Single and Polycrystal Resistivity Relationships
for Yttrium.

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Abstract

Different proposals for calculating polycrystal resistivities from single crystal values are applied to yttrium metal. It is shown that a simple average yielding $\rho_{\text{POLY}} = \frac{1}{3}(2\rho_{\perp} + \rho_{\parallel})$ gives the best fit to experimental data.

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INTRODUCTION

It is reasonable to expect that a general relation exists between the principal resistivities of a single crystal and the isotropic resistivity of a polycrystalline sample of the same substance. There is, however, little agreement in the literature as to the form of this relation; several methods of calculating the polycrystal resistivity have been proposed. Voigt¹ suggests

$$\sigma_{poly} = \frac{1}{3} (2\sigma_{\perp} + \sigma_{\parallel}) ,$$

which is the same as

(1)

$$\rho_{poly} = \frac{3\rho_{\perp}\rho_{\parallel}}{2\rho_{\parallel} + \rho_{\perp}} ,$$

where $\rho_{\parallel}$ and $\rho_{\perp}$ are the principal resistivities of a hexagonal crystal measured parallel and perpendicular to the axis of symmetry,

and where $\rho_{\perp} = \frac{1}{\sigma_{\perp}}$, $\rho_{\parallel} = \frac{1}{\sigma_{\parallel}}$, $\rho_{poly} = \frac{1}{\sigma_{poly}}$.

Andrade and Chalmers² obtain

$$\rho_{poly} = \frac{\sqrt{\rho_{\perp}(\rho_{\parallel} - \rho_{\perp})}}{\tanh^{-1} \sqrt{(\rho_{\parallel} - \rho_{\perp})/\rho_{\perp}}}$$

for the case $\rho_{\parallel} > \rho_{\perp}$. This is equivalent to

(2)

$$\rho_{poly} = \frac{\sqrt{\rho_{\perp}(\rho_{\perp} - \rho_{\parallel})}}{\tanh^{-1} \sqrt{(\rho_{\perp} - \rho_{\parallel})/\rho_{\perp}}}$$

for $\rho_{\perp} > \rho_{\parallel}$. Bruggeman³ proposes

$$\sigma_{poly} = \frac{1}{4} \left[\sigma_{\perp} + \sqrt{\sigma_{\perp} (8\sigma_{\parallel} + \sigma_{\perp})} \right],$$

or

$$(3) \quad \rho_{poly} = 4 / \left[\frac{1}{\rho_{\perp}} + \sqrt{\frac{1}{\rho_{\perp}} \left(\frac{8}{\rho_{\parallel}} + \frac{1}{\rho_{\perp}} \right)} \right].$$

Another method,⁴ which has been proposed more recently, yields

$$(4) \quad \rho_{poly} = \frac{1}{3} (2\rho_{\perp} + \rho_{\parallel}).$$

This relation is the result of an average over the total solid angle of the well-known expression,

$$\rho(\theta) = \rho_{\parallel} \cos^2 \theta + \rho_{\perp} \sin^2 \theta,$$

for the resistivity along an arbitrary direction of current flow in a hexagonal crystal.⁵

Several attempts^{2, 3} to resolve this question experimentally have been indecisive; if the anisotropy ratio, $\rho_{\perp}/\rho_{\parallel}$ is very near unity, the small difference between values given by any of equations 1-4 is masked by the probable error of the experiment. Recently Hall et al.⁶ have reported the principal single-crystal resistivities of yttrium (see Table II). Their values at 300°K lead to the exceptionally high anisotropy ratio of

$$\rho_{\perp}/\rho_{\parallel} = 2.07,$$

which suggests that a fair test of the various calculated ρ_{poly} might be made through a comparison with measurements on polycrystalline samples of this metal.

EXPERIMENTAL PROCEDURE

The polycrystalline yttrium data were obtained from three separate arc-melted samples produced in this Laboratory. The finished samples were cylinders 2 inches long and 3/16 inch in diameter. They were annealed at 900°C for two hours, cold swaged with a 10% reduction in diameter and reannealed at 800°C for one hour. These operations eliminated grain orientation and reduced the size of the grains. The samples were examined and found to have an average grain diameter of 0.14 mm. Analysis by back-reflection x-ray methods showed that the grains were randomly oriented. Table I lists the impurities found by spectrographic and vacuum fusion analyses on one of the samples, Y-II. Except for the silicon content which is unusually high, these values are representative of the other samples.

The resistivities of the three samples were measured using the method and apparatus described by Colvin et al.⁷

RESULTS AND CONCLUSIONS

The electrical resistivity of a representative sample (Y-II) in the temperature range 1.3°K to 300°K is shown in Fig. 1. Table II shows the residual resistivities together with the resistivities at 300°K (less residuals)

of the three polycrystalline samples and the single crystals. The calculated results of substituting the measured principal single-crystal resistivities into Eqs. 1-4 are also included. The last column of this table is depicted graphically in Fig. 2. Also shown are the estimated probable errors, taken to be $\pm 1\%$ for the experimental values and $\pm 2\%$ for the calculated values. It is clear that the best fit to the experimental data is obtained from Eq. 4. It therefore seems appropriate to use this relation for hexagonal metals.

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Table I.

Impurity Analysis of Sample Y-II. (Values in %).

Er $\leq$ 0.005; Ho $\leq$ 0.05; Dy $\leq$ 0.005; Tb $\leq$ 0.04; Gd $\leq$ 0.01; Sm $\leq$ 0.05;
Ca $<$ 0.02; Fe, 0.05; Mg $<$ 0.01; Si, 0.5; Ta $<$ 0.1; Cr, 0.1, O₂, 0.033;
Mn, Ti, Cu, Ni, Ca, Al, Cr, Mg, Si, Bi, trace present.

Table II.

Comparison of Experimental and Calculated Results.

	Residual resistivity, (ohm-cm x 10 ⁻⁶)	Resistivity at 300°K (Residual subtracted) (ohm-cm x 10 ⁻⁶)
Basal plane*	2.6	71.6
c-axis*	1.5	34.6
Y - I **	9.6	60.0
Y - II **	5.2	59.5
Y - III **	3.2	59.6
Eq. 1 ***	-	52.8
Eq. 2 ***	-	56.8
Eq. 3 ***	-	55.2
Eq. 4 ***	-	59.3

* Hall et al.⁶

** Present experimental results

*** Values obtained from substitution of Hall's
results into Eqs. 1-4.

FIGURE CAPTIONS

Fig. 1. The electrical resistivity of yttrium (Y-II) as a function of temperature.

Fig. 2. Graphical presentation of results showing probable errors.

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