1</td>
<td>$\leq$</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi^+\pi^-$</td>
<td>1.8</td>
<td>$\leq$</td>
</tr>
<tr>
<td>$\Lambda$ decays</td>
<td>$\leq$</td>
<td>1</td>
</tr>
</tbody>
</table>

The explicit form of the signal lifetime PDF is

$$R_{F,L}(t; \sigma_t) = f_{i1} \mathcal{D}(t; \sigma_t; S^T_F S_{Fs1}, t_0, \tau_L) + (1 - f_{i1}) [f_{i2} \mathcal{D}(t; \sigma_t; S^T_F S_{Fs2}, t_0, \tau_L) + (1 - f_{i2}) \mathcal{D}(t; \sigma_t; S^T_F S_{Fs3}, t_0, \tau_L)],$$

where $f_{i1}$ (with $i = 1,2$) parameterizes the contribution of each individual Gaussian, $s_i$ (with $i = 1,2,3$) is a scaling factor associated with each Gaussian, and $t_0$ is an offset of the mean of the resolution function. The function $\mathcal{D}(t; \sigma_t; s, t_0, \tau)$ is given by

$$\mathcal{D}(t; \sigma_t; s, t_0, \tau) = C_{\sigma_t} \int \exp\left(-\frac{t_{\text{true}}^2}{\tau}\right) \exp\left(-\frac{(t - t_{\text{true}} + t_0)^2}{2s^2\sigma_t^2}\right) dt_{\text{true}},$$

where the normalization coefficient $C_{\sigma_t}$ is chosen such that

$$\int \mathcal{D}(t; \sigma_t; s, t_0, \tau) dt = 1 \quad \text{for each } \sigma_t.$$

With this definition, the product $H^\text{sig}_{\sigma_t}(\sigma_t) \cdot \mathcal{D}(t; \sigma_t; s, t_0, \tau)$ is a properly normalized two-dimensional conditional PDF, where $H^\text{sig}_{\sigma_t}(\sigma_t)$ is a PDF characterizing the $\sigma_t$ distribution, described below. To account for small differences in the resolution function for the different final states we introduce additional mode-dependent scale factors $S_F$, $F = K\pi, K K, \pi\pi$. We also allow for differences in the resolution functions for tagged and untagged modes by means of scale factors $S^T_F, T = \text{tagged}$ (tagged) or unt (untagged). We fix $S_{K\pi}$ and $S^\text{unt}_T$ to 1.

The three lifetime parameters are $\tau_L = \{\tau^+, \bar{\tau}^+, \tau_{K\pi}\}$, where $\tau_{K\pi}$ is extracted from the tagged and untagged $K^\mp\pi^\pm$ modes, while $\tau^+$ and $\bar{\tau}^+$ are extracted from the tagged and untagged CP-even modes. Approximately 0.4% of the tagged CP-even samples contain correctly reconstructed $D^0$ candidates combined with an unrelated $\pi^+$; this fraction has been estimated from simulated events and verified in data by an earlier $\text{BABAR}$ analysis [1]. These candidates have the same resolution and lifetime behavior as those from correctly reconstructed $D^+$ decays, but about half of them will be tagged as the wrong flavor. Therefore, the tagged CP-even $D^0$ proper-time distributions are modeled as the weighted sum of PDFs for correctly tagged and untagged candidates, characterized by the lifetime parameters $\tau^+$ and $\bar{\tau}^+$, respectively, and a mistag fraction $f_{\text{tag}} = 0.2\%$. The tagged CP-even $\bar{D}^0$ proper-time distributions are modeled in a similar fashion, where now the correctly tagged and mistagged PDFs are characterized by the lifetime parameters $\bar{\tau}^+$ and $\tau^+$, respectively. The untagged $K^-K^+$ proper-time distribution is modeled as a weighted sum of two PDFs characterized by the lifetime parameters $\tau^+$ and $\bar{\tau}^+$, and a weighting fraction $f_{D^0} = 0.5$. These parameterizations assume no direct CPV, and allow for CPV in the interference between decays with and without mixing characterized by a mode-independent weak phase $\phi$. Both $f_{\text{tag}}$ and $f_{D^0}$ are varied as part of the systematic error estimate for $y_{cp}$ and $\Delta Y^*$. All five tagged and two untagged signal lifetime PDFs are explicitly given in Appendix B.

