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STRESS ANALYSES OF SPECIMENS FOR IN-PILE CREEP TESTS OF PARABOLIC GRAPHITE BEAMS

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TESTS OF PARABOLIC GRAPHITE BEAMS

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STRESS ANALYSES OF SPECIMENS FOR IN-PILE-CREEP TESTS OF PARABOLIC GRAPHITE BEAMS

J. M. Corum

Abstract

The irradiation-induced creep of graphite is being investigated in experiments that consist of loading parabolically shaped cantilever beams at the free end and measuring the resulting deflections with time. A series of stress analyses was made to verify the applicability of the elementary strength-of-materials approach for obtaining relations between stress and creep strain from the load-deflection data. The results of the analyses, which included a theory-of-elasticity solution, an evaluation of the effect of shear, and a bending analysis using an actual stress-strain diagram for graphite, show that an elementary strength-of-materials approach is adequate to predict the initial or elastic stresses. Preliminary results from the in-pile experiments indicate that the creep strains are linear with stress; thus, the initially linear bending stress distribution given by the elementary theory remains unchanged during creep.

Introduction

A cantilever-beam type of specimen was selected by the Metals and Ceramics Division for use in determining the creep behavior of EGCR-type AGOT graphite under irradiation.¹ This particular type of specimen was chosen because it could be tested easily and because the end displacement could be made large in comparison with the strains in the extreme fibers. The beam used has constant width and a thickness that varies with the square of the length from the free end. The elevation view is formed by a straight line and a parabola with its vertex at the free end. A beam of this kind has extreme-fiber stresses that are constant along the entire length when loaded by a concentrated force at the end. Hence, the stress and strain for a given force and end displacement are easily computed.

The characteristics of the beam under load are ascribed through the use of elementary strength-of-materials beam theory, which is based on pure bending of a prismatic beam and a linear stress-strain relationship.

These conditions are not satisfied in the case of a parabolic graphite beam. Despite the discrepancies between the conditions assumed and the actual conditions, however, the elementary method is sufficiently accurate for use in this case. This was verified by a series of theoretical analyses. The purpose of this report is to describe these analyses and the results obtained from them.

The first analysis was an examination of the initial or elastic stress distributions in the parabolic beams, based on a linear stress-strain relation. To determine the effect of the parabolic shape, a theory-of-elasticity analysis was used to predict the stress distributions, and these were compared with the distributions predicted by the elementary beam theory.

The stresses predicted by the strength-of-materials approach agreed well with those given by the more exact theory-of-elasticity analysis. Consequently, it was concluded that, based on a linear stress-strain relation, the simpler strength-of-materials approach predicts the initial or elastic stresses with sufficient accuracy.

The strength-of-materials beam theory is based on pure bending, and its application to beams with shear is based on the assumption that the beam is long enough for the shear effects to be negligible in comparison with the bending effects. For parabolic beams, the shear stress distribution is entirely different from the shear stress distribution for prismatic beams, and therefore the effect of shear on the end deflection was investigated in a second analysis. An energy approach was used, and it was found that the effect of the shear force on the deflection is only about 0.7%. Thus neglecting the shear effect in determining the bending strains from the end deflections of the creep specimens is justifiable.

The first two analyses verified the applicability of the strength-of-materials approach provided the stress-strain relation for the material of the parabolic beams is linear. The stress-strain relation for graphite, however, is nonlinear even at low stresses. Thus, a third investigation was a nonlinear bending analysis of the beams using an actual stress-strain curve for AGOT graphite. It was found that the maximum bending stress predicted by the elementary linear theory differed from that given by the more exact nonlinear theory by only 5.7%.

Together the three analyses indicated that the end deflections can be attributed almost entirely to bending and that the elementary strength-of-materials approach, which is based on a linear stress-strain relation, is adequate for predicting the initial stresses. Since the preliminary irradiation-induced creep results indicate that the creep strains are linear with stress, an initially linear stress distribution remains unchanged as creep occurs. Thus, the maximum stress is always given by the simple beam-theory formula. As a demonstration of these conclusions, an empirical creep law was derived from the preliminary irradiation-induced creep data and was used to predict the creep deflection of the beam with time. The resulting predictions agreed well with the original experimental data.

Analyses and Discussions

Comparison of Strength-of-Materials and Theory-of-Elasticity Analyses

The parabolic cantilever-beam in-pile creep specimen is shown in Fig. 1. The beam is 6 in. long and 1/2 in. square at the built-in end. The thickness h of the beam is given by

$$h = h_0 \left(\frac{x}{l} \right)^{1/2}, \quad (1)$$

where h_0 is the thickness at the built-in end and l is the length.

