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Checking the Operation of a Nuclear Counting Channel

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Checking the Operation of a Nuclear Counting Channel

Kenneth E. Relf

INTRODUCTION

"Instantaneous" information concerning the behavior of a reactor as well as "delayed" information such as neutron flux distributions, resonance escape probabilities, thermal utilizations, conversion ratios and fast fission factors depend upon the determination of numbers proportional to the disintegration rate of a radioactive material. These numbers are obtained from a nuclear counting channel consisting of an array of electronic gear to perform the following functions: (1) detection of the nuclear radiation, (2) pulse amplification, (3) pulse discrimination, (4) pulse registration, and (5) program control. The complexity of such a system demands special methods for determining its satisfactory operation. The usual test equipment consisting of oscilloscopes, vacuum tube voltmeters, and tube checkers is necessary but not sufficient. Temperature drifts of phototube and amplifier gain, discriminator levels and slit widths, and scintillator power-supply voltages cannot be detected with ordinary test equipment. Defective optical coupling between scintillator crystals and phototubes, imperfect crystals, aging Geiger tubes and ailing proportional counters also escape instrumental detection.

Conclusions drawn from experimental data depend upon the correctness of the original information; the precision with which the original information is obtained determines the ultimate confidence limits to be associated with a given

result. Since most of the design information for a nuclear reactor obtained from a critical facility depends in some way upon information obtained from a nuclear counting channel, correct operation of the channel is of the utmost importance. This paper presents quantitative criteria for judging the performance of a counting channel.

REVIEW OF METHODS

Quantitativeness is a necessity for judging the relative merits of most physical systems. Numbers are preferable to the qualitative descriptions "satisfactory," "unsatisfactory," "acceptable," and "unacceptable." Several methods for appraising the performance of a counting channel have been suggested, but each of them appears to lack decisiveness. Combining these procedures with statistical tests of significance into one straight-forward test, however, can strengthen the assurance of a properly operating channel.

The "statistical purity check" proposed by Riedel^{1/} offers an excellent method for evaluating graphically the statistical distribution of a set of counts obtained from a long-lived standard. He points out that those spurious pulses not detected by a cursory observation of a set of counts are usually distributed at random. The distribution remains Gaussian in nature, but it becomes broadened. This feature appears not as a random but rather as a systematic deviation of the experimental points from the expected "pure" normal line when the data are plotted on probability paper.^{2/} The difference in slope between the "line of best fit" drawn through the experimental points and the line drawn through the three theoretical points provides a numerical estimate of the actual standard deviation. The difficulties with the Riedel test are (1) the "line of best fit" is usually difficult to draw; and (2) a criterion is needed to specify when the difference between the actual and the theoretical standard deviations is too great for the channel to be considered operating properly. The Riedel purity check would seem to be lacking in precision, but would be valuable to illustrate graphically the fit of the experimental data to a normal distribution. The need for graphical inspection will be discussed later.

Comparison of the actual and the theoretical standard deviations has been proposed by de Lange.^{3/} The actual standard deviation is given by the relation:

$$\sigma_{\bar{R}} = \left[\sum_{i=1}^y \frac{(\bar{R} - R_i)^2}{y(y-1)} \right]^{1/2} \quad (1)$$

The theoretical standard deviation is expressed by the equation:

$$\sigma_s = \frac{\sigma_s}{y} = \frac{1}{y} \left[\sum_{i=1}^y \frac{N_i}{T_i^2} \right]^{1/2} \quad (2)$$

The terminology is that of de Lange and is defined as follows:

N_i = number of observed counts in a single count interval.

T_i = the time required to accumulate N_i counts.

R_i = the count rate = N_i/T_i .

σ_s = the sum of the standard deviations = $\left[\sum_{i=1}^y \sigma_{si}^2 \right]^{1/2}$,

$$\text{where } \sigma_{si} = \frac{\sqrt{N_i}}{T_i} = \left[\frac{R_i}{T_i} \right]^{1/2} = \left[\frac{R_i^2}{N_i} \right]^{1/2},$$

$$\sigma_{\bar{R}} = \frac{\sigma_s}{y}$$

$\sigma_{\bar{R}}$ = standard deviation of the mean count rate $\bar{R}$.

He states that the quantity $\sigma_{\bar{R}}$ is very sensitive to counting errors while σ_s is not. A fractional standard deviation can be defined for $\sigma_{\bar{R}}$ and σ_s as follows:

$$V_R = \frac{\sigma_{\bar{R}}}{\bar{R}} \times 100 \quad (3)$$

$$V_s = \frac{\sigma_s}{\bar{R}} \times 100 \quad (4)$$

The conclusions reached by de Lange are:

1. If $V_s^2 \geq V_R^2$, "it is highly unlikely that the apparatus can be faulty."
2. "...the apparatus can only be faulty if $V_s^2 < V_R^2$. The probability that there is a fault increases with the difference

$$V_R^2 - V_s^2."$$

It can be demonstrated that these conditions are equivalent to the following:

(See Appendix I)

$$\left(\frac{V_s}{V_R} \right)^2 \geq 1 \longrightarrow \frac{\sigma_{th}}{\sigma_{act}} \geq 1 \quad (5)$$

$$\left(\frac{V_s}{V_R} \right)^2 < 1 \longrightarrow \frac{\sigma_{th}}{\sigma_{act}} < 1. \quad (6)$$

The terms σ_{th} and σ_{act} are defined as follows:

$$\sigma_{th} = \text{the theoretical standard deviation} = \sqrt{m}$$

$$m = \text{mean of the } y \text{ counts, } N_1$$

$$\sigma_{act} = \text{the actual standard deviation} = \left[\frac{\sum_{i=1}^y (N_i - m)^2}{y - 1} \right]^{1/2}$$

(It is the opinion of the author that these definitions lessen the numerical work involved in an actual checkout.) These criteria are useful as indicators, but again there is a need for quantitativeness. Experience shows that in many cases $V_s < V_R$ but the channel, by all other tests, appears to be in working order. The standard deviation of the standard deviation has been overlooked in this case; hence a test of significance is desirable to dictate when the difference between σ_{th} and σ_{act} is important.