We determine the $t$ versus $\sigma_t$ misreconstructed-charm signal-like PDF shape parameters and yields by fitting simulated events in the lifetime-fit mass region and then fix these parameters in the lifetime fit to data. We vary the lifetimes and yields as part of the study of systematic effects.

The largest background in the lifetime-fit mass region is due to random combinations of tracks. The PDF describing the two-dimensional combinatorial background in $t$ and $\sigma_t$ in the lifetime-fit mass region is characterized as a weighted average of the 2-d PDFs extracted from the mass sideband regions. The weights for the low and high sidebands are obtained from simulated events. The $(t, \sigma_t)$ combinatorial PDF in each sideband and for each mode, except for the untagged $K^-K^+$ mode, is extracted as a 2-d histogram from the sideband samples. From these histograms we subtract the contribution of signal and misreconstructed-charm backgrounds, each of which is estimated from simulated events, to obtain the final combinatorial PDF in each sideband. For the untagged $K^-K^+$ mode, a similar procedure is used but, instead of histograms, analytic signal-like PDFs are used. For the background PDFs the offsets and the lifetimes are allowed to be different for each Gaussian. The signal and misreconstructed-charm PDF parameters are extracted by fitting simulated events and then fixed, along with the expected candidate yields, in the fit that extracts the combinatorial PDFs in each sideband.
For the untagged $K^-K^+$ mode both the expected signal and combinatorial yields are free parameters in the lifetime fit. The expected combinatorial background yields in the other modes are determined by integrating the total background PDF extracted from the mass fit in the lifetime-fit mass region, and then subtracting the expected misreconstructed-charm background yields, which are determined from samples of simulated events. A small bias on these fit yields is observed in fits to simulated events. To correct for this, we scale the data yields based on the simulated-event fits and vary the mode-dependent scale factors as a systematic uncertainty. Table II gives the event class yields plus uncertainties obtained from the lifetime fit and indicates the yields that are fixed.

<table>
<thead>
<tr>
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<th>Untagged Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^-\pi^+$</td>
<td>$K^-K^+$</td>
</tr>
<tr>
<td>Signal</td>
<td>65 430</td>
</tr>
<tr>
<td></td>
<td>±260</td>
</tr>
<tr>
<td>Comb. Bkgd.</td>
<td>3760</td>
</tr>
<tr>
<td></td>
<td>±1000</td>
</tr>
<tr>
<td>Charm Bkgd.</td>
<td>97</td>
</tr>
</tbody>
</table>

The simultaneous fit to all events in the lifetime-fit mass region has 20 floating parameters: the seven signal yields and three signal lifetimes; the yield of untagged $K^-K^+$ combinatorial candidates; the offset $t_0$; the parameters $f_{t1}$ and $f_{t2}$ characterizing the weight of each Gaussian in the signal resolution mode; and the proper-time error scaling parameters $s_1$, $s_2$, $s_3$, $S_{KK}$, $S_{\pi\pi}$, and $S'_{\text{tag}}$. After extracting the three signal lifetimes, using their reciprocals in the computation of $y_{\text{CP}}$ and $\Delta Y$ as defined in Eqs. (1) and (2), respectively, we find

\[
y_{\text{CP}} = [0.72 \pm 0.18(\text{stat})]\%,
\]

\[
\Delta Y = [0.09 \pm 0.26(\text{stat})]\%.
\]

The statistical errors are computed using the covariance matrix returned by the fit. The lifetime-fit mass region proper-time distributions and projections of the lifetime fit for the seven different decay modes are shown in Fig. 2.

**VI. CROSS CHECKS AND SYSTEMATICS**

We have performed numerous cross checks to search for potential problems, in addition to quantitative studies that yield the systematic uncertainties given in Table III, discussed below. Initially we tested the fit model by generating large ensembles of datasets randomly drawn from the underlying total PDF, and observed no biases in the $y_{\text{CP}}$ and $\Delta Y$ results obtained. In addition, we have fit an ensemble of four simulated datasets, each equivalent in luminosity to the data, and found no evidence of bias in $y_{\text{CP}}$ or $\Delta Y$. 