The strength-of-materials approach is based on the assumption that the axial bending stress σ_x is given by

$$\sigma_x = \frac{M\eta}{I},$$

where M is the moment at any axial location, I is the moment of inertia at that location, and η is the distance from the neutral axis. If this expression for the bending stress is related to the x,y coordinate system and M is replaced by Px and I by $bh^3/12$, the following expression is obtained:

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Fig. 1. Parabolic Cantilever-Beam In-Pile Creep Specimen.

$$\sigma_x = -\frac{12Py}{bh_0^3} \frac{l^{3/2}}{x^{1/2}} + \frac{6Pl}{bh_0^2} . \quad (2)$$

Here, P is the applied load and b is the width of the beam, which is $1/2$ in. For $y = 0$, the first term on the right side of Eq. (2) vanishes so that the bending stress at the top of the beam is independent of x . The bending stress along the bottom of the beam is also independent of x , as can be seen by substituting h , as given by Eq. (1), for y in Eq. (2).

Assuming that Eq. (2) is sufficiently accurate, the magnitude of the shearing stress τ_{xy} and the transverse stress σ_y can be determined by equilibrium requirements. The conditions of equilibrium in two dimensions are²

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

and (3)

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 .$$

With σ_x given by Eq. (2), the first of Eqs. (3) can be integrated, and, with the condition $\tau_{xy} = 0$ at $y = 0$, the resulting expression for τ_{xy} is

$$\tau_{xy} = -\frac{3Py^2}{bh_0^3} \left(\frac{l}{x} \right)^{3/2} . \quad (4)$$

Similarly, integration of the second of Eqs. (3) with the condition $\sigma_y = 0$ at $y = 0$, yields

$$\sigma_y = -\frac{3}{2} \frac{Py^3}{bh_0^3} \frac{l^{3/2}}{x^{5/2}} . \quad (5)$$

The stress components given by Eqs. (2), (4), and (5) satisfy all the

conditions of the theory of elasticity, including the boundary conditions, except the compatibility equation.

At distances from the loaded end greater than 2 or 3 in., the parabolic shape of the beam is closely approximated by a wedge. Since an exact analysis of the parabolic beam would be difficult, the theory of elasticity solution was based on a wedge approximation. Both the parabolic beam and the approximating wedge are shown in Fig. 2. The load P on the actual beam was replaced by an equivalent system of three loads acting on the end of the wedge, as shown. The solutions to each of the three problems corresponding to the three loads are given by Timoshenko and Goodier.³ The stresses in the beam are obtained by adding the three separate solutions, and, in polar coordinates, these stresses are

$$\sigma_r = - \frac{0.050P \cos \theta}{r \left(\alpha + \frac{1}{2} \sin 2\alpha \right)} + \frac{0.999P \sin \theta}{r \left(\alpha - \frac{1}{2} \sin 2\alpha \right)} - \frac{8P \sin 2\theta}{r^2 (\sin 2\alpha - 2\alpha \cos 2\alpha)}, \quad (6)$$

$$\sigma_\theta = 0,$$

$$\tau_{r\theta} = \frac{4P(\cos 2\theta - \cos 2\alpha)}{r^2 (\sin 2\alpha - 2\alpha \cos 2\alpha)}.$$

To compare the stresses given by the theory-of-elasticity analysis with those obtained from the strength-of-materials approach, the stresses given by Eqs. (6) were transformed to rectangular coordinates by means of the following transformation equations:

$$\sigma_x = \frac{1}{2} (\sigma_r + \sigma_\theta) + \frac{1}{2} (\sigma_r - \sigma_\theta) \cos 2(\alpha - \theta) + \tau_{r\theta} \sin 2(\alpha - \theta),$$

$$\sigma_y = \frac{1}{2} (\sigma_r + \sigma_\theta) - \frac{1}{2} (\sigma_r - \sigma_\theta) \cos 2(\alpha - \theta) - \tau_{r\theta} \sin 2(\alpha - \theta),$$

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Fig. 2. Wedge Approximation Used in Theory of Elasticity Analysis.

$$\tau_{xy} = \frac{1}{2} (\sigma_r - \sigma_\theta) \sin 2(\alpha - \theta) - \tau_{r\theta} \cos 2(\alpha - \theta) . \quad (7)$$

For the comparison, a load P of 1 lb was chosen, and the stresses were calculated at $x = 6$ in., which is at the built-in end. The axial bending stresses σ_x given by the strength-of-materials and the theory-of-elasticity approaches are shown in Fig. 3 as functions of the distance from the upper edge of the beam. The strength-of-materials prediction closely approximates the more exact theory-of-elasticity solution.