A synthesis of the good features of these two methods, a Fisher variance test, and a Chi-Square test can be made to provide a high level of probability that a counting channel is operating correctly. In addition, the magnitude of the instrumental errors can be estimated. The confidence level is set by the experimenter, hence the risk of error is quantitative and subject to control by the scientist.

SYNTHESIS OF A TESTING ROUTINE

Definiteness can be given to the Riedel purity check by subjecting the observed count rates to a Chi-Square test. This test provides a measure of the

goodness of fit of experimental data to a normal distribution. Chi-Square tables allow one to state that the observed counts follow (or do not follow) a normal distribution with a specified confidence limit. On the other hand, the Riedel plot complements the Chi-Square test since this test should not be used on a purely mechanical basis.

Quantitativeness can be given to the de Lange criterion by subjecting the actual and theoretical standard deviations to a Fisher variance test, usually referred to as an F test. This test indicates whether or not the two standard deviations are estimates of a common parent variance. The quantity $\bar{\sigma}_s$ ($\propto \sigma_{th}$) is an estimate of the standard deviation of a sample drawn from a population that is known to have a normal distribution, viz., pure random radioactive decay; the quantity $\sigma_{\bar{R}}$ ($\propto \sigma_{act}$) is an estimate of the standard deviation of a sample drawn from a population that may or may not have superposed upon it extraneous fluctuations. If such fluctuations exist, this sample would be characterized by a different value of the estimated standard deviation, usually larger. If there are no external disturbances to alter the normal distribution characterizing pure radioactive decay, $\bar{\sigma}_s$ and $\sigma_{\bar{R}}$ should be estimates of the same variance, this variance being characteristic of a common parent population from which the y determinations N_i are drawn. The quantities $\bar{\sigma}_s$ and $\sigma_{\bar{R}}$ will not, in general, be identically equal because they are two independent estimates computed by two different methods. The Fisher variance test states with what probability specified differences between $\bar{\sigma}_s$ and $\sigma_{\bar{R}}$ may be expected to occur. If the observed count rates belong to the set that represents pure random radioactive decay, the difference between the actual and theoretical standard deviations

will not be statistically significant; if the sample counts have superposed upon them other spurious counts, the actual standard deviation may differ significantly from the theoretical one. A set of counts exhibiting the theoretical standard deviation may then appear to be from a different population^{*} so that the actual and theoretical standard deviations would not be estimates of a common parent variance. The Fisher test lets the experimenter decide with whatever confidence level he desires whether or not the two standard deviations are derived from samples of the same population. If the actual standard deviation is the larger and is considered significant, the magnitude of the superposed standard error is easily computed, and a decision can be made as to whether or not the channel should be used.

The routine for investigating the behavior of a nuclear counting channel can be outlined as a series of steps which follows.

Step 1: Determine immediately if the channel is seriously defective by comparing the difference (statistically) between any two of the first few recorded count rates of a long-lived standard.^{4/} It could, for example, be the very first two.

(a) Let

$$\sigma_1 = \sqrt{N_1} \quad (7)$$

$$\sigma_2 = \sqrt{N_2} \quad (8)$$

^{*}A population refers to the infinite set of observations from which the sample is drawn. The implication here is that there are two possible populations--one is the infinite set representing pure random radioactive decay, the other is the set representing pure random decay upon which is superimposed other spurious counts of a random nature.

The standard error of the difference $N_1 - N_2$ is given by the expressions:

$$\sigma_{12} = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{N_1 + N_2} \quad (9)$$

(b) Compute the quantity k :

$$k = \frac{N_1 - N_2}{\sigma_{12}} \quad (10)$$

(c) Refer to Table.1^{4/} and estimate the probability of occurrence of this difference in count rates. A value of $k = 1.9$, for example, has a probability of occurrence of 0.020; i.e., once out of 50 times a difference as large as or larger between N_1 and N_2 might occur to give a value of $k = 1.9$. Such a probability might be considered indicative of trouble and a search could be made before proceeding with any further analysis. Certainly, if a value of $k = 3.1$ corresponding to a probability of 0.00097 (about 1 chance in 1000) were obtained, trouble shooting should begin immediately.

If, on the other hand, this quick test does not yield "significant" probabilities (say, none less than 0.05, i.e., 1 in 20) one should proceed to the more rigorous tests beginning with Step 2.