![FIG. 2: Proper-time $t$ distribution for each decay mode with the fit results overlaid. The combinatorial distribution (indicated as ‘Comb.’ in light gray) is stacked on top of the misreconstructed-charm background distribution (indicated as ‘Charm’ in dark gray). The normalized Poisson pulls for each fit are shown under each plot; “unt” refers to the untagged datasets. The bottom right plot shows the individual lifetimes (with statistical uncertainties only); the gray band indicates the PDG $D^0$ lifetime $1\sigma$ [27].](image-url)
In fitting the data, we find that the tagged and untagged extracted lifetimes for $K^-K^+$, and separately for $K^\mp\pi^\mp$, are compatible within the statistical uncertainties. We performed a simultaneous fit to the tagged channels, and a separate simultaneous fit to the untagged channels, and find the lifetimes to be compatible within the statistical uncertainties. We repeated the fit allowing the $K^-K^+$ and $\pi^-\pi^+$ final states to have separate $\pi^+$ and $\pi^-$ lifetimes, and observed no statistically significant difference between the $K^-K^+$ and $\pi^-\pi^+$ results. We estimated the effects of the SVT misalignment to be negligible.

We varied the lifetime-fit mass region width by ±4 and ±2 MeV/c². We adopt as the systematic uncertainty half the RMS of the differences $|\Delta[y_{CP}]|$ and $|\Delta[\Delta Y]|$ from the nominal fit central values. We also shifted the position of each mass region by centering each of them at the most probable value for the signal PDF obtained in the invariant mass fits. These systematic uncertainties are given in the first two lines of Table III.

For the untagged $K^-K^+$ mode, the combinatorial yield is a parameter determined in the lifetime fit. However, it is also needed to determine the signal $\sigma_t$ PDF. We first use the total background yield determined from the mass fit to extract a signal $\sigma_t$ PDF, which is employed in an initial simultaneous lifetime fit. The combinatorial yield from this fit is used to construct an improved $\sigma_t$ signal PDF and a second fit is performed (the nominal fit). We estimate the systematic error on $y_{CP}$ and $\Delta Y$ associated with the determination of the signal $\sigma_t$ PDF for the untagged $K^-K^+$ mode to be the difference in the values obtained from an additional iteration of the fit and the nominal fit.

We vary the nominal mistag rate of 0.2% by ±0.04%, a 20% relative variation, and find no significant change in the nominal fit values. Instead of assuming equal fractions of $D^0$ and $D^0$ in the untagged $K^-K^+$ mode, we adopt the latest CDF result for direct $CIV$ [32], and find negligible change in $y_{CP}$ and $\Delta Y$.

We rely on simulated events to determine both the PDF shapes and yields for the misreconstructed-charm backgrounds. To account for the model dependence, we vary the effective lifetime of these events by ±5%, except for the tagged $\pi^-\pi^+$ mode where the variation is ±15% due to the small number of simulated events that pass the selection criteria for this mode. We also vary the expected misreconstructed-charm yields by ±10% in the tagged channels, and ±5% in the untagged channels. Each variation is simultaneously applied to all modes. These are ±$2\sigma$ variations relative to the statistical uncertainties of the simulated datasets.

We vary the yields, weighting parameters, and fitting strategy used to obtain the 2-d lifetime PDF for combinatorial-background events in the lifetime-fit mass region from the mass sidebands. The yields for the tagged combinatorial-background events are varied by ~5% in the $\pi^-\pi^+$ mode, 15% in tagged $K^-K^+$, and 20% in $K^\mp\pi^\mp$. The untagged $K^\mp\pi^\mp$ combinatorial-background yield is varied using the value extracted from an alternative lifetime-fit model in which the yield is allowed to vary.

The weights given to the low- and high-mass sidebands in the data in order to derive the combinatorial PDF in the lifetime-fit mass region in data are extracted from simulated events. They are varied by plus and minus the statistical uncertainty derived from splitting the simulated dataset, which is equivalent to several times the nominal integrated luminosity, into datasets that numerically match the nominal luminosity.

We also apply the variations described above for the misreconstructed-charm background to vary the yields and shape of the PDF that describe the residual signal events in the sidebands. This is also done for the misreconstructed-charm PDF used in the sideband fits from which the 2-d combinatorial PDF is extracted. This yields the combinatorial PDF shape variation, which is then used in the nominal fit, to obtain the variation reported in Table III.