The shear stresses τ_{xy} given by the two approaches are shown in Fig. 4. Again, the results are very similar. The maximum shearing stress occurs at the lower edge of the beam, and, from the strength-of-materials solution, is given by

$$\tau_{\max} = -3 \frac{P}{bh_0} \left(\frac{l}{x} \right)^{1/2} .$$

At $x = l$, this is $2 \frac{1}{4}$ times the maximum shear stress that would occur in a prismatic beam with the same cross section. The latter stress occurs at the neutral axis and is given by

$$\tau_{\max} = \frac{4}{3} \frac{P}{bh_0} .$$

The transverse stresses σ_y given by the two methods are shown in Fig. 5, and the results agree closely. Figures 3, 4, and 5 show that the strength-of-materials approach is adequate for predicting the initial stresses in the parabolic beams provided the stress-strain relation is assumed to be linear.

Effects of Shear

In relating the end deflections measured in the in-pile experiments to the axial strain, only the action of the bending moment in causing deflection is taken into consideration. An additional deflection is produced by the shearing force in the form of mutual sliding of adjacent cross sections along each other. To evaluate the effect of the shearing

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Fig. 3. Axial Bending Stress Distributions at Built-in End of Parabolic Cantilever Beam for an End Load of 1 lb and a Unit Beam Width.

Fig. 4. Shear Stress Distributions at Built-in End of Parabolic Cantilever Beam for an End Load of 1 lb and a Unit Beam Width.

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Fig. 5. Transverse Stress Distributions at Built-in End of Parabolic Cantilever Beam for an End Load of 1 lb and a Unit Beam Width.

force on the deflection, the deflection due to bending alone and that due to shear alone were calculated by energy methods.

The strain energy in a beam due to bending is given by⁴

$$U_b = \int_0^l \frac{M^2}{2EI} dx, \quad (8)$$

where E is the modulus of elasticity. Substituting Px for M and $bh^3/12$ for I , and using Eq. (1) for h , the following expression is obtained for U_b :

$$U_b = \int_0^l \frac{6P^2x^2}{Ebh_0^3 (x/l)^{3/2}} dx = \frac{4P^2l^3}{Ebh_0^3}. \quad (9)$$

By Castigliano's theorem, the end deflection due to bending is

$$\delta_b = \frac{\partial U_b}{\partial P} = \frac{8Pl^3}{Ebh_0^3}. \quad (10)$$

The strain energy due to shear is given by⁵

$$U_s = \frac{(1 + \nu)}{E} b \int_0^l \int_0^h \tau_{xy}^2 dy dx, \quad (11)$$

where ν is Poisson's ratio. Using Eq. (4) for the shear stress τ_{xy} , the following expression for U_s is obtained from Eq. (11):

$$U_s = \frac{18(1 + \nu)P^2l}{5Ebh_0}. \quad (12)$$

By Castigliano's theorem, the end deflection due to shear alone is

$$\delta_s = \frac{\partial U_s}{\partial P} = \frac{36(1 + \nu)Pl}{5Ebh_0}. \quad (13)$$

For the graphite parabolic beams with $\nu = 0.15$, the deflection due to bending is 140 times the deflection due to shear. Consequently, the effect of the shearing force on the deflection is only about 0.7% and can be neglected.

Nonlinear Bending Analysis

Each of the preceding discussions was based on a linear stress-strain relation. Actually, graphite exhibits nonlinear stress-strain behavior even at low stresses. To evaluate the effects of the nonlinearity, a nonlinear bending analysis was used to determine the actual bending stress distribution in the graphite specimens. The analysis was based on a strength-of-materials approach in that a prismatic beam was assumed and the effects of shear were neglected. The method is described by Timoshenko.⁶

A typical stress-strain curve for AGOT graphite is shown by the solid curved line in Fig. 6. For convenience, the stress-strain relation was assumed to be the same in both tension and compression. Consequently, the neutral axis coincides with the centroidal axis of the deflected beam.