- Step 2: Accumulate a set of 50 counts ($y = 50$) from a long-lived source. The intensity of the source should be sufficient to yield a count rate as high as the highest expected count rate from the samples to be counted.
- Step 3: Calculate the average value of the 50 counts.

$$m = \frac{\sum_{i=1}^y N_i}{y} \quad (11)$$

- Step 4: Compute the theoretical standard deviation:

$$\sigma_{th} = \sqrt{m} \quad (12)$$

- Step 5:

- (a) Form the following sum and difference quantities:

$$\begin{aligned} \text{SUM: } m + 2\sigma_{th} \\ \text{DIFF: } m - 2\sigma_{th} \end{aligned} \quad (13)$$

- (b) Search the set of 50 counts for those that lie above and below the upper and lower limits computed in (a) above. Five per cent of the 50 counts (i.e., 2 or 3) can be expected to lie outside the limits since two standard deviations represent the 95% confidence limits in a normal distribution. Experience indicates that if 4 or more fall outside the limits the channel is usually defective, though it may not be seriously so.

Step 6: Compute the actual standard deviation:

$$\sigma_{\text{act}} = \left[\frac{\sum_{i=1}^y (N_i - m)^2}{y - 1} \right]^{1/2} \quad (14)$$

NOTE: To facilitate the computational procedure, use the following identity: (See Villars, ref. 10, p 21)

$$\sum_{i=1}^y (N_i - m)^2 = \sum_{i=1}^y N_i^2 - \frac{(\sum_{i=1}^y N_i)^2}{y} \quad (15)$$

Step 7: Apply the de Lange criterion:

- (a) If $\frac{\sigma_{\text{th}}}{\sigma_{\text{act}}} \geq 1$, the channel is probably functioning properly.
- (b) If $\frac{\sigma_{\text{th}}}{\sigma_{\text{act}}} < 1$, the channel may not be functioning properly.

Caution is necessary, however, in applying these criteria. Since both standard deviations are statistical quantities subject to random fluctuations, the difference between them may or may not be "significant". A "significance test" is required to give real meaning to the conditions (a) and (b); this is supplied by the Fisher test^{5/} for variances.

Step 8: Apply the Fisher test to the two standard deviations, σ_{act} and σ_{th} , by computing a value for F; two methods are available.

- (a) Method I - Double-tail test. The same confidence level is specified for both de Lange conditions (a) and (b).

$$F = \left(\frac{\sigma_{\text{larger}}}{\sigma_{\text{smaller}}} \right)^2 \quad (16)$$

Table II^{6/} lists the probabilities corresponding to several values of F. The table is interpreted as follows:

First, the assumption is made that σ_{act} and σ_{th} are equally good estimates of the standard deviation of a common parent population of which the set of y counts is a sample. For $y = 50$ (y-l-ry if y is large, say 20), the probability is 0.1 that the value of $F = 1.44$ will be exceeded, on the average. This means that, on the average, $\sigma_{\text{th}}/\sigma_{\text{act}} (= \sqrt{F})$ will equal or exceed 1.20 once out of every 10 times merely by chance. It also means that, on the average, $\sigma_{\text{act}}/\sigma_{\text{th}}$ will equal or exceed 1.20 once out of ten times. These statements can be combined into a single statement which constitutes the "double-tail" test, viz., that once out of 5 times, on the average, σ_{act} and σ_{th} will differ (either way) by as much as 20% or more due only to the play of chance. This can be easily demonstrated by the derivation below:

$$\sqrt{F} = \frac{\sigma_{\text{larger}}}{\sigma_{\text{smaller}}} \quad (\text{For double-tail test.}) \quad (17)$$

$$\sqrt{F} - 1 = \sigma_l / \sigma_s - 1 = \frac{\sigma_l - \sigma_s}{\sigma_s} \quad (18)$$

For a value of $F = 1.44$, one obtains the following result:

$$100 \times \frac{\sigma_l - \sigma_s}{\sigma_s} = 100 [\sqrt{F} - 1] = 100 (1.20 - 1) = 20\%.$$

To decrease the percentage difference for the same confidence limit, the number of accumulated counts must be increased. For example, if $y = 120$, $F = 1.26$ (defined by equation 16) will be exceeded 20 per cent of the time or once out of 5 times on the average. The standard deviations may differ by as much as 11.2 per cent or more (instead of 20 per cent for $y = 50$) once out of 5 times. Stated in a different way, 4 out of 5 times the standard deviations would not be expected to differ by more than 11.2%.

- (b) Method II - Single-tail tests. Separate confidence limits are specified for de Lange criteria (a) and (b).

If the actual standard deviation σ_{act} exceeds the theoretical value σ_{th} (case $\sqrt{b}$ of the de Lange criterion) a single tail Fisher test permits one to state in what percentage of cases the instrumental fluctuations can be expected to exceed some known fraction of the theoretical standard deviation. This can be demonstrated by the following development. The law governing the combination of independent errors permits the

following equation to be written:

$$\sigma_{\text{act}} = (\sigma_{\text{th}}^2 + \sigma_{\text{ins}}^2)^{1/2} \quad (19)$$

where σ_{ins} = standard error contributed by instrumental fluctuations.