Finally, we vary the $\sigma_t$ criteria by ±0.1 ps from the nominal $\sigma_t < 0.5$ ps, and take as the systematic uncertainty the RMS of the deviations from the nominal central value divided by $\sqrt{2}$. We also consider two variations in how multiple candidates sharing one or more daughter tracks are treated. In the first variation, we retain all multiple candidates, if each candidate passes all the other selection criteria. In the second variation, we reject all multiple candidates sharing one or more daughter tracks. We fit these datasets using the nominal fit model, and assign the largest observed deviation from the nominal $y_{CP}$ and $\Delta Y$ central values as the systematic uncertainty in Table III. The total $y_{CP}$ and $\Delta Y$ systematic uncertainties are calculated by summing the contributions from all sources in quadrature, and are reported in the last row of Table III.

| Fit Variation                      | $|\Delta[y_{CP}]|$ (%) | $|\Delta[\Delta Y]|$ (%) |
|-----------------------------------|------------------------|--------------------------|
| mass window width                 | 0.057                  | 0.022                    |
| mass window position              | 0.005                  | 0.001                    |
| untagged $KK$ signal $\sigma_t$ PDF | 0.022                  | 0.000                    |
| mistag fraction                   | 0.000                  | 0.000                    |
| untagged $KK$ $D^0$ fraction      | 0.001                  | 0.000                    |
| charm bkgd. lifetimes             | 0.042                  | 0.001                    |
| charm bkgd. yields                | 0.016                  | 0.000                    |
| comb. yields                      | 0.043                  | 0.002                    |
| comb. sideband weights            | 0.004                  | 0.001                    |
| comb. PDF shape                   | 0.066                  | 0.000                    |
| $\sigma_t$ selection              | 0.052                  | 0.053                    |
| candidate selection               | 0.028                  | 0.011                    |
| Total                             | 0.124                  | 0.058                    |
VII. CONCLUSIONS

In summary, we measured $y_{CP}$ and $\Delta Y$ to a precision significantly better than our previous measurements [2,3]. Both results are more precise than, and consistent with, the weighted average of all previous measurements [24], when the previous $B\bar{B}$ results are excluded. In particular, the $y_{CP}$ measurement is the most precise single measurement to date. We obtain

$$y_{CP} = [0.72 \pm 0.18 \text{(stat)} \pm 0.12 \text{(syst)}] \%,$$

$$\Delta Y = [0.09 \pm 0.26 \text{(stat)} \pm 0.06 \text{(syst)}] \% .$$

We exclude the null mixing hypothesis at 3.3$\sigma$ significance, and find no evidence for CPV. Our results are consistent with the world average value of the mixing parameter $y$ obtained from $D^0 \rightarrow K^0 h^- h^+$ (where $h = K, \pi$) [24], as expected in absence of CPV.

The value of $\Delta Y$ obtained here is consistent with our previously published result [2] when the same definition is used in both cases. The new $y_{CP}$ value is consistent with our previous result [3] with a probability of $\gtrsim 2\%$, assuming that the systematics for both the old and new measurements are fully correlated, and taking into account the fact that $\sim 40\%$ of the events in the current sample are also present in the samples used in the previous measurements [2,3]. The results here supersede the previous $B\bar{B}$ results for these modes [2,3].

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Appendix A: Mixing Formalism and Considerations on the Role of Direct CP Violation

The time evolution of the flavor eigenstates \( D^0 \) and \( \bar{D}^0 \) is governed by the Schrödinger equation:

\[
i \frac{\partial}{\partial t} \left( \frac{D^0(t)}{\bar{D}^0(t)} \right) = (M - \frac{i}{2} \Gamma) \left( \frac{D^0(t)}{\bar{D}^0(t)} \right).
\]  

(A1)

The mass eigenstates \( D_1 \) and \( D_2 \) are obtained from the diagonalization of the effective Hamiltonian \( \mathcal{H}_{\text{eff}} = M - \frac{i}{2} \Gamma \). Under the hypothesis of CPT conservation the two mass eigenstates can be written in terms of the flavor eigenstates as

\[
|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle,
|D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle,
\]

(A2)

where

\[
\left( \frac{q}{p} \right)^2 = \frac{M_{12}^2 - \frac{4}{3} \Gamma_{12}^2}{M_{12}^2 - \frac{4}{3} \Gamma_{12}^2} \quad \text{and} \quad |p|^2 + |q|^2 = 1.
\]

(A3)

We choose the positive root for \( q/p \); choosing the negative one just means exchanging \( D_1 \) with \( D_2 \). If \( CP|D^0\rangle = +|\bar{D}^0\rangle \), in case of no CPV, \( D_1 \) is the CP-even state and \( D_2 \) the CP-odd state.