The applied moment, $M = Px$, at any cross section must equal the internal resisting moment at that cross section. Denoting the maximum tensile strain in the extreme fibers of the beam by ϵ_1 and the maximum compressive strain in the extreme fibers on the opposite side of the beam by ϵ_2 , the above condition may be written as⁶

$$Px = \frac{bh^2}{(\epsilon_1 - \epsilon_2)^2} \int_{\epsilon_2}^{\epsilon_1} \sigma \epsilon \, d\epsilon .$$

With the relation for h given by Eq. (1), this becomes

$$P = \frac{bh_0^2}{(\epsilon_1 - \epsilon_2)^2 l} \int_{\epsilon_2}^{\epsilon_1} \sigma \epsilon \, d\epsilon . \quad (14)$$

As expected, the relation between the stress and strain distributions is the same at each cross section along the axis. The integral in Eq. (14)

Fig. 6. Bending Stress Distribution Based on an Actual AGOT Graphite Stress-Strain Curve.

represents the moment, with respect to the vertical axis through the origin, of the area under that portion of the stress-strain curve between ϵ_2 and ϵ_1 . For any given load P , the values of ϵ_2 and ϵ_1 , which are of equal magnitude but opposite sign, can be determined from Eq. (14) and the stress-strain curve by a trial-and-error procedure.

As a basis for comparison, the magnitude of the load P was chosen so that the maximum fiber stress calculated on a linearly elastic basis would be 1400 psi, which was one of the stress levels used in the creep experiments. The resulting nonlinear stress distribution, which is the same at each cross section, is depicted by the dark curve in Fig. 6. The abscissa of Fig. 6 is strain, but since plane sections must remain plane, the abscissa could, by a change of scale, represent distance from the top edge of the beam at any particular cross section. The maximum fiber stress for the nonlinear distribution is 1325 psi rather than 1400 psi as calculated on a linearly elastic basis. The error, however, for the linearly elastic calculation is only 5.7%. Consequently, it was concluded that the elementary strength-of-materials beam theory, which is based on a linear stress-strain relation, can be used to determine the initial stresses in the parabolic graphite beam specimens with sufficient accuracy.

Creep Analysis Based on Empirical Creep Law and a Strength-of-Materials Approach

Preliminary results obtained during the first reactor cycle of the in-pile creep experiments are shown by the data points and solid lines in Fig. 7. The results indicate that the irradiation-induced creep strains are approximately linear with stress. Consequently, if the bending stresses are assumed to be linearly distributed across the beam initially and if the shear effects are neglected, the stresses will remain linearly distributed as creep takes place. Using these assumptions, the creep bending behavior can easily be predicted from the data presented in Fig. 7. This was done, and the results were compared with the original load-deflection-time data as a final demonstration of the strength-of-materials approach.

Fig. 7. Comparison of Experimental Creep Data and Empirical Creep Relation.

It was found that the following empirical equation satisfactorily describes the preliminary experimental creep data:

$$\epsilon_c = 130 \times 10^{-9} \sigma t^{0.21} . \quad (15)$$

Here, ϵ_c is the creep strain and t represents time. The dashed curves shown in Fig. 7 were obtained from Eq. (15).

Since the stress distribution does not change with time, the factor $130 \times 10^{-9} t^{0.21}$ can be thought of as a time-dependent modulus that relates the time-dependent strains to the stresses just as the modulus of elasticity relates the initial strains to the stresses. Thus, Eq. (10) for the elastic bending deflection can also be used to predict the time-dependent creep bending deflections by substituting the factor $130 \times 10^{-9} t^{0.21}$ in place of $1/E$. Doing this and denoting the creep deflection at the end of the beam by δ_c , the following expression for the time-dependent deflection is obtained:

$$\delta_c = 8(130 \times 10^{-9}) t^{0.21} \frac{Pl^3}{bh_0^3} . \quad (16)$$

The calculated creep deflection for a load P that corresponds to an elastically calculated maximum bending stress of 1400 psi is shown in Fig. 8 as a function of time. The dashed curve represents the corresponding experimental data. The calculated curve agrees fairly well with the experimental curve. Better agreement could be obtained by taking additional terms in the empirical creep relation. The results of the calculation show, however, that the elementary strength-of-materials approach can be used to analyze the creep behavior of the parabolic beam specimens. This is possible only because the creep strains are linear with stress. For creep strains that are not linear with stress, the stress distributions for the beams become nonlinear, and it is much more difficult to relate the creep strains to stresses.

Fig. 8. Comparison of Calculated Creep Deflection and Actual Deflection.

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