Dividing both sides by σ_{th} and solving for σ_{ins} yields this relation:

$$\begin{aligned} \sigma_{\text{ins}} &= \sigma_{\text{th}} \left[(\sigma_{\text{act}}/\sigma_{\text{th}})^2 - 1 \right]^{1/2} \\ &= \sigma_{\text{th}} [F - 1]^{1/2} \end{aligned} \quad (20)$$

For $y = 50$, $F = 1.44$ at the 10 per cent level; i.e., once out of ten times, on the average, this value may be exceeded.

$$\sigma_{\text{ins}} = \sigma_{\text{th}} [1.44 - 1]^{1/2} = 0.664 \sigma_{\text{th}}$$

This result states that if the channel is operated under conditions such that $F = 1.44$, 9 out of 10 times, on the average, the instrumental error will not exceed 0.664 of the theoretical standard deviation. Suppose, then, that the average count rate of the samples to be counted has been estimated (or determined) to be 40,000 counts per count interval.

$$\sigma_{th} = \sqrt{m} = \sqrt{40,000} = 200 \text{ counts}$$

$$\sigma_{ins} = (0.664) (200) = 133.8 \text{ counts}$$

The theoretical relative standard error (commonly called the statistical error of counting) is given by the equation:

$$V_s = 100 \times \frac{\sigma_s}{\bar{R}} = 100 \times \frac{\sigma_{th}}{\bar{R}} = 0.50\%$$

The relative instrumental standard error is given by a similar expression:

$$V_{ins} = 100 \times \frac{\sigma_{ins}}{\bar{R}} = 0.33\%$$

The total relative standard error is:

$$\begin{aligned} V_{act} &= 100 \times \frac{\sigma_{act}}{\bar{R}} = 100 \times \frac{\sqrt{(200)^2 + (133.8)^2}}{40,000}^{1/2} \\ &= 0.601\% \end{aligned}$$

If the channel check yields a value of $F = (\sigma_{act}/\sigma_{th})^2 = 1.44$, therefore, nine out of ten times the instrumental error will contribute less than 0.1% error to the observed count rate. Part (b) of the de Lange criterion becomes quantitative when a confidence limit is specified, for then

a particular ratio of σ_{act}/σ_{th} defined as the critical value is set by the experimentalist according to the risk he believes acceptable for the work being done.

The case in which $\sigma_{th}/\sigma_{act} > 1$ (de Lange criterion, Part a) requires special consideration, for misleading conclusions could be deduced concerning the operation of the channel. If this ratio should be too large, it is possible that fluctuations not randomly distributed in time have been superposed on the genuine pulses representing radioactive decay. Riedel points out this possibility and enumerates three possible sources of non-random pulses: (1) amplifier oscillations, (2) pulse shapers which may be triggered into oscillation by genuine pulses, and (3) at high count rates, counter "deadtime". (Counter "deadtime" could give a superficial periodicity to random pulses.) For this reason, the ratio σ_{th}/σ_{act} should also be subjected to a single tail Fisher variance test and a critical value of F established as acceptable by the experimentalist. The choice of a critical value for F in this case is more difficult, but a suggested basis follows. Choose some reasonable significance level such as the 5% level. If the corresponding value of F is exceeded (meaning σ_{act} is too small), use an oscilloscope to search for oscillations in the amplifier and discriminator circuits. Be certain, too, that the count

rate is not exceeding the resolving powers of the circuitry. If no trouble is found, it might be assumed that the channel is, in fact, operating properly and that the F ratio obtained was one of the cases in which it was exceeded by chance. Another check could be made later. Suppose, then, that the experimentalist stipulates that 50 counts are to be recorded ($y = 50$) and that he wishes to consider the 5% level as significant. For $y = 50$, $F = 1.59$ at the 5% level.

$$\left[\frac{\sigma_{th}}{\sigma_{act}} \right]^2 = F = 1.59$$

$$100 \times \frac{\sigma_{th} - \sigma_{act}}{\sigma_{act}} = 100 \left[\sqrt{F} - 1 \right]$$

$$= 100 \left[1.26 - 1 \right] = 26\%$$

Once out of 20 times, then, the theoretical standard deviation can be expected to exceed the actual standard deviation by 26%. When it happens, however, the equipment could be investigated. If no apparent troubles were found, the high F value could be attributed to the play of chance alone.

Step 9: Apply the Chi-Square test to the differences between the average count rate and each individual count rate.^{4/} This will indicate

how well the set of counts fits a normal distribution.

$$\chi^2 = \frac{\sum_{i=1}^y (N_i - m)^2}{m} \quad (21)$$

Table III^{7/} lists values of χ^2 and the associated probabilities. The interpretation is the following: if the computed value of χ^2 does not exceed the value associated with a specified percentage point, the departure from the Poisson distribution is not significant at that level. For example, if the 10% level ($p = 0.10$) is chosen as the level of significance, the critical value of χ^2 is 63.167 for $y = 50$. If the calculated value of χ^2 from the data is 49.0, it is not significant at the 10% level (i.e., could be due to chance only) and the distribution of counts would be considered Poissonian.

- Step 10: (a) Determine the frequency distribution of the set of 50 counts and plot the results on probability paper in the manner outlined by Riedel.^{1/} (See Appendix II and Figures 1 and 2.)
- (b) Plot the confidence envelope in the manner suggested by Hill and Schmidt.^{12/} *

This step provides a check on the Chi-Square test of Step 8. Mills^{8/} warns against using the Chi-Square test on a purely mechanical basis because the distribution of the discrepancies $(N_i - m)$ is not accounted for. Further examination of the fit

*I should like to thank Mr. Donald Brown for suggesting the use of the confidence envelope.

is recommended. If the frequency distribution is plotted on Beckel-type probability paper and a confidence envelope constructed in the manner suggested, visual examination of the distribution of the counts is possible.

Completion of the computations outlined in these paragraphs permits the scientist to state quantitatively the quality of the operation of a counting channel with whatever confidence level may be desired or required.