It is traditional to quantify the size of \( D^0 - \bar{D}^0 \) mixing in terms of the parameters \( x \equiv \Delta m / \Gamma \) and \( y \equiv \Delta \Gamma / 2 \Gamma \), where \( \Delta m = m_1 - m_2 \) (\( \Delta \Gamma = \Gamma_1 - \Gamma_2 \)) is the difference in mass (width) of the states defined in Eq. A2 and \( \Gamma = (\Gamma_1 + \Gamma_2) / 2 \) is the average width. If either \( x \) or \( y \) is non-zero, mixing will occur. While most Standard Model expectations for the size of both are \( \lesssim 10^{-5} \) [10, 33], values as high as \( 10^{-2} \) or even higher are predicted by certain models [13, 15].

CP violation can manifest in \( D^0 \) decays in three ways:

- in decay, when \( |A_f/\bar{A}_f| \neq 1 \);
- in mixing, when \( r_m = |q/p| \neq 1 \);
- in the interference between decays with and without mixing, when the weak phase \( \phi_f \) of \( \lambda_f \equiv \frac{q}{p} \frac{A_f}{\bar{A}_f} \) is different from zero,

where \( A_f \) (\( \bar{A}_f \)) is the amplitude for \( D^0 \) (\( \bar{D}^0 \)) decaying into a final state \( f \), \( A_f \equiv \langle f|\mathcal{H}_D|D^0\rangle \) \( (\bar{A}_f \equiv \langle f|\mathcal{H}_D|\bar{D}^0\rangle) \).

The presence of mixing alters the exponential distribution for the \( D^0 \) decay into a final state \( f \). In particular we have

\[
\Gamma(D^0(t) \rightarrow f) = \frac{1}{2}|A_f|^2 e^{-\Gamma t} \left[ (1 + |\lambda_f|^2) \cos \gamma \left[ (1 - |\lambda_f|^2) \cos \gamma (1 - |\lambda_f|^2) \cos \gamma \right] \sinh \gamma \left[ (1 - |\lambda_f|^2) \cos \gamma (1 - |\lambda_f|^2) \cos \gamma \right] \right],
\]

(A4)

\[
\Gamma(\bar{D}^0(t) \rightarrow f) = \frac{1}{2}|\bar{A}_f|^2 e^{-\Gamma t} \left[ (1 + |\lambda_f|^2) \cos \gamma \left[ (1 - |\lambda_f|^2) \cos \gamma (1 - |\lambda_f|^2) \cos \gamma \right] \sinh \gamma \left[ (1 - |\lambda_f|^2) \cos \gamma (1 - |\lambda_f|^2) \cos \gamma \right] \right],
\]

(A5)

In this analysis we are interested in CP-even final states \( (f = h^+ h^-, h = K, \pi) \). If we neglect second-order terms in \( x \Gamma t \) and \( y \Gamma t \), the decay time distributions can be treated as exponentials with effective widths [34]:

\[
\Gamma(D^0(t) \rightarrow f) \propto e^{-\Gamma_{hh}^+ t} \quad \text{with} \quad \Gamma_{hh}^+ = \Gamma \left[ 1 + y \Re(\lambda_{hh}) - x \Im(\lambda_{hh}) \right],
\]

(A6)

\[
\Gamma(\bar{D}^0(t) \rightarrow f) \propto e^{-\Gamma_{hh}^- t} \quad \text{with} \quad \Gamma_{hh}^- = \Gamma \left[ 1 + y \Re(\lambda_{hh}^-) - x \Im(\lambda_{hh}^-) \right].
\]

(A7)

To better understand the effects of CP violation we introduce two more parameters, one describing CPV in decay \( (A_D^f) \) and one in mixing \( (A_M) \):

\[
A_D^f = \frac{|A_f/\bar{A}_f|^2 - |\bar{A}_f/A_f|^2}{|A_f/\bar{A}_f|^2 + |\bar{A}_f/A_f|^2},
\]

(A8)

\[
A_M = \frac{r_m^2 - r_m^2}{r_m^2 + r_m^2},
\]

(A9)
Since $f = h^+ h^-$ then $f = \bar{f}$. Noting that there is no strong phase in $\lambda_f$ since the final state is its own CP-conjugate, we can express $\lambda_{hh}$ in terms of $A_{hh}^h$, $A_M$ and the CP-violating phase $\phi_{hh}$:

$$\lambda_{hh} = \left[ 1 - A_{hh}^h \frac{1 + A_M}{1 - A_M} \right]^{1/4} e^{i \phi_{hh}}. \quad (A10)$$