RESETTING A CHANNEL TO A PREVIOUS STATE OF OPERATION

The measurement of resonance escape probabilities, absolute power calibrations and conversion ratios offer examples of work in which it is imperative that a counting channel be in the same state of operation over long periods of time (hours, days, weeks, perhaps). The "same state of operation" is defined by two conditions:

1. Sensitivity - The channel must yield the same number of counts per unit time for a particular long-lived standard regardless of when it is counted.
2. Standard Deviation - The actual standard deviation of a set of counts must remain the same (statistically speaking) at all times.

These two conditions can be realized utilizing statistical techniques.

A counting channel can be reset to the same sensitivity at any future time if a set of counts from a long-lived standard are accumulated at the time the channel is first set up. If the channel is checked out by the methods outlined

in the previous section, the set of 50 counts accumulated fulfill this need. To reset the channel to the same sensitivity, another set of 50 counts can be accumulated and the average value computed. The Student-t test^{10/} is used to compare the two averages. Table IV^{11/} lists the percentage points of the t-distribution. Again, the experimenter is free to choose his confidence limits according to the risk he wishes to take that the channel does not have the same sensitivity.

The procedure for comparing the sensitivity of a counting channel at two different times consists of determining the average count rate of the same long-lived sample at two different times as mentioned in the preceding paragraph. Assume, for example, that two sets of counts y_1 and y_2 have been accumulated at two different times; let their means be m_1 and m_2 . Compute the value of the parameter t using the following equations (ref. 5, p. 112):

$$t = \frac{m_1 - m_2}{\sigma_e} \left[\frac{y_1 y_2}{y_1 + y_2} \right]^{1/2} \quad (22)$$

$$\sigma_e^2 = \frac{y_1 s_1^2 + y_2 s_2^2}{y_1 + y_2 - 2} \quad (23)$$

$$y_1 s_1^2 = \sum_{i=1}^{y_1} (N_i - m_1)^2 \quad (24)$$

$$y_2 s_2^2 = \sum_{i=1}^{y_2} (N_i - m_2)^2 \quad (25)$$

Use Table IV to estimate the probability that this value of t might have occurred by chance alone. Suppose that $y_1 + y_2 - 2 = 100$ and that the computed value of t is 2.625. This value of t is found under the column $p = 0.01$, meaning that values of t greater than 2.625 occur by chance only one time in a hundred, on the average. If the scientist is satisfied with this confidence level (1 in 100), the value of t is just significant and the channel would probably be considered to have the same sensitivity in both cases. If, on the other hand, the scientist had selected his confidence level as 0.1, the corresponding level of t being 1.661, the t value computed (2.625) would be considered significant and the channel would be assumed to have changed sensitivity.

This procedure is not only useful for comparing channel sensitivities on different days, but it is also valuable if foils are being counted over periods of several hours. In this case, a long-lived standard can be counted several times before counting begins and the same standard counted 2 or 3 times at regular intervals during the time the other foils are being counted. The average of the 2 or 3 counts recorded at regular intervals can be compared to the original mean before counting began to determine if a change in sensitivity occurred.

A counting channel can be reset to yield the same standard deviation by utilizing the Fisher variance test to compare the original with the current standard deviation computed from the following equation:

$$\sigma_{\text{act}} = \left[\frac{\sum_{i=1}^y (N_i - m)^2}{y - 1} \right]^{1/2} \quad (26)$$

The Fisher test will indicate whether or not the standard deviations are significantly different. If they are significantly different, the trouble must be found, remedied, and another check made. The assumption, of course, is that the channel was operating properly originally.

APPENDIX I

The following demonstration will show that the de Lange criteria can be stated as follows:

$$1. V_s^2 \geq V_R^2 \text{ is equivalent to } \sigma_{th}^2 \geq \sigma_{act}^2 \text{ or } \frac{\sigma_{th}}{\sigma_{act}} \geq 1.$$

$$2. V_s^2 < V_R^2 \text{ is equivalent to } \sigma_{th}^2 < \sigma_{act}^2 \text{ or } \frac{\sigma_{th}}{\sigma_{act}} < 1.$$

The fractional standard deviations are defined in the following way:

$$1. V_R = \frac{\sigma_{\bar{R}}}{\bar{R}} \times 100$$

$$2. V_s = \frac{\sigma_{\bar{s}}}{\bar{R}} \times 100$$

The expression for $\sigma_{\bar{s}}$ is:

$$3. \sigma_{\bar{s}} = \frac{\sigma_s}{y} = \frac{1}{y} \left[\sum_{i=1}^y \frac{N_i}{T_i^2} \right]^{1/2}$$

In the checkout procedure, preset time is used so that $T_1 = T_2 = \dots = T_y$; hence, equation (3) becomes:

$$4. \quad \bar{\sigma}_s = \frac{1}{yT} \left[\sum_{i=1}^y N_i \right]^{1/2} = \frac{1}{\sqrt{yT}} \left[\sum_{i=1}^y \frac{N_i}{y} \right]^{1/2} = \frac{\sqrt{m}}{\sqrt{yT}} = \frac{\sigma_{th}}{\sqrt{yT}}$$

The term $\left[\frac{\sum_{i=1}^y N_i}{y} \right]^{1/2} = \sqrt{m} = \sigma_{th}$, the theoretical standard deviation.