Expanding Eqs. A6 and A7, and retaining only terms up to first order in $A_{hh}^h$ and $A_M$, we obtain

$$\Gamma_{hh}^+ \simeq \Gamma \left[ 1 + (y \cos \phi_{hh} - x \sin \phi_{hh}) + \frac{1}{2} (A_M - A_{hh}^h) (y \cos \phi_{hh} - x \sin \phi_{hh}) - \frac{1}{4} A_M A_{hh}^h (y \cos \phi_{hh} - x \sin \phi_{hh}) \right], \quad (A11)$$

$$\Gamma_{hh}^- \simeq \Gamma \left[ 1 + (y \cos \phi_{hh} + x \sin \phi_{hh}) - \frac{1}{2} (A_M - A_{hh}^h) (y \cos \phi_{hh} + x \sin \phi_{hh}) - \frac{1}{4} A_M A_{hh}^h (y \cos \phi_{hh} + x \sin \phi_{hh}) \right]. \quad (A12)$$

Combining the widths defined above we obtain the two observables $y_{\Delta CP}$ and $\Delta Y$, which, in general, depend on the final state because of the CPV parameters $A_{hh}^h$ and $\phi_{hh}$:

$$y_{\Delta CP}^{hh} = \frac{\Gamma_{hh}^+ + \Gamma_{hh}^-}{2\Gamma} - 1, \quad (A13)$$

$$\Delta Y^{hh} = \frac{\Gamma_{hh}^- - \Gamma_{hh}^+}{2\Gamma}. \quad (A14)$$

Other experiments characterize the CP-violating observable as $A_{\Gamma}$,

$$A_{\Gamma} = \frac{\Gamma_{hh}^+ - \Gamma_{hh}^-}{\Gamma_{hh}^+ - \Gamma_{hh}^-}. \quad (A15)$$

The relationship between $A_{\Gamma}$, $\Delta Y$ and $y_{\Delta CP}$ is

$$\Delta Y = (1 + y_{\Delta CP}) A_{\Gamma}. \quad (A16)$$

These quantities are directly related to the fundamental parameters that govern mixing and CPV in the charm sector:

$$y_{\Delta CP}^{hh} = y \cos \phi_{hh} - \frac{1}{2} [A_M + A_{hh}^h] x \sin \phi_{hh} - \frac{1}{4} A_M A_{hh}^h y \cos \phi_{hh}, \quad (A17)$$

$$\Delta Y^{hh} = -x \sin \phi_{hh} + \frac{1}{2} [A_M + A_{hh}^h] y \cos \phi_{hh} + \frac{1}{4} A_M A_{hh}^h x \sin \phi_{hh}. \quad (A18)$$

Both $y_{\Delta CP}$ and $\Delta Y$ are zero if there is no $D^0 - \bar{D}^0$ mixing. Otherwise, a non-zero value of $y_{\Delta CP}$ implies mixing and a non-zero value of $\Delta Y$ implies CPV.

In the charm sector, because the CKM elements involved belong to the Cabibbo submatrix, we can assume that the weak phase $\phi_{hh}$ does not depend on the final state: $\phi_{hh} = \phi [35]$. As stated earlier if direct CPV has a significant effect, then the values of $y_{\Delta CP}$ and $\Delta Y$ depend on the final state. In this analysis we assume that the effect of direct CPV is negligible in the decays to CP eigenstates; i.e., we assume $\Gamma_{KK}^+ = \Gamma_{\pi\pi}^+$ (and $\Gamma_{KK}^- = \Gamma_{\pi\pi}^-$). In Eqs. A11 and A12 this means neglecting the linear terms in $A_{hh}^h$. Assuming that $A_{hh}^h$ and $y$ are both $O(1\%)$ and $\phi_{hh} = 0$, the neglected term is $O(10^{-4})$, beyond any current experimental sensitivity.

Under the above assumptions, Eqs. A11 and A12 become

$$\Gamma^+ \simeq \Gamma \left[ 1 + (y \cos \phi - x \sin \phi) + \frac{A_M}{2} (y \cos \phi - x \sin \phi) \right], \quad (A19)$$

$$\Gamma^- \simeq \Gamma \left[ 1 + (y \cos \phi + x \sin \phi) - \frac{A_M}{2} (y \cos \phi + x \sin \phi) \right]. \quad (A20)$$
Inserting Eqs. A19 and A20 into Eqs. 1 and 2 yields

\[ y_{CP} = y \cos \phi - \frac{A_M}{2} x \sin \phi, \]  
\[ \Delta Y = -x \sin \phi + \frac{A_M}{2} y \cos \phi. \]  

(A21)  

(A22)

From the experimental point of view, we measure three lifetimes instead of the partial widths:

• \( \tau^+ \) for the \( D^0 \to K^-K^+, \pi^-\pi^+ \) decays,

• \( \bar{\tau}^+ \) for the \( \bar{D}^0 \to K^-K^+, \pi^-\pi^+ \) decays,

• \( \tau_{K\pi} \) for the \( D^0 \) and \( \bar{D}^0 \) \( \to K^+\pi^\pm \) decays (the Cabibbo favored \( K^-\pi^+ \) and the doubly Cabibbo suppressed \( K^+\pi^- \) decays are collected in the same sample),

and use their inverse to compute \( y_{CP} \) and \( \Delta Y \).

The measured observables constrain the parameters that govern mixing and indirect \( CPV \) in the charm sector.

Appendix B: Signal Lifetime PDFs

The explicit form of the signal lifetime PDFs based on the prototype PDFs presented in the main text are given below:

\[ \mathcal{P}^{D^+}_{\pi\pi}(t, \sigma_i) = (1 - f^+_{tag}) \mathcal{R}^{tag}_{\pi\pi}(t, \sigma_i; S_{\pi\pi} S'_\text{tag} s_i, t_0, \tau^+) + f^+_{tag} \mathcal{R}^{tag}_{\pi\pi}(t, \sigma_i; S_{\pi\pi} S'_\text{tag} s_i, t_0, \bar{\tau}^+), \]

\[ \mathcal{P}^{D^-}_{\pi\pi}(t, \sigma_i) = (1 - f^-_{tag}) \mathcal{R}^{tag}_{\pi\pi}(t, \sigma_i; S_{\pi\pi} S'_\text{tag} s_i, t_0, \tau^+) + f^-_{tag} \mathcal{R}^{tag}_{\pi\pi}(t, \sigma_i; S_{\pi\pi} S'_\text{tag} s_i, t_0, \bar{\tau}^+), \]

\[ \mathcal{P}^{D^+}_{KK}(t, \sigma_i) = (1 - f^+_{tag}) \mathcal{R}^{tag}_{KK}(t, \sigma_i; S_{KK} S'_\text{tag} s_i, t_0, \tau^+) + f^+_{tag} \mathcal{R}^{tag}_{KK}(t, \sigma_i; S_{KK} S'_\text{tag} s_i, t_0, \bar{\tau}^+), \]

\[ \mathcal{P}^{D^-}_{KK}(t, \sigma_i) = (1 - f^-_{tag}) \mathcal{R}^{tag}_{KK}(t, \sigma_i; S_{KK} S'_\text{tag} s_i, t_0, \tau^+) + f^-_{tag} \mathcal{R}^{tag}_{KK}(t, \sigma_i; S_{KK} S'_\text{tag} s_i, t_0, \bar{\tau}^+), \]

\[ \mathcal{P}^{D^+\pm}_{KK}(t, \sigma_i) = \mathcal{R}^{tag}_{KK}(t, \sigma_i; S_{KK} S'_\text{tag} s_i, t_0, \tau_{K\pi}), \]

\[ \mathcal{P}^{D^0}_{KK}(t, \sigma_i) = (1 - f_{D^0}) \mathcal{R}^{untag}_{KK}(t, \sigma_i; S_{KK} S'_\text{unt} s_i, t_0, \tau^+) + f_{D^0} \mathcal{R}^{untag}_{KK}(t, \sigma_i; S_{KK} S'_\text{unt} s_i, t_0, \bar{\tau}^+), \]

\[ \mathcal{P}^{D^0}_{\pi\pi}(t, \sigma_i) = \mathcal{R}^{untag}_{\pi\pi}(t, \sigma_i; S_{\pi\pi} S'_\text{unt} s_i, t_0, \tau_{K\pi}), \]

where \( f^+_{tag} = 0.2\% \), \( f_{D^0} = 0.5 \) and \( S_{KK} = S'_\text{unt} = 1 \) are fixed in the nominal fit.