The expression for $\sigma_{\bar{R}}$ is:

$$5. \quad \sigma_{\bar{R}} = \left[\frac{\sum_{i=1}^y (R - R_i)^2}{y(y-1)} \right]^{1/2}$$

Since $R_i = \frac{N_i}{T}$, $\bar{R}$ can be written as:

$$6. \quad \bar{R} = \frac{\sum_{i=1}^y \frac{N_i}{T}}{y} = \frac{1}{T} \sum_{i=1}^y \frac{N_i}{y}$$

Hence,

$$7. \quad \sum_{i=1}^y (\bar{R} - R_i)^2 = \sum_{i=1}^y \left(\sum_{i=1}^y \frac{N_i}{T} \cdot \frac{1}{y} - \frac{N_i}{T} \right)^2 = \frac{1}{T^2} \sum_{i=1}^y (m - N_i)^2$$

The equation for $\sigma_{\bar{R}}$ is then:

$$8. \left[\frac{1}{T^2} \sum_{i=1}^y \frac{(N_i - m)^2}{y(y-1)} \right]^{1/2} = \frac{1}{\sqrt{y} T} \left[\sum_{i=1}^y \frac{(N_i - m)^2}{y-1} \right]^{1/2}$$

The term $\left[\sum_{i=1}^y \frac{(N_i - m)^2}{y-1} \right]^{1/2}$ is equal to σ_{act} , and

therefore, one has the following result:

$$9. \sigma_{\bar{R}} = \frac{\sigma_{act}}{\sqrt{y} T}$$

Substitution of equations (4) and (9) into (1) and (2) yields this relation:

$$10. V_R = \frac{\sigma_{\bar{R}}}{\bar{R}} \times 100 = \frac{\sigma_{act} / \sqrt{y} T}{\bar{R}} \times 100$$

$$11. V_s = \frac{\sigma_s}{\bar{R}} \times 100 = \frac{\sigma_{th} / \sqrt{y} T}{\bar{R}} \times 100$$

The de Lange criteria as stated at the beginning follow immediately.

APPENDIX II

Example of a Channel Checkout

<u>Step</u> <u>Number</u>	<u>Equation</u> <u>Number</u>
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PROBLEM: TO DETERMINE THE QUALITY OF OPERATION OF A
NUCLEAR COUNTING CHANNEL

1

QUICK CHECK

DATA: $N_1 = 10,405$
 $N_2 = 10,219$

1(a)

COMPUTE σ_{12} :

9

$$\begin{aligned}\sigma_{12} &= (N_1 + N_2)^{1/2} \\ &= (20,624)^{1/2} \\ &= \underline{143.6} \quad \leftarrow \sigma_{12}\end{aligned}$$

1(b)

COMPUTE k :

10

$$\begin{aligned}k &= \frac{N_1 - N_2}{\sigma_{12}} \\ &= \frac{186}{143.6} \\ &= 1.295 \quad \leftarrow k\end{aligned}$$

Step Equation

1(c)

CONSULT TABLE 1

FOR $k = 1.3$, $p = 0.097$

THIS VALUE OF k IS PROBABLY DUE TO CHANCE.
THEREFORE, PROCEED TO STEP 2.

2

ACCUMULATE 50 COUNTS: ($y = 50$)

DATA:

10269	10228	10167	9986	10331
10490	10311	10352	10195	10179
10278	10201	10033	10380	10491
10228	9987	10252	10380	10124
10159	10041	10286	10229	10180
10248	10156	10183	10202	10232
10315	10199	10152	10362	10385
10252	10281	10232	10280	10108
10219	10317	10303	10334	10291
10100	10196	10437	10333	10381

3

11

CALCULATE THEIR AVERAGE VALUE:

$$m = \frac{\sum_{i=1}^y N_i}{y}$$

NOTES: 1. USE THE CALCULATOR SO THAT

$$\sum_{i=1}^y N_i \text{ AND } \sum_{i=1}^y (N_i)^2 \text{ ARE}$$

OBTAINED SIMULTANEOUSLY.
(SEE STEP 6.)

2. SUBTRACT 10,000 FROM EACH COUNT
TO SIMPLIFY COMPUTING.

$$\sum (N_i - 10,000) = 12,227$$

$$\sum (N_i - 10,000)^2 = 3,471,235$$

$$= 10,000 + \frac{12,227}{50}$$

$$= \underline{10,245} \quad \text{---} \quad m$$

Step Equation

4

COMPUTE THE THEORETICAL STANDARD DEVIATION:

12

$$\begin{aligned}\sigma_{th} &= (m)^{1/2} \\ &= (10,245)^{1/2} \\ &= \underline{101} \quad \leftarrow \sigma_{th}\end{aligned}$$

5(a)

FORM THE FOLLOWING SUM AND DIFFERENCE QUANTITIES:

13

$$\begin{aligned}\text{SUM} &= m + 2\sigma_{th} \\ \text{DIFF.} &= m - 2\sigma_{th} \\ \text{SUM} &= 10,447 \\ \text{DIFF.} &= \underline{10,043} \quad \leftarrow \begin{array}{l} \text{SUM} \\ \text{DIFF.} \end{array}\end{aligned}$$

5(b)

COUNT THE NUMBER OUTSIDE ABOVE LIMITS:

$$\begin{aligned}\text{NO. } m + 2\sigma &: 3 \\ \text{NO. } m - 2\sigma &: 0\end{aligned}$$

6

COMPUTE THE ACTUAL STANDARD DEVIATION:

14

$$\sigma_{act} = \left[\frac{\sum_{i=1}^y (N_i - m)^2}{y - 1} \right]^{1/2}$$

USE FOLLOWING IDENTITY:

$$\begin{aligned}\sum_{i=1}^y (N_i - m)^2 &= \sum_{i=1}^y N_i^2 - \frac{(\sum_{i=1}^y N_i)^2}{y} \\ &\quad \text{(SEE STEP 3)} \\ &= 3,471,235 - \frac{(12,227)^2}{50} \\ &= 481,244\end{aligned}$$

$$\begin{aligned}&= \left[\frac{481,244}{50 - 1} \right]^{1/2} \\ &= \underline{99} \quad \leftarrow \sigma_{act}\end{aligned}$$

<u>Step</u>	<u>Equation</u>
-------------	-----------------

7	APPLY THE DE LANGE CRITERION:
---	-------------------------------

$$\frac{\sigma_{th}}{\sigma_{act}} = \frac{101}{99} \gg 1 \quad \text{THE CHANNEL IS PROBABLY WORKING.}$$

8	APPLY THE FISHER TEST TO σ_{act} AND σ_{th} :
---	---

16

$$F = \left[\frac{\sigma_{larger}}{\sigma_{smaller}} \right]^2$$

$$= 1.04 \quad \leftarrow \text{F}$$

THIS IS NOT SIGNIFICANT AT THE 1% LEVEL.

9	APPLY THE CHI-SQUARE TEST:
---	----------------------------

21

$$\chi^2 = \frac{\sum_{i=1}^y (N_i - m)^2}{m}$$

$$= \frac{481,244}{10,245}$$

$$= 46.97 \quad \leftarrow \chi^2$$

THIS IS NOT SIGNIFICANT AT THE 1% LEVEL.

10(a)	DETERMINE THE FREQUENCY DISTRIBUTION OF THE SET OF 50 COUNTS.
-------	---

$$\begin{aligned} \text{RANGE} &= \text{HIGHEST COUNT} - \text{LOWEST COUNT} \\ &= 10,480 - 10,043 \\ &= 437 \end{aligned}$$

$$\begin{aligned} \text{GROUP SIZE} &= \frac{437}{10} \quad (\text{FOR 10 GROUPS}) \end{aligned}$$

$$\approx 44$$

Step Equation

CLASS			FREQUENCY		
NO.	LIMITS		NO.	%	SUM
1	2	3	4	5	6
1	10,043	- 10,087	2	4	4
2	10,088	10,132	6	12	16
3	10,133	10,177	4	8	24
4	10,178	10,222	10	20	44
5	10,223	10,267	13	26	70
6	10,268	10,312	3	6	76
7	10,313	10,357	4	8	84
8	10,358	10,402	4	8	92
9	10,403	10,447	1	2	94
10	10,448	10,492	3	6	100
TOTAL			50	100	

PLOT ON BECKEL-TYPE PROBABILITY INTEGRAL GRAPH PAPER
COLUMN 3 vs. 6. PLOT, ALSO, m AT 50%, $m - 2\sigma_{th}$ AT
2.3%, $m + 2\sigma_{th}$ AT 97.7%. (SEE FIG. 1)

10(b)

CONSTRUCT THE CONFIDENCE ENVELOPE.

For a sample of size y , the true mean of the
distribution lies within an interval

$$m \pm \frac{k \sigma_{th}}{\sqrt{y}} \quad \text{where } k \text{ is determined by the}$$

confidence level desired as shown below: *

Value of k	Confidence Level
0.6745	0.5
1.0	0.683
1.15	0.75
1.64	0.90
1.96	0.95
2.58	0.99
3.0	0.9973
4.0	0.99994

* From ref. 12.

Step Equation

The true standard deviation lies within an interval

$$\sigma_{th} \pm \frac{k \sigma_{th}}{\sqrt{2y}}$$

PLOT THE FOLLOWING POINTS.

POINT DESIGNATION	EQUATION	LOCATION ON GRAPH	VALUE
A	$(m - \sigma_{th}) + \frac{1.96\sigma_{th}}{\sqrt{2y}} + \frac{1.96\sigma_{th}}{\sqrt{y}}$	16% line	10,192
B	$m + \frac{1.96\sigma_{th}}{\sqrt{y}}$	50% line	10,273
C	$(m + \sigma_{th}) + \frac{1.96\sigma_{th}}{\sqrt{2y}} + \frac{1.96\sigma_{th}}{\sqrt{y}}$	84% line	10,394
D	$(m - \sigma_{th}) - \left[\frac{1.96\sigma_{th}}{\sqrt{2y}} + \frac{1.96\sigma_{th}}{\sqrt{y}} \right]$	16% line	10,096
E	$m - \frac{1.96\sigma_{th}}{\sqrt{2y}}$	50% line	10,217
F	$(m + \sigma_{th}) - \left[\frac{1.96\sigma_{th}}{\sqrt{2y}} + \frac{1.96\sigma_{th}}{\sqrt{y}} \right]$	84% line	10,298

Step . Equation

CONNECT THE POINTS BY STRAIGHT LINES AND EXTEND THEM TO THE LIMITS OF THE GRAPH PAPER AS FOLLOWS:

B $\rightarrow$ A, B $\rightarrow$ C.

E $\rightarrow$ D, E $\rightarrow$ F.

The envelope is formed. Except possibly for the two end points, all points should lie within the envelope.

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TABLE 1

Values of $k = \frac{N_1 - N_2}{\sigma_{12}}$ the probabilities of exceeding which are tabulated under the column titled p.

k	p	k	p	k	p	k	p
1.0	0.159	2.0	0.023	3.0	0.00135	4.0	0.0000317
1.1	0.136	2.1	0.018	3.1	0.00097	4.1	0.0000207
1.2	0.115	2.2	0.014	3.2	0.00068	4.2	0.0000133
1.3	0.097	2.3	0.011	3.3	0.00048	4.3	0.0000085
1.4	0.081	2.4	0.008	3.4	0.00034	4.4	0.0000054
1.5	0.067	2.5	0.006	3.5	0.00023	4.5	0.0000034
1.6	0.055	2.6	0.005	3.6	0.00016	4.6	0.0000021
1.7	0.045	2.7	0.003	3.7	0.00011	4.7	0.0000013
1.8	0.036	2.8	0.003	3.8	0.00007	4.8	0.0000008
1.9	0.020	2.9	0.002	3.9	0.00005	4.9	0.0000005

This table can be derived from Table 9, ref. 6, by computing the quantity $\sqrt{1 - P(t/\nu)}$ for values of t listed under the column above labeled k for $\nu = \infty$; i.e., the k of this table is the t of Table 9, Pearson and Hartley.

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TABLE II: Percentage Points of the F Distribution^{*}

The body of the table contains values of F the probabilities of exceeding which are given as column headings.

Degrees of Freedom $\nu = y-1 \approx y$	Single Tail	0.001	0.005	0.01	0.015	0.05	0.10	0.25
	Double Tail	0.002	0.01	0.02	0.03	0.10	0.20	0.50
20		4.29	3.32	2.94	2.46	2.12	1.79	1.36
30		3.22	2.63	2.39	2.07	1.84	1.61	1.28
40		2.73	2.30	2.11	1.88	1.69	1.51	1.24
^{**} 50		2.41	2.08	1.93	1.75	1.59	1.44	1.21
60		2.25	1.96	1.84	1.67	1.53	1.40	1.19
120		1.76	1.61	1.53	1.43	1.35	1.26	1.13

^{*}When a double-tail test is desired (as it is in this paper) the value of the percentage points is doubled; i.e., the 25% column becomes the 50% column, etc. (See ref. 6, page 39)

^{**}These values were computed using the interpolation formula of Lagrange applied to the three nearest points - $\nu = 30, 40, 60$.

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TABLE III: Percentage

Degrees of
FreedomThe body of the table contains values of χ^2

y	0.995	0.990	0.975	0.950	0.900	0.750	0.500
20	7.43386	8.26040	9.59083	10.8508	12.4426	15.4518	19.3374
30	13.7867	14.9535	16.7908	18.4926	20.5992	24.4776	29.3360
40	20.7065	22.1643	24.4331	26.5093	29.0505	33.6603	39.3354
50	27.9907	29.7067	32.3574	34.7642	37.6886	42.9421	49.3349
60	35.5346	37.4848	40.4817	43.1879	46.4589	52.2938	59.3347
70	43.2752	45.4418	48.7576	51.7393	55.3290	61.6983	69.3344
80	51.1720	53.5400	57.1532	60.3915	64.2778	71.1445	79.3343
90	59.1963	61.7541	65.6466	69.1260	73.2912	80.6247	89.3342
100	67.3276	70.0648	74.2219	77.9295	82.3581	90.1332	99.3341

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Points of the χ^2 Distribution

the probabilities of exceeding which are given as column headings.

0.250	0.100	0.050	0.025	0.010	0.005	0.001
23.8277	28.4120	31.4104	34.1696	37.5662	39.9968	45.315
34.7998	40.2560	43.7729	46.9792	50.8922	53.6720	59.703
45.6160	51.8050	55.7585	59.3417	63.6907	66.7659	73.402
56.3336	63.1671	67.5048	71.4202	76.1539	79.4900	86.661
66.9814	74.3970	79.0819	83.2976	88.3794	91.9517	99.607
77.5766	85.5271	90.5312	95.0231	100.425	104.215	112.317
88.1303	96.5782	101.879	106.629	112.329	116.321	124.839
98.6499	107.565	113.145	118.136	124.116	128.299	137.208
109.141	118.498	124.342	129.561	135.807	140.169	149.449

Pearson and Hartley, pp. 130-131, ref. 6.

TABLE IV: Percentage Points of the t Distribution

The body of the table contains values of t the probabilities of exceeding which are given as column headings.

Degrees of Freedom $\nu = y_1 + y_2 - 1$	p0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05	0.02	0.01	0.001
20	0.127	0.257	0.391	0.533	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
30	0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.126	0.255	0.388	0.529	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
*50	--	--	--	--	0.680	0.849	--	1.299	1.676	2.008	--	2.678	3.496
60	0.126	0.254	0.387	0.527	0.679	0.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
*70	--	--	--	--	0.678	0.847	--	1.294	1.667	1.994	--	2.648	3.435
*80	--	--	--	--	0.678	0.847	--	1.293	1.665	1.989	--	2.638	3.416
*90	--	--	--	--	0.678	0.846	--	1.291	1.662	1.986	--	2.631	3.402
*100	--	--	--	--	0.677	0.846	--	1.290	1.661	1.982	--	2.625	3.390
120	0.126	0.254	0.386	0.526	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373

*These values are reproduced by permission from Dr. Bernard Ostle, "Statistics in Research," page 451, The Iowa State College Press (1954). All other values are reprinted by permission from R. A. Fisher and Frank Yates, Statistical Tables For Biological, Agricultural and Medical Research, Fifth Edition, page 44, Oliver and Boyd. (1957).

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